



# COMPUTER AIDED DESIGN OF BRUSHLESS SYNCHRONOUS GENERATOR USING MATLAB

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## ABSTRACT

This paper deals with the presentation of a new formulation to solve combined magnetic and electric problems including the motional electromotive force in a rotating electric machine. In this analysis, the boundary element method and finite element method are applied to the brushless synchronous generator. This method takes the effect of the relative motion between the field winding and the armature winding into account, this technique is used in order to analyze the motion induced current in the armature windings. We use this method to predict the dynamic behavior of electric machines that is gaining in popularity because of the availability of high performance microcomputers and the development of new features, such as the integration of the rotor's motion and the machine's supply circuitry in the finite element solution.

**Keywords:** *Computer Aided Design (CAO), Brushless Synchronous Generator (BSG), Matlab.*

## 1. INTRODUCTION

Magnetic and electric field analysis of a brushless synchronous generator is presented in this paper. This kind of machine has armature windings which are connected to the diodes. The static magnetic field generated by additional field windings induces speed electromotive force in the closed armature windings, connected diodes rectify alternating currents, then they can provide d.c armature currents without brushes.

To model magnetic phenomena in 2D, the current density must usually be known. But, people often use voltage sources, so they are compelled to make some iteration to calculate the final voltage from the current density in order to try with a new value of current density. To avoid such problems, the idea is to solve simultaneously the magnetic and electric equations. In the literature, we find two types of approach, the integro-differential method [1] and the direct method [2]-[5], this last method seems to be more efficient.

Building a machine model based on simplifying assumptions allows a fast prediction of the system's behavior using commercial circuit simulation software.

When a linear model of the machine is assumed, this method is fast but often inaccurate because the

magnetic material's nonlinear behavior, as well as the influence of spatial harmonic's and eddy currents, cannot be included in the machine model without leading to long computation time and complexity.

One way to improve the simulation's accuracy to directly compute the drive system's dynamic using the numerical method.

Numerical methods, such as the boundary method and the finite element method has become established tools for electric machine analysis. However, the application of conventional methods to the electric generator is difficult. The reasons are as follows:

- (1) The current induced by the relative motion has not been possible to determine.
- (2) The governing equation including the motional electromotive force becomes a convection-diffusion equation. It is well-known that the high Peclet number produces unstable solutions.
- (3) When using FEM, or BEM the mesh subdivisions should be modified as the rotor moves.

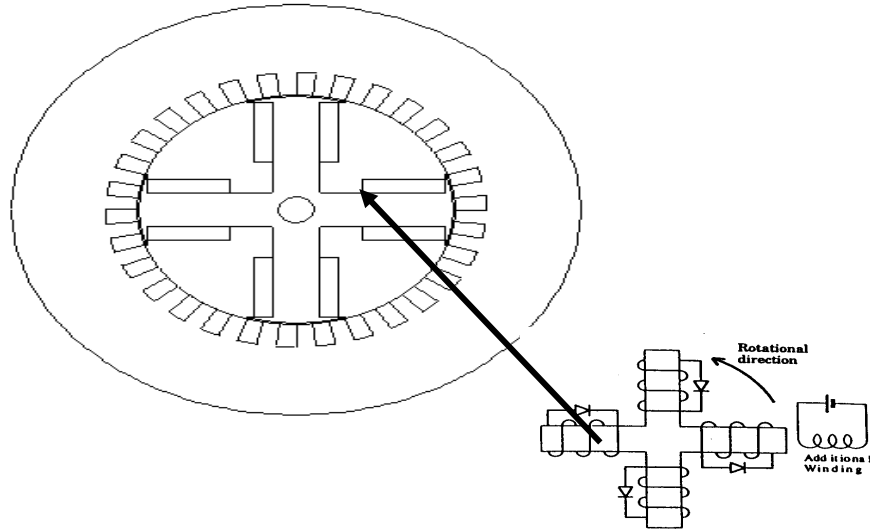


Figure 1. Model of brushless synchronous generator

In a rotating machine, the effect of the relative motion is of considerable importance in some practical application. In this paper, the boundary element and the finite element methods taking the external power source is developed [6]-[7], so that the motion induced current can be determined. This method is applied to the brushless synchronous generator as shown in Figure 1, and clarifies the behavior of the magnetic field when a load is applied.

Additionally, we expand the finite element method to apply a brushless synchronous generator. The functioning of this apparatus depends on the nonlinear characteristics of the diodes connected to the armature windings. It is necessary to analyze the apparatus taking into account not only speed electromotive force but also nonlinear electronics circuits. With this method, the behavior of the magnetic field due to the armature reaction and the contributions of diode rectifiers can be clarified.

## 2. FORMULATION

The Maxwell equations play a fundamental role in the well established formulation of the electromagnetic theory [8]. These equations lead to the derivation of precise mathematical models useful in many applications in physics and engineering. The Maxwell equations involve only the integer-order calculus and, therefore, it is natural that the resulting classical models adopted in electrical engineering reflect this perspective. Recently, a closer look of some phenomena present

in electrical systems, such as motors, transformers and lines, and the motivation towards the development of comprehensive models, seen to point out the requirement for fractional calculus approach.

In an alternative perspective several authors have verified that well-known expressions for the magnetic potential are related through integer-order integral and derivatives and have proposed its generalization based on the concept of fractional-order poles. Nevertheless, the mathematical generalization towards calculus approach lacks a comprehensive method for its practical implementation.

In the stationary coordinate system Maxwell's equations are always valid and expressed as follows:

$$(1) \begin{cases} \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} = \mathbf{J} \\ \nabla \cdot \mathbf{B} = 0 \\ \nabla \cdot \mathbf{D} = 0 \end{cases}$$

Where  $\mathbf{E}$  is the electric field intensity,  $\mathbf{B}$  the magnetic flux density,  $\mathbf{H}$  the magnetic field intensity,  $\mathbf{J}$  the current density, and  $\mathbf{D}$  the electric flux density. It is assumed that the electric charges and the displacement currents are negligible.



The Galilean space-time transformation may be applied to equations (1) to give Maxwell's equations in a moving coordinate system [7]:

$$(2) \quad \begin{cases} \nabla' \times (\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla' \times \mathbf{H} = \mathbf{J} \\ \nabla' \cdot \mathbf{B} = 0 \\ \nabla' \cdot \mathbf{D} = 0 \end{cases}$$

where primes denote quantities expressed in the moving coordinate system, and  $\mathbf{v}$  is the velocity of a point in the moving coordinate system  $(x', y', z', t')$ .

The quantities  $\mathbf{H}$ ,  $\mathbf{B}$ ,  $\mathbf{D}$  and  $\mathbf{J}$  are not altered in both coordinate systems, only the electric field intensity vector  $\mathbf{E}$  is modified as:

$$(3) \quad \mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}$$

In two-dimensional analysis, substituting  $\mathbf{B} = \text{rot } \mathbf{A}$  into equation (3) yields:

$$(4) \quad \mathbf{E}' = -\left(\frac{\partial \mathbf{A}}{\partial t}\right)' - \text{grad } \varphi' - v_x \left(\frac{\partial \mathbf{A}}{\partial x}\right)' - v_y \left(\frac{\partial \mathbf{A}}{\partial y}\right)'$$

Where  $\mathbf{A}$  is the magnetic vector potential,  $\varphi$  is the scalar potential,  $v_x$  and  $v_y$  are  $x$ - and  $y$ -directional components of the velocity, respectively.

On the other hand, in the stationary coordinate system, the electric field intensity  $\mathbf{E}$  is:

$$(5) \quad \mathbf{E} = \frac{\partial \mathbf{A}}{\partial t} - \text{grad } \varphi$$

In the case of a generator, the electric field vector intensity is induced in the windings. The terminal voltage  $U_T$  of the windings is expressed as follows:

$$(6) \quad U_T = -n_w \frac{d\Phi}{dt} = n_w \int_1 \mathbf{E} dl$$

Where  $n_w$  is the number of turns of winding,  $\Phi$  is the interlinkage flux, and  $l$  is the length around the winding. Assuming the windings are connected to the external electric circuit as shows in Figure 2, the circuit equations are obtained as follows:

In a stationary winding:

$$(7) \quad U_1 = n_w \int \frac{dA_{i1}}{l dt} dl + R_1 I_1$$

In a moving winding:

$$(8) \quad U_2 = n_w \int \left( \frac{dA_{i2}}{l dt} + v_x \frac{dA_{i2}}{dx} + v_y \frac{dA_{i2}}{dy} \right) dl + R_2 I_2 + U_{diode}$$

The subscript  $i$  means the value at the winding,  $U$  is the voltage of the external power source,  $R$  the resistance,  $I$  the current and  $U_{diode}$  the terminal voltage of the connected diode.

When the rotation is assumed to be around the  $z$ -axis at an angular frequency  $\omega_r$ ,  $v_x$  and  $v_y$  are given by:

$$(9) \quad v_x = -\omega_r y, \quad v_y = -\omega_r x$$

The boundary integral equation including induced current is expressed as follows:

$$(10) \quad C_i A_i + \int_{\Gamma} q^* A d\Gamma = \int_{\Gamma} A^* q d\Gamma + \mu_0 \frac{nw}{S} \int_{\Omega} A^* I d\Omega$$

In order to determine the unknown current  $I$ , the circuit equation including the external electric circuit should be combined with the field equation (10).

In general,  $A$  and its derivatives are unknown, however, they can be given by the boundary integral equations:

$$(11) \quad A_i = \frac{1}{C_i} \left( -\int_{\Gamma} q^* A d\Gamma + \int_{\Gamma} A^* q d\Gamma + \mu_0 \frac{nw}{S} \int_{\Omega} A^* I d\Omega \right)$$

After the discretizing process, the field equations (10), (11) and equations (7), (8) are combined to solve the equations.

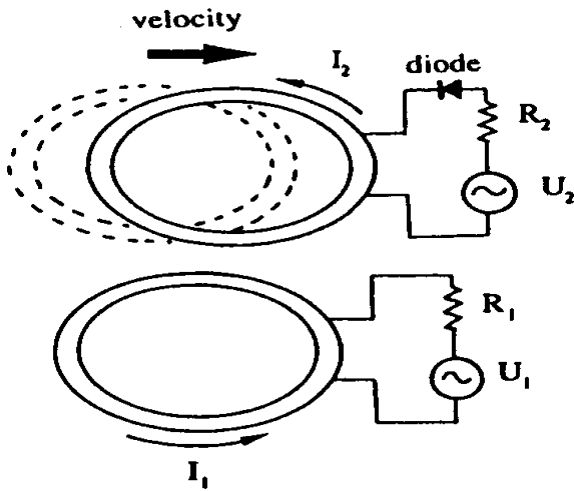


Figure 2. Relative motion of windings

This method has been extensively used in solving problems in electrostatics, high voltage, electrical machines and electromagnetic fields. In most cases the problem to be solved can be modelled in terms of the Laplace equation in either two or three dimensions. The accuracy of the solution obtained from this analysis depends on the error introduced due to (i) Approximation of geometry, (ii) Function interpolation and (iii) Numerical integration. Many approaches have been reported in the literature to address some of these problems in order to get a better approximation [9].

Consider a region  $D$  bounded by a closed surface, from Green's theorem we can derive the boundary integral equation:

$$\theta(p) \cdot \Phi(p) = [\theta(p) \cdot \Phi(p)]_1 - [\theta(p) \cdot \Phi(p)]_2 \tag{12}$$

with:

$$[\theta(p) \cdot \Phi(p)]_1 = \frac{1}{4\pi S} \int \frac{\partial \Phi(q)}{\partial n_q} \cdot \frac{1}{|\mathbf{q} - \mathbf{p}|} dq \tag{13}$$

$$[\theta(p) \cdot \Phi(p)]_2 = \frac{1}{4\pi S} \int \Phi(q) \frac{\partial}{\partial n_q} \cdot \frac{1}{|\mathbf{q} - \mathbf{p}|} dq \tag{14}$$

Here  $\mathbf{p}$  is the field point where the solution is desired,  $\mathbf{q}$  is the source point on the surface  $S$ .

$$\theta(p) = 1 \text{ if } \mathbf{p} \in D$$

$\theta(p) = \Omega / (4\pi)$  if  $\mathbf{p} \in S$  ( $\Omega$  is the solid angle subtended at the field point).

$\theta(p) = 0$  elsewhere.

If the solution domain is the infinite space outside  $S$ , we consider an imaginary sphere of large radius tending to infinity centred around the field point. The new formulation is now applied to the annular region between  $S$  and the sphere at infinity, this leads to:

$$\theta(p) \cdot \Phi(p) = [\theta(p) \cdot \Phi(p)]_1 - [\theta(p) \cdot \Phi(p)]_2 + \Phi_\infty \tag{15}$$

Here  $\Phi_\infty$  is the potential at infinity, which need not be zero.

In this new formulation we divide the surface  $S$  into a mesh elements. The unknown potential in the second term in equation (12) and the unknown normal gradient of the potential in the first term of equation (12) are linearly interpolated within each element in terms of the values at the nodes of the triangle. When the field points are chosen to be one of the nodes of the mesh, the left hand side of equation (12) can be written in terms of the unknown nodal potentials. This leads to the following matrix equation:

$$\sum_j (C_i \delta_{ij} - h_{ij}) \Phi_j = \sum_j g_{ij} \phi_j \tag{16}$$

If equation (15) is solved we have a similar equation but with one more unknown  $\Phi_\infty$ . For this we need an additional equation, this is obtained from the following condition:

$$\int \frac{\partial \Phi(q)}{\partial n_q} dq = 0 \tag{17}$$

In equation (16),  $C_i$  is the value of  $\theta(p)$  defined after equation (12) and is related to the solid angle at the field point  $\mathbf{p}$ , which is one of the nodes  $\mathbf{n}$ . The matrix elements are given by:

$$g_{ij}^{(e)} = \frac{|\mathbf{t}_1 \times \mathbf{t}_2|}{4\pi} \int_0^1 \int_0^{1-\xi} \frac{N_j(\xi, n)}{|\mathbf{n} - \sum_k N_k \mathbf{r}_k|} d\xi dn \tag{18}$$

and

$$h_{ij}^{(e)} = \frac{-1}{4\pi} \int_0^1 \int_0^{1-\xi} \frac{N_j(\xi, n) [\mathbf{r}_i - \sum_k N_k \mathbf{r}_k]}{|\mathbf{r}_i - \sum_k N_k \mathbf{r}_k|} d\xi dn \quad (19)$$

The superscript is used in equation (18) and (19) to emphasise that the integral is carried over an element.

In these integrals  $\mathbf{t}_1$  and  $\mathbf{t}_2$  are the vectors along the triangle edges and  $N_i$  are the interpolation functions.

### 3. BUILDING A MODEL

The recent high growth of the power of the computer systems, development of the software's to solve complicated problems made it possible to use methods of calculations in electric machinery to obtain the magnetic field analysis.

The most spread nowadays finite element method still demands too much computing power to obtain the results in a short time. In this case, we have developed in our institution a software which is used to build a different machines by Matlab®. The structure and the strategy of used this software to build a brushless synchronous generator is given by Figure 3.

This work is articulated around the CAO of brushless synchronous generator in static and dynamic mode using Matlab® software.

Matlab® is a general purpose mathematical utility program which provides editing, plotting, debugging and graphics capabilities, as well as access to extensive and sophisticated library of powerful computational algorithms, and is becoming widely used throughout the engineering community.

#### 3.1. Constructing the geometry

The geometry of an object as shown in Figure 1 is constructed using points and lines. The surface, shell and pinpoint regions are constructed from these entities. The user has access to parameters which can help in both the construction of the geometry and in modifying the geometry (modifying the shape, displacing an object....),.

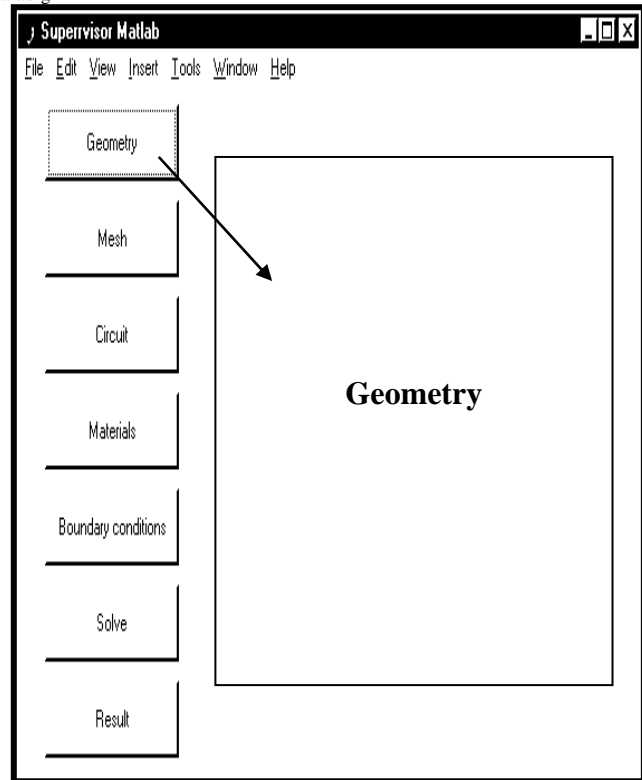


Figure 3. Supervisor Matlab

The normal method consists of first defining the parameters which will be used. A parameter is defined with a name, a comment and an arithmetic expression.

The second step to follow consists of defining the points, they are determined by their coordinates (constant numerical values, parameters, arithmetic expression) or by duplicating other points (duplication is done by modifying the values of parameters, and not by applying geometric transformations to the points).

The third step is the definition of the lines, there are three types: straight line, arc and circle.

The last step in the geometry construction consists of defining the regions, the surface regions are constructed from one or several closed surfaces limited by the lines.

#### 3.2. Meshing the geometry

The finite elements method requires the subdivision of the study domain into elements to create what is called the mesh as shown in Figure 4. The mesh can be created with the help of the automatic mesh generator or the assisted mesh generator which allows a very precise control of the

### 3.3. Describing the circuit

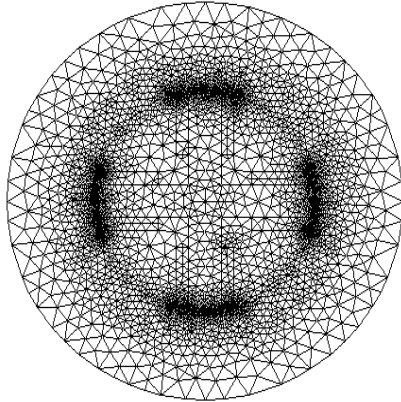


Figure 4. Meshing the geometry

mesh obtained. These mesh generators generate second order finite elements.

The automatic mesh generator generates triangular mesh elements, using the subdivisions on those lines having been used for the geometry definition. The user can increase or decrease the number of subdivisions on a line or choose the distance between two subdivisions. The subdivisions can be linearly or non-linearly distributed. If required the mesh generator can increase by dichotomy the number of nodes on a line.

The assisted mesh generator allows you to control precisely the shape and quality of the mesh, which is very useful in analyzing certain problems (for example a small skin depth).

The user can subdivide those surface regions enclosed by three or four lines, regions that make up the elementary mesh. The user chooses one by one the mesh elements to subdivide and for each element the type of subdivision and the number of subdivisions. The resultant mesh can be refined by further dividing previously generated mesh elements. The mesh elements can be triangular or rectangular in shape.

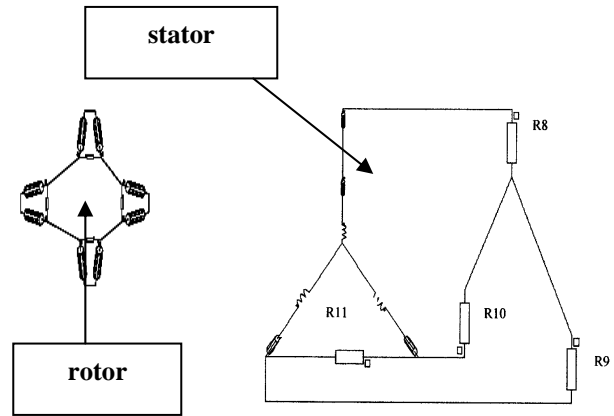


Figure 5. Circuit model

The following section gives a brief and general approach to solve electromagnetic field problems. To solve the problem of the brushless synchronous generator, the circuit equations must be coupled with the field equations and Figure 5 shows the circuit model used.

Matlab<sup>®</sup> computes the back emf of the stator winding by connecting this winding to an external open circuit load and rotating the rotor over one electric cycle. The three phase stator winding for the one pole numerical model of the motor (1/4 of the machine) is star connected. The finite-element formulation equations are coupled with the circuit equations which leads to a system of equations that is solved taking the non-linearities into account by means of a Newton-Raphson method and using a time stepping technique. The time step must be small enough to assure good results and the choice of this value is critical.

### 3.4. Defining the physical properties

Each material as shown in Figure 6, is described by a name, a comment and one or several physical properties.

A property can be defined by a model which indicates the variable(s) on which it depends, by the form of this dependence and by numerical values (for example an electric resistivity can be described by a function linear with temperature; in which case you need to supply two numerical values).

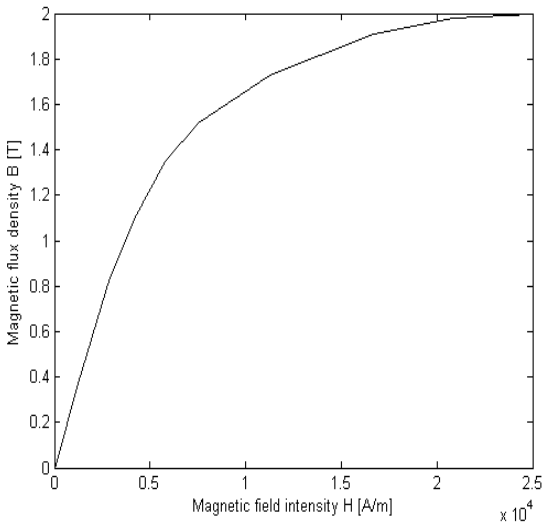


Figure 6.  $B(H)$  Curve

### 3.5. Boundary conditions

The boundary conditions are set on nodes belonging to lines: external or internal shell regions, internal pinpoint regions, parts of external boundaries. They allow you to set limits to the study domain by setting the value of the variable, to take into account physical symmetries, to impose the value of the variable inside the study domain Matlab® offers the following boundary conditions: *Neumann condition* and *Dirichlet condition*.

### 3.6. Solving the problems

Solver, which uses Matlab® functions, allows you, to define the values of the geometrical and physical parameters to be taken into account during the solving process, and to carry out the solving process it self.

### 3.7. Results analysis

This program allows you to process the results of a problem by using Matlab® functions (isovalues, color shaded plots, 2D and 3D curves, computation of local or global quantities, animations, parametric analysis ...).

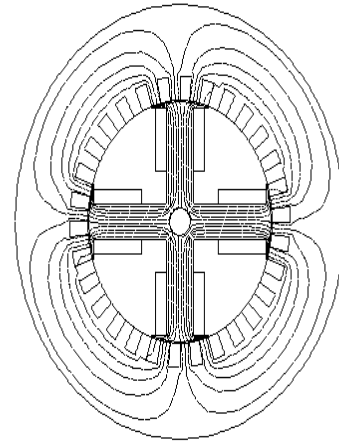


Figure 7a. Distribution of the magnetic potential vector

The magnetic vector potential  $\mathbf{A}$  presented in Figure 7a and Figure 7b, is assumed to vary sinusoidal with time at the frequency  $f$  or angular frequency  $\omega$  and is represented by its complex phasor notation.

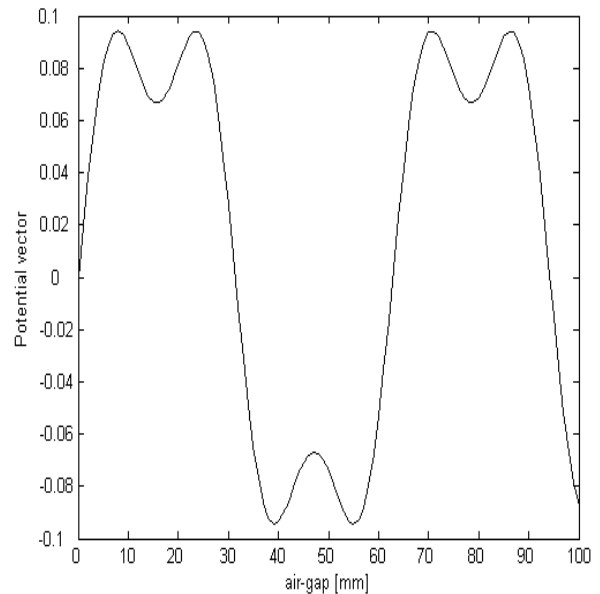


Figure 7b. The magnetic potential vector along the air gap

The rotating air gap is a circular region situated inside the air gap of a rotating machine presented in Figure 8. It allows you to turn the rotor without changing the geometry and the mesh of the machine in the geometry module. When solving the problem

and for each rotor position, the coordinates of the nodes of the rotor are recalculated and the finite elements of the rotating air gap are reconstructed.

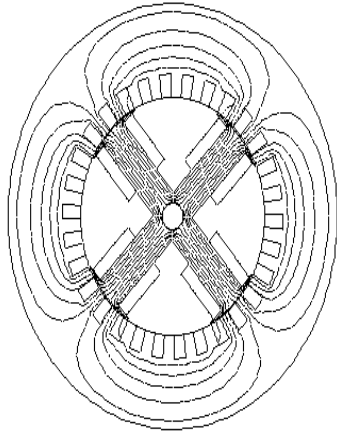


Figure 8. Rotating the machine

field....) can be represented in the form of vectors on a region as shown in Figure 9. The vectors are computed in each finite element, in the integration point where the quantity has a maximum value. Figure 10 indicate the flux density components  $B_x, B_y$

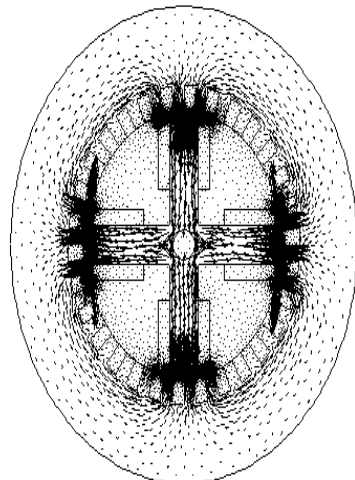


Figure 9. Magnetic flux density vectors

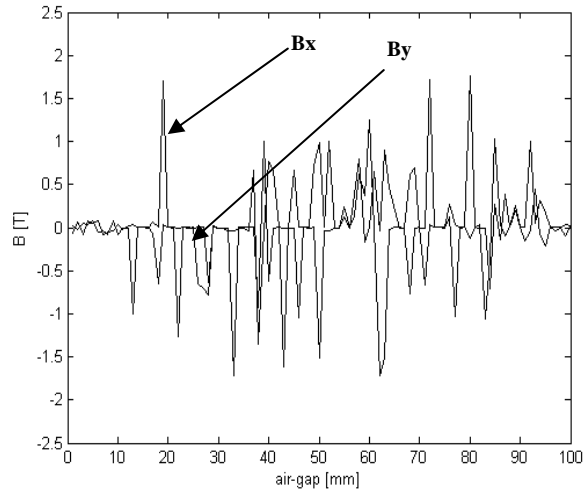


Figure 10. The flux density components  $B_x$  et  $B_y$  along the air-gap

Here are presented the results of the transient analysis in the case of three phase bolded short circuit applied on the terminals of the synchronous generator.

The responses of phase currents, field current and the electromechanical torque are obtained when magnetic field solutions are coupled with simulations.

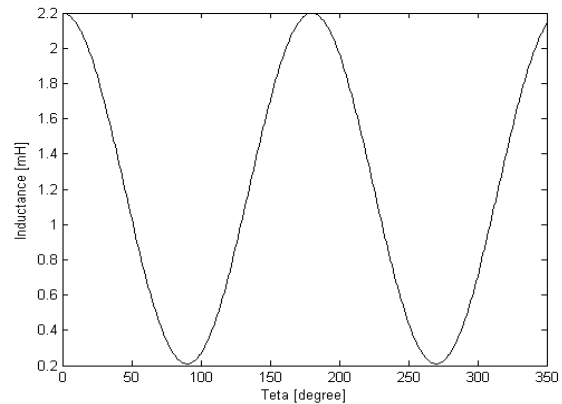


Figure 11. Variation of the inductance with the position



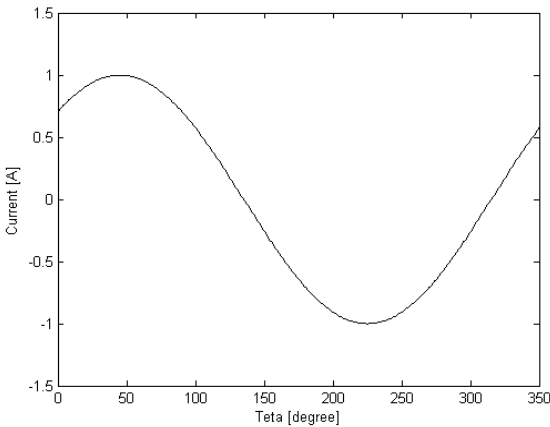


Figure 12. Variation of the current with the position

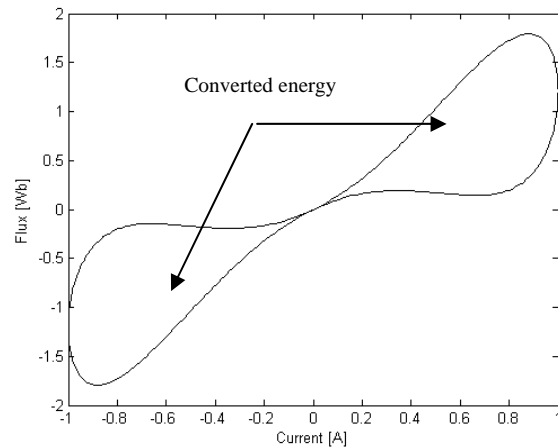


Figure 14. Characteristic flux-current of the machine

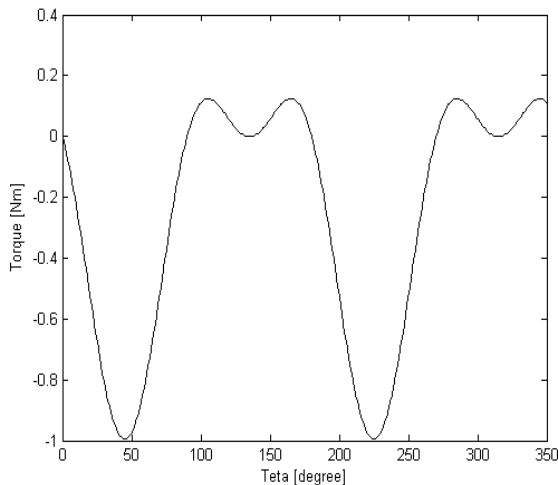


Figure 13. Variation of the torque with the position

The Figures 11 to 13 respectively, shows the variation of inductance, the current and the torque of the machine. The torque is maximum for  $45^\circ$  mechanical when the current and the derivative of inductance are maximum.

Moreover, there is creation of a engine torque parasites between  $90^\circ$  and  $180^\circ$  mechanical which causes the reduce of the average Torque.

The Figure 14 shows the characteristic flux-current of the machine at the time of a cycle of operation. It is noticed that converted energy is reduced compared with DC machines.

#### 4. CONCLUSION

In this paper the results of an analysis electromagnetic design of brushless synchronous generator using numerical methods as the boundary element and the finite element method were presented and discussed. The results comprehended the analysis of the proposed design of the generator done by the software Matlab.

This new method is used in order to analyze the induced current in the armature windings. This technique take the external power source into account including the motional electromotive force in a rotating of the electric machines.

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