



INSTANTANEOUS POWER THEORY BASED ACTIVE POWER FILTER: A MATLAB/ SIMULINK APPROACH

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ABSTRACT

Power quality standards (IEEE-519) compel to limit the total harmonic distortion (THD) within acceptable range caused by rapid usage of power electronic equipment. This paper envisages on the simulation of instantaneous active and reactive theory based shunt active filter with MATLAB/ Simulink, as a better solution for reduction of the harmonics.

Keywords: Active Filters, THD, Instantaneous Power Theory.

1. INTRODUCTION

In a modern power system, due to broader applications of nonlinear loads such as power electronic equipment or arc furnaces, the degree of waveform distorted is increasingly serious now. These loads may cause poor power factors, lead to voltage notch, or result in a high degree of harmonics. Such cases have brought the power quality as an increasing concern. Moreover, from economical viewpoints, a utility's revenue may get affected at a higher cost. Therefore, efficient solutions for solving these pollution problems have become highly critical for both utilities and customers.

The amount of distortion in the voltage or current waveform is quantified by means of an index called the *total harmonic distortion* (THD). The THD in current is defined as

$$\%THD = 100 \times \sqrt{\sum_{n \neq 1} \left(\frac{I_{sn}}{I_{s1}} \right)^2}$$

Conventional methods are of harmonics/ current reference and classified either as time or frequency-domain and are limited steady-state analysis. This paper envisages on the instantaneous power theory validating both the steady and transient-state analysis. The compensation command signals are obtained from the instantaneous active power and the

instantaneous reactive power. This method does not require the phase synchronization.

2. INSTANTANEOUS POWER THEORY

Akagi et al [1] proposed a theory based on instantaneous values in three-phase power systems with or without neutral wire, and is valid for steady-state or transitory operations, as well as for generic voltage and current waveforms called as Instantaneous Power Theory or Active-Reactive (p-q) theory which consists of an algebraic transformation (Clarke transformation) of the three-phase voltages in the *a-b-c* coordinates to the $\alpha\text{-}\beta\text{-}0$ coordinates, followed by the calculation of the p-q theory instantaneous power components:

$$\begin{bmatrix} v_o \\ v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} \frac{1}{\sqrt{2}} & 1 & 0 \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} v_o \\ v_\alpha \\ v_\beta \end{bmatrix} \quad (2)$$



where v_a, v_b, v_c are phase voltages. Identical relations hold for line currents $i_a, i_b,$ and i_c .

The instantaneous three-phase power is given by:

$$\begin{aligned} p_{3\phi}(t) &= v_a i_a + v_b i_b + v_c i_c = v_\alpha i_\alpha + v_\beta i_\beta + v_0 i_0 \\ &= p_\alpha(t) + p_\beta(t) + p_0(t) = p_\alpha(t) + p_\beta(t) + p_0(t) \\ &= p(t) + p_0(t) \end{aligned} \quad (3)$$

where $p = p_\alpha + p_\beta$ is instantaneous real power; and $p_0(t) = v_0 i_0$ is the instantaneous zero-sequence power.

One advantage of using the transformation of α - β -0 is to separate the zero-sequence component of the system.

The reactive power measurement can be give by $q(t) \approx v_\alpha i_\beta - v_\beta i_\alpha$

$$(4)$$

Rewritten in terms of a-b-c components as

$$q = -[(v_a - v_b)i_c + (v_b - v_c)i_a + (v_c - v_a)i_b] / \sqrt{3} \quad (5)$$

The powers p and q can be rewritten as

$$\begin{bmatrix} p \\ q \end{bmatrix} = \begin{bmatrix} v_\alpha & v_\beta \\ -v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} \quad (6)$$

From this matrix equation, for $\Delta = v_\alpha^2 + v_\beta^2$

$$\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \frac{1}{\Delta} \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} \quad (7)$$

Separating the Active and Reactive parts

$$\begin{aligned} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} &= \frac{1}{\Delta} \left\{ \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} p \\ 0 \end{bmatrix} + \begin{bmatrix} v_\alpha & -v_\beta \\ v_\beta & v_\alpha \end{bmatrix} \begin{bmatrix} 0 \\ q \end{bmatrix} \right\} \\ &\cong \begin{bmatrix} i_{\alpha p} \\ i_{\beta p} \end{bmatrix} + \begin{bmatrix} i_{\alpha q} \\ i_{\beta q} \end{bmatrix} \end{aligned} \quad (8)$$

where, the current components are

$$i_{\alpha p} = v_\alpha p / \Delta, i_{\alpha q} = -v_\beta q / \Delta \quad (9)$$

$$i_{\beta p} = v_\beta p / \Delta, i_{\beta q} = v_\alpha q / \Delta \quad (10)$$

Power in phases α and β can be separated as

$$\begin{aligned} \begin{bmatrix} p_\alpha \\ p_\beta \end{bmatrix} &= \begin{bmatrix} v_\alpha & i_\alpha \\ v_\beta & i_\beta \end{bmatrix} = \begin{bmatrix} v_\alpha & i_{\alpha p} \\ v_\beta & i_{\beta p} \end{bmatrix} + \begin{bmatrix} v_\alpha & i_{\alpha q} \\ v_\beta & i_{\beta q} \end{bmatrix} \\ &\cong \begin{bmatrix} p_{\alpha p} \\ p_{\beta p} \end{bmatrix} + \begin{bmatrix} p_{\alpha q} \\ p_{\beta q} \end{bmatrix} \end{aligned} \quad (11)$$

where, the power components are

$$p_{\alpha p} = v_\alpha i_{\alpha p} = v_\alpha^2 p / \Delta \quad (12)$$

$$p_{\alpha q} = v_\alpha i_{\alpha q} = -v_\alpha v_\beta q / \Delta \quad (13)$$

$$p_{\beta p} = v_\beta i_{\beta p} = v_\beta^2 p / \Delta \quad (14)$$

$$p_{\beta q} = v_\beta i_{\beta q} = v_\alpha v_\beta q / \Delta \quad (15)$$

Therefore the three phase active power can be rewritten

$$\begin{aligned} p_{3\phi}(t) &= p_\alpha + p_\beta + p_0 \\ &= p_{\alpha p} + p_{\alpha q} + p_{\beta p} + p_{\beta q} + p_0 \\ &= p_{\alpha p} + p_{\beta p} + p_0 \end{aligned} \quad (16)$$

Thus from equations (13) and (15)

$$p_{\alpha q} + p_{\beta q} = 0 \quad (17)$$

Thus, $p_{\alpha p}$ - α axis instantaneous active power.

$p_{\beta p}$ - β axis instantaneous active power.

$p_{\alpha q}$ - α axis instantaneous reactive power.

$p_{\beta q}$ - β axis instantaneous reactive power.

It is observed that the reactive power corresponds to the parts of instantaneous power, which is dependent on the instantaneous imaginary power q , in each independent phase and vanishes when added ($p_{\alpha q} + p_{\beta q} = 0$), in a two-phase ($\alpha - \beta$) system.

Instantaneous real power p , gives the net energy per second being transported from source to load and vice-versa at any time, which is

dependent only on the voltage and currents in phases α and β and has no zero-sequence present.

2.2 Non-linear Load

The three phase sinusoidal voltages supplying a non-linear load are represented as

$$\begin{aligned} v_a &= \sqrt{2}V \sin \omega t \\ v_b &= \sqrt{2}V \sin(\omega t + 120^\circ) \\ v_c &= \sqrt{2}V \sin(\omega t - 120^\circ) \end{aligned} \quad (18)$$

and the currents being

$$\begin{aligned} i_a &= \sum_{n=1}^{\infty} \sqrt{2}I_n \sin(n\omega t - \phi_n) \\ i_b &= \sum_{n=1}^{\infty} \sqrt{2}I_n \sin[n(\omega t + 120^\circ) - \phi_n] \\ i_c &= \sum_{n=1}^{\infty} \sqrt{2}I_n \sin[n(\omega t - 120^\circ) - \phi_n] \end{aligned} \quad (19)$$

Then,

$$i_\alpha = \sum_{n=1}^{\infty} \frac{2}{\sqrt{3}} I_n \sin(n\omega t - \phi_n) [1 - \cos(n120^\circ)] \quad (20)$$

$$i_\beta = \sum_{n=1}^{\infty} 2I_n \cos(n\omega t - \phi_n) \sin(n120^\circ) \quad (21)$$

$$\begin{aligned} i_0 &= \frac{1}{\sqrt{3}} (i_a + i_b + i_c) \\ &= \sum_{n=1}^{\infty} \sqrt{6}I_{3n} \sin(3n\omega t - \phi_{3n}) \end{aligned} \quad (22)$$

The power components p , q , p_0 and $p_{3\phi}$ are

$$\begin{aligned} p &= v_\alpha i_\alpha + v_\beta i_\beta = p_{\alpha p} + p_{\beta p} \\ &= 3VI_1 \cos \phi_1 \\ &\quad - 3VI_2 \cos(3\omega t - \phi_2) + 3VI_4 \cos(3\omega t + \phi_4) \\ &\quad - 3VI_5 \cos(6\omega t - \phi_5) + 3VI_7 \cos(6\omega t + \phi_7) - \dots \end{aligned} \quad (23)$$

$$\begin{aligned} q &= v_\alpha i_\beta - v_\beta i_\alpha = 3VI_1 \sin \phi_1 \\ &\quad - 3VI_2 \sin(3\omega t - \phi_2) + 3VI_4 \sin(3\omega t + \phi_4) \\ &\quad - 3VI_5 \sin(6\omega t - \phi_5) + 3VI_7 \sin(6\omega t + \phi_7) - \dots \end{aligned} \quad (24)$$

$$p_0 = v_0 i_0 = 0 \text{ and } p_{3\phi} = p \quad (25)$$

Thus, these expressions are

$$p = \bar{p} + \tilde{p} \text{ and } q = \bar{q} + \tilde{q} \quad (26)$$

Each of these expressions represents the mean-value and alternating components with mean-value equal to zero.

From (23) and (24) it can be concluded that

$$\bar{p} = P_{3\phi} \text{ and } \bar{q} = Q_{3\phi} \quad (27)$$

Thus the harmonic power is given by

$$H = \sqrt{\tilde{P}^2 + \tilde{Q}^2} \quad (28)$$

where \tilde{P} and \tilde{Q} are the RMS values of \tilde{p} and \tilde{q} , respectively.

3. COMPENSATION STRATEGY

The reactive and harmonic compensation is carried by injecting appropriate currents into the circuit through a compensator i.e., shunt active filter as shown in Fig. 1.

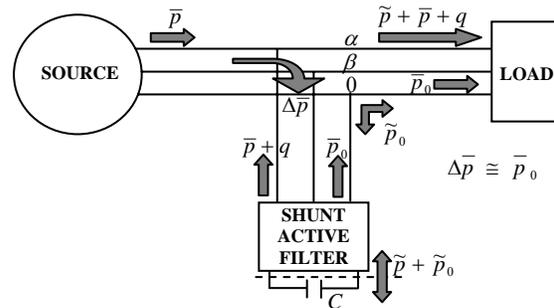


Fig. 1 Strategy of Instantaneous Power Theory

In order to compensate $p_{\alpha q}$ and $p_{\beta q}$, currents $i_{\alpha c}$ and $i_{\beta c}$ are injected equivalent to reactive currents as

$$i_{\alpha c} = i_{\alpha q}, \text{ and} \quad (29)$$

$$i_{\beta c} = i_{\beta q} \quad (30)$$

where, $i_{\alpha q}$ and $i_{\beta q}$ are given by (9) and (10).

The current source i_{ac} is in shunt with the voltage source v_{α} , thus supplies the power $P_{\alpha q} = v_{\alpha} i_{\alpha q}$. Similarly, the current source $i_{\beta c}$ supplies $P_{\beta q} = v_{\beta} i_{\beta q}$. Thus, the voltage sources v_{α} and v_{β} need to supply only $P_{\alpha p}$ and $P_{\beta p}$. From (17), the power necessary to compensate for $i_{\alpha q}$ is equal to the negative of the power necessary to compensate for $i_{\beta q}$.

The current sources $i_{\alpha c}$ and $i_{\beta c}$ represent active power filters, that are generated from the VSI inverter controlled to generate $i_{\alpha q}$ and $i_{\beta q}$. As such, no DC source is necessary and also no large energy storage element is necessary to compensate the reactive powers. Instantaneously, the reactive power required by one phase can be supplied by the other one. This means that, the size of the capacitor does not depend on the amount of reactive power to be compensated. In fact, in actual systems only a small capacitor is used because the switching of the inverters.

As stated in (26), p can be decomposed in two parts as \bar{p} and \tilde{p} . As \bar{p} is the actual working power, only \tilde{p} has been compensated.

Thus (12) and (14) are modified to:

$$P_{\alpha \tilde{p}} = v_{\alpha} i_{\alpha \tilde{p}} = v_{\alpha}^2 \tilde{p} / \Delta \quad (31)$$

$$P_{\beta \tilde{p}} = v_{\beta} i_{\beta \tilde{p}} = v_{\beta}^2 \tilde{p} / \Delta \quad (32)$$

The above power terms have mean-value equal to zero but their summation is not zero at every instant, that is, $P_{\alpha \tilde{p}} + P_{\beta \tilde{p}} \neq 0$. The capacitor receives energy when \tilde{p} is negative and supplies when \tilde{p} is positive.

4. SIMULATION RESULTS

The simulation of the proposed instantaneous power theory is carried on MATLAB/ Simulink as represented in the Fig. 2.

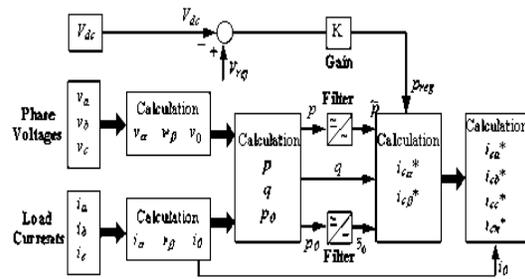


Fig. 2 Proposed power control strategy

The source supply is designed with amplitude of 360 volt and frequency of 315 rad/sec with a phase difference of 2.0944 rad between phases.

The load is simulated to include harmonic distortion by injecting the diode current generated from a three phase uncontrolled diode rectifier of 36 kW and a three phase fully controlled thyristor rectifier of 12 kW with 60° firing delay angle.

A 4th order Butterworth Low Pass Filter with 25 Hz cut off frequency delaying the rise in the dc quantities.

A VSI inverter is instantaneous and infinitely fast to track current reference and is implemented as a current amplifier with unity gain block with unity value.

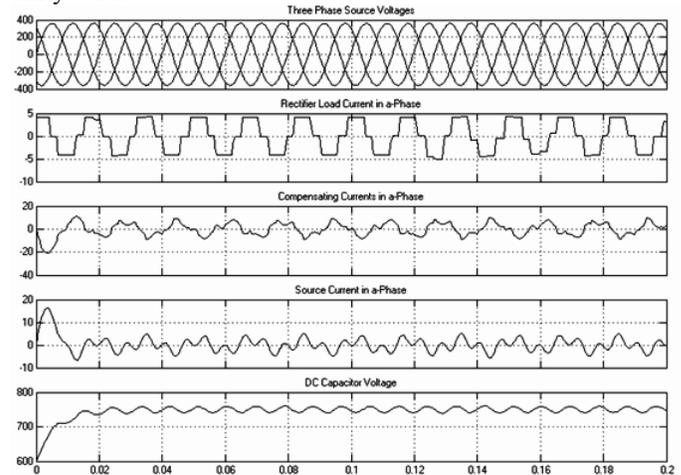


Fig. 3 Simulation Results of Ideal Source Voltage with Diode Rectifier Load

The THD were seen as spectral analyses with 512 samples per second were observed so as to show the distortion up to 25th harmonic.

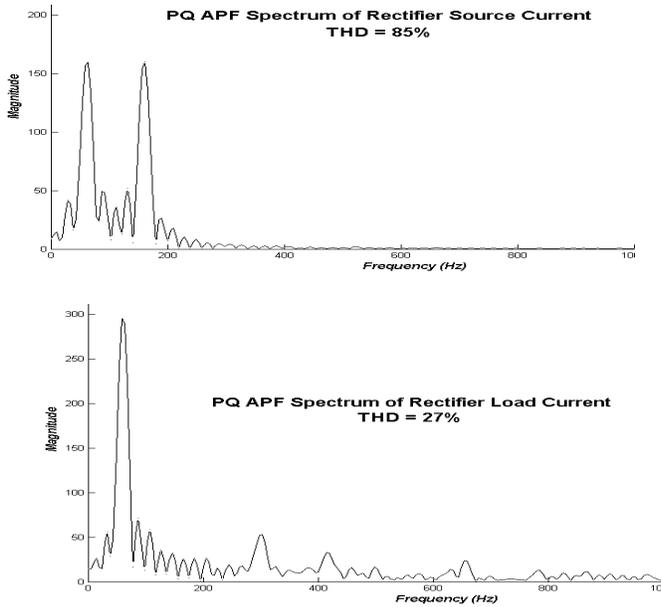


Fig. 4 Spectral Analysis with Total Harmonic Distortion of Source and Load Currents for Diode Rectifier Load with Ideal Source

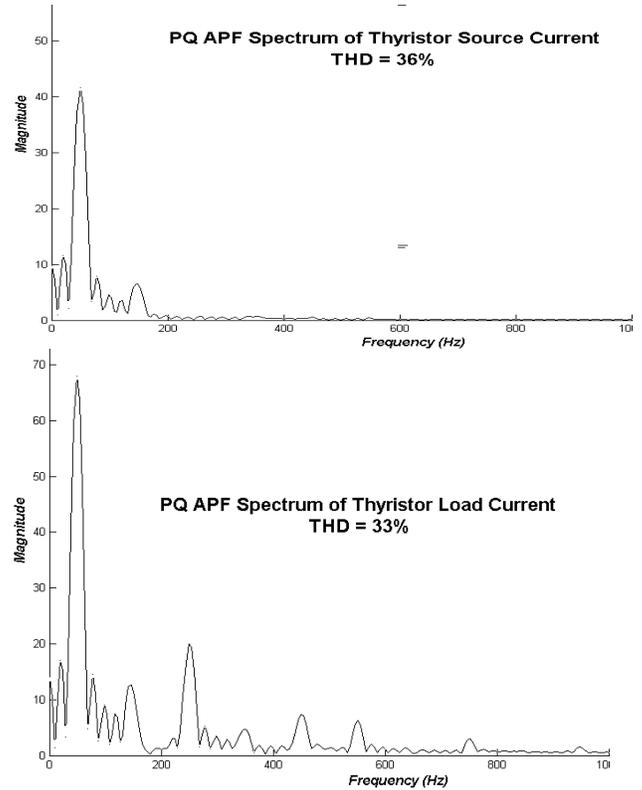


Fig. 6 Spectral Analysis with Total Harmonic Distortion of Source and Load Currents for Thyristor Rectifier Load with Ideal Source

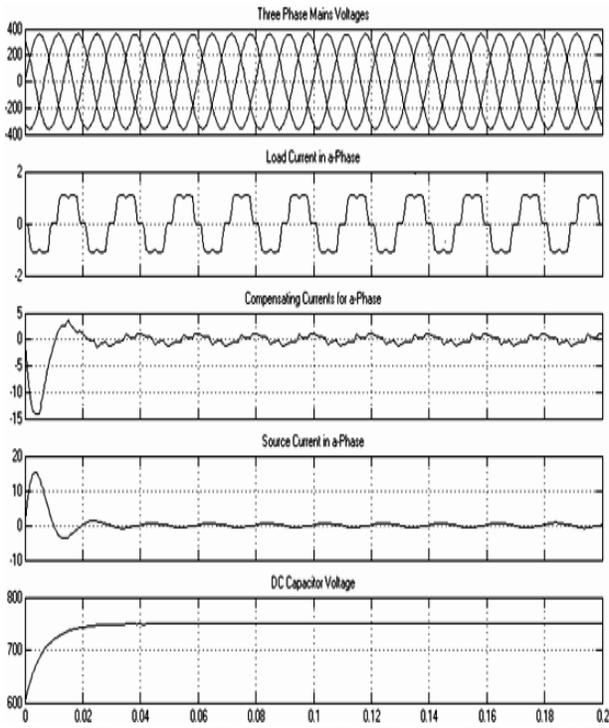


Fig. 5 Simulation Results of Ideal Source Voltage with Thyristor Rectifier Load

The spectral analyses were also obtained with load simulated as thyristor rectifier load with fire angle control to vary load THD.

A comparative study on the above brings that, the THD of 85% has been reduced drastically to 27% with un-controlled rectifier diode bridge in the load with ideal source voltages and in controlled thyristor rectifier diode bridge from 36% to a nominal 33% approving the chipping in of the designed active power filter in the power system prone to the harmonics.

5. CONCLUSIONS

Instantaneous power theory gives a piecemeal approach in analysis and control of the active and reactive components of the harmonic load and introduces the active power filter for appropriate corrective measure for the total harmonic distortion for improvement of the power quality as per the scheduled standards.

Energy efficient power supplies incorporating active power supplies shall govern the future in the electrical power quality standardizations.

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