

WAVELET THRESHOLDING AND FUSION FOR ULTRASOUND MEDICAL IMAGE DE-NOISING

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ABSTRACT

Ultrasound medical images are a source of visual information regarding the health condition of human body. This non invasive diagnostics technique is most widely used by doctors around the globe has been in use for quite some time now. However, the ultrasound medical image quality is severely hampered due to the presence of speckle components that invade the ultrasound image. Noise presence in ultrasound images greatly reduces the visual information related to internal body parts. This research proposes wavelet based soft and hard thresholding techniques for removing speckle noise from ultrasound medical images. Thresholding operation is a global processing technique which will damage the objects of interest in the US images. To restore the lost visual quality due to global thresholding, wavelet based fusion of original ultrasound image and thresholded ultrasound image is performed. This procedure restores the lost object information in the thresholded ultrasound images. Evaluation of fusion rules for the proposed process is done using fusion factor and fusion symmetry. The overall performance of the proposed techniques is computed using mean square error (mse) and peak signal to noise ratio (psnr).

Keywords: *Ultrasound Medical Image, Speckle Noise, Discrete Wavelet Transform (DWT), Soft and Hard Thresholding, Wavelet based Fusion.*

1. INTRODUCTION

Ultrasound Imaging [1]-[3] has been in extensive use in medical analysis for diagnostics of internal human body parts without causing pain. The advantages offered by using Ultrasound imaging mechanism are low cost, portability and less amount of time required for diagnostics. Due to the above cited reasons ultrasound imaging has been in wider acceptance in low per capital income countries around the world over other high technology modalities such as Computed Tomography (CT) and Magnetic Resonance Imaging (MRI). With the arrival of marketable Ultrasound scanners and development of the technology in the framework of scanner materials and construction, acquisition and visualization, computational capability and digital processing, there is a mounting awareness among researchers to explore the efficacy of ultrasound imaging [4]-[7].

Image quality is a prime factor of worry in ultrasound imaging due to multiple reflections of ultrasound signals that are picked up by the receiver from the hard tissues in the human body[8]-[9]. Ultrasound images are inflated by many types of artifacts, making it complicated for a viewer to

understand and investigate the images in order to obtain quantitative information from them. In spite of the limitations of ultrasound imaging, it is still the safest and inexpensive imaging modality in many clinical applications. Efforts have been put to augment medical ultrasound images for the purpose of getting valid and correct information for study and diagnosis [10]-[12].

De-noising is often a obligatory preprocessing to be performed before utilizing the attained data [13]-[14]. Despite many advantages listed, medical ultra sonographic images are of poor visibility, resulting from speckle noise [15] which occurs especially in the images of fetus of pregnant woman less than 12 to 14 weeks, whose underlying structures are too small to be resolved by large wavelength [16]. The presence of speckle results in degradation of image quality and makes it difficult for human interpretation and diagnosis. Thus speckle reduction (de-speckling) is an important aspect for analysis of ultrasound images. Many algorithms have been developed on despeckling.

Conventional Spatial filters have been normally used for removing noise from images and signals [17]. Spatial filters usually smooth the data to reduce the noise, and also blur the data.

Numerous new techniques have been reported in the last few years which improve on spatial filters by removing the noise more effectively while preserving the edges in the data. Some of these techniques used the concepts of partial differential equations and computational fluid dynamics such as level set methods, total variation methods [16], nonlinear isotropic and anisotropic diffusion. Various other techniques combine impulse removal filters with local adaptive filtering in the transform domain to remove not only white and mixed noise, but also their mixtures [17][13]. In order to reduce the presence of noise in medical images many techniques are available like digital filters (FIR or IIR), adaptive filtering methods etc. However, digital filters and adaptive methods can be applied to stationary signal. Recently the wavelet transform has been proven to be a useful tool for non-stationary signal analysis [18][19]. Researchers have proposed many de-noising algorithms on wavelet framework effectively but they suffer from shortcomings such as oscillations, shift variance, aliasing, and lack of directionality.

Pre-processing and Post-processing techniques are the two fundamental modules of Ultrasound enrichment processes. Pre-processing techniques deal with image degradation related issues linked to the physical properties of the signals involved and consist of modifications in the mechanism of signal generation and/or image acquisition stage/s. On the other hand, Post-processing algorithms use signal conditioning/processing techniques to enhance the images after they have been captured.

This research proposes to use wavelet transform [21]-[23] based hard and soft thresholding [24] operations to de-noise ultrasound medical images. These two techniques operate globally on images damaging high frequency content of the objects in an ultrasound image. Wavelet based fusion of original ultrasound image with the thresholded ultrasound image is performed to restore the quality of the image. The ultrasound medical image is noise cleaned and enhanced for quality viewing for better diagnostics. Haar mother wavelet at level-1 is used for the all the transformation from spatial domain to wavelet domain. Performance of the proposed techniques is evaluated visually and computed using mean squared error (mse) and peak signal to noise ratio (psnr).

The rest of the paper is organized as follows. Section 2 briefly introduces 2D discrete wavelet transform. Section 3 discusses the soft and

hard thresholding techniques used for noise reduction. Enhancements to ultrasound images using wavelet based fusion are proposed in section 4. Section 5 presents experimental outcomes for numerous test subjects procured from a gynaecologist clinic in Vijayawada, Andhra Pradesh, INDIA. Conclusions are formulated in section 6 based on the discussions in section 5.

2. DISCRETE WAVELET TRANSFORM

From an engineering perspective, the discrete wavelet analysis [22] is a two channel digital filter bank structure consisting of the low pass and the high pass filters, replicated on the low pass output. The low pass filtering gives an approximation of a signal (on a given scale), while the high pass (band pass) filtering gives the details that bring out the difference between the two successive approximations. Figure 1 shows an illustration of a n-level wavelet decomposition of an image $I(x,y)$. Where n is a positive integer. A family of wavelets is then associated with the band pass and a family of scaling functions with the low pass filters.

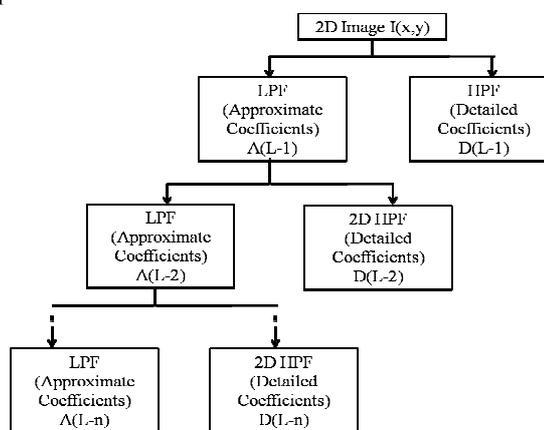


Figure 1: A n-Level wavelet decomposition of an Image $I(x,y)$

A unique prototype function, called a mother wavelet[23], is the basic source of the wavelet family. Given a real variable x , the function $\psi(x)$ is called a mother wavelet provided it oscillates, averaging to zero $\int_{-\infty}^{\infty} \psi(x) dx = 0$

and which decreases rapidly to zero when $|x|$ tends to infinity as shown in eq.14. In practice, applications impose additional requirements among which, a given number of vanishing moments N_v

$$\int_{-\infty}^{\infty} x^k \Psi(x) dx = 0; \quad \forall 0 < k \leq N_v - 1 \quad (1)$$

The mother wavelet $\psi(x)$, generates the other wavelets $\psi_{a,b}(x)$, $a > 0$, $b \in \mathbb{R}$, of the family by change of scale and by change of position b ,

$$\Psi_{a,b}(x) = \frac{1}{\sqrt{a}} \Psi\left(\frac{x-b}{a}\right) \quad \forall a > 0, b \in \mathbb{R} \quad (2)$$

By translation and dilation of the wavelet ψ we define the atoms of the wavelet transform as depicted in eq.2. The wavelet transform has been used for the decomposition of the signal into high and low frequency components. The wavelet coefficient represents a measure of similarity in the frequency content between a signal and a chosen wavelet function. These coefficients are computed as a convolution of the signal and the scaled wavelet function, which can be interpreted as a dilated band-pass filter because of its band-pass like spectrum (Valens ;Rioul and Vetterli 1991)[23].

From the discrete wavelet transform of figure1 can be formulated as approximate coefficients and detailed coefficients. The DWT approximate coefficients for a 2D signal $I(x,y)$ is formulated as

$$A^L = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} I(x,y) \psi_{ab}^L(x,y) \quad (3)$$

And the detailed coefficients are formulated as

$$D^L = \sum_{x=0}^{N-1} \sum_{y=0}^{M-1} I(x,y) \psi_{a,b}^L(x,y) \quad (4)$$

Wavelet decomposition level L can be iteratively used to putrefy the image into various frequency planes.

3. DE-NOISING WITH WAVELETS

Wavelet transform operates with variable size time scales for changing frequencies. Hence it is the choice of researchers for denoising varying noises like speckle noise that affects ultrasound medical images. In ultrasound medical images noise energy is mostly concentrated in small number of wavelet dimensions. The image object coefficients are relatively outsized compared to noise coefficients whose energy is stretched over large number of coefficients. Hence by thresholding smaller coefficients to near zero and preserving the larger coefficients, speckle noise can be partially eliminated from ultrasound images. The de-noising is performed on amplitudes of coefficients instead of frequency, low frequency noise can be eliminated.

Thresholding is performed using four different methods using approximate and detailed coefficients to remove de-noise ultrasound medical images.

3.1 Hard Thresholding

Thresholding is mainly performed on the detailed coefficients of wavelet transformed ultrasound image. The detailed coefficients of wavelet decomposition primarily constitutes of noise coefficients. Hard Thresholding is the simplest form of thresholding where the threshold value is chosen by the user. Hard thresholding is applied on the detailed coefficients using the formulation

$$D_m^L(i,j) = \begin{cases} D(i,j) & \text{if } |D(i,j)| > \xi \\ 0 & \text{if } |D(i,j)| \leq \xi \end{cases} \quad (5)$$

Where $D_m(i,j)$ are the modified or thresholded coefficients at level L at location (i,j) . ξ is the soft threshold value. In this research the soft threshold value is computed using the following equation.

$$\xi^L = \frac{\max(\max(D^L))}{M} \quad (6)$$

Threshold value changes with wavelet level of decomposition. M is the maximum number of gray levels in the original image. ξ is the maximum value in the detailed coefficients.

3.2 Soft Thresholding – 1

Soft thresholding – 1, according to [25], the threshold value is computed as

$$\xi = \gamma \sqrt{2 \log(M)} \quad (7)$$

Where M is the number of pixels in the image and γ is estimated as

$$\gamma = \frac{|\text{median}(I(x,y))|}{0.6745} \quad (8)$$

From eq.9 numerator gives the absolute of median values of original Ultrasound medical image under consideration.

3.3 Soft Thresholding – 2

Soft thresholding – 2 is similar to that of 1, with a change in γ value which is estimated as

$$\gamma = \frac{|\text{median}(D^L)|}{0.6745} \quad (9)$$

In soft thresholding – 2, γ is estimated as absolute median values of detailed coefficients.

3.3 Soft Thresholding – 3

Soft thresholding – 3 is computed on detailed coefficients of wavelet transformed ultrasound medical image using the following expression

$$D_m^L(i,j) = \begin{cases} \text{sgn}(D^L(i,j) \times (|D^L(i,j)| - \xi)) & \text{if } |D(i,j)| > \xi \\ 0 & \text{if } |D(i,j)| \leq \xi \end{cases} \quad (7)$$

Where sgn() is a signum function. Where $D_m(i, j)$ are the modified or thresholded coefficients at level L at location (i,j). ξ is the hard threshold value.

4. FUSION WITH WAVELETS

Thresholding relatively is global operation on the entire image. Apart from reducing speckle from ultrasound medical images these global thresholding operations also target the quality of the images. They greatly reduce the edge information as high frequency coefficients are a part of it. To improve the quality of Ultrasound images by restoring edge information is principal aim of wavelet based image fusion [26]-[28] in our research. The fusion is accomplished on wavelet transformed original ultrasound medical image and denoised medical image.

There are many fusion rules in literature and for de-noising it is chosen as max-mean fusion rule after trial and error from 8 fusion rules such as max-max, min-max, min-min, mean-mean, mean-max, mean-min and max-mean. Max-Mean method is presented below.

Max-Mean fusion rule evaluates the average values at each position of detailed coefficients of original ultrasound image and denoised ultrasound image and Max values at each position for both the images. Max-Mean wavelet based fusion rule is shown in figure 2.

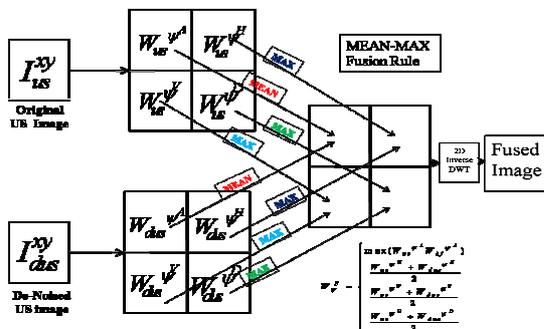


Figure2: Max-Mean Fusion Rule for Ultrasound Image De-Noising

Max-Mean fusion rule is computed using the expression

$$W_{\psi}^F = \begin{cases} \max(W_{us}^{\psi^A} W_{hf}^{\psi^A}) \\ \frac{W_{us}^{\psi^H} + W_{dus}^{\psi^H}}{2} \\ \frac{W_{us}^{\psi^V} + W_{dus}^{\psi^V}}{2} \\ \frac{W_{us}^{\psi^D} + W_{dus}^{\psi^D}}{2} \end{cases} \quad (8)$$

5. DE-NOISING RESULTS

The choice of mother wavelet in this research is cut across orthogonal and bi-orthogonal functions. ‘Haar’, ‘db2’, ‘sym3’ and ‘bior1.1’ are 4 mother wavelets that are used for experimentation in this work. For this part the decomposition level is kept at one. The mother wavelet functions used for the purpose of de-noising are presented in figure 3.

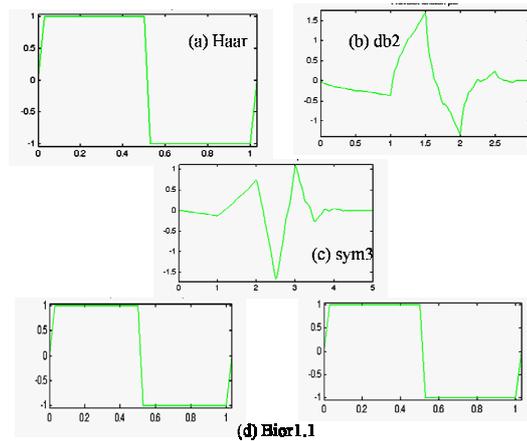


Figure 3: Mother wavelet functions used (a) haar (b) db2 (c) sym3 (d) bior1.1

The set of ultrasound images on which the following experimentation is carried out are shown in figure 4. The images are obtained in .png format from AMMA Hospital, Ultrasound Lab, Vijayawada, Andhra Pradesh, INDIA. Figure 4(a) is ultrasound image of baby body of a healthy pregnant female. Similarly figures 4(b), 4(c) and 4(d) are ultrasound scans of baby face, legs and spinal cord. The considered pregnant female is around six months pregnant.

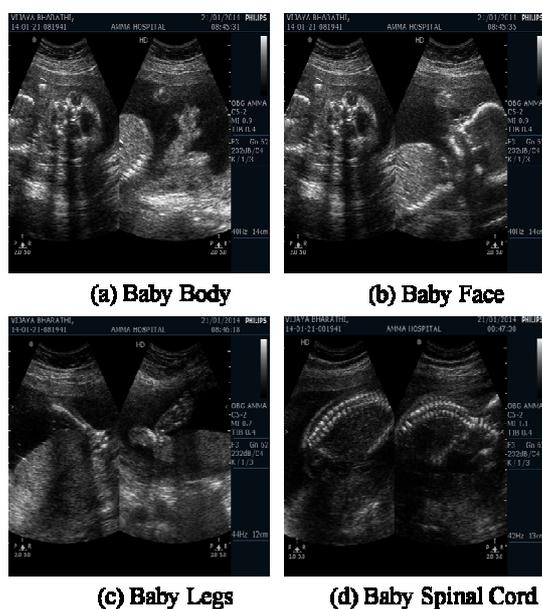


Figure 4: Ultrasound Test Images used in this experimentation

Figure 5,6,7 and 8 show wavelet transformed ultrasound images with mother wavelets 'Haar', 'db2', 'sym3' and 'bior1.1' respectively.

Mother wavelet-haar DWT components

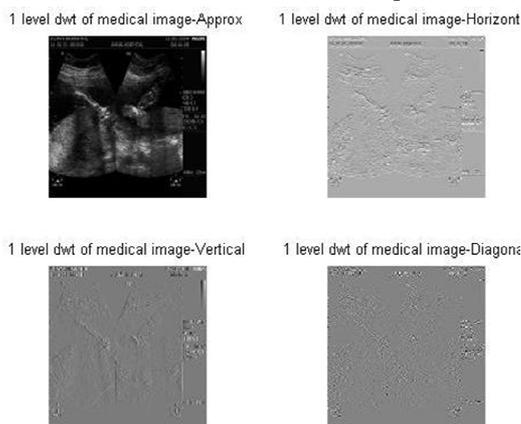


Figure 5: Haar Wavelet on Ultrasound Baby image at Level-1

Mother wavelet-db2 DWT components

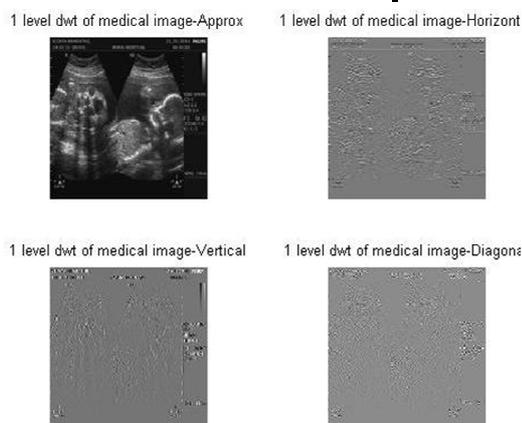


Figure 6: db2 Wavelet on Ultrasound Baby image at Level-1

Mother wavelet-sym3 DWT components

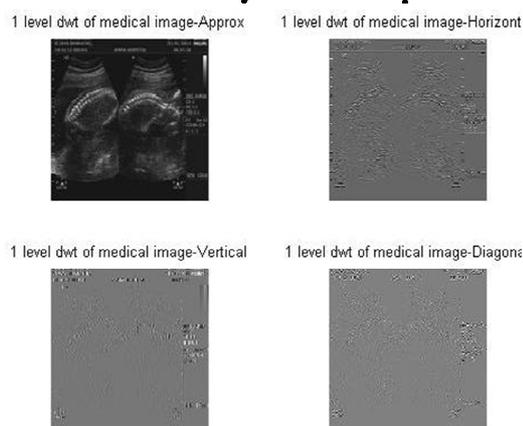


Figure 7: sym3 Wavelet on Ultrasound Baby image at Level-1

Mother wavelet-bior1.1 DWT components

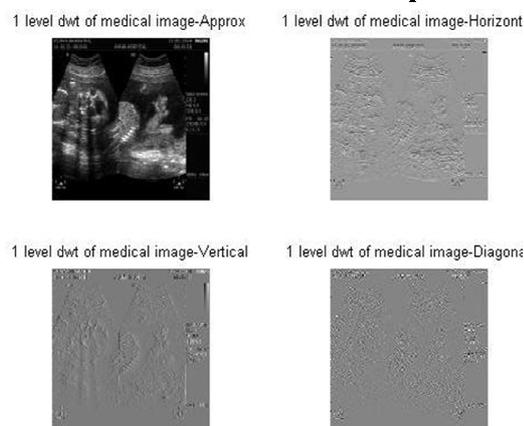


Figure 8: Haar Wavelet on Ultrasound Baby image at Level-1

The following figures from 8-11 show the response of the proposed thresholding paradigms on various ultrasound medical images. The first row is original ultrasound image. The second row consisting of thresholding techniques applied on original ultrasound image. The third row is fused ultrasound images. The figures show a improved quality in visual content compared to original ultrasound image.

Mother wavelet-haar Level-1

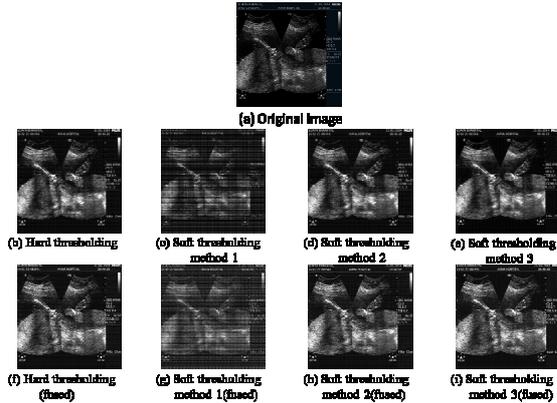


Figure 8: HAAR Wavelet (a) Original Ultrasound Image (b) Hard Threshold (c) Soft Thresholding -1 (d) soft Thresholding -2 (e) Soft Thresholding -3. (f) Hard Threshold with fusion (g) Soft Thresholding -1 with fusion (h) soft Thresholding -2 with fusion (i) Soft Thresholding -3 with fusion.

Mother wavelet-db2 Level-1

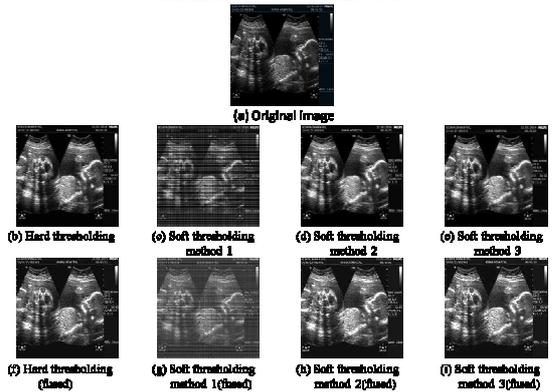


Figure 9: db2 Wavelet (a) Original Ultrasound Image (b) Hard Threshold (c) Soft Thresholding -1 (d) soft Thresholding -2 (e) Soft Thresholding -3. (f) Hard Threshold with fusion (g) Soft Thresholding -1 with fusion (h) soft Thresholding -2 with fusion (i) Soft Thresholding -3 with fusion.

Mother wavelet-sym3 level-1

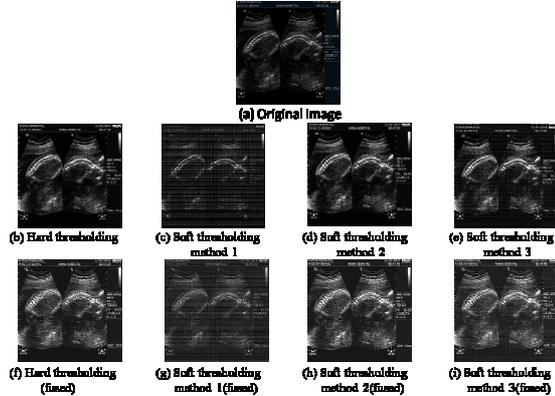


Figure 10: sym 3 Wavelet (a) Original Ultrasound Image (b) Hard Threshold (c) Soft Thresholding -1 (d) soft Thresholding -2 (e) Soft Thresholding -3. (f) Hard Threshold with fusion (g) Soft Thresholding -1 with fusion (h) soft Thresholding -2 with fusion (i) Soft Thresholding -3 with fusion.

Mother wavelet-bior1.1 Level-1

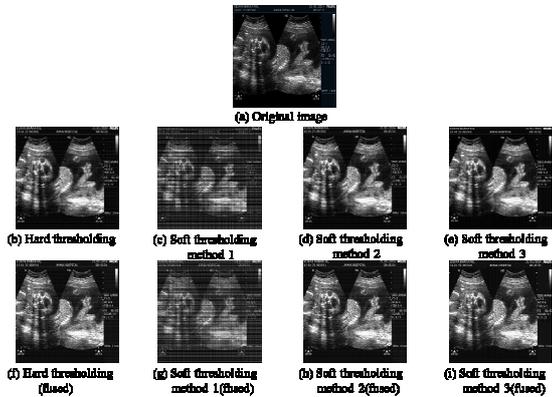


Figure 11: bior 1.1 Wavelet (a) Original Ultrasound Image (b) Hard Threshold (c) Soft Thresholding -1 (d) soft Thresholding -2 (e) Soft Thresholding -3. (f) Hard Threshold with fusion (g) Soft Thresholding -1 with fusion (h) soft Thresholding -2 with fusion (i) Soft Thresholding -3 with fusion.

Visually the experimental results show a very good prospect for the proposed algorithms for ultrasound image denoising. The radiologist at AMMA hospital was indeed impressed with the results and the processed images helped doctors access the condition of the patient faster than the usual. Figures 12 and 13 show experiments conducted on different parts of the baby.

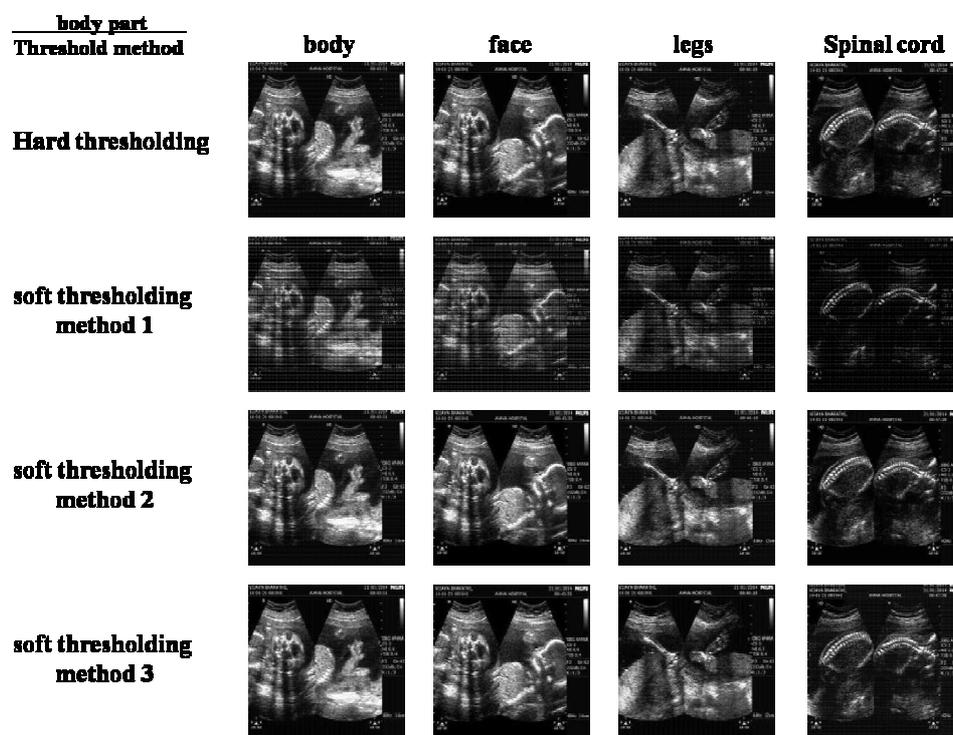


Figure 12: De-Noised Ultrasound Medical Images with 'db2' wavelet at level-1 using the proposed techniques

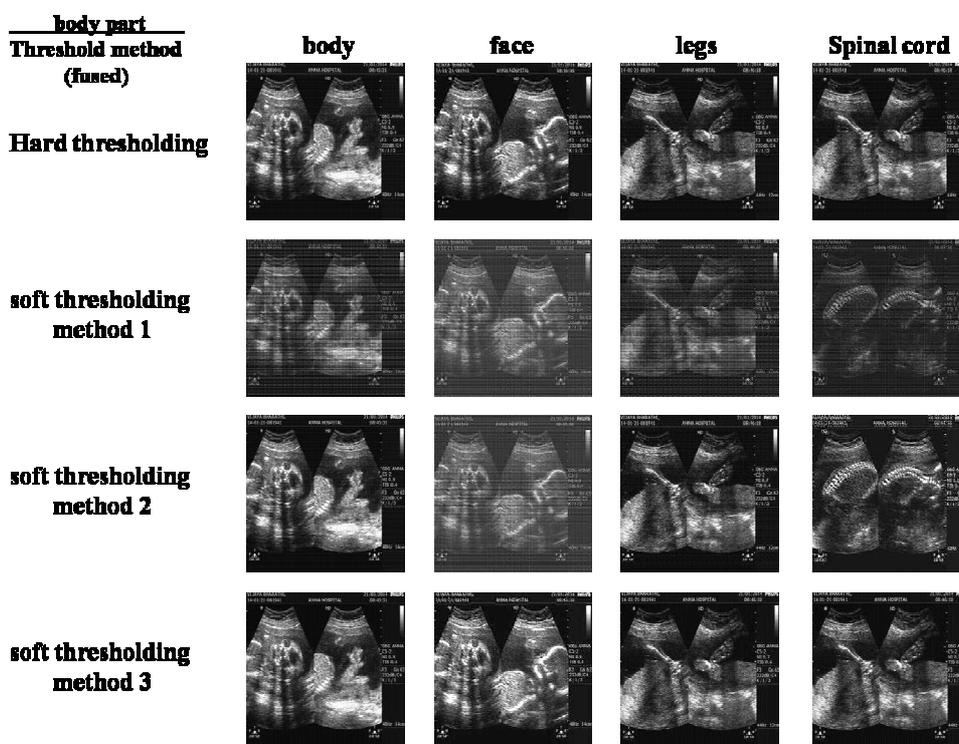


Figure 13: De-Noised Ultrasound Medical Images with 'db2' wavelet at level-1 using the proposed techniques. Visually good results for the proposed techniques prompted us to test the quantitatively

using math models such as mean square error(mse), signal to noise ratio(snr) and peak signal to noise ratio(psnr). Table 1 provide the values computed between the proposed de-noising techniques for 'db2' wavelet at level -1.

Table-1 Performance Evaluation of Proposed De-Noising Methods

	Mean Square Error	SNR	PSNR
Hard Thresholding	199.82	106.38	25.15
Hard Thresholding Fused	71.2499	110.85	29.63
Soft Thresholding Method 1	4.67E+03	92.69	11.47
Soft Thresholding Fused Method 1	3.58E+03	93.84	12.62
Soft Thresholding Method 2	199.82	106.38	25.15
Soft Thresholding Fused Method 2	71.24	110.85	29.63
Soft Thresholding Method 3	124.71	108.42	27.20
Soft Thresholding Fused Method 3	5.56	121.93	40.70

From table-1 it can be observed that the values are far advanced than most of the traditional medical image de-noising techniques.

6. CONCLUSION

This paper focuses on de-noising methods of ultrasound medical images using discrete wavelet transform. Ultrasound images are obtained and are transformed into wavelet domain. The hard thresholding and three types of soft thresholding models are applied on detailed components of decomposed ultrasound images. Four types of mother wavelets are used in this process. The visual quality is greatly improved by wavelet based fusion technique. Wavelet fusion restores the quality of the de-noised ultrasound images using the unprocessed ultrasound images. The performance of various de-noising techniques is qualitatively by using mse, snr and psnr values. It was found that 'db2' wavelet with soft thresholding -3 outperformed rest of the thresholding modalities for de-noising ultrasound medical images.

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