HYBRIDIZATION GRADIENT DESCENT SEARCH WITH ARTIFICIAL BEES COLONY ALGORITHM IN GENERAL GLOBAL OPTIMIZATION PROBLEMS

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ABSTRACT

General gradient-based optimization techniques such as the steepest descent method, Newton's method, and quasi-Newton method, often fail in globally solving non-convex optimization problems (multi-modal functions). The main reason is that once a local solution has been determined, these methods do not know how to pass a hill to obtain another better local solution. Therefore, a new gradient type method so-called ABCED Steepest Descent Method has been introduced in this paper which does not have the weakness been mentioned above. The ABCED SD method is a hybrid from a modified steepest descent method and the Artificial Bee Colony (ABC) algorithm. ABCED SD method developed through the ABC framework by replacing the exploitation search phase with modified steepest descent method, therefore the global optimum solution is obtained by the modified steepest descent algorithm. Reported numerical results shown that ABCED SD method able to locate the global optimum solution for the benchmarked general global optimization problems and the comparison results with the original Artificial Bee Colony algorithm also shown that ABCED SD method able to obtain the global optimum solution with less iterations. Besides that, ABCED SD method also does not require any initial point and it will not only determine the local minimizer as in the classical steepest descent method, but it manage to determine the global minimizer of the general global optimization problems.

Keywords: *Multi-Modal Function, Non-Convex Optimization, Steepest Descent Method, Artificial Bee Colony Algorithm*

1. INTRODUCTION

Mathematicians believe that every daily problem that we face can be modeled into a mathematical model entirely. In mathematical terms, the goal of solving those models in the "best" way is called optimization. These might mean maximize profit, minimize loss, maximize efficiency or minimize the risk in running a business; minimize weight or maximize strength in designing a bridge and minimize the time or fuel use in selecting an aircraft flight plan.

There are several gradient-based optimization techniques that have been proposed to solve those mathematical models, such as the steepest descent method, Newton's method, and the quasi-Newton method. These methods are well-performed to determine local solutions or once say globally determined the solution when solving convex optimization problems, in which there have only one local solution and can also be called a global solution.

However, most of our daily problems happen as non-convex optimization problems, which may contain a multi-local optimum solution. Most of the time, the local solution is greatly different and meaningless when compared to the global one. Therefore, the most important objective and challenge in solving these non-convex optimization problems are how to determine the optimum value among all the local optimum solutions in the domain or we call it a globally optimum solution. By the way, those well-performed methods mentioned above always lose their efficiency when applied to the global minimizer for non-convex problems.

The artificial Bee Colony (ABC) algorithm is one of the most recent swarm intelligence-based algorithm proposed by Dervis Karaboga in the year 2005 [8]. It is a biological-inspired optimization algorithm. ABC is inspired by the foraging behavior of honeybee swarm. The process of the swarm of bees searching for a food source is the process used to find an optimal solution [8]. Exploration and exploitation are two important mechanisms in ABC. The exploitation process starts when the employed bees approach to food sources. After determining the nectar amounts of the food sources by the employed bees, the onlooker bees will go to the highest probability value of the source and determine the nectar amount. When the source is exhausted, it indicated the end of the exploitation process. Meanwhile, the exploration process begins when scouts are sent to search for new food sources randomly. However, there are some insufficiencies regarding ABC. ABC perform better during the exploration stage but weaker at the exploitation stage [2][4][11][13].

In the ABC algorithm, abandoned food source is replaced with a new food source by scout bee [9]. This is realized by creating a random position. The scout bee will go for a new food source. If a position cannot be developed in the number of control parameters called as “limit”, it is assumed that this food source is abandoned. [4] try to overcome this issue by introduced the ABC algorithm based on information learning. Karaboga and Kaya have presented a novel solution generating mechanism by utilizing arithmetic crossover and adaptive neighborhood radius [10]. Yang et al. introducing a single equation unifying multiple strategies [12]. In addition, a chaotic strategy is employed for parameter adaption to balance the proportion of exploration and exploitation in different stages.

Goh and his team introduced a Simplexed ABC algorithm that improves the accuracy and efficiency of the ABC in solving global optimization problems. The finding of the lead this research to the new direction of the investigation. A Nelder-Mead simplex method is a derivative-free approach in which the order of convergent is much slower compared to the gradient-based method. However, the enhancement of Simplexed ABC has indicated that even with a smaller number of colony involvement, its ability to approximate optimum solution is much better than the original ABC [6]. After that, Goh et al. proposed a ABCED Conjugate Gradient Method which hybrid the artificial bees colony algorithm with the most effective deterministic local optimization approaches, the experiment numerical results shows that the proposed hybridization method able to obtain better global optimization solution with less bees colony size and number of cycles involved [7]. Therefore, this research has led to a new path that will enhance the original ABC with Steepest Descent Method [3][5].

In this paper, we have introduced a method so-called ABCED Steepest Descent Method (ABCED SD) which its algorithm is a hybrid from the modified steepest descent method [5] into the Artificial Bees Colony (ABC) algorithm for solving general global optimization problems. The main idea of the ABCED SD method is replacing the exploitation process in the original ABC with the modified steepest descent. The performance of the exploitation process will be improved by the efficiency of the steepest descent method. Besides that, via this hybridization process, the ability of the steepest descent method also improved to able to determine the global optimal solution for non-convex optimization problems.

This paper is organized as follows. In Section 2, we define several basic definitions of global optimization and the properties of the gradient type method which must be understood before discussing the development of the ABCED SD method in more detail. The algorithm of the ABCED SD method has been shown in section 3. The numerical results and the comparison results which reflect the effectiveness of the ABCED SD method in

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solving general global optimization problems have been presented in the following section. Finally, the conclusion which ends this paper will be presented in Section 5.

2. ABCED STEEPEST DESCENT METHOD

The steepest descent method is the simplest gradient-based method. The enhancement of the gradient-based method begins with this Steepest Descent method is to make sure it possible to work with another gradient-based method. The enhanced steepest descent method so-called ABCED Steepest Descent Method (ABCED SD) which its algorithms is hybrid from the modified steepest descent method [5] into the Simplexed ABC algorithm for solving general global optimization problems. The main idea of the ABCED Steepest Descent method is replacing the Nelder-Mead algorithm in Simplexed ABC with the modified steepest descent method. The exploitation process of the Simplexed ABC will be improved by the efficiency of the steepest descent method. Besides that, via this hybridization process, the ability of the steepest descent method also improved to able to determine the global optimal solution for non-convex optimization problems.

The advantages of the steepest descent method compared to the Nelder-Mead algorithm are the requirement of the number of the initial point and the guaranteed descent direction of the steepest descent. The Steepest Descent method only required one initial point to begin its searching algorithm. This requirement has made the steepest descent method can be engaged by any employed bee for their exploitation searching process at their respective food source. The steepest descent direction has enabled the employed bee to exploit the food source and obtain the optimum solution faster than the Nelder-Mead algorithm.

Goh has reported that the steepest descent method with the Armijo line search [1] showed its efficiency in locally solving non-convex optimization problems [5]. Therefore, the Modified Steepest Descent with Armijo line search has been selected to improve in this research. The Armijo line search rule [1] is described as follows.

Given $s > 0$, $\beta \in (0,1)$, $\sigma \in (0,1)$ and $\lambda_k = \max\{s, s\beta, s\beta^2, \ldots\}$ such that

$$f(x_k + \lambda_k d_k) - f(x_k) \le \sigma \lambda_k g_k^T d_k \quad (1)$$

where $d_k = -g_k = -\nabla f(x_k)$.

Algorithm 2.1 (Steepest descent method with Armijo line search)

Input: Initial point $x_0 \in \mathbb{R}^n$, Function to be minimized $f : \mathbb{R}^n \to \mathbb{R}$, and Tolerance $\varepsilon \in \mathbb{R}$.

1. $k = 0$
2. $d_k = -\nabla f_k$
3. while $\left(\|d_k\| \le \varepsilon\right)$ do
   4. $\lambda_k = 2,\ \beta = 0.618,\ \sigma = 0.8$
   5. while $\left(f(x_k + \lambda_k d_k) - f(x_k) \le \sigma \lambda_k g_k^T d_k\right)$ do
      6. $\lambda_k = \lambda_k \beta$.
   7. $x_{k+1} = x_k + \lambda_k d_k$.
   8. $k = k + 1$.
   9. $d_k = -\nabla f_k$
10. x_k is a minimizer.

3. ALGORITHM OF ABCED STEEPEST DESCENT METHOD

Initialize the population of solutions x_{ij}, *i=1, 2...SB*, *j= 1, 2...n*, *trial= 0* is the non-improvement number of the solution x_{ij}, used for abandonment

Evaluate the population.

Set Cycle= 1

Repeat

{Produce a new food source population for employed bees}

for *i*=1 to *SN* **do**

Produce a new food source v_i for the employed bee of the food source x_i using **Algorithm 2.1**

Apply a greedy selection process between v_i, x_i and select the better one.
If solution x_i does not improve $trial_i = trial_i + 1$, otherwise, $trial_i + 1 = 0$
end for
Calculate the probability values p_i by

$$p_i = \frac{fit_i}{\sum_{j=1}^{SN} fit_j}$$ for the solutions

Where the fitness values

$$fit_i = \begin{cases} \dfrac{1}{1+f_i} & \text{if } f_i \geq 0 \\ 1 + abs(f_i) & \text{if } f_i < 0 \end{cases}$$

{Produce a new food source population for onlooker bees}
t=0, i=1
repeat
if random<p_i then
Produce a new v_i food source by **Algorithm 2.1** for onlooker bee
Apply a greedy selection process between v_i and x_i and select the better one
If solution x_i does not improve $trial_i = trial_i + 1$, otherwise $trial_i = 0$
$t = t+1$
end if
until ($t=SN$)
{Determine scout}
if max ($trial_i$)> limit **then**
Replace x_i with a new randomly produced solution by
$x_i^j = x_{\min}^j + rand(0,1)(x_{\max}^j - x_{\min}^j)$
end if
Memorize the best solution achieved so far
cycle= cycle+1
until (cycle= Maximum Cycle Number)

4. NUMERICAL RESULTS

The ABCED Steepest Descent algorithm discussed in section 3 had been programmed into C++ and tested to 21 different kinds of global optimization problems. These 21-different kinds of global optimization problems are selected because they possess different kinds of challenges in global optimization problems. The list of test problems given in Appendix A and the graph of performance in Appendix B. With the assessment of these 21 global optimization problems, the enhanced ABCED Steepest Descent method will be able to solve any kind of global optimization problems after this.

The numerical results reported in **Appendix A** Table 4.1 demonstrated the efficiency of the enhanced ABCED Steepest Descent Method in solving various types of global optimization benchmark problems. From the numerical results, it shows that the ability of the original Steepest Descent has been improved to successfully solving global optimization problems. Besides that, the original Steepest Descent requires the initial point to start the approximation had been overcome with the enhancement. The weakness of the original steepest descent method often trapped in local optimum point unable to move to the global solution had been eliminated and successfully obtain the global optimum solutions.

4.1 Comparison results of ABCED SD with original ABC

The numerical performance of ABCED Steepest Descent has been comparing to the original ABC in solving 10 selected global optimization problems (Yang, 2014). The comparison results are presented in the following graph of the logarithm of the global solution with the number of iterations. The comparison results show that the ABCED Steepest Descent post better convergent ability than the original ABC. In all selected problems, the ABCED Steepest Descent method obtained a better global optimum solution compare to the original ABC. These comparisons result in Figure 4.1 (see **Appendix B**) have been proved the performance for the enhanced method in solving global optimization problems. From the comparison results, ABCED SD method able to obtain global solution with less than 10 iterations. The convergent rate to the global solutions, ABCED

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SD shows a drastic improvement compared to the original ABC by approaching global minimizer much slower than ABCED SD method.

5. CONCLUSION AND DISCUSSION

In this paper, the enhancements had produced a new gradient-based method called ABCED Steepest Descent. The numerical results reported in section 4 show that the ability of the original local optimizations has been improved their ability in globally solving multimodal global optimization problems. The results indicated that the proposed algorithm is capable of obtaining the global optimal solution effectively compared to the original ABC algorithm except for the Both function. However, the convergence ability of the proposed algorithm is still better than the original ABC algorithm. In summary, the proposed algorithm achieved outstanding performance by ABCED Steepest Descent solved all the various selected global optimization problems had verified their ability been improved. The performance of the original ABC algorithm and selected test function in terms of iterations number is presented in **Appendix B** Figure 4.1. These figures show that the logarithm of global optimum value and the number of iterations. All the considered algorithms converged to the optimal solution, however overall the original ABC algorithm requires a greater number of iterations compared to the ABCED Steepest Descent algorithm. The numerical results in Figure 4.1 also showed that ABCED SD method perform better than the 5 variants of ABCED Conjugate Gradient (Goh,2018B), this also prove that those local optimizations that possess the global convergent ability like steepest descent method will perform better than those local convergent method in globally solving those selected multimodal global optimization problems. Therefore, this improvement can lead be applied to Quasi Newton Method which also possess the global convergent in solving multimodal global optimization problems.

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Appendix A

Table 4.1 Numerical Results of ABCED Steepest Descent Method

No.	Problem	Mean of 20 Runs
1	Griewank(5 variable) $f(x)=1+\frac{1}{4000}\sum_{i=1}^{5}x_i^2-\prod_{i=1}^{5}\cos\left(\frac{x_i}{\sqrt{i}}\right)$	4.493174e-003
2	Sphere (5 variable) $f(x)=\sum_{i=1}^{5}\left[x_i^2\right]$	7.897967e-012
3	Rosenbrock (5 variable) $f(x)=\sum_{i=1}^{4}\left[100(x_{i+1}-x_i^2)^2+(x_i-1)^2\right]$	6.683427e-003
4	Rastrigin function (5 variable) $f(x)=50+\sum_{i=1}^{5}\left[x_i^2-10\cos(2\pi x_i)\right]$	5.124035
5	Rastrigin function (2 variable) $f(x)=20+\sum_{i=1}^{2}\left[x_i^2-10\cos(2\pi x_i)\right]$	0.04974795
6	Ackley's function $f(x_1,x_2)=-20\exp\left[-0.2\sqrt{0.5(x_1^2+x_2^2)}\right]$ $-\exp\left[0.5\left(\cos(2\pi x_1)\cos(2\pi x_2)\right)\right]+e+20$	2.473979e-006
7	Beale's function $f(x_1,x_2)=(1.5-x_1+x_1x_2)^2+(2.25-x_1+x_1x_2^2)^2$ $+(2.625-x_1+x_1x_2^3)^2$	2.631804e-005
8	Goldstein–Price function $f(x_1,x_2)=\left[1+(x_1+x_2+1)^2\left(19-14x_1+3x_1^2-14x_2+6x_1x_2+3x_2^2\right)\right]$ $\left[30+(2x_1-3x_2)^2\left(18-32x_1+12x_1^2+48x_2-36x_1x_2+27x_2^2\right)\right]$	3.000000
9	Booth's function $f(x_1,x_2)=(x_1+2x_2-7)^2+(2x_1+x_2-5)^2$	1.819610e-011
No.	**Problem**	**Mean of 20 Runs**
10	Matyas function $f(x_1,x_2)=0.26(x_1^2+x_2^2)-0.48x_1x_2$	4.588312e-009
11	Lévi function N.13 $f(x_1,x_2)=\sin^2(3\pi x_1)+(x_1-1)^2\left(1+\sin^2(3\pi x_2)\right)$ $+(x_2-1)^2\left(1+\sin^2(2\pi x_2)\right)$	3.634035e-007
12	Three-hump camel function $f(x_1,x_2)=2x_1^2-1.05x_1^4+\frac{x_1^6}{6}+x_1x_2+x_2^2$	1.376839e-012
13	Easom function $f(x_1,x_2)=-\cos(x_1)\cos(x_2)\exp\left(-\left((x_1-\pi)^2+(x_2-\pi)^2\right)\right)$	-1.000000e+00

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14	Adjiman function $f(x_1,x_2)=\cos(x_1)\sin(x_2)-\frac{x_1}{x_2^2+1}$	-5.004024
15	bird function $f(x_1,x_2)=\sin(x_1)e^{(1-\cos(x_2))^2}+\cos(x_2)e^{(1-\sin(x_1))^2}+(x_1-x_2)^2$	-106.7645
16	Bohachevsky 1 Function $f(x_1,x_2)=x_1^2+2x_2^2-0.3\cos(3\pi x_1)-0.4\cos(4\pi x_2)+0.7$	3.087730e-010
17	Bohachevsky 2 Function $f(x_1,x_2)=x_1^2+2x_2^2-0.3\cos(3\pi x_1)\cdot 0.4\cos(4\pi x_2)+0.3$	0.18
18	Bohachevsky 3 Function $f(x_1,x_2)=x_1^2+2x_2^2-0.3\cos(3\pi x_1+4\pi x_2)+0.3$	9.183535e-010
19	Branin RCOS function 1 $f(x_1,x_2)=\left(x_2-\frac{5.1x_1^2}{4\pi^2}+\frac{5x_1}{\pi}-6\right)^2+10\left(1-\frac{1}{8\pi}\right)\cos(x_1)+10$	0.3978874
20	Branin RCOS function 2 $f(x_1,x_2)=\left(x_2-\frac{5.1x_1^2}{4\pi^2}+\frac{5x_1}{\pi}-6\right)^2$ $+10\left(1-\frac{1}{8\pi}\right)\cos(x_1)\cos(x_2)\ln(x_1^2+x_2^2+1)+10$	-9.489830
21	Bukin 2 function $f(x_1,x_2)=100(x_2-0.01x_1^2+1)+0.01(x_1+10)^2$	-396.2484

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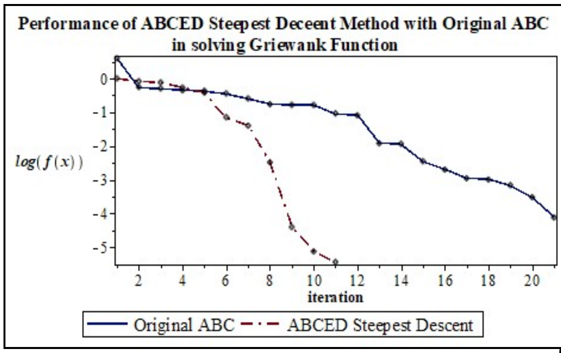
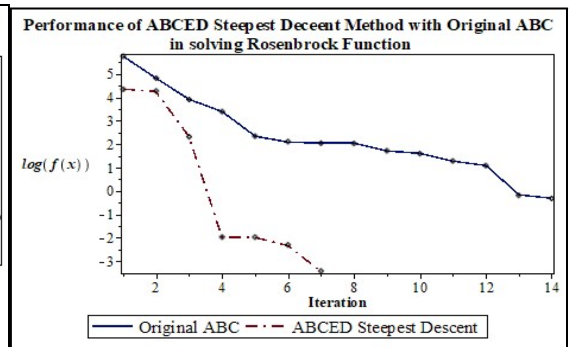
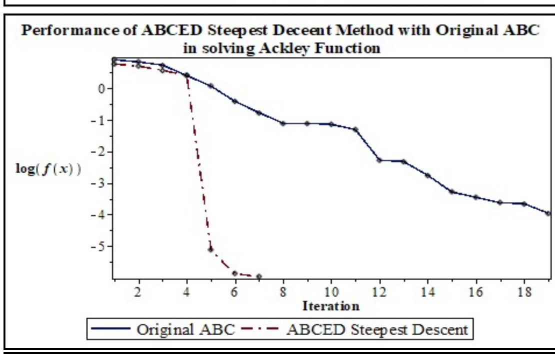
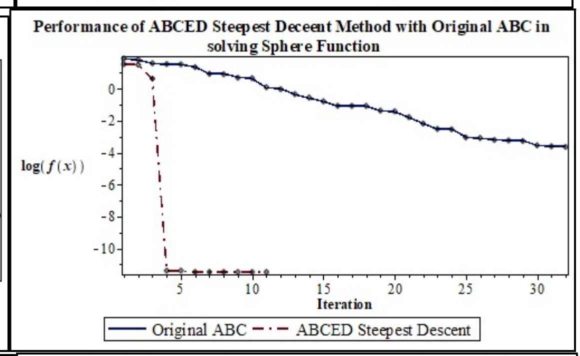
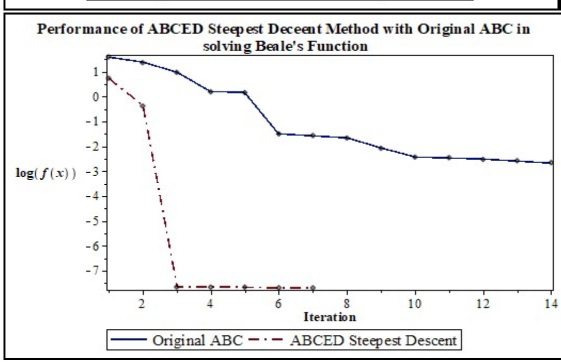
Appendix B: The performance of new proposed method

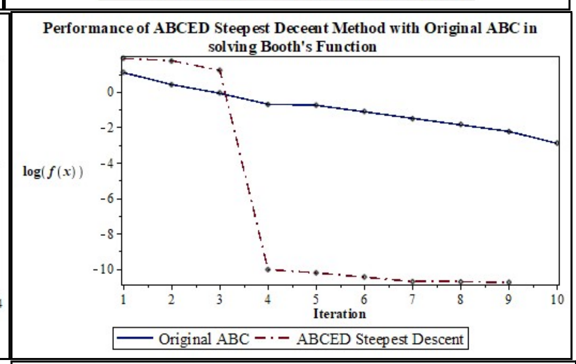


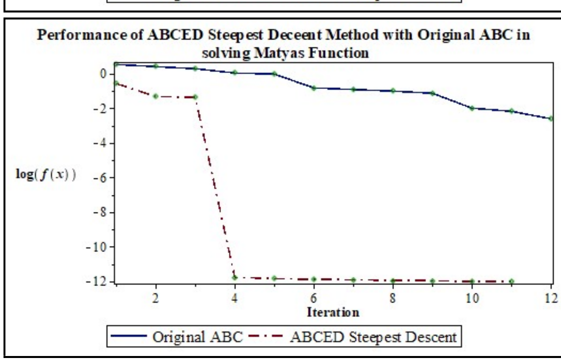


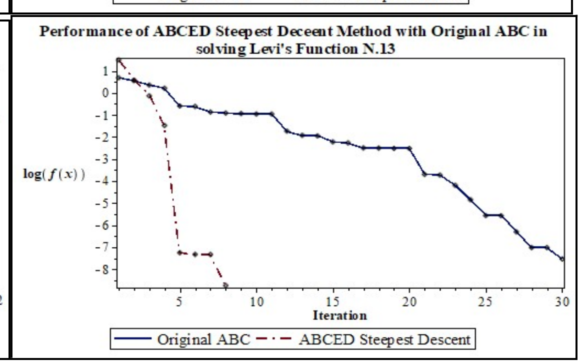












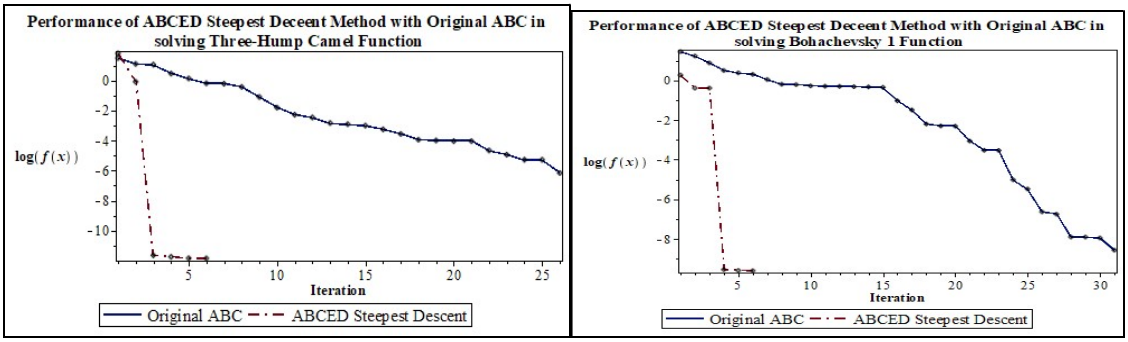


Figure 4.1: Comparison ABCED Steepest Descent with Original ABC