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NEW APPROACH FOR RESOLUTION OF MAXWELL EQUATIONS USING GALERKIN METHOD AND DIRICHLET CONDITIONS IN FINITE ELEMENT ANALYSIS

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ABSTRACT

The Galerkin method combined with the Dirichlet conditions provides a powerful way of solving the Maxwell equations. This approach allows for a detailed description of the behavior of the electric and magnetic fields in the problem domain and can provide accurate solutions for complex engineering problems. The method can be used in conjunction with Finite Element Analysis to solve various types of problems, such as those involving loads, force, displacement, vibration, and fluid flow. The use of the Dirichlet conditions to impose boundary conditions on the system also ensures accuracy in the results obtained. This helps engineers to better understand the physical behavior of the system, assess any discrepancies in the data, and make more informed decisions about the overall design. FEA combined with Dirichlet conditions offer an efficient and accurate way of predicting design behavior and allow engineers to make changes to the design quickly and efficiently. The proposed numerical solution for MAXWELL equations using the GALERKIN method and DIRICHLET conditions is presented by a Finite Element Analysis mesh distribution to calculate the flux and the magnetic field into the air gap of the electrical motor.

Keywords: *Finite Element Analysis, Galerkin, Dirichlet, Maxwell Equations, Electric Motors*

1. INTRODUCTION

The Finite Element Analysis (FEA) is a numerical technique used to solve complex problems in civil engineering, mechanical engineering, and other physics-based engineering. It is used to solve problems involving loads, force, displacement, vibration, and fluid flow. The FEA is based on the discretization of the problem domain into small finite elements connected and an understanding of the behavior of these elements under different types of boundary conditions. The Maxwell equations are a set of four partial differential equations, which describe the behavior of electric and magnetic fields. The resolution of these equations is essential in the design of electrical and electromechanical components. In this paper, we propose to address the resolution of the Maxwell equations using the Galerkin method and Dirichlet conditions in Finite Element Analysis.

The goal of FEA is to provide an approximation to the exact solution of a given problem. In FEA, the problem is typically divided into small regions or elements, which are then analyzed both theoretically and numerically, with the use of Dirichlet conditions. These conditions determine the nature of the problem and can be used to define boundary conditions, material properties, and other parameters. FEA can also be used to determine the optimal design of a structure or system by factoring in various design parameters. Additionally, FEA is used to predict how a structure or system will respond to certain environmental stresses and loads. By combining FEA with Dirichlet conditions, engineers and scientists can obtain more accurate solutions that include both effects from external forces and those from the physical properties of the

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structure or system. FEA is often used to quickly determine the optimal design for a given structure or system, helping to reduce the time and cost of design and fabrication.

The Galerkin method is an approximate solution technique which relies on an underlying mathematical structure and a set of discrete numerical approximations. The method is used in two major branches of FEA, namely, the Galerkin Method and the Finite Difference Method. In the Galerkin Method, the problem domain is subdivided into cell-like finite elements, with each element characterized by its own stiffness and localized material properties. Moreover, this method allows for a wide variety of element types and hundreds of parameters that can be used to describe problems of interest. The stiffness matrix and local material properties are then used to derive the system of equations. These equations are then solved using an iterative process.

Dirichlet conditions are boundary conditions that prescribe that the values of a quantity or vector field of interest should be equal to a given value at the boundaries of the problem domain. They are usually used when solving a system of partial differential equations. The Dirichlet conditions are used to specify the correct boundary conditions for a particular problem. For example, in the case of Maxwell's equations, the Dirichlet conditions impose that the components of the electric and magnetic fields should be equal to the prescribed local field values on the boundaries of the problem domain.

Electrical machines are electromagnetic devices with various shapes and sometimes very complex geometries and are often equipped with physical phenomena whose analysis requires knowledge and know-how. The modeling of such machines requires a perfect mastery of the MAXWELL equations combined with the physical properties linked to the behavior of the manufacturing materials. Therefore, it is necessary to take into consideration the electrical, magnetic, mechanical, and dynamic equations of electrical machines [1]. The use of these equations highlights many parameters whose knowledge is essential for the rest of this publication.

We proceeded to exploit the MAXWELL equations to examples of structures well known in the literature. Such a study shows that the presence of an axis of symmetry or of revolution simplifies the resolution of the MAXWELL equations. And in the presence of a closed circulation of the flow and when the conditions of DIRICHLET are verified, the resolution of these equations would be of advantage simplified by the exploitation of the method of numerical resolution of GALERKIN. An example of an application on an air gap has made it possible to illustrate the effectiveness of such a method of resolution.

2. MAXWELL AND MACHINE MODELLING THEORY USING GALERKIN METHOD

The Galerkin method is based on the technique of approximation. Essentially, it involves approximating the solution of a PDE by some arbitrary function of a finite number of variables. A finite difference scheme is then used to construct a numerical solution of the PDE.

The first step of the Galerkin method is to select the basic functions that will be used in the approximation. The basic functions must satisfy a certain set of properties for the approximation to be valid. These properties include linearity, continuity, differentiability, and so on. Once the basic functions have been chosen, they are then used to build a linear system of equations. This system can then be solved to obtain an approximate solution of the PDE.

The key concept of the Galerkin method is the idea of using trial solutions to search for the true solution of a PDE. To do this, a few predetermined trial solutions are constructed and tested against the PDE to determine which one provides the closest approximation. Once the trial solution is chosen, it is then used as the input to a numerical scheme. The numerical scheme is then used to obtain a numerical solution of the PDE.

The Galerkin method is a powerful tool for finding numerical solutions to PDEs. However, it is important to note that the method is not always the most efficient method to use. If a PDE can be solved analytically, then it is usually more efficient to solve it analytically than to use the Galerkin method. Additionally, the Galerkin method requires a number of assumptions, such as linearity and continuity, in order to provide accurate results. Therefore, it is important to choose the appropriate set of assumptions before applying the Galerkin method to a given problem.

The electromagnetic modelling theory of electric machines is described by the differential equations of MAXWELL [2]. The displacement current (D) will be neglected due to the low frequency emitted

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by the electrical source [3]. In table 1, we introduce the basic parameters of Maxwell's equations.

Table 1: Basic Parameters of MAXWELL's Equation

The MAXWELL's equations are given by:

$$\nabla \times H = J \quad (1)$$

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (2)$$

$$\nabla . B = 0 \quad (3)$$

On the other hand, the equations of electric and magnetic fields, noted respectively E and B, are related to the properties of materials by:

$$B = \mu_0 \mu_r H = \mu H \quad (4)$$

$$J = \sigma . E \quad (5)$$

In the frequency domain and taking EDDY currents into consideration, via equations (2) and (3), the fields E and B are expressed as a function of the field vector potential A and the electric scalar potential V by:

$$E = -\frac{\partial A}{\partial t} - \nabla . V \quad (6)$$

$$B = \nabla \times A \quad (7)$$

Equations (1) and (7), lead to the electric current density equation [4] [5].

$$\nabla \times \left(\frac{\nabla \times A}{\mu_0 \mu_r} \right) = J \quad (8)$$

The efficient exploitation of the electromagnetic equations for a safe design of electrical machines requires a prior recognition of the magnetic field flux distribution in the different regions of their magnetic circuits. On the other hand, the complexity of the geometry of the new machines and the non-linearity of their behaviour, mainly due to the saturation of the manufacturing materials of their magnetic circuits, added to the nature of the coupling of the electric and magnetic equations tend to make the system of more complex equations [6].

3. APPLICATIONS OF ELECTROMAGNETIC EQUATIONS TO INDUCTION MACHINES

Induction machines are quasi-static electromagnetic systems [7] formed of a stator containing the field windings and a rotor housing the induced windings. In fact, they consist of a rotor with short-circuited conductors. The movement of the rotating field induces currents in the conductors which in turn develop a magnetic field and consequently a mechanical torque able to turn the rotor.

In our case, the machine studied and schematized by figure 1 has two pairs of poles, 36 stator slots, and is controlled by a three-phase network with a frequency of 50Hz. The squirrel-cage rotor consists of short-circuited copper bars welded at each end to two aluminum rings [8] [10]

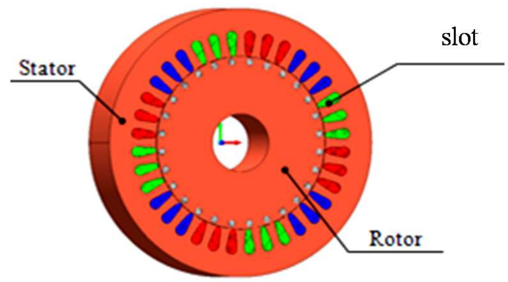

Figure 1. Three-phase asynchronous machine

The number of revolutions of the machine in [rpm] is expressed by: [8]

Symbol.	Notation	Unit
$\nabla .$	Gradient of a scalar field	
$\nabla \times$	Divergence of a vector field	
E	Electric field	V/m
B	Magnetic induction	Tesla
H	Magnetic field strength	A/m
J	Electric current density	A/m2
µ0	Magnetic permeability of air	H/m
µr	Magnetic permeability of the material	constant
□	Electrical conductivity	ms/cm
A	Vector potential of the magnetic field	
V	Electric scalar potential	V
D	Displacement Current	
U	Voltage	V
R	Resistance	Ω

$$n_s = \frac{60f}{p} \quad (9)$$

Where f is the frequency, p is the number of pole pairs and ns the synchronous speed. According to the configuration of the machine studied, the determined number of revolutions n is equal to 1425 rpm.

The slip s of a three-phase M.A.S is the difference between the synchronous speed and that of the rotor expressed relative to the synchronous speed, given by relation 10:

$$s = \frac{n_s - n}{n_s} \tag{10}$$

By analyzing, the induction machine by the equations (1), (2), (4) and (5) and by cancelling the current of displacements D in front of the density of the electric current J [11], we can write that:

$$\frac{\partial D}{\partial t} \ll J \tag{11}$$

The permeability of the laminated iron of the machine depends on the magnetic flux. The expression linking the field vector potential A and the magnetic induction B given by 7 and equation 2 leads us to:

$$E = -\frac{\partial A}{\partial t} - \nabla . V \tag{12}$$

$$\nabla \times (\mu \nabla \times A) = J \tag{13}$$

The density of the electric current is therefore:

$$J = -\sigma . \frac{\partial A}{\partial t} - \sigma . \nabla . V \tag{14}$$

Equation (13) satisfies the relation:

$$\nabla . J = 0 \tag{15}$$

By replacing equation (13) in equation (12) and (14), we obtain:

$$\nabla \times (\mu \nabla \times A) + \sigma \frac{\partial A}{\partial t} + \sigma \nabla . V = 0 \tag{16}$$

$$\nabla \left(\sigma \frac{\partial A}{\partial t} \right) + \nabla (\sigma \nabla . V) = 0 \tag{17}$$

The solution of equation (16) requires that a value must be given to the divergence of the vector potential A. The COULOMB gauge will be used in this case [7] [12]:

$$\nabla . A = 0 \tag{18}$$

If the structure of the machine to be studied has an axis of symmetry, the expressions of equations (15) and (17) will be simpler. When we consider that z is an axis of symmetry, for example, space relates to a plane (x, y) and the field vector potential A and the electric scalar potential V will be expressed by the system (19)

$$\begin{cases} A = A(x, y, t) e_z \\ J = J(x, y, t) e_z \end{cases} \tag{19}$$

Where x, y, t and ez are respectively: the abscissa axis, the ordinate axis, the time, and the unit vector along the z axis. By replacing equation (18) in (13), we find that the scalar potential V is linear along the z axis.

$$V = V_1 z + V_0 \tag{20}$$

Where V1 and V0 are respectively the slope of the affine line and the ordinate at the origin.

For an electrical conductor of length L:

$$U = \int_0^L -\nabla . V . dl = -V_1 L \tag{21}$$

The total current I which runs through the conductor of section S is given by:

$$I = \int_S J dS = \int_S \left(-\sigma \frac{\partial A}{\partial t} - \sigma \nabla . V \right) dS$$

$$I = -\int_S \sigma \frac{\partial A}{\partial t} dS + U \frac{1}{L} \int_S \sigma dS \tag{22}$$

In direct current, the resistance R of the conductor is:

$$R = \frac{1}{\int_S \sigma dS} \tag{23}$$

The voltage across the conductor:

$$U = RI + R \int_S \sigma \frac{\partial A}{\partial t} dS \tag{24}$$

Figure 2 [9] shows a cross section of the induction machine where the flux distribution induced by a three-phase current shifted by [7]

The number of notches per phase q is expressed by relation 25: [13]

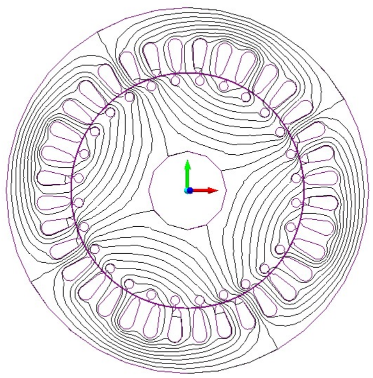

Figure 2. Flux distribution in the three-phase asynchronous machine

$$q = \frac{Z}{2pm} \tag{25}$$

Where Z is the number of slots, p is the number of pole pairs and m the number of phases. Another characteristic is the pole pitch which translates the number of empty slots to leave between two consecutive lines of the same phase. The pole pitch is expressed by equation 26: [13]

$$\tau = \frac{Z}{2p} \tag{26}$$

Once the design of the induction machine is studied, a three-phase current is injected. It is obvious to point out the importance of the mesh in the air gap, schematized by figure 3 [16], and of the material of construction chosen to obtain a flux in the air gap, as shown in figure 4 [16] of the same form as the source from which it originated, namely the mains voltage.

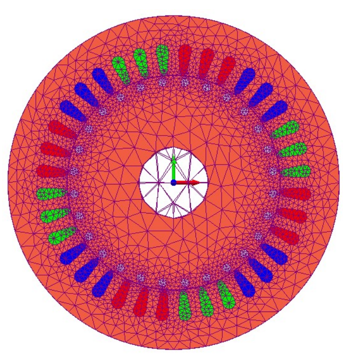

Figure 3. Mesh quality in the induction machine

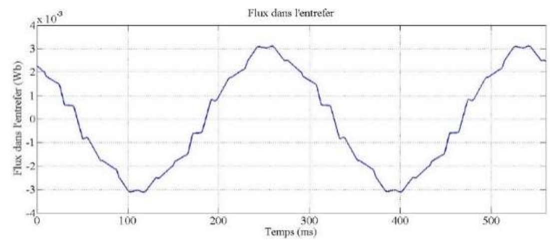

Figure 4. Flux distribution in the air gap

The finite element method is a numerical technique for solving equations resulting from complex geometric models such as electrical machines, which present properties of linear and non-linear magnetic materials that are quite difficult to solve analytically.

The basic principle of the finite element method is to cut any geometric structure into small elements called finite elements, which are interconnected at common points called nodes [14]. In a cartesian coordinate system, a group of nodes forms a mesh. These deniers take a triangular shape if we proceed on a plane (x, y), but on a volume of coordinates (x, y, z), the meshes take the form of a tetrahedron [15].

Thus, the geometrical structure once discretized, the equations of MAXWELL applied are solved for all the meshes containing the structure there. After the discretization of the geometric model, the solutions retained for the finite element analysis relate to the magnetic scalar potential V and the magnetic vector potential A. From these two potentials, based on the Maxwell equations, all the quantities of electrical machines are derived. These parameters are the magnetic field B, the intensity of the magnetic field H, and the flux from the winding.

The study of the different materials constituting the casing of electrical machines is of great importance during finite element analysis. Indeed, ferromagnetic materials depend on the induction at the level of the coils; for weak inductions, the magnetic permeability is a function linking the B and H fields. level of induction, the relation between B and H becomes nonlinear, and the value of the reluctance will then depend on the induction of the coils. [17]. The electrical and magnetic properties of materials are characterized by electrical permittivity, magnetic permeability, and electrical conductivity. [18]

The materials used for the machines impose a limit that stems from saturation, current density, temperature, insulation, and mechanical properties.

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Indeed, a good alloy allows us to guarantee a strong magnetic permeability to avoid saturation. [19]
Once the proper materials for the design of the machine are fixed [20], it is obvious to note the importance of using finite element analysis. The mastery of this calculation tool consists in designing a fair distribution of the meshes over the entire model of the machine, but in particular in the air gaps. We then demonstrate that the magnetic parameters () will depend on the length and the mesh of the air gap.

4. CONCLUSION

The resolution of the Maxwell equations by the GALERKIN method and the DIRICHLET technique of electrical machines in general by finite elements requires in-depth multidisciplinary knowledge in order to properly exploit the results during finite element analysis. The safe design phase of electrical machines requires perfect mastery of field calculation software, the refinement of the meshes of which in the air gap zone must be dense and equitably distributed throughout the air gap, especially since this zone is the seat of the different magnetic parameters such as field and flux distribution.
FEA is often combined with boundary condition techniques such as Dirichlet Condition to effectively simulate the real-world behavior of the components of an electric motor. With the combination of FEA and DC, the design process can be optimized and adequate performance can be ensured in real-world applications with respect to certain criteria. This combination not only ensures that the electric motor components undergo enough stress to meet the requirements, but also reduces the wastage of electricity to unsafe levels. This means efficient energy utilization and minimizes losses due to ill-constructed motor components. Moreover, components such as bearings can be evaluated for their lifetime effectiveness and an accurate prediction of demise can be made. FEA and Dirichlet conditions employed together improve the overall performance and lifespan of an electric motor in an effective manner.

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