ABSTRACT

This paper introduces an analysis of the phenomena of stability of synchronous machines under small perturbations by examining the case of a single machine connected to a large system through external impedance, and uses robust control $H_\infty$ techniques to design stabilizer for electric power system. $H_\infty$ techniques that are used are $H_\infty$ -optimal controller synthesis, $H_\infty$ - mixed sensitivity controller synthesis and $H_\infty$ - loop shaping controller synthesis.

Keywords: Stability Of Synchronous Machines, Single Machine, Robust Control, Power System Stabilizer, $H_\infty$ -Optimal Controller Synthesis, $H_\infty$ - Mixed Sensitivity Controller Synthesis And $H_\infty$- Loop Shaping Controller Synthesis.

1. INTRODUCTION

The phenomenon of stability of synchronous machine has received a great deal of attention in the past and will receive increasing attention in the future. As economies in the system design are achieved with larger unit sizes and higher per unit reactance generating and transmission equipment designs, more emphasis and reliance is being placed on the controls to provide the required and the reliance is being placed on controls to provide the required compensating effects with which to offset the reductions in stability margins inherent from these trends in the equipment design [1].

The electric power system is a complex system with highly non-linear dynamics. Its stability depends on the operating conditions of the power system and its configuration. Low frequency oscillations are a common problem in large power systems. Excitation control or Automatic Voltage Regulator (AVR) is well known as an effective means to improve the overall stability of the power system. Power System Stabilizers (PSS) are added to excitation systems to enhance the damping during low frequency oscillations. The output of the PSS is applied as a supplementary control signal to the machine voltage regulator terminal. Oscillations of small magnitude and low frequency often persist for long periods of time and in some cases can cause limitations on the power transfer capability.

The Power System Stabilizer (PSS) is a device that improves the damping of generator electromechanical oscillations. Stabilizers have been employed on large generators for several decades, permitting utilities to improve stability-constrained operating limits. The input signal of conventional PSS is filtered to provide phase lead at the electromechanical frequencies of interest (ie, 0.1 Hz to 5.0 Hz). The phase lead requirement is site-specific, and is required to compensate for phase lag introduced by the closed-loop voltage regulator.

The PSS conventional and the PSS control based on root locus and eigenvalue assignment design techniques have been widely used in power systems. Such PSS ensure optimal performance only at a nominal operating point and do not guarantee good performance over the entire range of the system operating conditions due to exogenous disturbances such as changes of load and fluctuations of the mechanical power. In practical power system networks, a priori information on these external disturbances is always in the form of a certain frequency band in which their energy is concentrated. Remarkable efforts have been devoted to design appropriate PSS with improved performance and robustness. These have led to a variety of design methods using optimal control [2] and adaptive control [3]. The shortcoming of these model-based control strategies
is that uncertainties cannot be considered explicitly in the design stage. More recently, robust control theory has been introduced into PSS design which allows control system designers to deal more effectively with model uncertainties [4, 5, 6 and 7]. H∞ based control approach is particularly appropriate for plants with unstructured uncertainty. In this paper, a PSS based on H∞ robust control techniques is introduced and results are displayed in time response approach for studying stability of electric power system under different conditions.

2. SYSTEM DESCRIPTION

The power system considered in this study is modelled as a synchronous generator connected through a transmission line to infinite busbar. A simplified model that describing the system dynamics used in this study is given by the following state space equations [6,8].

\[ x' = Ax(t) + B_1w(t) + B_2u(t) \quad (1) \]
\[ z(t) = C_1x(t) + D_{11}w(t) + D_{12}u(t) \quad (2) \]
\[ y(t) = C_2x(t) + D_{21}w(t) + D_{22}u(t) \quad (3) \]

where \( u \) represents the PSS output added to the voltage set points \( \Delta V_{ref} \), \( \omega \) is an external disturbance represented by the mechanical power \( \Delta P_m \). The matrices \( A, B_1, B_2 \), the vector \( z, y \) and the state vector \( x \) are defined by

\[
A = \begin{bmatrix}
0 & 314 & 0 & 0 \\
-K_e/M & 0 & -K_t/M & 0 \\
-K_i/M & 0 & -1/K_eT_{do} & 1/T_{do} \\
-(K_e/T_a)K_1 & 0 & -(K_t/T_a)K_0 & -1/T_{a} \\
\end{bmatrix}
\] (4)

\[
B_1 = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ 0 \end{bmatrix}^T, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ K_e/T_a \end{bmatrix}^T
\]

\[
Z = \begin{bmatrix} \Delta P_e \end{bmatrix}^T, \quad Y = \begin{bmatrix} \Delta V_e \end{bmatrix}^T
\]

\[
x = [\Delta \delta \ \Delta \omega \ \Delta E''_f \ \Delta E''_d \ \Delta P_e]^T
\]

where \( \delta, \omega, E''_f, E''_d, P_e \) and \( V_i \) are respectively the torque angle, the angular velocity the internal machine voltage, the excitation voltage, power output and generator terminal voltage. \( T_{do} \) is the open-circuit transient time constant, \( \Delta \) represents a small deviation around the operation point. The operating conditions for the above systems are completely defined by the values of the real (P) and reactive (Q) powers at the generator terminals and the transmission line impedance \( X_e \). A detailed block diagram of the power system (open loop) is shown in Fig. 1.

Fig. 1 block diagram for open loop

We note that the constants \( K_i (i = 1, ..., 6) \) are uncertain and depend upon the network parameters, the quiescent operating conditions and the infinite bus voltage[8].

3. ROBUST CONTROL

Feedback control is well understood for large classes of nonlinear systems with single inputs. For general multi-input nonlinear systems, however, feedback control and especially robustness issues are still research topics, the urgency of which has been rendered more acute by the recent development of machines with challenging nonlinear dynamics. The basis for control design and stability analysis is a dynamical model that captures prominent features of the system under consideration. To account for unnoticeable and unknown aspects of the real system in the mathematical model, one often uses the notion of uncertainty. Uncertainty denotes any obscure element in the dynamics of the real system. Possible uncertainties include unknown parameters, unknown functions, disturbances, and unmodeled dynamics. In general uncertainties can be either stochastic or deterministic and control design and performance analysis must be done accordingly. Uncertainties can also be classified as either “structured” or “unstructured” [9].

Structured Uncertainty represents parametric variation in the plant dynamics, for example:

- Uncertainties in certain entries of state-space matrices \( (A, B, C, D) \).
- Uncertainties in specific poles and/or zeros of the plant transfer function
- Uncertainties in specific loop gains/phases.

Unstructured uncertainty may be used to represent frequency-dependent elements such as actuator saturations and unmodeled structural modes in the high frequency range or plant disturbances in the low frequency range.
The challenge in robust multi-variable feedback control system design is to synthesize a control law which maintains system response and error signals to within pre-specified tolerances despite the effects of uncertainty on the system. Depending on the nature of the uncertainties, different designs can be used to achieve effective control.

With the birth of robust control in the late seventies though, $H^\infty$ proved to be superior in terms of their robustness [10, 11].

The linear quadratic regulator, Kalman filter, and linear quadratic gaussian problems these (optimization problems) can be alternatively posed using the system $H^\infty$-norm as a cost function. The $H^\infty$-norm is the worst-case gain of the system and therefore provides a good match to engineering specifications, which are typically given in terms of bounds on errors and controls [11]. The terms $H^\infty$ norm and $H^\infty$ control are not terms which convey a lot of engineering significance. $H^\infty$ is considered a design method which aims to minimize the peak(s) of one or more selected transfer functions. The $H^\infty$ norm of a stable scalar transfer function $F(s)$ is the peak value of $|f(j\omega)|$ as a function of frequency, that is

$$\|F(s)\|_\infty = \max_{\omega} |f(j\omega)| \quad (5)$$

Strictly speaking, “max” (the maximum value) should be replace by “sup” (supremum, the least upper bound) because the maximum may only be approached as $\omega \to \infty$ and may therefore not actually be achieved. The symbol $\infty$ comes from the fact that the maximum magnitude over frequency may be written as

$$\max_{\omega} |f(j\omega)| = \lim_{P \to \infty} \left( \int_{-\infty}^{\infty} |F(J\omega)|^p d\omega \right)^{\frac{1}{p}} \quad (6)$$

Essentially, by raising $|F|$ to an infinite power, we pick out its peak value. $H^\infty$ is the set of transfer functions with bounded $H^\infty$-norm, which is the set of stable and proper transfer functions [12].

3-a. $H^\infty$ optimal controller problem formulation:

Given a proper continuous time linear time-invariant plant $P$ mapping exogenous inputs $w$ and control inputs $u$ to controlled outputs $z$ and measured outputs $y$. That is

$$\begin{pmatrix} z(s) \\ y(s) \end{pmatrix} = P(s) \begin{pmatrix} w(s) \\ u(s) \end{pmatrix}$$

and given some dynamic output feedback law $u = K(s)y$ and with the partitioning

$$P(s) = \begin{pmatrix} P_{11}(s) & P_{12}(s) \\ P_{21}(s) & P_{22}(s) \end{pmatrix}$$

the closed-loop transfer function from disturbance $\omega$ to controlled output $z$ is:

$$F(P, K) = P_{11} + P_{12}K(I - P_{22}K)^{-1}P_{21}$$

The overall control objective is to minimize the $H^\infty$ norm of the transfer function from $w$ to $z$. This is done by finding a controller $K$ which, based on the information in $y$, generates a control signal $u$ which counteracts the influence of $w$ on $z$, thereby minimizing the closed-loop norm from $w$ to $z$.

In practice, we calculate the suboptimal rather than optimal solution. The sub-optimal $H^\infty$ control problem of parameter $\gamma$ consists of finding a controller $K(s)$ such that:

- The closed-loop system is internally stable.
- The $H^\infty$ norm of $F(P, K)$ (the maximum gain from $w$ to $z$) is strictly less than $\gamma$, where $\gamma$ is some prescribed performance level.

It might be noticed here that the term “suboptimal” is used rather than “optimal”. The reason for that is that it is often not necessary and sometimes even undesirable to design an optimal controller. A suboptimal controller may also have nice properties (e.g., lower bandwidth) over the optimal ones. However, knowing the achievable optimal (minimum) $H^\infty$-norm may be useful theoretically since it sets a limit on what can be achieved.

3-b. $H^\infty$ mixed sensitivity controller:

Mixed sensitivity $H^\infty$ design approach does not necessarily look for an optimum solution but rather looks for a solution which satisfies many requirements or specifications at once. Shaping the sensitivity function ($S = (J + GX)^{-1}$) along with one or more other closed-loop transfer functions such as $KS$ or the complementary sensitivity function ($T = I - S$) provides a direct and effective way of achieving multi-variable loop shaping.
Fig. 3 depicts block diagram for mixed sensitivity controller:

![Block Diagram](image)

In the problem formulation, disturbance attenuation specifications, stability margin specifications as well as other specifications can be combined into a single infinity norm specification of the form:

$$\begin{bmatrix} W_1S \\ W_2K \\ W_3T \end{bmatrix}$$

and this is usually called the mixed-sensitivity cost function which is to be minimized.

Where \( G \) is the open loop system, \( W_1, W_2 \) and \( W_3 \) are weighting functions.

**weighting function selection [13]:**

Selection of the weighting function is very important in design. Fortunately, the relation between the open loop frequency domain and time domain performance is well understood. Typically, we select weights \( W_1 \) and \( W_2 \) such that the open loop has the following conflict properties:

1. Achieving good performance tracking and good disturbance rejection require large open loop gain normally at a low frequency range.
2. Achieve good robust stability and sensor noise rejection requires a small open loop gain normally at a high frequency range.

**3-c. \( H^\infty \) loop shaping controller:**

\( H^\infty \) loop-shaping control, proposed by McFarlane and Glover [14], is an efficient way to design a robust controller and has been applied to a variety of control problems. Uncertainties in this approach are modeled as co prime factor uncertainty. This uncertainty model does not represent actual physical uncertainty, which, in fact, is unknown.

This approach requires only a desired open loop shape in the frequency domain. Two weighting functions, \( W_1 \) (pre-compensator) and \( W_2 \) (post-compensator), are specified to shape original plant \( G \) so that the desired open loop shape is achieved. Fig. 4 shows a block diagram for \( H^\infty \) loop-shaping control.

![Block Diagram](image)

In this approach, the shaped plant is formulated as a normalized co prime factor that separates plant \( G_s \) into normalized nominator \( N_s \) and denominator \( M_s \) factors. In any plant model \( G \), the shaped plant \( G_s \) is formulated as [14]:

$$G_s = W_2GW_1 \Rightarrow \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$G_s = (N_s + \Delta N_s)(M_s + \Delta M_s)^{-1}$$

Where \( A, B, C, D \) represent plant in the state-space, form \( \Delta N_s, \Delta M_s \) \( \leq \varepsilon \), \( N_s \) and \( M_s \) are nominator and denominator normalized co prime factors. \( \Delta N_s \) and \( \Delta M_s \) are uncertainty transfer functions in nominator and denominator factors. \( \varepsilon \) is an uncertainty boundary, called a stability margin. To obtain these normalized co prime factors, the following equation is applied [15]:

$$[N_s \quad M_s] = \begin{bmatrix} A+HC & B+HD \\ R^{-1/2}C & R^{-1/2}D \end{bmatrix}$$

Where

\( H = -(BD^T+ZC)^{-1} \), \( R = I + DD^T \) and matrix \( Z \geq 0 \) is the unique positive definite solution to the algebraic Riccati equation

\[
(A - BS^{-1}D^T C)Z + Z(A - BS^{-1}D^T C)^T - ZC^TR^{-1}CZ + BS^{-1}B^T = 0
\]

Once the desired loop shape is achieved, the \( H^\infty \)-norm of the transfer function from disturbances \( w \) to states \( z \) is subjected to be minimized over all stabilizing controllers \( K \).

**4. RESULTS**

The following figures depict the step response for system under study and at different operating points.
with different transmission lines impedance \((x_e)\), where weighting functions that are chosen to ensure satisfactory performance of the closed loop system at high frequencies are:

\[
W_1 = \frac{0.04(S + 0.2)}{S + 0.004}, \\
W_3 = \frac{0.75(S + 0.33)}{S + 600}, \\
W_2 = [\text{ ]}
\]

The following figure depicts frequency response for weighting functions.

It is noted that weighting function intersected near frequency 30 rad/sec.

4-a. Result of system without stabilizer:

4-b. Result of \(H^\infty\) optimal controller:

From previous figures, it can be conclude that system at open loop only stable at light load (fig.6) and unstable for normal and heavy loading (fig.7, 8) respectively.

Fig. 7 \(\Delta \omega\) at \(P=1\) p.u. and \(Q=0.015\) p.u.

Fig. 8 \(\Delta \omega\) at \(P=1.25\) p.u. and \(Q=0.3\) p.u., \(x_e=0.997\) p.u.

Fig. 9 \(\Delta \omega\) at \(P=0.05\) p.u. and \(Q=-0.225\) p.u.
Fig. 10 $\Delta \omega$ at $P=1$ p.u. and $Q=0.015$ p.u.

Fig. 11 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.997$ p.u.

Fig. 12 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.7$ p.u.

Fig. 13 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.45$ p.u.

Fig. 14 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.2$ p.u.

4-c. Result of $H^\infty$ mixed sensitivity controller:

Fig. 15 $\Delta \omega$ at $P=0.05$ p.u. and $Q=-0.225$ p.u.

Fig. 16 $\Delta \omega$ at $P=1$ p.u. and $Q=0.015$ p.u.

Fig. 17 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.997$ p.u.
4-d. Result of $H^\infty$ loop shaping controller:

Fig. 18 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.7$ p.u.

Fig. 19 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.45$ p.u.

Fig. 20 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.2$ p.u.

Fig. 21 $\Delta \omega$ at $P=0.05$ p.u. and $Q=-0.225$ p.u.

Fig. 22 $\Delta \omega$ at $P=1$ p.u. and $Q=0.015$ p.u.

Fig. 23 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.997$ p.u.

Fig. 24 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.7$ p.u.

Fig. 25 $\Delta \omega$ at $P=1.25$ p.u. and $Q=0.3$ p.u., $xe=0.45$ p.u.
These results are obtained by using MATLAB, Robust Control Toolbox functions [16].

4-e. Transfer function results:

Transfer function of the system has very important role for studying the stability of the any system by obtaining characteristics equation for the system. The transfer function of the system under study without stabilizers has 4th order, but when adding $H_\infty$ optimal controller, $H_\infty$ mixed sensitivity controller, and $H_\infty$ loop shaping controller to system, the order of transfer function of the system with controllers will be 11th, 11th, and 15th respectively and the parameters of the system will be changed by changing the loading conditions.

Example form results of transfer function for system with and without controllers at heavy load ($P=1.25$ p.u., $Q=0.3$ p.u., $X_e=0.997$):

Transfer function for original model without any controller: (4th order):

$$1000s^3 + 253.9s^2 - 7795s + 3061$$

Transfer function by using $H_\infty$ mixed sensitivity syntheses (11th order):

$$6.468e004s^{13} + 7.988e008s^{12} + 3.306e012s^{11} + 4.652e015s^{10} + 2.862e017s^9 + 7.076e018s^8 + 9.117e019s^7 + 6.579e020s^6 + 2.624e021s^5 + 5.278e021s^4 + 4.315e021s^3 + 7.963e020s^2 + 1.941e017s + 4.74e004$$

Transfer function for all system after using loop shaping syntheses (15th order):

$$6.468e004s^{13} + 7.988e008s^{12} + 3.306e012s^{11} + 4.652e015s^{10} + 2.862e017s^9 + 7.076e018s^8 + 9.117e019s^7 + 6.579e020s^6 + 2.624e021s^5 + 5.278e021s^4 + 4.315e021s^3 + 7.963e020s^2 + 1.941e017s + 4.74e004$$

5. CONCLUSION

In this paper the design and evaluation of power system stabilizers based $H_\infty$ techniques has been considered. The simulation results presented demonstrate the effectiveness of these control techniques to improve the stability and transient response of power systems under a variety of operating conditions. The robustness of the controller has been evaluated with respect to model uncertainties of the power system. $H_\infty$ techniques which were used are : $H_\infty$ optimal controller, $H_\infty$ mixed sensitivity controller, and $H_\infty$ loop shaping controller. From the simulation results that were obtained it was clear that $H_\infty$ optimal controller gave the best results between them, it gave faster damping, with less overshooting, also it is noted that the order of the transfer function of the system is increased by adding controllers.

6. REFERENCES


BIOGRAHY:

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