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EVALUATION OF TRANSFORMER FAULTS USING DOUBLE FOURIER SERIES – A FASTEST METHOD FOR FIELD COMPUTATIONS

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ABSTRACT

Transformers are essential and important elements of power systems. In the past few years, there has been an increasing concern about the occurrence of turn-to-turn faults in power transformers due to the high costs that unexpected outages cause. It is not always possible to analyze the transformer behavior under such faults under rated conditions, since the tests are largely destructive. Therefore, in order to avoid severe damage to machines, models are used for the analysis. Thus, many attempts were made to develop a more accurate method to compute the fault currents, among which Finite Element Method (FEM)-based approaches are the most accurate. Unfortunately, the time taken to model and solve by such methods is more, especially when the number of elements considered are more. This tempts the designer to think of an accurate and quickest method of evaluation of fault currents. One such method is presented in this article. Here, a field-based method is proposed for the analysis of internal winding faults of a Power transformer. The transformer with turn-to-turn fault is modeled using an analytical method based on the concept of "Application of Double Fourier Series for irregular flux distribution" [1]. The normal and various faulty cases are presented, which will help in the study of the terminal behavior of the transformer.

Keywords: Power transformer, Double Fourier series method, a field-based computation, reactance evaluation, and inter turn or earth faults.

1. INTRODUCTION

Evaluation of Transformer design and improvement is becoming a vital and challenging job for the designer, especially in view of avoiding outages due to internal short circuit faults [2]. When short circuit occurs at any point in a system, the short circuit current is limited by the impedance of the system up to the point of the fault. In many situations, the impedances limiting the fault currents are largely reactive and total reactance obtained through calculations exceeds three times the resistance [3] therefore the resistance is usually neglected. Thus reactance of the system plays an important role in deciding the severity of the short circuit current [4]. Many attempts were made to develop more accurate methods to compute the fault currents from the reactance value, among which Finite Element Method (FEM) is the most accurate. Unfortunately, the time required for modeling and for providing solution by such methods is more, especially when the number of elements considered is more.

Thus, in the proposed method, a field-based and more accurate method is chosen to evaluate reactance of the transformer, from which fault currents are calculated using the Double Fourier Series. To reduce the computational time involved in the exhaustive calculations, a computer code is developed for the proposed method by which the output values of fault current for the given input conditions can be obtained very quickly. . Hence, the proposed method is flexible, quick, and simple. This method involves two major steps viz. Evaluation of Leakage Reactance and Evaluation of Fault Currents and these steps are explained further in detail.

2. LEAKAGE REACTANCE EVALUATION

There are various methods to calculate the reactance of the transformer. But each method has

its own advantages and disadvantages. Various methods were discussed and it was concluded that

Double Fourier Series Method is an effective and less approximated method. By doubly harmonic series, we mean a series with terms such as sin mx × sin ny. This is more general than the single series used by Rogowski. The heavy mathematical calculations associated with this type of method have restricted its wide application. However, due to the invention of advanced computing machines and methods, this method of reactance calculation is now widely adopted. Although laborious computations are involved, the method is relatively simple in theory and is of wide applicability, even though not quite universal. The method of field analysis by double harmonics (such as sin mx x sin ny) is applied in this approach to calculate the leakage reactance of transformer winding. The advantage of this approach is that it is applicable to any standard type of transformer winding and also to the irregular distribution of windings. The arrangement of the windings in the core window may be entirely arbitrary, but is divisible into rectangular blocks, each block having a uniform current density within itself. This analysis applies strictly to that portion of the winding which is in the window and is approximate for the rest. This analysis of evaluation of leakage reactance involves the following steps:

i) Harmonic concept of current density distribution in the core window

ii) Types of harmonics involved in core window

iii) Calculation of flux harmonics

iv) Calculation of leakage reactance from flux harmonics

These steps are well explained in detail in the reference. However, it is briefed for a quick clarification about the methodology behind the proposed method of evaluation of leakage reactance using Double Fourier Series method for irregular flux distributions. The assumptions made in the evaluation of leakage reactance of the transformer are as follows:

Assumptions:

a) The core window is considered as π radians wide and π radians high, regardless of its absolute dimensions X_0, Y_0 (as shown in fig. 1)

b) The position of each rectangular coil group or block in the window is marked in angular measure as $\theta_{x1=}(x/x_0)$ π radians, $\theta_{y2=}$ (y/y_0) π radians etc.

c) The area A_k of the k^{th} block in sq. radians is given by, $A_k = (\theta_{x2} - \theta_{x1})(\theta_{y2} - \theta_{y1})$

d) The ampere-turns of the k^{th} block divided by its angular area A_k gives average ampere-turn density i_k in the cross-section of that block in ampere-turns per sq. radian. $i_k = \{NI/A\}_k$ ampere-turns per sq. radian. This i_k is constant over the entire area of block k, different for different blocks and it may be positive or negative.

e) The ampere-turn density distribution $i(\theta_x, \theta_y)$ as the flux distributions are conceived of as consisting of components that vary harmonically along both the x- and y-axes (such as I_{mn} sin $m\theta_x$ sin $n\theta_y$), 'm' representing the order of the harmonics in the x direction and 'n' that in the y direction, where I_{mn} are peaks of harmonics of ampere-turn density.

The current density (or ampere-turn density), which is a periodic quantity in two independent directions, such as x and y, demands terms that are doubly harmonic. These may be written in various forms, two of which are as follows.

$$i(x, y) = \sum_m \sum_n I^{I}_{mn} \sin m\pi inm\pi_0) \sin n\pi(y/y_0)$$
$$+ \sum_m \sum_n I^{II}_{mn} \sin m\pi inm\pi_0) \cos n\pi(y/y_0)$$
$$+ \sum_m \sum_n I^{III}_{mn} \cos m\pi osm\pi_0) \sin n\pi(y/y_0)$$
$$+ \sum_m \sum_n I^{IV}_{mn} \cos m\pi osm\pi_0) \cos n\pi(y/y_0) \text{......(1)}$$

where m, n = -∞,..., 0,.....,∞.

Equation (1) can be simplified for our problem by judicious choice of axes and can be written in the following form,

$$i(x, y) = \sum_m \sum_n I_{mn} \cos m\theta_x \cos n\theta_y \text{ ....(2)}$$

The great virtue of a given ampere-turn density distribution as a series of harmonics as given in equation (2) is that the flux linkages associated with it are found to be exactly of the same form as,

$$\varphi(x, y) = \sum_m \sum_n I_{mn} \cos m\theta_x \cos n\theta_y \text{ ....(3)}$$

where m, n = -∞,...,0,.....,∞.

Here ϕ_{mn} is the amplitude of the mn^{th} space harmonic (effective value in time). The advantages of equation (3) are first, that it can be used for all irregular winding distributions in a rectangular window; and second, that it shows how to calculate the set of I_{mn}'s when ampere-turn distribution is given.

Calculation of Coefficient of Current Density Harmonics:

The mn^{th} current density harmonic is of the form

$$i(\theta_x,\theta_y) = I_{mn}\cos m\theta_x \cos n\theta_y \text{.........(4)}$$

For a given i (θ_x, θ_y), the coefficient I_{mn} are obtained using the following equation.

$$I_{mn} = \frac{h}{\pi^2}\int_{\theta_{y1}}^{\theta_{y2}}\int_{\theta_{x1}}^{\theta_{x2}} i(\theta_x,\theta_y)\cos m\theta_x \cos n\theta_y \, d\theta_x \, d\theta_y \text{...(5)}$$

where m, n = -∞,..., 0,.....,∞. And the factor 'h' is included to make the equation adaptable to modifications.

Calculation of Flux Harmonics:

The flux ϕ at a point (x, y) here indicates the flux linkages per cm. Length of a current filament flowing perpendicularly to the (x, y) plane at that

$$\Rightarrow \phi_{mn} = \frac{4\pi}{10}\times\frac{x_0}{y_0}\times\frac{I_{mn}}{m^2+n^2\left(\frac{x_0}{y_0}\right)^2} \text{.......(6)}$$

$$\varphi_{mn} = \frac{4\pi}{10}\times\frac{I_{mn}\, x_0\, y_0}{m^2 y_0^2 + n^2 x_0^2} \text{.......(7)}$$

Equation (6) is interpreted more conveniently for concentric designs and (7) for others.

Calculation of Leakage Reactance from Flux Harmonics:

Consider the voltage e_{mn} that the flux ϕ_{mn} induces in one centimetre length of the filament.. At the time frequency 'ω', this voltage will be

$$e_{mn} = \varpi\phi_{mn}10^{-8}\cos(m\vartheta_x)\cos(n\vartheta_y) \text{......(8)}$$

In the above equation i.e. Eq. (8), the voltage calculated is per cm. length of filament. As this voltage is associated with the current filament $di_{mn} = I_{mn}\cos m\theta_x \cos n\theta_y \, d\theta_x \, d\theta_y$), the contribution of each such filament to the reactive volt-amperes, I^2X, of the field for the mn^{th} harmonic will be

$$d(I^2X)_{mn} = e_{mn}\, di_{mn}$$

$$(I^2X)_{mn} = \omega\varphi_{mn}I_{mn}\times 10^{-8}\int_0^\pi\int_0^\pi \cos^2 m\theta_X \cos^2 n\theta_Y \, d\theta_X \, d\theta_Y$$

$$= \frac{\pi^2}{10}\omega\phi_{mn}I_{mn}10^{-8}$$

$$= \omega\phi_{mn}I_{mn}\times 10^{-8}\cos^2 m\theta_X \cos^2 n\theta_Y \text{.....(9)}$$

Or simply

$$\left(I^2X\right)_{mn} = \left(\frac{\pi^2\omega 10^{-8}}{h}\right)\varphi_{mn}I_{mn} \text{..........(10)}$$

where 'h' is same as that defined previously. Equation (10) represents reactive volt-amperes of whole field for mn^{th} harmonic to a depth of one cm. For complete depth (i.e. mean length of turn), and for all harmonics, reactive volt-amperes will

$$I^2X = \left(\frac{\pi^2\omega p}{h}\right)10^{-8}\sum_m\sum_n \phi_{mn}I_{mn} \text{.........(11)}$$

be

Substituting the value of $Ø_{mn}$ from equation (6), we get,

$$I^2X = \left(\frac{4\pi^2\omega p}{h}\right)10^{-9}\left(x_0 y_0\right)\sum_m\sum_n\frac{I^2_{mn}}{m^2y_0^2+n^2x_0^2} \text{........(12)}$$

$$I^2X = \left(\frac{4\pi^2\omega p}{h}\right)10^{-9}\left(\frac{x_0}{y_0}\right)\sum_m\sum_n\frac{I^2_{mn}}{m^2+n^2\left(\frac{x_0}{y_0}\right)^2} \text{........(13)}$$

(or)

To obtain per unit reactance, I^2X is divided by the out put per phase VA.

However, it is very important to discuss the merits of this method in evaluating the leakage reactance. This method involves flexibility of giving inputs analytically, so it provides the scope to input the m.m.f. per turn, which also allows analyzing the incipient faults. To make this method simple, a C-language-based code is developed to evaluate the leakage reactance. The following example explains how the leakage reactance of a transformer under normal conditions can be evaluated.

In the same way, the reactance of the transformer under turn-earth and turn-turn fault can also be evaluated by wisely choosing the inputs, which is explained in the next section.

Case I: Evaluate the leakage reactance of the power transformer having the following data: 31.5 MVA, 132 KV, 50Hz transformer with primary and secondary ampere-turns 126882.4 AT and -126882.4 AT.

Solution: The Input format should be as in Table 1.

Table 1: Input data

1	No. of Harmonics	50	
2	Capacity in MV A (per phase)	31.5	
3	Number of winding columns	2	
4	Mean perimeter of windings (cm)	257.3	
5	Width of window (in cm)	662.67	
6	Height of window (in cm)	1520	
		Block1	Block2
7	Number of sandwich Blocks	1	1
8	Yoke to Winding distance (cm)	130	130
9	Width of winding (cm)	71	96
10	Height of Winding (cm)	1260	1260
11	Core to Winding Distance (cm)	73	142
12	Ampere Turns (AT)	126882	-126882

Note:

1. In the Table 1, the fields 8, 9, 10, 11, and 12 are repeated as many number of times as the number of sandwich blocks.
2. In Table 1, the fields 7,8,9,10,11, and 12 are repeated as many number of times as the number of concentrating windings.

By giving the above input format, we get the output reactance value as 11.93% (output percentage reactance). Thus the leakage reactance for various transformer configurations are calculated under normal and fault conditions [5]. Table 2 illustrates the comparison of the percentage leakage reactance values obtained by the proposed method of Double Fourier series Technique and the corresponding practical values.

From the results given in Table 2, it should be noted that in the proposed method, the leakage flux is calculated for the winding portion and is approximated for the rest of the window portion; the probability of error are high when the windings cover less portion of the window as in the case of sixth configuration. But in practical cases, most of the winding covers the widow portion as in the rest of the cases illustrated in Table 2; therefore, the error is very less. So this method can be applied with more accuracy and speed to the calculation of leakage reactance because it is developed with a computer code. And even it can be done at a glance using the code without any knowledge of this method.

Case	Capacity (MVA)	Voltage Rating (KV)	Value of % X	
			Proposed (D.F.S.) method	Experimental methods
I	31.5	132	11.93	11.89
II	100	66	13.56	13.6
III	315	400	12.46	12.86
IV	100	38.1	15.15	14.61
V	6.35	3.3	13.51	13.68
VI	0.5	7.61	9.65*	15.3
VII	16.65	34.5	172.0	171.5
VIII	9.064	11	175.18	178.56
IX	0.184	0.433	3.48	4.2

Table 2 – Percentage Reactance of Various Cases

3. EVALUATION OF INTERNAL WINDING FAULT CURRENTS

The continuous increase in demand of power has resulted in the addition of more generating capacity and interconnections in power systems. Both these factors have contributed to an increase in short circuit capacity of networks, making the short circuit duty of transformers more severe. Failure of transformers due to short circuits is a major concern for transformer users [6]. When short circuit occurs at any point in a system, the short circuit current is limited by the impedance of the system up to the point of fault. In many situations, the impedances limiting the fault currents are largely reactive. Of course, every conductor has its own resistance, but it is considerably negligible when compared with the reactance, so it is neglected in the calculation of short circuit current. The error introduced by this assumption will not exceed 1%. Four fault types were studied: turn-to-earth fault on the primary side, turn-to-earth fault on the secondary side, turn-to-turn fault on the primary winding, and turn-to-turn fault on the secondary winding[7].

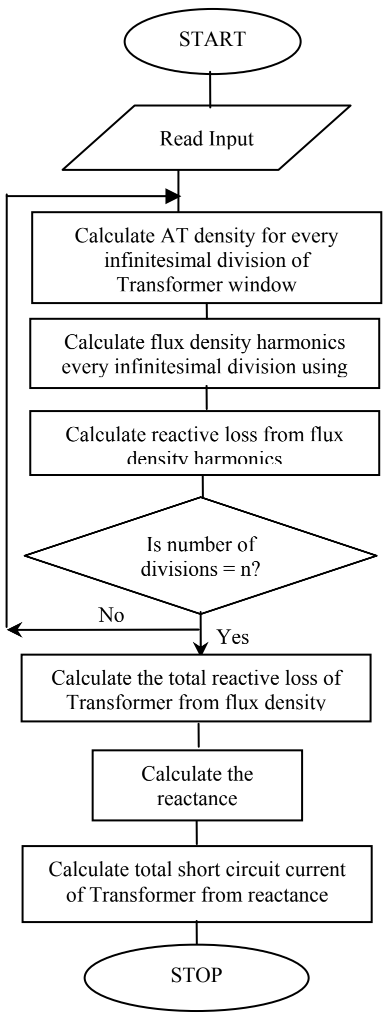

Fig 1: A generalized flow chart for inter turn / earth fault

The m.m.f of the faulty portion of the winding is assumed as zero, and thus the leakage reactance of the transformer under various faulty conditions is evaluated using the above method, which is illustrated in section II. This gives an accurate value of output for the irregular flux distributions, which contributes to the case of faulty conditions, because magnetically both are having the same characteristics [8]. It is easy to calculate the short circuit current from the leakage reactance of the transformer by following the steps shown in the flow chart in fig.1. And the formulae used for the calculation of internal fault current or short circuit current from the reactance are as follows [9]:

Percentage reactance is the percentage of the total phase voltage dropped in the circuit when full load current is flowing.

i.e. $\%X = (IX/V) \times 100$.

where 'I' is full load current,
'V' is phase voltage and
'X' is reactance in ohms per phase.

Also, $\%X = (KVA \times X) / (10 \times KV^2)$ and

$I_{SC} = V/X = I \times (100/\%X)$.

The advantage of using percentage reactance instead of ohmic reactance in calculations involving short circuit is that the percentage reactance values remain unchanged as they are referred through transformers, unlike ohmic reactance which become multiplied by the square of transformer ratio. Percentage reactance at base KVA = (Base KVA/rated KVA)×percentage reactance at rated KVA. And *short circuit KVA* is the product of normal system, and short circuit at point of fault expressed in KVA is known as short circuit KVA i.e. $I_{SC}=I\times(100/\%X)$, for three phase, $I_{SC}=3VI_{SC}/1000$. Thus, short circuit KVA = base KVA × (100/%X)= (3VI/1000)×(100/%X). The evaluation of internal fault currents is carried out for various cases and the results are summarized in the next section.

4. RESULTS AND SUMMARY

The comparisons between the results evaluated by the C program [10] of Double Fourier Series Method (DFSM) and simulation results obtained from the results of Finite Element Method (FEM) are listed in Table 1. In the table, P or S represents whether the fault is on the primary winding or on the secondary winding.

The number 0 represents the ground. Thus P0_15 represents a turn-to-earth case where the 15th turn on the primary winding was connected to earth and P337_364 represents a turn-to-turn case where the 337th and the 364th turns on the primary winding are connected together. No. is the number of the shorted turns. V_1 and V_2 are primary and secondary voltages, respectively, and I_1 and I_2, are primary and secondary currents, respectively; I_S represents the circulating current. All the values are peak values. For the terminal voltages and currents, the difference between the simulation and the field test is quite small. For the circulating current, the results obtained from the simulation and field tests are of the same magnitude. The comparison results show that in most cases the transformer model for an internal fault using this method provides quite an accurate representation of the actual behavior.

Table 3 – Results Comparison

Case		V_1(V)	V_2(V)	I_1(A)	I_2(A)	I_g(A)
Norm	FEM	10496	347.5	5.05	150.8	---
	DFSM	10496	346.5	5.20	151.7	---
	Diff	---	0.28%	2.9%	0.7%	---
P(0-15)	FEM	10224	337.6	21.6	146.5	863.2
	DFSM	10224	335.2	22.4	154.7	982.8
	Diff	---	0.71%	3.7%	5.6%	2.3%
P(0-50)	FEM	10253	338.8	97.4	147.1	1314.7
	DFSM	10253	336.0	96.1	150.2	1318.8
	Diff	---	0.83%	1.3%	2.1%	0.31%
P(337-364)	FEM	10282	340.2	42.5	147.6	1087.3
	DFSM	10282	338.0	43.3	149.0	1096.6
	Diff	---	0.64%	1.9%	0.9%	0.85%
S(0-2)	FEM	10442	321.9	17.5	139.7	4982.0
	DFSM	10442	325.4	18.2	145.1	5380.0
	Diff	---	1.08%	4.0%	3.8%	7.98%
S(2-4)	FEM	10400	338.2	17.8	146.8	5040.5
	DFSM	10400	332.2	18.9	147.7	5043.9
	Diff	---	1.77%	6.17%	0.61%	0.067%
S(10-13)	FEM	10401	332.1	31.4	144.1	6907.2
	DFSM	10401	323.6	30.2	146.9	6646.2
	Diff	---	2.55%	3.8%	1.87%	3.77%

5. CONCLUSION

In this article, a method to study the internal short-circuit winding faults of a power transformer using Double Fourier series is presented. On the basis of the information of the physical properties of the transformer, this method is implemented for a normal transformer or a transformer with an internal fault. And on the basis of this method, the computer code is developed in 'C' language. Experimental results from previous work were used for obtaining comparison leakage reactance values (Table 1). Also, from the observations of the simulation and experimental results, we see that this analytical method can provide an accurate estimation of the terminal values of an internal winding fault for a distribution transformer. When an internal fault occurs, the leakage flux of the transformer increases. The values evaluated using this method and those obtained by simulation are compared. The comparison results validated the analytical method developed for simulation of internal faults in power transformers. In future work, a similar procedure can be applied to model and study incipient internal winding faults, at very minor level, by wisely characterizing them in field point of view.

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