TRACKING CONTROL OF 3-DOF ROBOT MANIPULATOR USING GENETIC ALGORITHM TUNED FUZZY PID CONTROLLER

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ABSTRACT

Control of an industrial robot includes nonlinearities, uncertainties and external perturbations that should be considered in the design of control laws. This paper presents the Genetic algorithm tuned Fuzzy PID controller (GAFPID) to trace the desired trajectory for a three degree of freedom (DOF) robot arm. Numerical simulation using the dynamic model of three DOF robot arm shows the effectiveness of the approach in trajectory tracking problems. Comparative evaluation with respect to PD, PID and Fuzzy PID controls are presented to validate the controller design. The results presented emphasize that a satisfactory tracking precision could be achieved using the proposed controller than conventional controller.

Keywords: Fuzzy PID, Genetic algorithm, robot control, Degree of Freedom (DOF)

1. INTRODUCTION

The basic problem in controlling robots is to make the manipulator follow a desired trajectory. In the robotics literature, this control problem is referred to as the motion control problem or the trajectory tracking problem [1]. Conventional robot control methods depend heavily upon accurate mathematical modeling, analysis, and synthesis. These approaches are suitable for the control of robots that operate in structured environments. However, operations in unstructured environments require robots to perform much more complex tasks without an adequate analytical model. The most challenging problem in this field is that there are always uncertainties in the unstructured environments. These uncertainties are primarily due to sensor imprecision and unpredictability of the environment characteristics and its dynamics. Adaptive control algorithms [2-4] have been proposed for this type of control systems. Generally, most of the existing adaptive algorithms are proposed to control the specific systems with known model characteristics and unknown parameters. Hence, developing model-free adaptive control structure has become an interesting research topic.

The advent of fuzzy set techniques provides us with a powerful tool for solving demanding real-world problems with uncertain and unpredictable environments [5]-[7]. Fuzzy controller can characterize better behavior comparing with classical linear PID controller because of its non linear characteristics. Recently, fuzzy-logic and conventional-techniques have been combined (hybrid) to design FL controllers which pave to appropriate solution for controlling the robot manipulators [8-10].

The core of designing a Fuzzy logic controller is the selection of high performance membership functions that represents the human expert's interpretation of the linguistic variables. The existing iterative approaches for choosing the membership functions are basically a manual trial-and-error process and lack learning capability and autonomy. Therefore, the more efficient and systematic genetic algorithm (GA) [11], which acts on the survival-of-the-fittest Darwinian principle for reproduction and mutation, has been applied to FLC design for searching the poorly understood, irregular and complex membership function space with improved performance.

GAs have proven to be a useful method for optimizing the membership functions of the fuzzy sets Karr, for example, has used a GA to generate membership functions for a PH control process [12] and cartpole problem [13]. Mohammadian and Stonier developed a fuzzy logic controller and optimized the membership functions by genetic algorithm [14]. Mester in [15] developed a

In our approach, a Fuzzy PID Controller (FPID) is designed and then genetic algorithm is applied to tune and optimize membership functions and scaling factors of designed fuzzy controller in such a manner that a high precision controller for trajectory tracking of three DOF robot is obtained. Comparative evaluation with respect to PD, PID and Fuzzy PID controls are presented to validate the controller design. Organization of the paper is as follows. Section 2 introduces the three degrees of freedom robot arm and its dynamic model. Section 3 describes the design of fuzzy PID controller. Section 4 introduces the genetic algorithm concepts, Section 5 deal with the use of GA to Optimize Fuzzy Logic Controller and Section 6 provides numerical simulation results to demonstrate the effectiveness of the approach and comparative evaluation with respect to PD, PID and Fuzzy PID controls is performed. And Section 7 discusses the benefits of the studied fuzzy control law and conclusions are presented.

2. ROBOT ARM DYNAMICS AND MODELING

Three Degree of freedom (DOF) robot manipulator used for numerical simulation for this approach is shown Fig 1.

Consider a robot manipulator with a degree of freedom (DOF). The torque to be applied is defined by [1],

$$T_i = \frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial \dot{L}}{\partial q_i} \right], \quad i = 1, 2, 3 \ldots n$$

(1)

where T is the torque, L is the Lagrange function, \(q_i\) is the generalized coordinate of \(i^{th}\) joint, is the generalized velocity of \(i^{th}\) joint.

Lagrange function is described by kinetic (K) and Potential energy (V) as follows:

$$L = K - V$$

(2)

The potential energy can be calculated by

$$L = \frac{1}{2} \sum_{i,j} \dot{q}_i \dot{q}_j - V(q)$$

(3)

If equation (3) is replaced in to equation (2), the below equations can be obtained
The equations of motion for the robot system can be derived from (5) and are given by

\[ M = D(q)\ddot{q} + h(q, \dot{q}) + G(q) \]

(6)

Where \( G(q) \) is the \( ix1 \) vector that is formed matrixes occurred due to gravity, \( D(q) \) is the \( ix1 \) Inertia matrix, is the \( ix1 \) vector shown Centrifugal and Coriolis Torques, \( h(q, \dot{q}) \)

Consequently, the Torque of three DOF robot arm is given by

\[ T_j = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} + \begin{bmatrix} A & B & C \\ D & E & F \\ G & H & K \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} + \begin{bmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{bmatrix} \]

(7)

The coefficients of \( d \) matrix and \( A, B, C, D, E, F, G, H, K \) variables depend on mass, length and angle of joints are given below.

The coefficients of \( d \) matrix are:

\[ d_{11} = m_1 l_1^2 + m_1 (l_1^2 + l_2^2 \sin^2 \theta_2) + m_1 \left[ l_3 \cos \theta_2 \sin \theta_3 - l_3 \sin \theta_2 \sin \theta_3 \right]^2 \]

\[ d_{12} = d_{13} = -m_1 l_2 \cos \theta_2 + l_3 \sin \theta_2 \sin \theta_3 \]

\[ d_{22} = m_2 l_2^2 + m_3 (l_2^2 + l_3^2 + \sin^2 \theta_3) + I_2 + l_2 \]

\[ d_{23} = d_{32} = -m_3 l_2 l_3 \cos \theta_3 + 1, \]

\[ d_{33} = m_3 l_3^2 + I_3 \]

\[ A = c_{121} \cdot q_2 + c_{131} \cdot q_3 \]

\[ B = c_{211} \cdot q_1 + c_{221} \cdot q_2 + c_{231} \cdot q_3 \]

\[ C = c_{311} \cdot q_1 + c_{321} \cdot q_2 + c_{331} \cdot q_3 \]

\[ D = c_{112} \cdot \dot{q}_1 + c_{132} \cdot \dot{q}_3 \]

\[ E = c_{212} \cdot \dot{q}_1 \]

\[ F = c_{312} \cdot \dot{q}_1 + c_{322} \cdot \dot{q}_2 + c_{332} \cdot \dot{q}_3 \]

\[ G = c_{113} \cdot \dot{q}_1 + c_{123} \cdot \dot{q}_2 \]

\[ H = c_{213} \cdot \dot{q}_1 + c_{223} \cdot \dot{q}_2 \]

\[ K = 0 \]

and \( c \) coefficients are found as

\[ c_{111} = 0 \]

(8)

\[ c_{121} = c_{211} = m_1 \frac{l_2^2}{2} \sin \theta_2 \cos \theta_2 \]

\[ + m_3 \left( \frac{l_3^2}{2} \sin \theta_2 \cos \theta_2 + l_3 \sin \theta_2 \sin \theta_3 \cos \theta_3 \right) \]

\[ c_{131} = c_{311} = -m_3 \left( \frac{l_3^2}{2} \sin \theta_3 \cos \theta_3 \sin \theta_2 + l_3 \sin \theta_2 \cos \theta_3 \right) \]

\[ c_{221} = c_{222} = m_1 l_2^2 \sin \theta_2 \sin \theta_2 \sin \theta_3 \]

\[ c_{231} = c_{312} = -m_3 \left( \frac{l_3^2}{2} \sin \theta_2 \cos \theta_2 + l_3 \sin \theta_2 \cos \theta_3 \right) \]

\[ c_{331} = c_{332} = -m_3 \left( \frac{l_3^2}{2} \sin \theta_3 \cos \theta_3 + l_3 \sin \theta_2 \cos \theta_3 \right) \]

\[ c_{112} = -m_3 \frac{l_3^2}{2} \sin \theta_2 \cos \theta_2 \sin \theta_3 \]

\[ c_{122} = c_{222} = 0 \]

\[ c_{313} = c_{333} = m_3 l_3^2 \sin \theta_3 \]

\[ c_{113} = c_{132} = m_3 \left( \frac{l_3^2}{2} \sin \theta_2 \sin^2 \theta_2 + l_3 \sin \theta_2 \cos \theta_2 \sin \theta_3 \sin^2 \theta_3 \right) \]

\[ c_{133} = c_{313} = m_3 l_3^2 \sin \theta_3 \cos \theta_3 \]
\[
\begin{align*}
c_{123} &= c_{213} = -m_3 \left( l_c^3 \sin \theta_2 \sin \theta_3 + l_c^2 \cos \theta_2 \sin \theta_3 \right) \\
c_{223} &= -m_3 l_c^2 \sin \theta_3 \cos \theta_3
\end{align*}
\]

\[
c_{133} = c_{213} = 0
\]

\[
c_{233} = c_{232} = 0, \quad c_{333} = 0
\]

\[
\begin{align*}
\dot{\phi}_1, \dot{\phi}_2, \dot{\phi}_3, \text{ are the potential energy values of each joint and given Equation (6).}
\end{align*}
\]

\[
\begin{align*}
\dot{\phi}_1 &= 0 \\
\dot{\phi}_2 &= -m_3 g l_c \cos \theta_2 - m_3 g (l_c \cos \theta_2 \sin \theta_3 - l_3 \sin \theta_2) \\
\dot{\phi}_3 &= m_3 g l_c \sin \theta_2 \cos \theta_3
\end{align*}
\]

where \( m \) is the mass, \( g \) is the gravity acceleration; \( l_c \) is the half of the length of joints. \( T_1, T_2, T_3 \) are replaced into the DC motor equation.

The Parameters and details of the robotic arm used in this paper are given in Table 1.

Table 1. Parameters of 3-DOF robotic arm

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of arm</td>
<td>0.7 Kg</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>( 5 \times 10^{-10} ) ( \text{kg m}^2 )</td>
</tr>
<tr>
<td>Length of arm</td>
<td>0.8 m</td>
</tr>
<tr>
<td>Gravity acceleration</td>
<td>9.8 ( \text{m}^2 / \text{s} )</td>
</tr>
</tbody>
</table>

3. Fuzzy PID Controller Design

This section describes the design of three-input Fuzzy PID controller which is used in simulation. The general discrete – time form of PID controller is given by,

\[
n(nT) = u(nT - T) + \hat{K}_p \dot{e}(nT) + \hat{K}_i (\epsilon(nT) - \epsilon(nT - T)) + \hat{K}_d (\epsilon(nT) - \epsilon(nT - T))
\]

Equation (12) can now be written as

\[
\Delta u = e_p + e_i + e_d
\]
and in incremental form as
\[ \Delta u_{cmd} = u_{cmd}(nT) - u_{cmd}(nT - T) = e_i + e_p + e_L \]
(17)

These three errors are converted into three different input membership functions. The simplest membership functions used for \( e_i, e_p \) and \( e_L \) are the triangular membership functions with a threshold limit \( L \). This limit specifies the maximum and minimum values for the fuzzification process. Two inputs positive denoted by ‘p’ and negative denoted by ‘n’ are used to specify the input membership functions. This is shown in Figure 3.

![Figure 3. Input membership functions](image)

Similarly, four output membership functions are specified to represent positive (p), negative (n), negative large (nl) and positive large (pl). Parameter \( L \) defines maximum and minimum outputs, but there are two new output terms centers at +/- \( L/3 \) respectively. This is shown in below Figure 4.

![Figure 4. Output membership functions](image)

With the above membership functions, the inference composition rules are defined as follows.
*IF \( e_p \) is Negative, \( e_i \) is Positive \& \( e_D \) is Positive, Then \( \Delta u_{is} \) is Positive*
*IF \( e_p \) is Negative, \( e_i \) is Positive \& \( e_D \) is Negative, Then \( \Delta u_{is} \) is Negative*
*IF \( e_p \) is Positive, \( e_i \) is Positive \& \( e_D \) is Negative, Then \( \Delta u_{is} \) is Positive*
*IF \( e_p \) is Positive, \( e_i \) is Positive \& \( e_D \) is Positive, Then \( \Delta u_{is} \) is Positive*

4. GENETIC ALGORITHMS

GA is a stochastic global search optimization technique based on the mechanisms of natural selection. Recently, GA has been recognized as an effective technique to solve optimization problems and compared with other optimization techniques; GA is superior in avoiding local minima which is a common aspect in nonlinear systems. GA starts with an initial population containing a number of chromosomes where each one represents a solution of the problem which performance is evaluated by a fitness function. Figure 5 shows the flow chart of the genetic algorithm process.
Figure 5. Flow chart of the genetic algorithm

GA’s consist of three basic operations: reproduction, crossover, and mutation. Reproduction is the process where members of the population reproduced according to the relative fitness of the individuals, where the chromosomes with higher fitness have higher probabilities of having more copies in the coming generation. There are a number of selection schemes available for reproduction, such as “roulette wheel,” “tournament scheme,” “ranking scheme,” etc. [17-18]. Crossover in GA occurs when the selected chromosomes exchange partially their information of the genes, i.e., part of the string is interchanged within two selected candidates. Mutation is the occasionally alteration of states at a particular string position. Mutation is essentially needed in some cases where reproduction and crossover alone are unable to offer the global optimal solution. Further discussion on GA’s can be obtained in [17&18].

5. USING GA’S TO OPTIMIZE FUZZY LOGIC CONTROLLERS

The performance of Fuzzy logic controllers for controlling multi input-multi output and complex systems strongly depends on the membership function parameters. GA’s are stochastic, robust, and global search algorithms with the ability to find near optimal solutions in complex search spaces without derivative information. Since GA’s require no prior knowledge about the system’s behavior to formulate a set of functional control rules through learning, they have been applied widely to fuzzy-control design for unknown plants. Many genetic fuzzy systems, i.e. fuzzy systems augmented by a learning process based on GA’s, have been proposed [19-20]. Figure 6 shows the structure of Genetic tuned Fuzzy control system. Genetic algorithms can be used to optimize these parameters, by taking input parameters as scaling factors, membership functions or control rules etc. and outputs desired objective function [21]. GA is used to calculate optimum values of FPID controller.

Fitness functions play a major role in genetic algorithms because the direction of search totally depends on these functions. Because the purpose of the optimization problem is to minimize the fitness of individuals, performance measures of the system are compulsory elements of the fitness function. In robotic manipulators fitness can be considered for small errors, smooth torques, and fast response time. The fitness considered in this paper is integral absolute error.

![Figure 6. Structure of Genetic tuned Fuzzy control system.](image)

Fitness value =

\[
\frac{k_p}{k_2}\sum_{i=1}^{n} e_i(t) + \frac{k_p}{k_2}\sum_{i=1}^{n} \dot{e}_i(t) + \frac{k_d}{k_2}\sum_{i=1}^{n} \ddot{e}_i(t) + \frac{k_d}{k_2}\sum_{i=1}^{n} \dddot{e}_i(t) + \frac{k_i}{k_2}\sum_{i=1}^{n} \dddot{e}_i(t)
\]

where, \(k_p\), \(k_d\), and \(k_i\) are the proportional, derivative and integral controller gains of Fuzzy PID controller which are to be tuned. \(k_p, k_d, k_i\)
6. SIMULATION RESULTS AND DISCUSSION

Trajectories which are used in this work are shown in Table 2. Figures 7-9 below show joint positions following desired trajectories with PD, PID, Fuzzy PID and Figure 10 shows the error profile of GA tuned Fuzzy PID controllers for the three joints.

<table>
<thead>
<tr>
<th>Table 2. Set Trajectory for simulation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 joint</td>
</tr>
<tr>
<td>sin(πt/20)</td>
</tr>
</tbody>
</table>

Figure 7 (a)

Figure 7 (b)

Figure 7 (c)

Figure 8 (a)
Figure 8 (b) Tracking using PID controller for various joints

Figure 9 (a) Tracking using PID controller for various joints

Figure 9 (b) Tracking using PID controller for various joints

Figure 9 (c) Tracking using PID controller for various joints

Table 3. Positional errors with different controllers

<table>
<thead>
<tr>
<th>Controller</th>
<th>IAE</th>
<th>ISE</th>
<th>ITAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PD</td>
<td>0.4051</td>
<td>0.2714</td>
<td>0.2766</td>
</tr>
<tr>
<td>PID</td>
<td>0.3944</td>
<td>0.2642</td>
<td>0.2604</td>
</tr>
<tr>
<td>FPID</td>
<td>0.2332</td>
<td>0.1596</td>
<td>0.1596</td>
</tr>
<tr>
<td>GAFPID</td>
<td>0.2113</td>
<td>0.1436</td>
<td>0.1021</td>
</tr>
</tbody>
</table>
Figure 10. Error Profile of GAFPID controller for three joints

Fuzzy PID controller performs well compared to PD and PID controllers, but only problem faced with these controllers is overcome by tuning of scaling factors with Genetic Algorithm. Optimum values found for kp, kd and ki values for three joints up to 30 iterations are, 
\[ k_p^i = 423.496257, \quad k_d^i = 3.217461, \quad k_i^i = 0.078799 \]
and 3 shows the comparison of Positional errors with different controllers.

7. CONCLUSION

Due to the strong nonlinear characteristics and parameter variations in real environments, tracking control of a robot arm system is difficult. Fuzzy PID controller performs well compared to PD and PID controllers, but only problem faced with these controllers is overcome by tuning of scaling factors with Genetic Algorithm. And Genetic Algorithm are systematic, do not rely heavily on the designer but they involve some processing time. Tuning of Fuzzy PID controller with fuzzy PID tuner in which sequential quadratic programming algorithms can be further studied.

REFERENCES


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