A ROBUST VARIABLE STRUCTURE POSITION CONTROL OF DC MOTOR

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ABSTRACT

Variable structure control (VSC) is a controlling method in which the structure of feedback is altered to get desired output for DC motor. The position of DC motor is controlled using VSC. However, the VSC technique has not had wide spread acceptance due to some limitations in form of chattering which is ever-existing in any VSC system. The chattering problem is much more serious in low moment of inertia systems because of finite amount of discontinuous surface. The DC motors have not only low inertia but also small time constants. The chattering is main problem for such type motor. The concept of switching lines is used to reduce chattering. In this paper the concept of three lines switching technique (TLST) is employed to eliminate the chattering in VSC controlled DC motor. In this technique, switching lines are used to delay the reversal of direction of the representative point (RP). The biggest advantage of this technique is that it results in invariant systems and makes the system total robust.

Keywords: Variable structure control, Chattering, Representing point, Three lines switching technique, Sliding mode.

Nomenclature

U : Control input
K_a : Power amplifier gain
r : Reference input
X_1 : Angular Position measured by transducer
K_1 : Position switching gain
K_p : Position constant
K_2 : Velocity switching gain
K_v : Velocity constant
θ : Angular position of output shaft
E_a : voltage input to the motor
s : Laplace variable
K_m : Motor gain constant
T_m : Motor time constant
X_2 : Angular velocity measured by transducer
α : Position transducer gain
β : Velocity transducer gain
w : Angular velocity
S : Switching variable
t_h : Hitting time (Second)
C : Slope of the switching line
t : Time (Second)

1. INTRODUCTION

VSC consists of a set of continuous sub-system together with the suitable switching logic [1-3]. It means plant remains same but the characteristics of system are changed. For this, the gains and other parameters are changed to alter the characteristics of system; it means that a program of a model is divided into subprogram. This subprogram is changed by increasing/ decreasing gain and switching as positive or negative according to required condition, than change in subprogram is called VSC. Advantageous properties are the result of changing structure according to switching logic.

By changing the structure of this system one can resolve the conflict between static accuracy and speed of response. The motion of the representing point (RP) along a line (switching line), which is not a trajectory of any of these structures, is sliding mode. Variable structure system operating in sliding mode has much advantage [4-6]. For most among motion in sliding mode is insensitive to variation in the plant parameters and external disturbance. However, these invariance properties are exhibited only during the so-called sliding mode. The control laws are so designed that the system trajectory always reaches the sliding mode.

In the design of VSC system, it is assumed that the control can be switched from one value to another and will be infinitely fast. Practically it is impossible to achieve high switching control due to presence of finite time delays for control computations, limitations of physical actuator, switching delays due to non-idealities such as hysteresis, and neglected time constants etc. So system trajectory oscillates about the switching line. These high frequency oscillations about the switching line in sliding mode and about the desired equilibrium point in the steady state mode of VSC system is termed as chattering [7]. In many VSC systems chattering causes harmful effects, such as torque pulsation, unmodelled undesired high frequency dynamics and acoustic noise. Thus an important problem is to reduce chattering. Problem is much more serious in low moment of inertia system because of finite amount of discontinuous surface, and thus helps to smoothen the movement of the RP around the switching line. A chattering reduction technique is used that is three lines switching technique. It reduces the chattering and increase the efficiency of system.

2. VARIABLE STRUCTURE CONTROL

The block diagram of VSC for the position control of DC motor in sliding mode based on the control law [4, 5, 6] is shown in "Figure 1". The output from position tracking is fed back to reference input. The output of summation is given to VSC. The control input U after variable structure control is fed to DC motor can be written as:

$$U = K_a [(r - X_1) K_1 K_p + d/dt (r - X_1) K_2 K_v] \quad (1)$$

The DC motor dynamics can be written as [8]

$$\theta(s)/E_a(s) = K_m/s(1+sT_m)$$

$$s^2 T_m \theta(s) + s\theta(s) = K_m E_a(s)$$

Taking inverse Laplace

$$\ddot{\theta} T_m + \dot{\theta} = K_m E_a \quad (2)$$

To convert this is into State Space, it can be written as

$$\theta = X_1$$

$$\dot{\theta} = X_2 = \dot{X}_1$$

$$\ddot{\theta} = \dot{X}_2$$

Output of transducers are used for position and velocity measurement is

$$X_1 = \alpha\theta,\ X_2 = \beta w$$

$$\theta = X_1 / \alpha \quad \text{and}$$

$$w = \dot{\theta} = X_2 / \beta = \dot{X}_1 / \alpha \quad (3)$$

From equation (2) and (3)

$$T_m (\dot{X}_2 / \beta) + (X_2/ \beta) = K_m U$$

Where U (= E_a) is control input voltage to the motor.

Since $X_2 = \beta \dot{X}_1 / \alpha$, motor dynamics can be written as

$$(T_m \beta/\alpha) \ddot{X}_1 + (\beta/\alpha) \dot{X}_1 = \beta K_m U \ \ddot{X}_1 + (1/T_m) \dot{X}_1 = (\alpha K_m / T_m) \quad (4)$$

From equations (1) & (4) when r = 0

$$\ddot{X}_1 + (1/T_m + \alpha K_m K_a K_v K_2/T_m) \dot{X}_1 + (\alpha K_m K_a K_p K_1/T_m) X_1 \quad (5)$$

For the given system under consideration values of various parameter and gains are:

T = 0.04 sec, K_p = 10, α = 2.3 V/rad, K_m = 0.39, β = 0.25 V-sec/rad, K_a = 2, K_v = 0.1

By putting these values in equation (5)

$$\ddot{X}_1 + (25+22.425K_2) \dot{X}_1 + 2242.5 K_1 X_1 = 0 \quad (6)$$

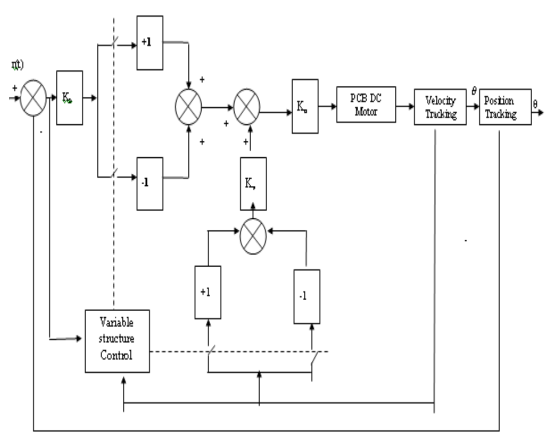

Figure1. Block diagram of VSC in sliding mode

3. THREE LINES SWITCHING TECHNIQUE

In the simulation of conventional VSC, It has been seen that whatever combination of slope and gains, we may choose; considerable amount of chattering still persists [7]. To tackle this problem, the concept of three switching lines is used. The basic aim in using three switching lines is to delay the reversal of direction of the representative point. In this technique, there are three lines S_1, S_0 and S_2 with slopes C_1, C_0 and C_2 respectively. Where S_0 is the main line around which sliding motion takes place, and it is named under supervisory line. The other two lines S_1 & S_2 are used to check the armature voltage and back electromagnetic force (EMF) of motor which introduce symmetrical result in different acceleration, deceleration and high switching. These three lines are:

$S_1 = C_1 X_1 + X_2 = 0$
$S_0 = C_0 X_1 + X_2 = 0$
$S_2 = C_2 X_1 + X_2 = 0$

where $C_2 > C_0 > C_1 > 0$ and the ratio C_2/C_1 should not be made large for robustness requirement.

The RP follows a motion which is shown in "Figure 2", from A to F through B, C, D and E. The same sequence is repeated until the point reaches within a ball of small radius. Then a simple proportional controller with suitable gains can be applied in this way chattering is reduced considerably. The switching is not required to be at very fast speed, which gives sufficient time for control computation. Also in each structure damping is introduced to smooth the motion of RP around switching line. The increased damping also reduces the speed of actuator at each switching instant.

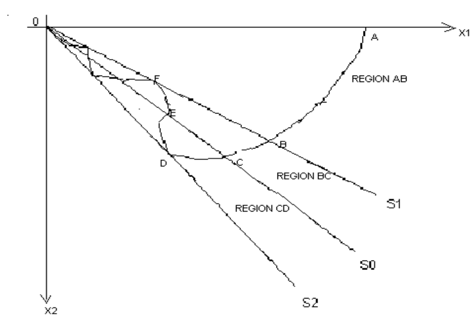

Figure 2. Chattering Reduction with Three Line Switching (with $S_1 = C_1X_1 + X_2 = 0$, $S_2 = C_2X1 + X_2 = 0$, $S_0 = C_0X_1 + X_2 = 0$, where $C_2 > C_0 > C_1 > 0$, and the ratio C_2/C_1 should not be made large for robustness requirement).

The gain condition for the motion of RP through different regions is derived from the control law as described here-

Region AB:-
Gains are kept maximum (within the limit of control input) at start to minimize the reaching time. For deciding the gain the required law is
$S_1\, dS_1/dt < 0$ and
$S_1 X_1 \geq 0,\ S_0 X_1 > 0,\ S_2 X_1 > 0$

Region BC
$S_0\, dS_0 / dt < 0$ and
$S_1 X_1 < 0,\ S_0 X_1 > 0,\ S_2 X_1 > 0$

Region CD
$S_2\, dS_2/dt < 0$ and
$S_1 X_1 < 0,\ S_0 X_1 < 0,\ S_2 X_1 > 0$

Region after S_2 is hit direction of the RP is changed, using the Law
$S_2\, dS_2/dt < 0$ and
$S_1 X_1 < 0,\ S_0 X_1 > 0,\ S_2 X_1 < 0$
The gain in this region is kept less to have a quick change of direction.

Region DE
The RP moves towards E (or S_0) line hence
$S_0\, dS_0 / dt < 0$ and
$S_1 X_1 < 0,\ S_0 X_1 > 0,\ S_2 X_1 > 0$

Region EF
RP moves towards S_1 line hence
$S_1\, dS_1/dt < 0$ and
$S_1 X_1 < 0,\ S_0 X_1 > 0,\ S_2 X_1 > 0$

As soon as the line S_1 is hit i.e. $S_1 = CX_1 + X_2$ become greater then zero. The direction of the RP is changed again by the law describing motion in AB. in this way the system guarantees one cycle of motion of the RP around supervisory line.

To reduce the complexity of the computation constant velocity feedback has been taken – 1 i.e ($K_2 = - 1$) in equation (6), then

$$\ddot{X}_1 + 2.6\,\dot{X}_1 + 2242.5\, K_1 X_1 = 0 \qquad (7)$$

The switching is considered only in position loop. The switching K_{11} during motion AB, K_{12} during BC, K_{13} during CD, K_{14} at D for reversing the direction after S_2 is hit, K_{15} for DE and K_{16} for motion EF towards line S_1. As in the same zone two gains are provided. One for hitting the line ahead of it, and other to be hit back, a flag is used

in simulation program indicating the direction of motion. The flag is set at the start in its forward path from A to D. but is reset as soon as the RP reverses its direction at the point D. The switching gain (K_1) for equation (7) can be derived as follows:
For existence condition

$$S_1 \dot{S}_1 < 0$$

Where $S_1 = C_1 X_1 + X_2$

$$dS_1 / dt = C_1 \dot{X}_1 + \dot{X}_2$$
$$= (C_1 X_2 - 2.6 X_1 - 2242.5 K_{11} X_1)$$

Hence $S_1 \, dS_1 / dt < 0$ implies

$$(C_1 X_1 + X_2)(C_1 X_2 - 2.6X_2 - 2242.5k_{11}X_1) < 0$$

$$(C_1^2 2.6C_1 - 2242k_{11}X_1X_2 - 2242X_1^2 + (C_1 - 2.6)X_2^2) < 0$$

$$(C_1^2 - 2.6C_1 - 2242k_{11})X_1(S_1 - C_1X_1) - 2242C_1k_{11}X_1^2 + (C_1 - 2.6)(S_1 - C_1X_1^2) < 0$$

$$(C_1^2 - 2.6C_1 - 2242k_{11})X_1S_1 + (C_1 - 2.6)S_1^2 < 0 \qquad (8)$$

If $S_1X_1 > 0$; then sufficient condition for it to be true is $(- C_1^{\ 2} + 2.6 C_1 - 2242k_{11}) < 0$,

$$K_{11} > (2.6C_1 - C_1^2) / 2242$$

If $C_1 > 2.6$ it would be mean K_{11} is much grater than $(2.6 C_1 - C_1^2)$. Similarly, other control laws can be derived as

$$\left.\begin{array}{l} K_{12} > (2.6 C_0 - C_0^2)/2242, \\ K_{13} > (2.6C_2 - C_2^2)/2242, \\ K_{14} > (2.6C_2 - C_2^2)/2242, \\ K_{15} > (2.6 C_0 - C_0^2)/2242, \\ K_{16} > (2.6 C_1 - C_1^2)/22424 \end{array}\right| \qquad (9)$$

The last three gains are for the return journey of the RP when the flag is reset to zero. It is worth mentioning that if switching in velocity loop is introduced then the result can be improved by increased damping at each successive change of gain values.

4. RESULTS AND ANALYSIS

4.1 Conventional VSC Simulation

In the simulation of conventional VSC, whatever combinations of slopes and gains are chosen considerable amount of chattering still persists as shown below "Figures 3-6" in simulation with different values of slope (C). Because of unreliability of infinitely fast switching, system trajectory oscillates about the switching line. These oscillations are termed as chattering.

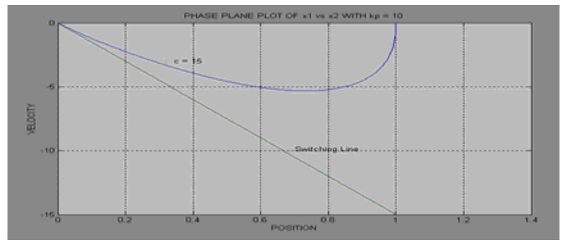

Figure 3. Phase trajectory of conventional VSC at C = 15

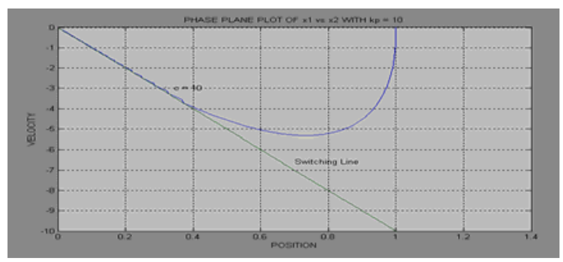

Figure 4. Phase trajectory of conventional VSC at C = 10

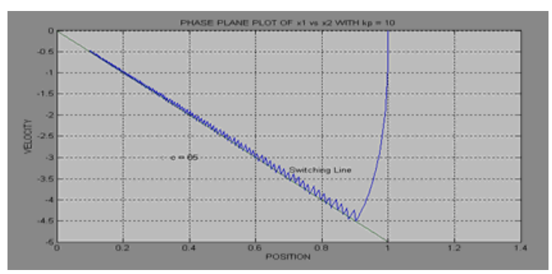

Figure 5. Phase trajectory of conventional VSC at C = 05

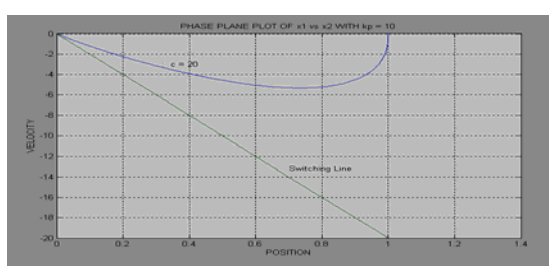

Figure 6. Phase trajectory of conventional VSC at C=20

The responses ("Figures 3-6") of system at different parameters are summarized as bellow:
The presence of chattering can be seen in response curves which exist because the RP cannot switch about the sliding line at infinite frequency. The fast response of variable structure controller compared to other types of controller is shown in above figures. Larger the switching line slope C, is the response is faster in sliding mode. But C cannot be increased as desired because of response capability of physical system. Smaller value of C result in small hitting time (t_h) and faster response in reaching mode but slower response in sliding mode. Therefore, rise time and setting time increases.

Increasing of gains (k_p and k_v) implies considerable decrease in hitting time (t_h) and makes the over all response faster. But increase in gains, proportionately increase the control input required and may exceed the rated value. For the same gain values, chattering is more at lower value of C. This is so because at low values of slope, the response time in the sliding mode is more and control input required is much less. The system response has been verified to be quite robust and insensitive to parametric variations. Even with 100% variation on either side, in the parameters of the system. There is no significant change in the response on the sliding line. Response on hitting zone however, remains sensitive to the parameter variations.

4.2 Three Lines Switching Simulation

In the simulation of VSC based DC motor using three lines switching technique, chattering handling as shown in following simulation results as shown in "Figures 7-9" for combinations of different values of C.

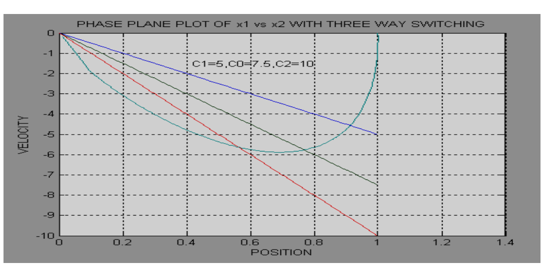

Figure 7. Phase Plane Plot X_1 vs X_2 with Three Lines Switching

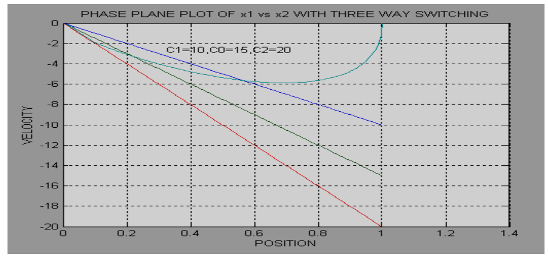

Figure 8. Phase Plane Plot of X_1 vs X_2 with Three Lines Switching

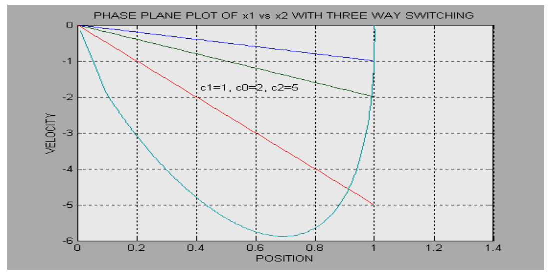

Figure 9. Phase Plane Plot of X_1 vs X_2 with Three Lines Switching

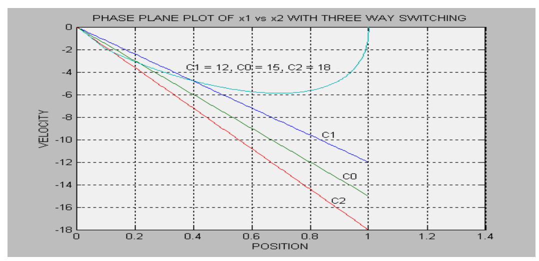

Figure 10. Phase Plane Plot of X_1 vs X_2 with Three Lines Switching

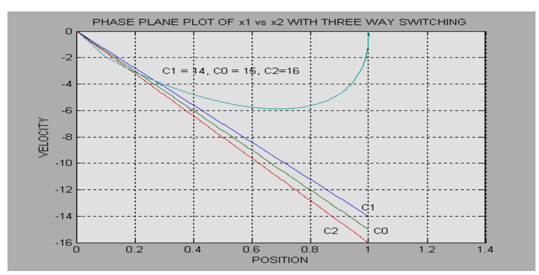

Figure 11. Phase Plane Plot of X_1 vs X_2 with Three Lines Switching

Various observations from the response characteristics ("Figures 7-9") can be summarized as follows: Chattering is substantially handling by this technique. In this simulation, within two to three oscillations, the RP achieves the desired point. The technique can be applied to all types of the systems and is not restricted to low moment of inertia systems. As slope of switching lines increases $|K_{1i}| > (2.6C_i\text{-}Ci^2{}_y/2242$; i =1, 2, 3 increases and $|K_{1i}| < (2.6C_j\text{-}C_j^2{}_j/2242$; j = 4, 5, 6

To satisfy the reachability conditions. Gain is decreasing further, for the initial hitting K_1 greater than K_{12} to minimize the hitting time. But K_1 cannot be increased much because of the limitation on U from equation (1)

$$U \alpha (K_1 K_p) \text{ and } U \alpha (K_2 K_v)$$

As it is assumed that $X_1(0) = 1$ and $X_2(0) = 0$, the input is given by

$$U = K_1 K_p K_a = 20 K_1$$

With other parameter remaining same, as the ratio C_2/C_1, increases, chattering further reduce and response improves with decrease in rise time and settling time as shown in "Figures 7 & 9". But minimum is the ratio of C_2/C_1, chattering increases but system is more robust as shown in Figs. 8, 10, & 11, practically 10% of variation in C_1 and C_2 with respect to C_0, and has shown good robustness and considerable reduction in chattering.

4.3 Comparisons of Conventional VSC and TLST

At lower value of C the chattering easily seen in the graphs of simulation of conventional VSC as shown in "Figures 3 & 4" at C = 05 & C = 10 respectively. As C increases the chattering is eliminated but the response is so faster that it is not moving on switching line as shown in "Figures 2 & 5" at C = 15 & C = 20 respectively. In the simulation of TLST, at different combinations values of C. as the ratio C_2/C_1, increases, chattering further reduce and response improves with decrease in rise time and settling time as shown in "Figures 7 & 9". But minimum is the ratio of C_2/C_1, more robust is the system as shown in "Figures 8, 10, & 11", and chattering reduces. Practically 10% of variation in C_1 and C_2 with respect to C_0, and has shown good robustness and considerable reduction in chattering.

5. CONCLUSION

The chattering can be reduced using three lines switching techniques in the presented work. The basic aim of using three switching lines is to delay the reversal of direction of the representative point (RP). This is very useful technique and can be applied to all types of the systems and is not restricted to low moment of inertia systems. In the simulation of this technique, chattering is reduced within two to three oscillations, after it the RP achieves the desired point.

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