

# ANALYSIS OF ASYMMETRIC COPLANAR WAVEGUIDE WITH FINITE-EXTENT GROUND AND FINITE GROUND BACKING

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## ABSTRACT

Quasi-static characteristics of the asymmetric coplanar waveguide (CPW) with finite-extent ground and finite ground backing are studied using the conformal mapping method (CMM); closed form expressions for the effective relative permittivity and the characteristic impedance are derived. Various structures can be analyzed from the original structure. The effect of various parameters are studied using the expressions obtained. In addition, the structure is analyzed using finite difference method (FDM). The results obtained using CMM are compared to previous work, FDM and commercial software and have good agreement.

**Keywords:** *Coplanar Waveguide, Asymmetric Coplanar Waveguide, Conformal Mapping Method, Finite Difference Method, Finite-Extent Ground, Finite Ground Backing.*

## 1. INTRODUCTION

The coplanar waveguide (CPW) was first introduced by C. P. Wen in 1969 [1] Coplanar waveguides are finding extensive applications in microwave integrated circuits [2], [3], [4]. Inclusion of coplanar waveguides in microwave circuits adds to the flexibility of circuit design and improves the performance of some circuit functions.

Many structures have been analyzed using conformal mapping method; the symmetric and asymmetric coplanar waveguide (CPW) [5], [6], [7], asymmetric CPW (ACPW) with conductor backing [8], [9], [10], conductor-backing ACPW with one lateral ground plane [11], asymmetrical cylindrical coplanar waveguide (ACCPW) was analyzed using conformal mapping method in [12], [13], [14], [15]. Effect of finite ground-plane widths on quasi-static parameters of asymmetrical coplanar waveguides was investigated in [16]. The characteristics of coplanar waveguide with finite ground-planes by the Method of Lines was analyzed in [17]. Recently, design an ultra-wideband coplanar strip (CPS)-to-parallel stripline (PSL) transition is presented, this design is based on an analytical transition model obtained by conformal mapping [18]. In [19], the authors analyzed the Coplanar Waveguide applied in photonics based on analytical formulas, which were delivered and modified from conformal mapping

technique. Analytic expressions of the coupling of a coplanar waveguide resonator to a coplanar waveguide feedline using a conformal mapping technique were obtained in [20]. An expression for the characteristic mode impedances and coupling coefficients of an asymmetric multi-conductor transmission line was derived.

In this paper, the quasi-static characteristics of the asymmetric CPW with finite-extent ground and finite ground backing will be studied using the conformal mapping method (CMM), closed form expressions for the effective relative permittivity and the characteristic impedance are derived. Various structures can be analyzed from the original structure such as coplanar waveguide (CPW), asymmetric coplanar waveguide (ACPW), conductor backed CPW, conductor backed ACPW, microstrip line, and other geometries. All the above geometries are special cases of the proposed geometry and the parameters for these geometries can be evaluated using the expressions derived for this geometry.

The original structure will also be analyzed also using the finite difference method (FDM) [21], [22]. The formulation of the problem is based on the solution of Laplace's equation subject to appropriate boundary conditions, and the use of Taylor's series expansion to approximate the first and second order derivatives in Laplace's equation.

Numerical results from the two methods will be compared and discussed. The effect of various parameters on the characteristics of asymmetric CPW with finite-extent ground and finite ground-backing and elevated coplanar waveguide will be studied using the expressions obtained.

2. THEORY

2.1 Conformal Mapping Method

The structure of asymmetric CPW with finite-extent ground and finite ground backing is shown in Figure 1. The quasi-static conformal mapping method is used to derive closed form expressions for the effective permittivity and the characteristic impedance of this structure.

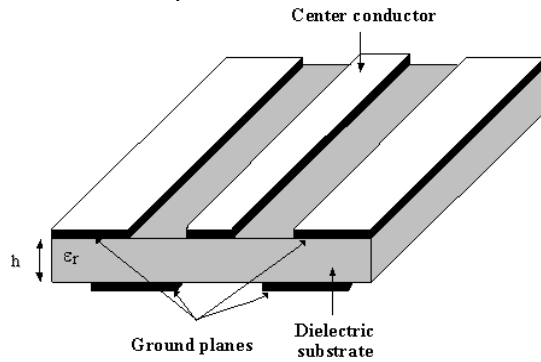


Figure 1: Geometry Of Asymmetric CPW With Finite-Extent Ground And Finite Ground Backing.

Figure 2a shows the asymmetric CPW with finite-extent ground and finite ground backing structure to be transformed. In the analysis, it is assumed that all the conductors have zero thickness; the substrate has a thickness h with relative permittivity εr. The structure is divided into 4 regions. The overall capacitance will be the sum of the capacitances of the four regions.

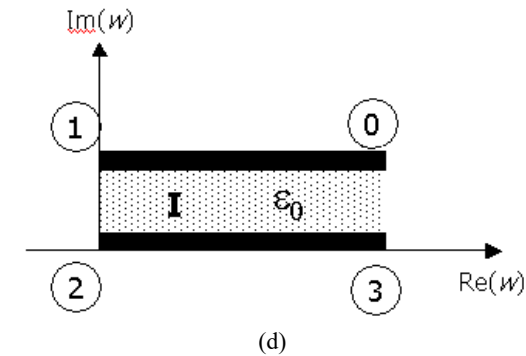
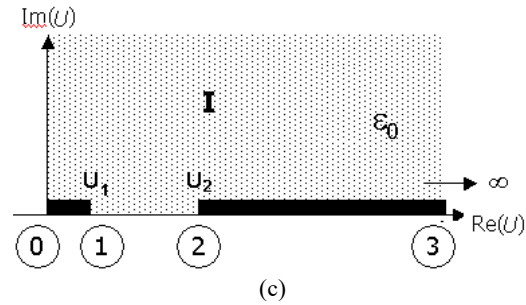
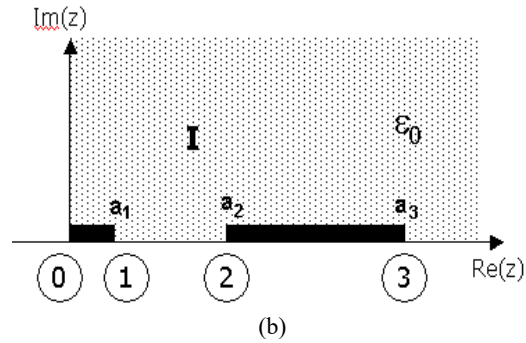
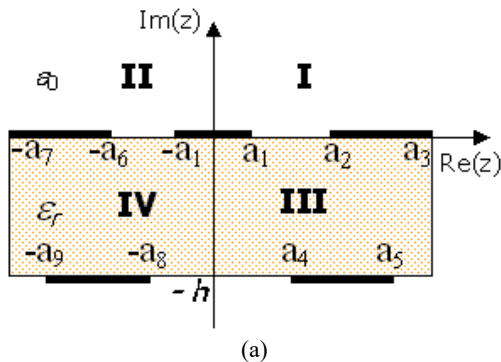


Figure 2: (A) Original Structure (B) Original Structure (Region I) (C) Intermediate U Plane (D) Final Mapping Into A Plane-Parallel Capacitor.

First, region I of the original structure (Figure 2b) is transformed into the U-plane (Figure 2c) using the transformation

$$U = \frac{z}{\sqrt{a_3^2 - z^2}} \tag{1}$$

where

$$U_1 = \frac{a_1}{\sqrt{a_3^2 - a_1^2}} \tag{2a}$$

$$U_2 = \frac{a_2}{\sqrt{a_3^2 - a_2^2}} \tag{2b}$$

Then it is transformed into the interior of a rectangle (Figure 2d) in the w-plane using the Schwartz-Christoffel transformation [6]

$$w = \int_{u_0}^u \frac{dU}{\sqrt{U(U-U_1)(U-U_2)}} \quad (3)$$

As a result, the capacitance per unit length of region I is

$$C_I = \varepsilon_0 \frac{K(k_1)}{K(k_1')} \quad (4)$$

where

$$k_1 = \sqrt{\frac{a_1^2(a_3^2 - a_2^2)}{a_2^2(a_3^2 - a_1^2)}}, \quad k_1' = \sqrt{1 - k_1^2} \quad (5)$$

and K is the complete elliptic integral of the first kind.

Using the same transformations, the capacitance of region II can be found as

$$C_{II} = \varepsilon_0 \frac{K(k_2)}{K(k_2')} \quad (6)$$

where

$$k_2 = \sqrt{\frac{a_1^2(a_7^2 - a_6^2)}{a_6^2(a_7^2 - a_1^2)}}, \quad k_2' = \sqrt{1 - k_2^2} \quad (7)$$

Second, to find the capacitance in the substrate region, region III is mapped to the upper half of the u plane (Figure 3b) using the transformation

$$u = \cosh^2 \frac{\pi z}{2h} \quad (8)$$

where

$$u_1 = \cosh^2 \frac{\pi a_1}{2h} \quad (9a)$$

$$u_2 = \cosh^2 \frac{\pi a_2}{2h} \quad (9b)$$

$$u_3 = \cosh^2 \frac{\pi a_3}{2h} \quad (9c)$$

$$u_4 = \sinh^2 \frac{\pi a_4}{2h} \quad (9d)$$

$$u_5 = \sinh^2 \frac{\pi a_5}{2h} \quad (9e)$$

Then it is transformed to the upper half of the v plane (Figure 3c) using the transformation

$$v = \frac{(u - u_4)}{\sqrt{(u_3 - u)(u - u_5)}} \quad (10)$$

where

$$v_1 = \frac{(1 - u_4)}{\sqrt{(u_3 - 1)(1 - u_5)}} \quad (11a)$$

$$v_2 = \frac{(u_1 - u_4)}{\sqrt{(u_3 - u_1)(u_1 - u_5)}} \quad (11b)$$

$$v_3 = \frac{(u_2 - u_4)}{\sqrt{(u_3 - u_2)(u_2 - u_5)}} \quad (11c)$$

Then it is transformed into the interior of a rectangle (Figure 3d) in the w-plane using the Schwartz-Christoffel transformation

$$w = \int_{v_0}^v \frac{dt}{\sqrt{v(v - v_1)(v - v_2)(v - v_3)}} \quad (12)$$

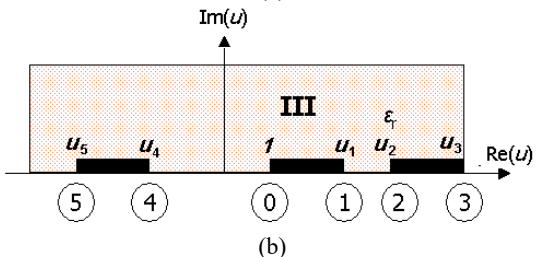
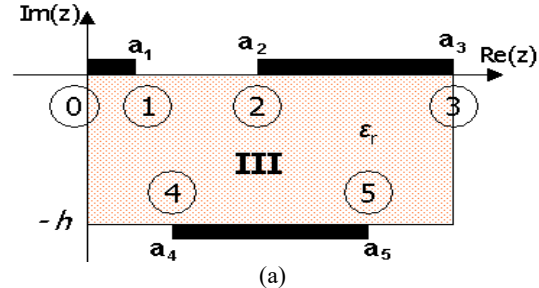
As a result, the capacitance per unit length of region III is

$$C_{III} = \varepsilon_0 \varepsilon_r \frac{K(k_3)}{K(k_3')} \quad (13)$$

where

$$k_3 = \sqrt{\frac{v_3(v_2 - v_1)}{v_2(v_3 - v_1)}}, \quad k_3' = \sqrt{1 - k_3^2} \quad (14)$$

and K is the complete elliptic integral of the first kind.



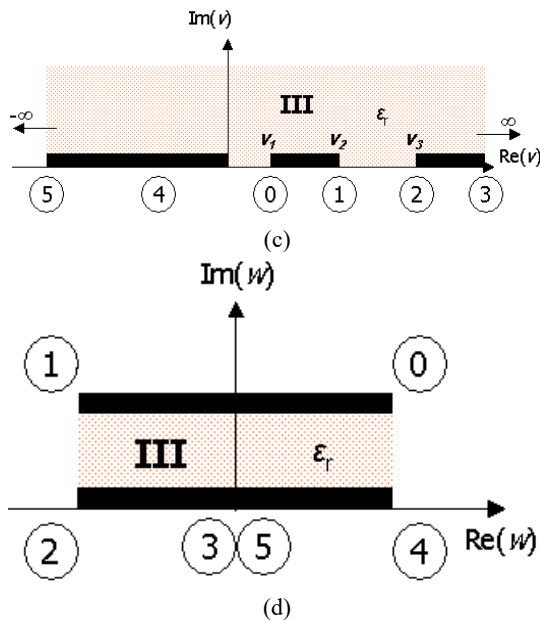


Figure 3: (a) Original structure (region III) (b) conformal mapping to \$u\$ plane (c) intermediate transformed \$v\$ plane (d) final mapping into a plane-parallel capacitor.

Using the same transformations, the capacitance of region IV can be found as

$$C_{IV} = \epsilon_0 \epsilon_r \frac{K(k_4)}{K(k_4')} \quad (15)$$

where

$$k_4 = \sqrt{\frac{v_6(v_5 - v_4)}{v_5(v_6 - v_4)}}, \quad k_4' = \sqrt{1 - k_4^2} \quad (16)$$

and

$$v_4 = \frac{(1 - u_8)}{\sqrt{(1 - u_7)(1 - u_9)}} \quad (17a)$$

$$v_5 = \frac{(u_1 - u_8)}{\sqrt{(u_1 - u_7)(u_1 - u_9)}} \quad (17b)$$

$$v_6 = \frac{(u_6 - u_8)}{\sqrt{(u_6 - u_7)(u_6 - u_9)}} \quad (17c)$$

$$u_6 = \cosh^2 \frac{\pi a_6}{2h} \quad (17d)$$

$$u_7 = \cosh^2 \frac{\pi a_7}{2h} \quad (17e)$$

$$u_8 = \sinh^2 \frac{\pi a_8}{2h} \quad (17f)$$

$$u_9 = \sinh^2 \frac{\pi a_9}{2h} \quad (17g)$$

Therefore, the overall capacitance per unit length of the original structure is

$$C = C_I + C_{II} + C_{III} + C_{IV} = \epsilon_0 \left[ \frac{K(k_1)}{K(k_1')} + \frac{K(k_2)}{K(k_2')} \right] + \epsilon_0 \epsilon_r \left[ \frac{K(k_3)}{K(k_3')} + \frac{K(k_4)}{K(k_4')} \right] \quad (18)$$

and the overall capacitance per unit length of the original structure in the absence of the substrate is (free space capacitance)

$$C_0 = \epsilon_0 \left[ \frac{K(k_1)}{K(k_1')} + \frac{K(k_2)}{K(k_2')} \right] + \epsilon_0 \left[ \frac{K(k_3)}{K(k_3')} + \frac{K(k_4)}{K(k_4')} \right] \quad (19)$$

The effective relative permittivity can be defined as

$$\epsilon_{eff} = \frac{C}{C_0} = \frac{\left[ \frac{K(k_1)}{K(k_1')} + \frac{K(k_2)}{K(k_2')} \right] + \epsilon_r \left[ \frac{K(k_3)}{K(k_3')} + \frac{K(k_4)}{K(k_4')} \right]}{\left[ \frac{K(k_1)}{K(k_1')} + \frac{K(k_2)}{K(k_2')} \right] + \left[ \frac{K(k_3)}{K(k_3')} + \frac{K(k_4)}{K(k_4')} \right]} \quad (20)$$

The characteristic impedance can be obtained from

$$Z_0 = \frac{\sqrt{\epsilon_{eff}}}{C v_0} = \frac{1}{\sqrt{\epsilon_{eff}} C_0 v_0} \frac{120\pi}{\sqrt{\epsilon_{eff}} \left[ \frac{K(k_1)}{K(k_1')} + \frac{K(k_2)}{K(k_2')} \right] + \left[ \frac{K(k_3)}{K(k_3')} + \frac{K(k_4)}{K(k_4')} \right]} \quad (21)$$

where \$v\_0\$ is the speed of light in free space.

## 2.2 Finite Difference Method

The quasi-static characteristics of the asymmetric CPW with finite-extent ground and finite ground backing are investigated using the finite-difference method (FDM) [12], [21]. This method is based on the solution of Laplace's

equation subject to appropriate boundary conditions and the use of Taylor's expansion to approximate the first and second order derivatives in Laplace's equation. To truncate the finite difference mesh three artificial boundaries are considered: perfect electric conductor (PEC), first order approximate boundary condition (ABC1), and second order approximate boundary condition (ABC2) [21], [22].

To find the characteristic impedance and effective relative permittivity of the original structure, it is required to calculate the capacitance per unit length. Referring to Figure 4, we can write

$$C = 2Q \tag{22}$$

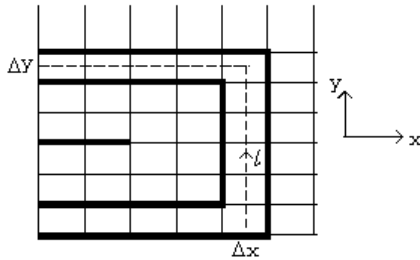


Figure 4: The path L used in calculating charge enclosed.

where Q is half the total charge on the center conductor. Thus, the problem is reduced to finding the charge Q per unit length. Note that the potential on the center strip is one volt. Applying Gauss's law to a closed path L enclosing this strip (Figure 4), we have

$$Q = \oint_L \vec{D} \cdot d\vec{l} = \oint_L \epsilon \frac{\partial V}{\partial n} dl \tag{23}$$

The charge Q can be written as [21]

$$Q = \epsilon_0 \sum \epsilon_{ri} V_i \frac{\Delta l}{\Delta n}$$

for nodes i on external loop with corners not counted

$$- \epsilon_0 \sum \epsilon_{ri} V_i \frac{\Delta l}{\Delta n}$$

for nodes i on inner loop with corners counted twice (24)

where  $V_i$  and  $\epsilon_{ri}$  are the potential and dielectric constant at the  $i$ th node. If  $i$  is on a dielectric interface,  $\epsilon_{ri}$  is taken to be the average between the two relative dielectric constants. Also, if  $i$  is on the line of symmetry, we use  $V_i/2$  instead of  $V_i$  to avoid including  $V_i$  twice in (24).

Finally, the effective relative permittivity and characteristic impedance can be obtained using

(25) and (26) where the free space capacitance  $C_0$  is obtained by removing the dielectric substrate.

$$\epsilon_{eff} = C/C_0 \tag{25}$$

The characteristic impedance can be obtained from

$$Z_0 = \frac{\sqrt{\epsilon_{eff}}}{C v_0} = \frac{1}{\sqrt{\epsilon_{eff}} C_0 v_0} \tag{26}$$

### 3. RESULTS AND DISCUSSION

Some results for the quasi-static characteristics of the asymmetric CPW with finite-extent ground and finite ground backing (shown in Figure 1) using conformal mapping method compared to the results obtained in literature, finite difference method and commercial software will be given.

#### Case 1 [4]: Asymmetric coplanar waveguide:

The characteristic impedance for asymmetric coplanar waveguide (ACPW) as a function of  $a_1/(a_6+a_2)$  for  $a_2/a_6 = 1.0$  is shown in Figure 5. The characteristic impedance decreases as  $a_1/(a_6+a_2)$  increases. The results obtained in [4] are shown as small circles. From the figure, it can be seen that the results are the same.

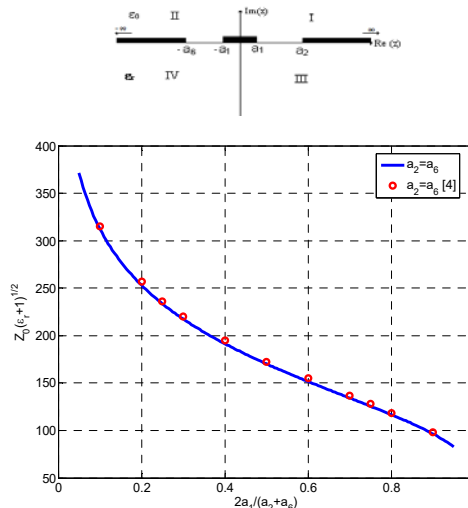
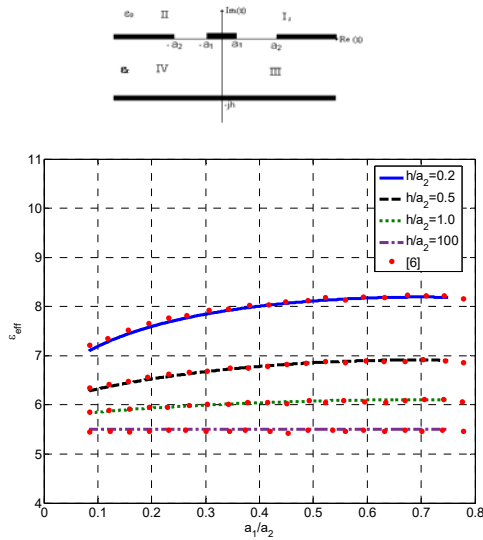


Figure 5:  $Z_0 \sqrt{\epsilon_r + 1}$  for ACPW as a function of  $2a_1/(a_2+a_6)$ .

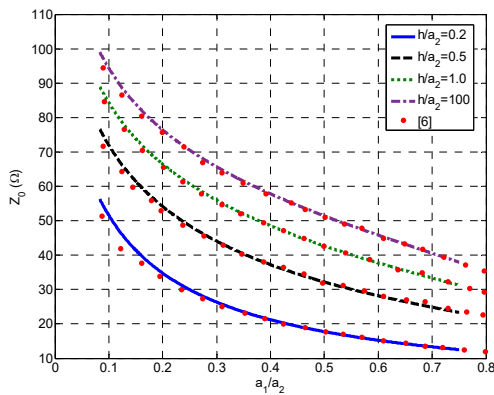
#### Case 2 [6]: Conductor-backed symmetric coplanar waveguide:

The effective relative permittivity and characteristic impedance for conductor-backed symmetric coplanar waveguide as a function of  $a_1/a_2$ ,

for different values of  $h/a_2$  for substrate permittivity  $\epsilon_r = 10$  are shown in Figures 6a and 6b. The effective relative permittivity increases as  $h/a_2$  decreases and the characteristic impedance decreases. The same results were obtained in [6] (shown as dots).



(a)



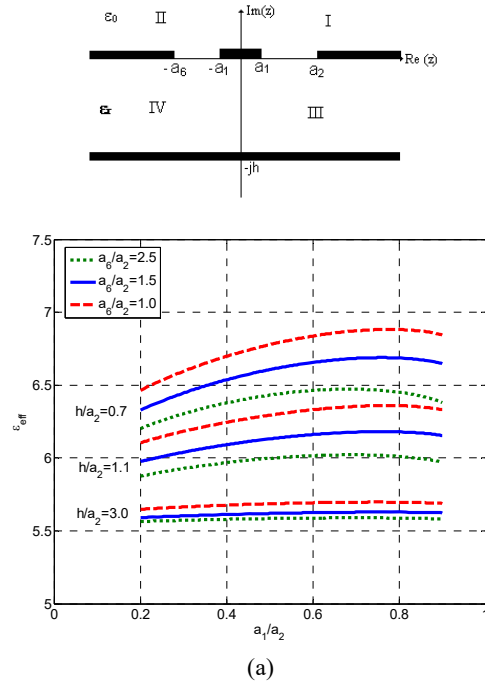
(b)

Figure 6: (a) Characteristic impedance (b) effective relative permittivity as a function of  $a_1/a_2$  for different values of  $h/a_2$ ,  $\epsilon_r = 10$  for CB-CPW.

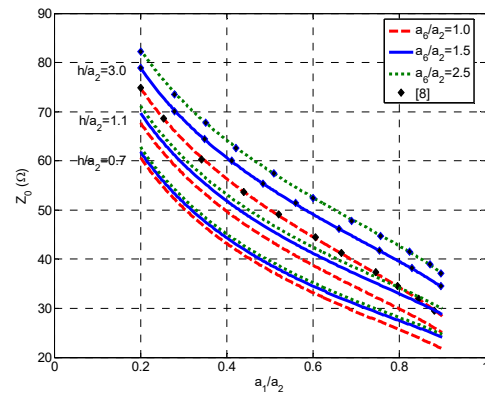
**Case 3 [8]: conductor-backed asymmetric coplanar waveguide:**

The effective relative permittivity and characteristic impedance for conductor-backed asymmetric coplanar waveguide as a function of  $a_1/a_2$ , for different values of  $h/a_2$  and  $a_6/a_2$  for substrate permittivity  $\epsilon_r = 10$  are shown in Figures 7a and 7b. The effective relative permittivity increases as  $h/a_2$  decreases and the characteristic impedance decreases. The results obtained in [8] are not correct, so we did not compare with them. Also, the characteristic impedance decreases as  $a_1/a_2$

increases. The same results were obtained in [8] for the characteristic impedance.



(a)

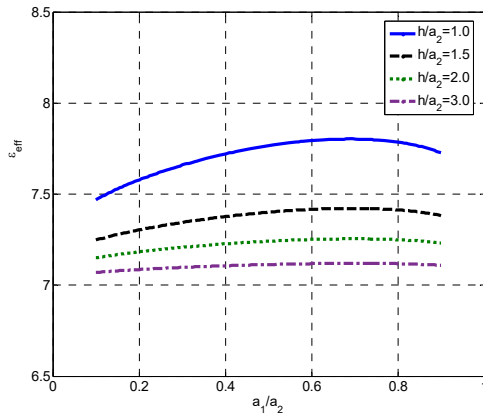


(b)

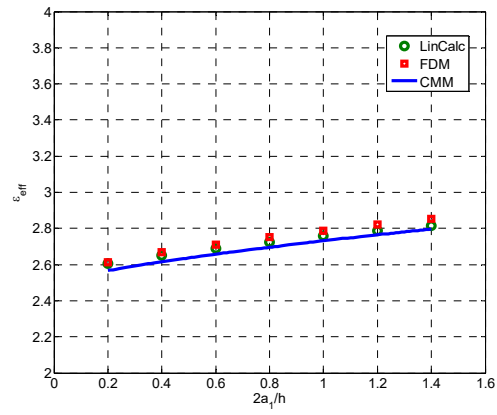
Figure 7: (a) Effective relative permittivity (b) characteristic impedance as a function of  $a_1/a_2$  for different values of  $h/a_2$  or  $a_2/a_6$ ,  $\epsilon_r = 10$  for CB-ACPW.

**Case 4 [7]: Conductor-backed asymmetric coplanar waveguide:**

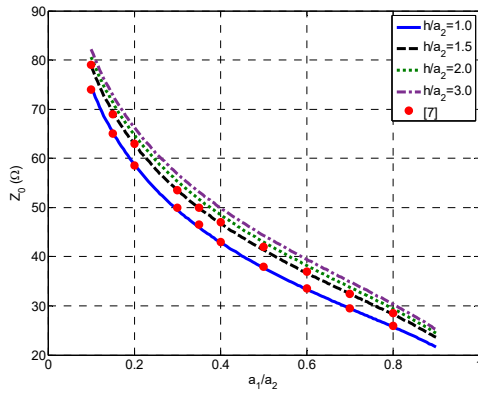
The effective relative permittivity and characteristic impedance for conductor-backed asymmetric coplanar waveguide as a function of  $a_1/a_2$ , for different values of  $h/a_2$  for substrate permittivity  $\epsilon_r = 13$  are shown in Figures 8a and 8b. The results are compared to those obtained in [7] and they are identical.



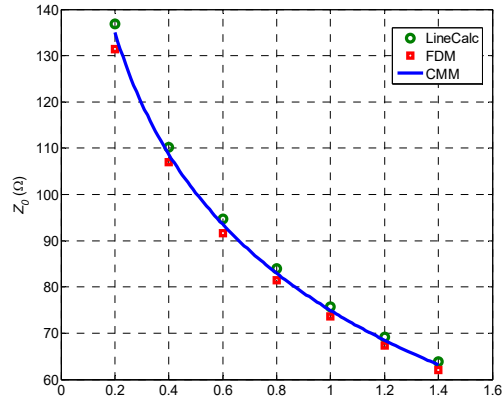
(a)



(a)



(b)



(b)

Figure 8: (a) Effective relative permittivity and (b) characteristic impedance as a function of  $a_1 / a_2$  for different values of  $h/a_2$ ,  $\epsilon_r = 13$  for CB-ACPW.

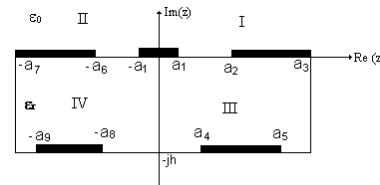
Figure 9: (a) Effective relative permittivity (b) characteristic impedance of a microstrip line.

**Case 5: Microstrip Line:**

The effective relative permittivity and characteristic impedance for the microstrip line with  $\epsilon_r = 3.38$  and  $h = 2\text{mm}$  are shown Figures 9a and 9b. The results are compared to the finite difference method (FDM) and LineCalc (advanced design system ADS tools) [23] results. The difference between the results is less than 3%.

**Case 6: Finite conductor backed coplanar waveguide:**

The characteristic impedance and relative effective permittivity for the finite conductor backed coplanar waveguide are shown in Figures 10a and 10b, as a function of  $a_1/a_2$  for different values of  $a_4$ ,  $\epsilon_r = 3.8$ ,  $h = 2\text{ mm}$ . In this case, the structure is symmetric ( $a_6 = a_2$ ,  $a_7 = a_3$ ,  $a_8 = a_4$ ,  $a_9 = a_5$ ). Two cases are compared to the results obtained using high frequency structure simulator (HFSS) [24], which are  $a_4 = 1\text{mm}$  and  $a_4 = 3\text{mm}$ , the results are in good agreement. It can be noticed that as  $a_4$  increases  $Z_0$  increases and  $\epsilon_{eff}$  decreases.





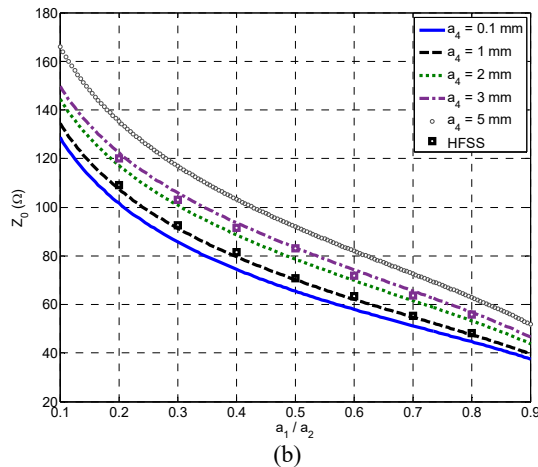
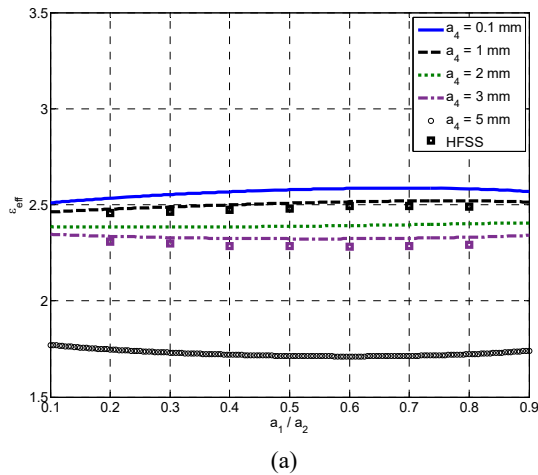


Figure 10: (a) Characteristic impedance and (b) effective relative permittivity as function of  $a_1/a_2$  for different values of  $a_4$ ,  $\epsilon_r = 3.8$ ,  $h = 2$  mm for CPW with two infinite backed ground planes.

#### Case 7: Asymmetric CPW with finite-extent ground and finite ground backing:

The characteristic impedance and relative effective permittivity for the finite asymmetric conductor backed coplanar waveguide are listed in Table 1 for different cases. In this case it is assumed that  $\epsilon_r = 10$ ,  $h = 2$  mm,  $a_1 = 1$  mm,  $a_3 = a_7$ ,  $a_5 = a_9$ . The results are compared to those obtained using

high frequency structure simulator (HFSS), it is noticed that the results are in good agreement; the error is less than 5% for the impedance and 2% for the relative permittivity.

#### 4. CONCLUSIONS

The quasi-static characteristics of the asymmetric CPW with finite-extent ground and finite ground backing is analyzed using the conformal mapping method (CMM). Closed form expressions for the effective relative permittivity and the characteristic impedance are derived. Various structures are analyzed and compared to published papers and commercial software.

Microstrip line is analyzed using the finite difference method (FDM). The formulation of the problem is based on the solution of Laplace's equation subject to appropriate boundary conditions, and the use of Taylor's series expansion to approximate the first and second order derivatives in Laplace's equation. The results obtained with FDM, CMM, and ADS Lincalac are approximately the same (the difference is less than 3%).

The results for the finite conductor backed coplanar waveguide, which is not available in literature, are obtained and compared to the full wave analysis results obtained by HFSS. The results are in good agreement. It is noticed that as the separation between the ground planes increases, the characteristic impedance increases, while the effective relative permittivity decreases.

#### ACKNOWLEDGEMENTS

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Table 1. The characteristic impedance and relative effective permittivity for the finite asymmetric conductor backed coplanar waveguide.

a <sub>2</sub>	a <sub>6</sub>	a <sub>4</sub>	a <sub>8</sub>	Z <sub>0</sub>			ε <sub>eff</sub>		
				CMM	HFSS	Error %	CMM	HFSS	Error %
2	2	0	0	42.9	45.3	5.3	6.08	5.98	1.7
2	3	0	0	44.7	47.2	5.3	6.22	6.13	1.5
2	4	0	0	45.4	47.7	4.8	6.32	6.24	1.3
2	5	0	0	45.8	48	4.6	6.39	6.27	1.9
2	2	0	1	44.3	45.8	3.3	5.96	5.94	0.3
2	2	1	1	46.1	48.5	4.9	5.85	5.74	1.9
2	2	0	2	46.9	46.8	0.2	5.79	5.85	1
2	2	1	2	48.7	50.8	4.1	5.66	5.56	1.8
2	2	2	2	52.1	52.8	1.3	5.36	5.23	1.9
2	2	0	3	48.7	48	1.5	5.67	5.72	0.87
2	2	1	4	50.7	53.5	5.2	5.54	5.44	1.8
2	2	0	5	48.7	48.5	0.4	5.67	5.70	0.53

## REFERENCES:

- [1] C. P. Wen, "Coplanar Waveguide: A Surface Strip Transmission Line Suitable for Nonreciprocal Gyromagnetic Device Applications," *IEEE Trans. Microw. Theory Tech.*, vol. 17, no. 12, pp. 1087–1090, Dec. 1969, doi: 10.1109/TMTT.1969.1127105.
- [2] S. Reza, "Coplanar Waveguide and Applications," in *Wiley Encyclopedia of Electrical and Electronics Engineering*, American Cancer Society, 2019, pp. 1–6.
- [3] K. C. Gupta, K. C. Gupta, R. Garg, I. Bahl, and P. Bhartia, *Microstrip Lines and Slotlines*. Artech House, 1996.
- [4] V. Fouad Hanna and D. Thebault, "Analysis of asymmetrical coplanar waveguides," *Int. J. Electron.*, vol. 50, no. 3, pp. 221–224, Mar. 1981, doi: 10.1080/00207218108901250.
- [5] N. B. and K. Porkumaran, "Conformal Mapping Analysis of Various Waveguide Structures," *ICTACT J. Commun. Technol.*, vol. 03, pp. 532–535, 2012, doi: 10.21917/ijct.2012.0074.
- [6] G. Ghione and C. Naldi, "Parameters of coplanar waveguides with lower ground plane," *Electron. Lett.*, vol. 19, no. 18, pp. 734–735, Sep. 1983, doi: 10.1049/el:19830500.
- [7] G. Ghione and C. U. Naldi, "Coplanar Waveguides for MMIC Applications: Effect of Upper Shielding, Conductor Backing, Finite-Extent Ground Planes, and Line-to-Line Coupling," *IEEE Trans. Microw. Theory Tech.*, vol. 35, no. 3, pp. 260–267, Mar. 1987, doi: 10.1109/TMTT.1987.1133637.
- [8] Shao-Jun Fang and Bai-Suo Wang, "Analysis of asymmetric coplanar waveguide with conductor backing," *IEEE Trans. Microw. Theory Tech.*, vol. 47, no. 2, pp. 238–240, Feb. 1999, doi: 10.1109/22.744300.
- [9] C. Veyres and V. Fouad Hanna, "Extension of the application of conformal mapping techniques to coplanar lines with finite dimensions," *Int. J. Electron.*, vol. 48, no. 1, pp. 47–56, Jan. 1980, doi: 10.1080/00207218008901066.
- [10] Asem Al-Zoubi, Ahmad Kishk, "Analysis of printed transmission lines," presented at the Mississippi Academy of Sciences., University of Mississippi, 2005, [Online]. Available: <https://msacad.org/journal/jan06journal/mas06.pdf>.
- [11] Adnan Görür , Ceyhun Karpuz, "Analytic formulas for conductor-backed asymmetric CPW with one lateral ground plane," *Microw. Opt. Technol. Lett.*, vol. 22, no. 2, pp. 123–126, Jun. 1999, doi: [https://doi.org/10.1002/\(SICI\)1098-2760\(19990720\)22:2<123::AID-MOP13>3.0.CO;2-1](https://doi.org/10.1002/(SICI)1098-2760(19990720)22:2<123::AID-MOP13>3.0.CO;2-1).
- [12] Asem Al-Zoubi, "A study of the characteristics of the cylindrical coplanar waveguide (CCPW)," Master Thesis, Jordan University of Science and Technology, Jordan, 1998.
- [13] A. S. Al-Zoubi, "Quasi-static analysis of asymmetric cylindrical coplanar waveguides with finite-extent ground," *Int. J. Electron.*, vol. 87, pp. 185–198, 2000.
- [14] A. Al-Zoubi and N. Dib, "CAD model of gap in cylindrical coplanar waveguide," *Electron. Lett.*, vol. 35, no. 21, pp. 1857–1858, Oct. 1999, doi: 10.1049/el:19991283.
- [15] A. Al-Zoubi, K. Obeidat, and N. Dib, "Quasi-static analysis of asymmetric cylindrical coplanar waveguide," in *IEEE Antennas and*

- Propagation Society International Symposium. 1999 Digest. Held in conjunction with: USNC/URSI National Radio Science Meeting (Cat. No.99CH37010)*, Jul. 1999, vol. 3, pp. 1824–1827 vol.3, doi: 10.1109/APS.1999.788310.
- [16] A. Gorur and C. Karpuz, “Effect of finite ground-plane widths on quasistatic parameters of asymmetrical coplanar waveguides,” *IEE Proc. - Microw. Antennas Propag.*, vol. 147, no. 5, pp. 343–347, Oct. 2000, doi: 10.1049/ip-map:20000676.
- [17] M. Wang, B. Gao, Y. T., and T. Ling, “Analysis of Characteristics of Coplanar Waveguide with Finite Ground-planes by the Method of Lines,” *Piers Online*, vol. 6, pp. 46–50, 2010, doi: 10.2529/PIERS090903044952.
- [18] G. H. Lee, D. H. Kim, W. Mohyuddin, S. Kumar, H. C. Choi, and K. W. Kim, “Design of an ultra-wideband coplanar strip-to-parallel stripline transition using an analytical model based on conformal mapping,” *Microw. Opt. Technol. Lett.*, vol. 63, no. 4, pp. 1054–1060, 2021, doi: <https://doi.org/10.1002/mop.32707>.
- [19] S. Kula, “Parametrization of CPW for Photonics Application with the Use of Closed Form Formulas,” *Period. Polytech. Electr. Eng. Comput. Sci.*, vol. 64, no. 3, pp. 286–290, 2020, doi: 10.3311/PPee.14894.
- [20] I. Besedin and A. P. Menushenkov, “Quality factor of a transmission line coupled coplanar waveguide resonator,” *EPJ Quantum Technol.*, vol. 5, no. 1, p. 2, Jan. 2018, doi: 10.1140/epjqt/s40507-018-0066-3.
- [21] M. N. O. Sadiku, *Numerical Techniques in Electromagnetics with MATLAB, Third Edition*, 3rd ed. USA: CRC Press, Inc., 2009.
- [22] A. Z. Elsherbeni, Y. Ying, C. E. Smith, and V. Rodriguez-Pereyra, “Finite difference analysis of cylindrical two conductor microstrip transmission line with truncated dielectrics,” *J. Frankl. Inst.*, vol. 333, no. 6, pp. 901–928, 1996, doi: [https://doi.org/10.1016/0016-0032\(96\)00048-8](https://doi.org/10.1016/0016-0032(96)00048-8).
- [23] *ADS, Advanced Design System*. Keysight Technologies Inc., 2017.
- [24] *HFSS (High Frequency Structure Simulator), Based on the Finite Element Method*. Version 15.1, ANSYS Corporation, 2008.