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NON-DOMINATED RANKED GENETIC ALGORITHM FOR SOLVING MULTI-OBJECTIVE OPTIMIZATION PROBLEMS: NRGA

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ABSTRACT

Multi-objective evolutionary algorithms (EAs) that use non-dominated sorting and sharing have been criticized. Mainly for their: 1- $O(MN^3)$ computational complexity (where M is the number of objectives and N is the population size). 2- Non-elitism approach; 3-the need for specifying a sharing parameter. In this paper, a method combining the new Ranked based Roulette Wheel selection algorithm with Pareto-based population ranking Algorithm is proposed, named Non-dominated Ranking Genetic Algorithm (NRGA), which alleviates most of the above three difficulties. A two tier ranked based roulette wheel selection operator is presented that creates a mating pool from the parents' population by selecting the best (with respect to fitness and spread) solutions stochastically. Simulation results on benchmark test problems show that the proposed NRGA, in most of the problems, is able to find much better spread of solutions and faster convergence near the true Pareto-optimal front compared to NSGA-II other elitist MOEA that pay special attention to creating a diverse Pareto-optimal front. Much better performance of NRGA is observed.

Keywords: Elitism, Genetic Algorithms, Multi-Criterion Decision Making, Multi-Objective Optimization, Pareto-Optimal Solutions.

1. INTRODUCTION

The Presence of multiple objectives in a problem, in principle, gives rises to not only single optimal solution but a set of optimal solutions (largely known as Pareto-optimal solutions). In the absence of any further information, one of these Pareto-optimal solutions cannot be said to be better than the other. This entails a user to find as many Pareto-optimal solutions as possible. Classical optimization methods (including the multi-criterion decision-making methods) suggest converting the multi-objective optimization problem to a single-objective optimization problem by emphasizing one particular Pareto optimal solution at a time. When such a method is to be used for finding multiple solutions, it has to be applied many times, with a view of finding a different solution at each simulation run. Over the past two decades, a number of multi-objective evolutionary algorithms (MOEAs) have been suggested [1], [2], [4], [5], [6], [7], and [16]. The primary reason for this is their ability to find multiple Pareto-optimal solutions in one single simulation run. Since evolutionary algorithms (EAs) work with a population of solutions, a simple EA can be extended to maintain a diverse set of solutions; with an emphasis on moving toward the true Pareto-optimal region. In this paper, a new algorithm named Non-dominated Ranking Genetic Algorithm (NRGA) is proposed. From the simulation results on a number of benchmark test problems, NRGA outperforms NSGA-II other elitist MOEA.

In the remainder of the paper, section 2 briefly lists a number of existing elitist MOEAs. Thereafter, in Section 3 the proposed NRGA algorithm is discussed. Section 4 presents simulation results of NRGA and compare them with NSGAII. Finally, the conclusion of this paper is outlined in section 5.

2. ELITIST MULTI-OBJECTIVE EVOLUTIONARY ALGORITHMS

During 1993−2005, a number of different EAs were suggested to solve multi-objective optimization problems. Of them, MOGA-III [4], SPEA2 [16], NSGA-II [10], Srinivas and Deb NSGA [6], and Horn et al. NPGA [5], Fonseca and Fleming MOMGA [17], for detailed information about other MOEA algorithms readers are encouraged to refer to [1] and [2]. The [17],[6] and [5] algorithms demonstrated the necessary additional operators for converting a simple EA to a MOEA. Two common features on all three operators were the following: 1) assigning fitness to population members based on non-dominated sorting; 2) preserving diversity among solutions of the same non-dominated front. Although they have been shown to find multiple non-dominated solutions on many test problems and a number of engineering design problems, researchers realized the need of introducing more useful operators to solve multi-objective optimization problems better. Particularly, the interest has been to introduce elitism to enhance the convergence properties of a MOEA. Elitism helps in achieving better convergence in MOEAs as shown in [8]. Among the existing elitist MOEAs, Zitzler and Thieles SPEA [7], [16], Knowles and Cornes Pareto-archived PAES [9], MOMGA-III [3], PAES [19], PAES-II [18], NSGA-II [10] are well studied. For details, readers are advised to refer to the original studies. In the following section, the proposed non-dominated ranking GA approach is presented.

3. ELITIST NON-DOMINATED RANKING GENETIC ALGORITHM

The following sections describe in brief the algorithms used in NRGA. Algorithms in section 3.1 and 3.3 are embedded in the NRGA algorithm for the sake of comparing with NSGAII, where any other sorting algorithm and diversity mechanism can be used.

3.1. Sorting Algorithm

In this study the fast non-dominated sorting approach from [10] is used for two reasons, because of the comparison with NSGA-II, and its $O(MN^3)$ computations.

3.2. Ranked Based Roulette Wheel Selection

The authors of [14] and [15] use modified roulette wheel selection algorithm where each individual is assigned a fitness value equal to its rank in the population; the highest rank has the highest probability to be selected (in case of maximization).

The probability is calculated as illustrated in the following equation:

$$Pi = \frac{2*Rank}{N*(N+1)} \qquad (1)$$

Where N is the number of individuals in the population. In this study the individuals in a front are ranked based on their crowding distance, and the fronts ranked based on the non-dominated rank.

3.3. Diversity Mechanism

Along with convergence to the Pareto-optimal set, it is desired that an EA maintains a good spread of solutions in the obtained set of solutions. In NSGA-II the crowded-comparison approach is used alone with the crowded-comparison operator. This approach does not require any user-defined parameter for maintaining diversity among population members. Also, the suggested approach has a better computational complexity. Readers are encouraged to refer to [10] for more information about both the crowding-comparison approach and operator. NRGA maintains the diversity by ranking the solutions in each non-dominated Pareto-front using their crowding distance.

3.4. Survival Selection (elitism)

After evaluating the offspring's fitness (non-dominated rank, crowding distance), parents and offspring fight for survival as Pareto dominance is applied to the combined population of parents and offspring. Then the least dominated N solutions survive to make the population of the next generation.

3.5. NRGA

Initially, a random parent population P is created. The sorting of the population is based on the non-domination. Each solution is assigned a fitness (or rank) equal to its non-domination level (1 is the best level, 2 is the next-best level, and so on).

Thus, minimization of fitness is assumed. At first, the usual Ranked based Roulette wheel selection, recombination, and mutation operators are used to create an offspring population Q of size N. Since elitism is introduced by comparing current population with previously found best non-dominated solutions, the procedure is different after the initial generation. First the i^{th} generation of the proposed algorithm as shown in algorithm 1 is described.

The algorithm 1 shows that NRGA is simple and straightforward. First, a combined population $P \cup Q$ is formed. The combined population is of size $2N$ then, the combined population is sorted according to non-domination. Since all previous and current population members are included in the combined population elitism is ensured. This procedure will select N solutions out of $2N$.

Algorithm 1 NRGA

```
Initialize Population P
  { Generate random population − size N
    Evaluate Objective Values
     Assign Rank (level) Based on Pareto dominance − sort }
Generate Child Population Q
  { Ranked based Roulette Wheel Selection
    Recombination and Mutation }
for i = 1 to g do
    for  for each member of the combined population (P ∪ Q)  do
        Assign Rank (level) based on Pareto - sort
        Generate sets of non-dominated fronts
        Calculate the crowding distance between members on each front
    end for
    (elitist) Select the members of the combined population based on least dominated N solution to make the population of the next generation. Ties are resolved by taking the less crowding distance.
    Create next generation
      { Ranked based Roulette Wheel Selection
        Recombination and Mutation }
end for
```

The new population of size N is used for selection. Now, two tiers ranked based roulette wheel selection [14] and [15] is applied, one tier to select the front and the other to select solution from the front, here the solutions belonging to the best non-dominated set F_1 have the largest probabilities to be selected. Thus, solutions from the set F_2 are chosen with less probability than solutions from the set F_1 and so on. Then crossover and mutation are applied to create a new population P of size N. The diversity among non-dominated solutions is introduced by the second tier of ranked based roulette wheel selection, which ranks the solutions (in the same front) based on their crowding distance. The solutions with least crowding distance will have the higher probabilities.

Since solutions compete with their crowding distance (a measure of density of solutions in the neighborhood), no extra niching parameter (such as σ_{share} needed in the NSGA) is required. Although the crowding distance is calculated in the objective function space, it can also be implemented in the parameter space, if so desired [11]. However, the objective function space niching is used in all simulations performed in this study.

4. SIMULATION RESULTS

In this section, first the test problems used to compare the performance of NRGA with NSGA-II are described, second the performance metrics are illustrated, and finally the results are discussed. The identical parameter setup for both algorithms NRGA and NSGA-II is maintained for the comparison purpose.

4.1. Test Problems

In 1999, Zitzler et al. [8] followed the suggested systematic way of developing test problems for multi-objective optimization by [11] and suggested well known six test problems five of them are chosen. These problems called ZDT1, ZDT2, ZDT3, ZDT4, and ZDT6. All of them have two objective functions. None of these problems have any constraint. Reader can refer to [1], [2], [8] for more information about the benchmark test problems. Table 1 describes these problems; also the table shows the number of variables, their bounds, the Pareto-optimal solutions, and the nature of the Pareto-optimal front for each problem.

The simulated binary crossover (SBX) operator and polynomial mutation [12] are used. The crossover probability of $P_c = 0.9$ and a mutation probability of $P_m = 1/n$ (where n is the number of decision variables) are set. Distribution indexes [12] for crossover and mutation operators as η_c = 20 and η_m = 20, respectively are specified. The population size was fixed to 100 in all the problems, for 350 generations in ZDT1 and 400 generations in the remaining problems.

4.2. Performance Measures

Unlike in a single-objective optimization, there are two goals in a multi-objective optimization: 1) convergence to the Pareto-optimal set and 2) maintenance of diversity in solutions of the Pareto-optimal set. These two tasks cannot be measured adequately with one performance metric. Many performance metrics have been suggested [1], [2], [13], and [20]. Here, two running performance metrics to understand the behavior of the algorithm from [20] are used in evaluating each of the above two goals, reader recommended to refer to the original study [20] for more details about the used running (convergence and diversity) metrics. In the experiments the number of grids is equal to the population size and $f_2 = 0$ plane to project the points.

4.3. Discussion of the Results

Figures 1 − 5 show the final generation, figure 1 shows all non-dominated solutions obtained after 350 generations with NRGA and NSGA-II on ZDT1 problem. The Pareto-optimal region also is shown in the figure. This figure demonstrates the abilities of NRGA in converging to the true front and in finding diverse solutions in the front. In both aspects of convergence and distribution of solutions, NRGA performed better than NSGA-II in this problem. Since NSGA-II could not converge to the true Pareto-optimal front in the final generation. Next, the non-dominated solutions on ZDT2 problem are shown in figure 2. This problem has a non-convex Pareto-optimal front. The performance of NRGA is better than NSGA-II. Although NRGA get closer to the true Pareto-optimal front than NSGAII, NRGA have found a better spread and more solutions in the entire Pareto-optimal region than NSGA-II. The problem ZDT3 has disconnected Pareto-optimal front, NRGA converged and distributed uniformly on each part of the Pareto-optimal front than NSGA-II see figure 3. The problem ZDT4 has 21^9 or $7.94\left(10^{11}\right)$ different local Pareto-optimal fronts in the search space, of which only one corresponds to the global Pareto-optimal front. Figure 4 shows that both NSGAII and NRGA get stuck at different local Pareto-optimal sets, but the convergence and ability to find a diverse set of solutions are definitely better with NSGA-II. Since NRGA converges poorly on this problem (see figure 4). Finally, Figure 5 shows that NRGA finds a better converged distributed set of non-dominated solutions in ZDT6 compared to NSGA-II algorithm.

But the previous figures will not give a clear picture about the behavior of the algorithms, for that the figures 6 − 15 illustrate the convergence and diversity behavior of both NRGA and NSGA-II algorithms. Figure 6 shows that the convergence metric of NRGA on problem ZDT1, quickly moves to zero faster than NSGA-II, thereby implying that starting from a random set of solutions NRGA quickly approach the Pareto-optimal front faster than NSGA-II. A zero value of convergence metric implies that all non-dominated solutions match the chosen Pareto-optimal solutions. After about 20 generations, NRGA population comes very close to the Pareto-optimal front, whereas NSGA-II took too much oscillating to get closer to the Pareto-optimal front. The same happens in figures 7, 8, 9, and 10. Figure 11 explains the diversity metric, which increases exponentially till 50 generations in NRGA and till 200 generations in NSGA-II after that the diversity remains more or less the same. Although the obtained solutions are very close to the chosen Pareto-optimal front, the diversity metric oscillates near a stable value. NRGA in generation 50 gets a very good diversity value than NSGA-II which reaches this value at generation 200, the same behavior of the diversity metric in the remaining figures.

From above NRGA outperforms NSGA-II in most of the test problems. NRGA with the adoption of two tiers ranked based roulette wheel selection is promising for these types of problems, and it is able to find a reasonably better spread and faster convergence of solutions than NSGA-II algorithm.

5. CONCLUSIONS

The new elitist ranked based MOEA proposed (NRGA) tested on five benchmark test problems, and shows that NRGA was able to converge significantly faster than NSGA-II, while maintaining reasonably better spread of solutions compared to NSGA-II, without specifying any additional parameter like σ_{share}, NRGA maintains the diversity among the solutions by controlling dynamically the crowding distance. With the properties of two tiers ranked based roulette wheel selection, a fast non-dominated sorting procedure, and elitist strategy. NRGA alleviates most of the difficulties of non-dominated sorting and sharing evolutionary algorithms.

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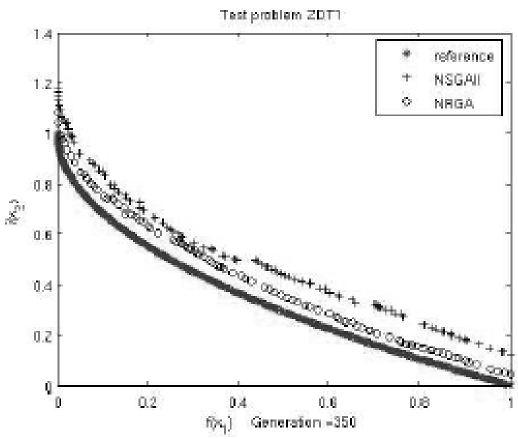

Figure 1: ZDT1 final generation

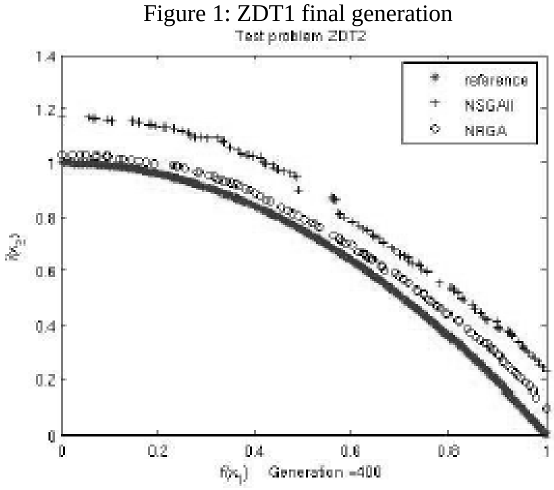

Figure 2: ZDT2 final generation

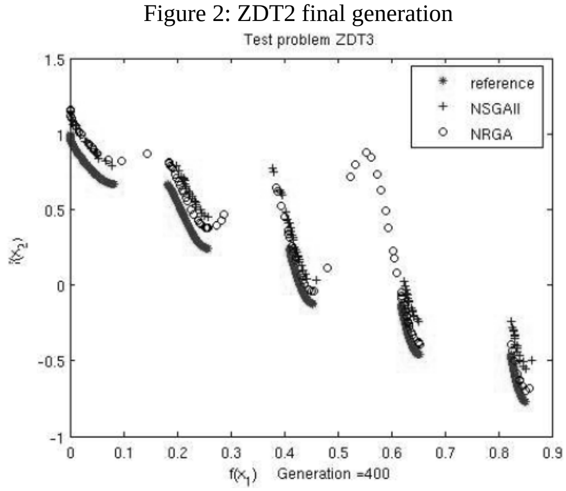

Figure 3: ZDT3 final generation

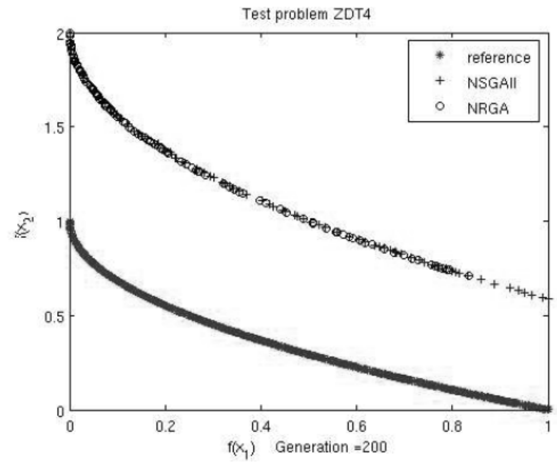

Figure 4: ZDT4 final generation

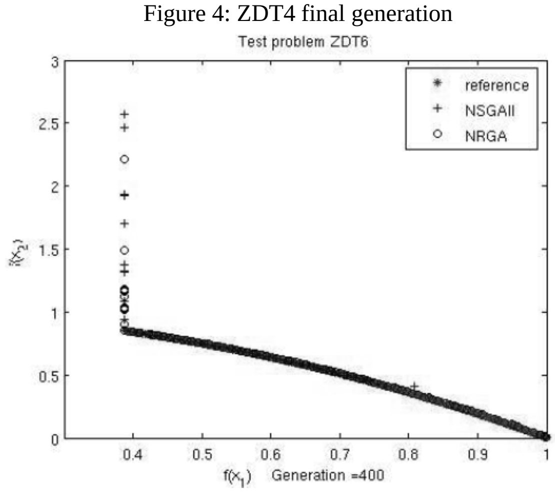

Figure 5: ZDT6 final generation

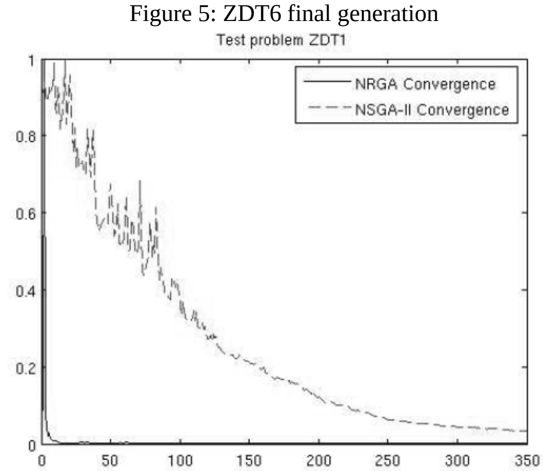

Figure 6: ZDT1 convergence

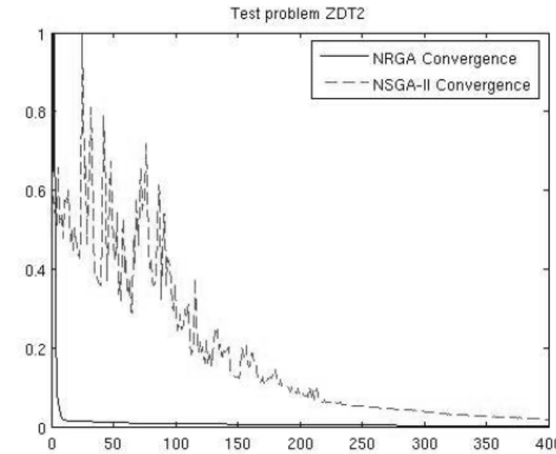

Figure 7: ZDT2 convergence

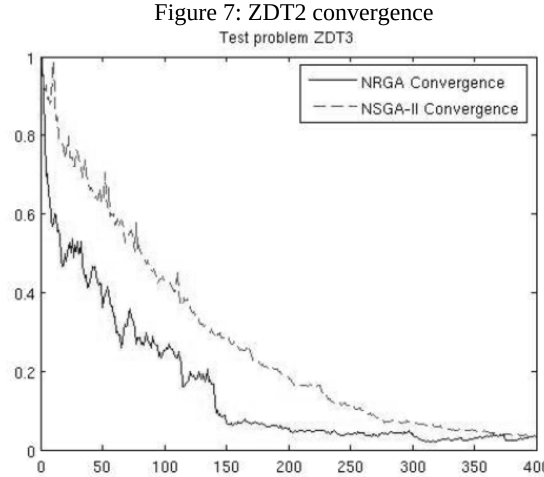

Figure 8: ZDT3 convergence

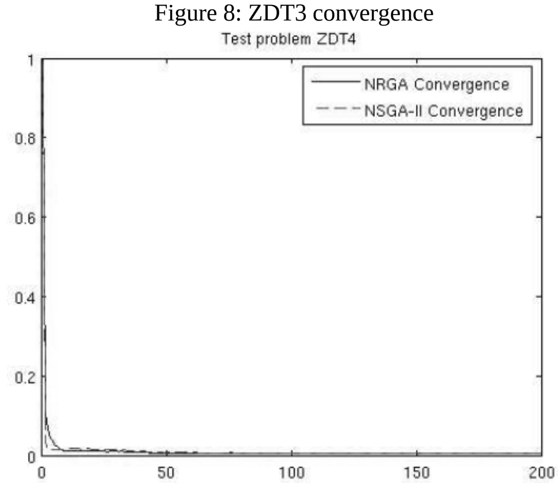

Figure 9: ZDT4 convergence

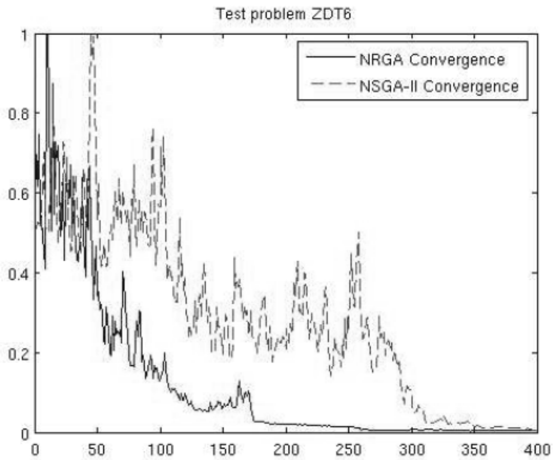

Figure 10: ZDT6 convergence

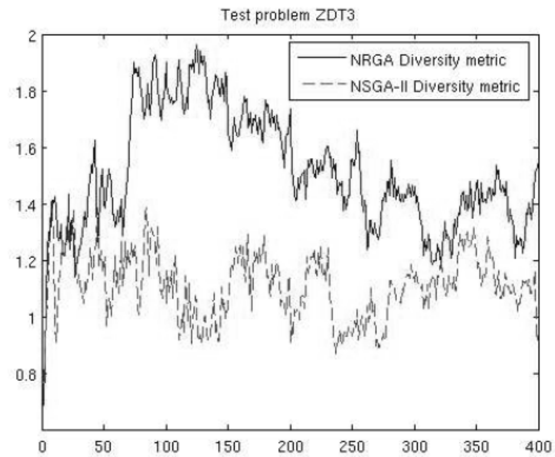

Figure 13: ZDT3 diversity

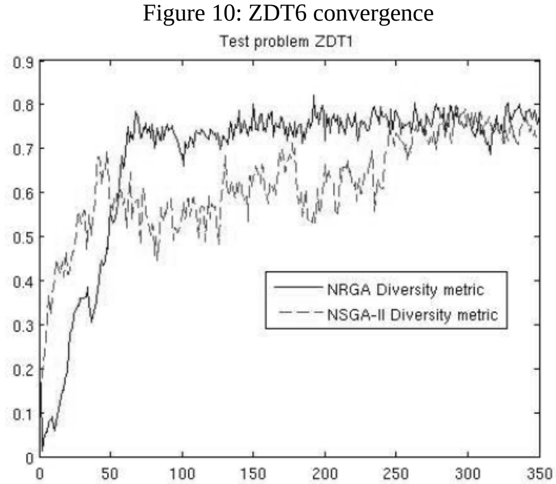

Figure 11: ZDT1 diversity

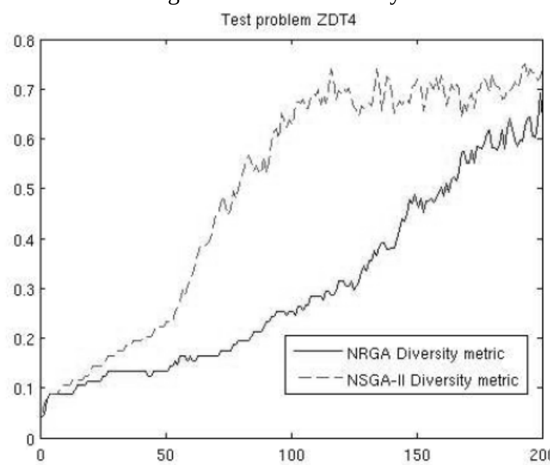

Figure 14: ZDT4 diversity

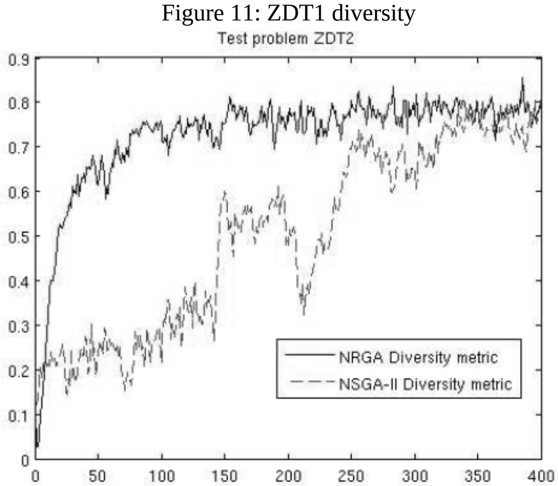

Figure 12: ZDT2 diversity

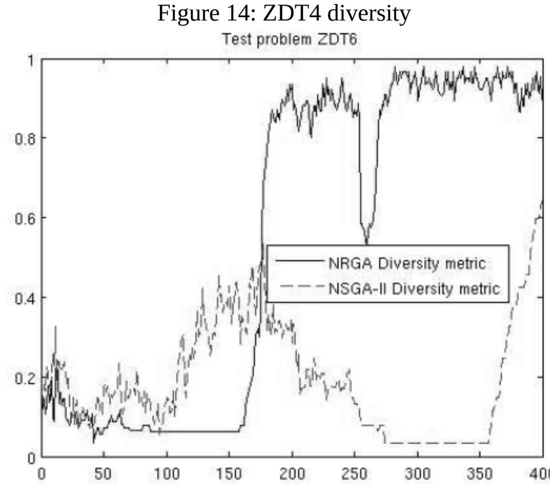

Figure 15: ZDT6 diversity

Table 1: TEST PROBLEMS USED IN THIS STUDY

Problem	n	Variables bounds	Objective functions	Optimal solution	Comments
ZDT1	30	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x)\left[1 - \sqrt{(x_1/g(x))}\right]$ $g(x) = 1 + 9(\sum_{i=1}^{n} x_i)/(n-1)$	$x \in [0,1]$ $x_i = 0$ $i = 2, ..., n$	convex
ZDT2	30	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x)\left[1 - (x_1/g(x))^2\right]$ $g(x) = 1 + 9(\sum_{i=1}^{n} x_i)/(n-1)$	$x \in [0,1]$ $x_i = 0$ $i = 2, ..., n$	nonconvex
ZDT3	30	[0,1]	$f_1(x) = x_1$ $f_2(x) = g(x)\left[1 - \sqrt{(x_1/g(x))} - \frac{x_1}{g(x)} sin(10\pi x_1\right]$ $g(x) = 1 + 9(\sum_{i=1}^{n} x_i)/(n-1)$	$x \in [0,1]$ $x_i = 0$ $i = 2, ..., n$	convex, disconnected
ZDT4	10	$x_1 \in [0,1]$ $x_i \in [-5,5]$ $= 2, ..., n$	$f_1(x) = x_1$ $f_2(x) = g(x)\left[1 - \sqrt{(x_1/g(x))} - \frac{x_1}{g(x)} sin(10\pi x_1\right]$ $g(x) = 1 + 10(n-1) + \sum_{i=2}^{n}\left[x_i^2 - 10cos(4\pi x_i)\right]$	$x \in [0,1]$ $x_i = 0$ $i = 2, ..., n$	nonconvex
ZDT6	10	$[0,1]$	$f_1(x) = 1 - exp(-4x_1)sin^6(6\pi x_1)$ $f_2(x) = g(x)\left[1 - (f(x_1)/g(x))^2\right]$ $g(x) = 1 + 9\left[\sum_{i=2}^{n} x_i/(n-1)\right]^{0.25}$	$x \in [0,1]$ $x_i = 0$ $i = 2, ..., n$	nonconvex, nonuniformly spaced