A COMPARATIVE ANALYSIS ON THE PERFORMANCE OF FINITE FAILURE NHPP SOFTWARE RELIABILITY MODEL BASED ON RAYLEIGH-TYPE LIFETIME DISTRIBUTION

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ABSTRACT

In this study, after applying the inverse Rayleigh and Rayleigh distributions which are widely used in the field of reliability to the finite failure NHPP software reliability model, we analyzed the reliability performance together with Goel-Okumoto basic model. For this, software failure time data was used, parametric estimation was applied to the maximum likelihood estimation method, and nonlinear equations were solved by a numerical method. As a result, in the analysis of the intensity function, the Rayleigh model is efficient because the failure occurring rate decreases with the failure time and the mean square error (MSE) is the smallest. In the analysis of the mean value function, all the proposed models showed an overestimated value compared to the true value, but the Rayleigh model showed the smallest error value. As a result of evaluating the software reliability after putting the mission time in the future, the Rayleigh model was stable and high together with the inverse Rayleigh model, but the Goel-Okumoto basic model showed a decreasing tendency. In conclusion, we found that the Rayleigh model has the best performance among the proposed models. In this study, the reliability performance of the inverse Rayleigh and Rayleigh distribution model without the existing research case was newly analyzed, and it is expected that it can be used as a basic guideline for the software developers to exploring the optimal software reliability model.

Keywords: *Goel-Okumoto, Inverse-Rayleigh, Lifetime Distribution, Finite Failure NHPP, Rayleigh, Software Reliability*

1. INTRODUCTION

Software technology, which is the core of the digital convergence era, has spread rapidly in industrial application fields, and the research for software development that can process various big data quickly and accurately without failures is also increasing. For this problem, software developers are still doing a lot of research to improve software reliability. That is, for software developers, solving problems to improve software reliability is a very important research topic. For this reason, software reliability models using the non-homogeneous Poisson process (NHPP) have been extensively studied to improve software quality. In particular, the NHPP software reliability model using the intensity function and the mean value function has been proposed to estimate the reliability performance attributes such as the failure occurring rate [1]. Concerning the NHPP reliability model, Goel and Okumoto [2] proposed an exponential software reliability model, Huang [3] explained the software reliability attributes using the mean value function, Shyur [4] proposed a generalized reliability model using change-point, and Kim [5] analyzed the statistical process control of software reliability model based on NHPP Rayleigh distribution model. Also, Kim and Moon [6] proposed a new comparative evaluation result of software reliability model using Exponential-exponential and Burr-Hatke-exponential lifetime distribution, and Voda [7] proposed that various types of lifetime distributions can be explained by the inverse Rayleigh distribution, Yang [8] also analyzed the new performance property of NHPP software reliability model based on Weibull family distribution.

Therefore, in this study, after applying the inverse Rayleigh and Rayleigh distribution widely used in the reliability field to the finite-fault NHPP model, we analyze the reliability performance of the proposed model and will present the optimal software reliability model through the analysis results.

2. RELATED RESEARCH

2.1 NHPP Software Reliability Model

N(t) is the cumulative number of failures of the software detected up to time t, m(t) is a mean value function when λ(t) is expressed by an intensity function, the cumulative failure number N(t) follows a Poisson probability density function having a parameter m(t). The software reliability model of the non-homogeneous Poisson process (NHPP) is a model that measures the reliability by using the average failure rate function around the number of failures generated per unit time.

That is,

$$P\{N(t)=n\}=\frac{[m(t)]^n\cdot e^{-m(t)}}{n!} \qquad (1)$$

Note that $n = 0,1,2,\cdots\ \infty$.

The mean value function m(t) and the intensity function λ(t) of the NHPP model are as follows.

$$m(t)=\int_0^t \lambda(s)ds \qquad (2)$$

$$\frac{dm(t)}{d(t)}=\lambda(t) \qquad (3)$$

In terms of software reliability, the mean value function represents a software failure occurrence expected value, the intensity function is the failure rate function, and means the failure occurrence rate per defect. Also, the time domain NHPP models are classified into a finite failure that the failure does not occur at the time of repairing the failure, and an infinite failure that the failure occurs at the time of repairing failure.
In this study, we will analyze the software reliability performance based on finite failure cases. That is, in the finite-failure NHPP model, if the expected value of the failure that can be found up to time [0, t] is θ, then the mean value function and the intensity function are as follows.

$$m(t|\theta,b)=\theta F(t) \qquad (4)$$

$$\lambda(t|\theta,b)=\theta F(t)'=\theta f(t) \qquad (5)$$

Considering Eq. 4 and Eq. 5, the likelihood function of the finite-failure NHPP model is derived as follows.

$$L_{NHPP}(\Theta|\underline{x})=\left(\prod_{i=1}^{n}\lambda(x_i)\right)exp[-m(x_n)] \qquad (6)$$

Note that $\underline{x}=(x_1,x_2,x_3\cdots x_n)$

2.2 Finite Failure NHPP: Goel-Okumoto Basic Model

The Goel-Okumoto model is a well-known basic model in the software reliability field. Let f(t) and F(t) for the Goel-Okumoto model be a probability density function and a cumulative density function, respectively. If the failure expected value in the observation point [0, t] is θ, the finite failure strength function and the mean value function are as follows.

$$m(t|\theta,b)=\theta F(t)=\theta(1-e^{-bt}) \qquad (7)$$

$$\lambda(t|\theta,b)=\theta f(t)=\theta b e^{-bt} \qquad (8)$$

Note that $\theta > 0,\ b > 0$.

By Eq. 7 and Eq. 8, the likelihood function of the finite-failure NHPP model is derived as follows.

$$L_{NHPP}(\theta,b|\underline{x})=\left(\prod_{i=1}^{n}\theta b e^{-bx_i}\right) exp[-\theta(1-e^{-bx_n})] \qquad (9)$$

Note that $\underline{x}=(0\le x_1\le x_2\le\cdots\le x_n)$.

The log-likelihood function, if using Eq. 9, is simplified to the following log conditional expression.

$$\ln L_{NHPP}(\theta|\underline{x})=n\ln\theta+n\ln b-b\sum_{k=1}^{n}x_k-\theta(1-e^{-bx_n}) \qquad (10)$$

Therefore, the maximum likelihood estimator $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ satisfying the following the Eq. 11 and Eq. 12 can be estimated by a numerical method.

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$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n} = 0 \quad (11)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_n - \hat{\theta} x_n e^{-\hat{b}x_n} = 0 \quad (12)$$

2.3 Finite Failure NHPP: Inverse-Rayleigh Distribution Model

The inverse Rayleigh distribution is widely used in several reliability fields. The probability density function and the cumulative distribution function considering the scale parameter(b) are as follows [9].

$$f(\mathrm{t}) = \frac{2b}{t^3} exp\left(-\frac{b}{t^2}\right) \quad (13)$$

$$F(\mathrm{t}) = exp\left(-\frac{b}{t^2}\right) \quad (14)$$

Note that $b > 0,\ t \in [0, \infty]$, b is the scale parameter.

Therefore, the mean value function and the intensity function of the finite fault NHPP Inverse Rayleigh model are as follows.

$$\mathrm{m}(\mathrm{t}|\theta, \mathrm{b}) = \theta F(\mathrm{t}) = \theta exp\left(-\frac{b}{t^2}\right) \quad (15)$$

$$\lambda(\mathrm{t}|\theta, \mathrm{b}) = \theta \mathrm{f}(\mathrm{t}) = \theta\left[\frac{2b}{t^3} exp\left(-\frac{b}{t^2}\right)\right] \quad (16)$$

The log-likelihood function to maximum likelihood estimation (MLE) by using Eq. 15 and Eq. 16 is derived as follows.

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln2 + nln\theta + nlnb$$

$$+b\sum_{i=1}^{n} \ln\left(\frac{1}{x_i^3}\right) - b\sum_{i=1}^{n}\frac{1}{x_i^2} - \theta exp\left(-\frac{b}{x_n^2}\right) \quad (17)$$

Note that $\underline{x} = (0 \le x_1 \le x_2 \le \cdots \le x_n)$, Θ is parameter space.

The partial derivatives of the parameters θ and b are as follows.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - exp\left(-\frac{\hat{b}}{x_n^2}\right) = 0 \quad (18)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} + \sum_{i=1}^{n} \ln\left(\frac{1}{x_i^3}\right) - \sum_{i=1}^{n}\frac{1}{x_i^2} + \frac{\hat{\theta}}{x_n^2} exp\left(-\frac{\hat{b}}{x_n^2}\right) = 0 \quad (19)$$

Note that $x = (x_1, x_2, x_3 \cdots x_n)$.

Therefore, the maximum likelihood estimator $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ satisfying the following the Eq. 18 and Eq. 19 can be estimated by a numerical method.

2.4 Finite Failure NHPP: Rayleigh Distribution Model

The Weibull lifetime distribution is widely known as a suitable model for life test and reliability measurements. The probability density function and the cumulative distribution function considering the shape parameter(α) are as follows [10].

$$\mathrm{f}(\mathrm{t}) = \frac{t^{\alpha-1}}{\beta^2} e^{-\frac{t^\alpha}{2\beta^2}} \quad (20)$$

$$F(\mathrm{t}) = \left(1 - e^{-\frac{t^\alpha}{2\beta^2}}\right) \quad (21)$$

Note that $\beta > 0,\ t \in [0, \infty]$.

To simplify Eq. 20 and Eq. 21, if substitution by the equation $\frac{1}{2\beta^2} = b$ is as follows.

$$\mathrm{f}(\mathrm{t}) = 2\mathrm{b}t^{\alpha-1}e^{-bt^\alpha} \quad (22)$$

$$F(\mathrm{t}) = \left(1 - e^{-bt^\alpha}\right) \quad (23)$$

Note that $b > 0,\ t \in [0, \infty]$.

In the Weibull distribution equation such as Eq. 22 and Eq. 23, an exponential distribution is obtained when the shape parameter(α) is 1, and a Rayleigh distribution is obtained when the shape parameter (α) is 2.

Therefore, the mean value function and the intensity function of the finite fault NHPP Rayleigh model are as follows.

$$\mathrm{m}(\mathrm{t}|\theta, \mathrm{b}) = \theta F(\mathrm{t}) = \theta(1 - e^{-bt^2}) \quad (24)$$

$$\lambda(\mathrm{t}|\theta, \mathrm{b}) = \theta \mathrm{f}(\mathrm{t}) = 2\theta \mathrm{bt}e^{-bt^2} \quad (25)$$

Note that $\theta > 0$, $b > 0$.

The log-likelihood function to Maximum Likelihood Estimation (MLE) by using the Eq. 24 and Eq. 25 is derived as follows.

$$\ln L_{NHPP}(\Theta|\underline{x}) = n ln2 + n ln\theta + n ln b + \sum_{i=1}^{n} ln x_i - b\sum_{i=1}^{n} x_i{}^2 - \theta\left(1 - e^{-bx_n^2}\right) \quad (26)$$

Note that $\underline{x} = 0 \le x_1 \le x_2 \le \cdots \le x_n$, Θ is the parameter space.

The partial derivatives of the parameters θ and b are as follows.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n^2} = 0 \quad (27)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_i^2 - \hat{\theta}x_n{}^2 e^{-\hat{b}x_n^2} = 0 \quad (28)$$

Note that $x = (x_1, x_2, x_3 \cdots x_n)$.

Therefore, the maximum likelihood estimator $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ satisfying the following the Eq. 27 and Eq. 28 can be estimated by a numerical method.

3. RELIABILITY PERFORMANCE ANALYSIS USING SOFTWARE FAILURE TIME

We will compare and analyze the reliability property of the proposed models using the software failure time data as shown in Table 1 [11]. This software failure time is the data that was occurred 30 times in 187.35 unit time. Therefore, these simulation results can be utilized for reliability analysis in various software development fields.

Laplace trend test was utilized to confirm the software failure time data as shown in Fig 1.

Table 1: Software Failure Time Data.

Failure Number	Failure Time (hours)	Failure Time (hours) $\times 10^{-1}$	Failure Number	Failure Time (hours)	Failure Time (hours $\times 10^{-1}$
1	4.79	0.479	16	107.71	10.771
2	7.45	0.745	17	109.06	10.906
3	10.22	1.022	18	111.83	11.183
4	15.76	1.576	19	117.79	11.779
5	26.10	2.61	20	125.36	12.536
6	35.59	3.559	21	129.73	12.973
7	42.52	4.252	22	152.03	15.203
8	48.49	4.849	23	156.40	15.64
9	49.66	4.966	24	159.80	15.98
10	51.36	5.136	25	163.85	16.385
11	52.53	5.253	26	169.60	16.96
12	65.27	6.527	27	172.37	17.237
13	69.96	6.996	28	176.00	17.6
14	81.70	8.17	29	181.22	18.122
15	88.63	8.863	30	187.35	18.735

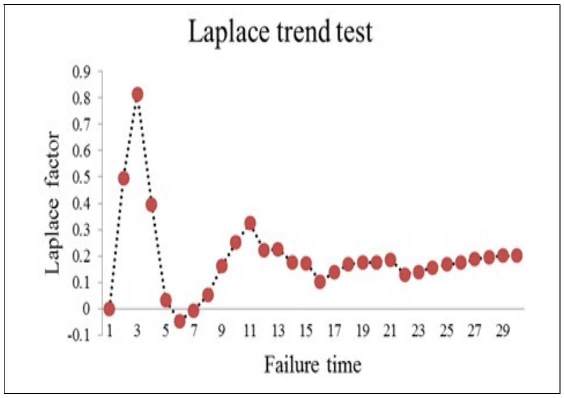

Figure 1: Estimation Results of Laplace Trend Test

Therefore, it is possible to apply this data because there is no extreme value. In general, if the Laplace factor estimates are distributed between -2 and 2, the data are reliable because the extreme values do not exist and are stable. As a result, the estimated v alue of the Laplace factor was distributed between 0 and 2, as shown inFigure. 1.

The calculation method of the nonlinear equations was solved by a numerical method, and the results are shown in Table 2.

As the basis for determining the efficient model, the mean square error(MSE) is defined as follows.

$$MSE = \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{n-k} \quad (29)$$

Note that $m(x_i)$ is the total accumulated number of errors observed within time is $(0, x_i)$, $\hat{m}(x_i)$ is the estimated error number at time x_i obtained from the fitting mean value function, n is the number of observations, and k is the number of parameters to be estimated. When selecting an efficient model, the smaller the mean square error(MSE), the more efficient the model [13].

The coefficient of determination (R^2) is a measuring value to the explanatory power of the difference between the true value and the observed value.

Type	NHPP Model	MLE (Maximum Likelihood Estimation)		Model Comparison	
				MSE	R^2
Basic	Goel-Okumoto	$\hat{\theta} = 32.9261$	$\hat{b} = 0.1297$	32.9379	0.8956
Rayleigh-Type Lifetime Distribution	Inverse-Rayleigh	$\hat{\theta} = 30.0100$	$\hat{b} = 1.6520$	165.7504	0.4747
	Rayleigh	$\hat{\theta} = 30.0412$	$\hat{b} = 1.8835$	32.1798	0.8980

Table 2: Parameter Estimate of The Proposed Models.

In this paper, the Maximum Likelihood Estimation (MLE) was used to perform parameter estimation. And numerical conversion data (Failure time[hours] $\times 10^{-1}$) to facilitate the parameter estimation was used [12].

When selecting an efficient model, the larger the value of the decision coefficient, the more efficient the model because the error is relatively small [14]. It is defined as

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{\sum_{i=1}^{n}\left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \quad (30)$$

As shown in Table 2, the Rayleigh model has the largest coefficient of determination and the smallest mean square error is more efficient than the other models. Also, Fig 2 shows the transition of mean square error (MSE) according to each failure number. That is, in this figure, the Rayleigh model shows better estimates than the other models in the total range of failure numbers. In this Fig 2, the mean squared error of the Rayleigh model showed a trend of the smallest error as time passed [15].

Figure 3 shows trends in the strength function, which is the instantaneous failure rate. Also, in the analysis of the intensity function, the intensity function estimations ($\lambda(t)$) of the proposed models are shown in Table 3. The Rayleigh and the inverse Rayleigh model showed the increasing and decreasing tendency as the failure time passes same as the actual failure phenomenon, indicating that it is an efficient model. But, only the Goel-Okumoto model showed a decreasing pattern [16].

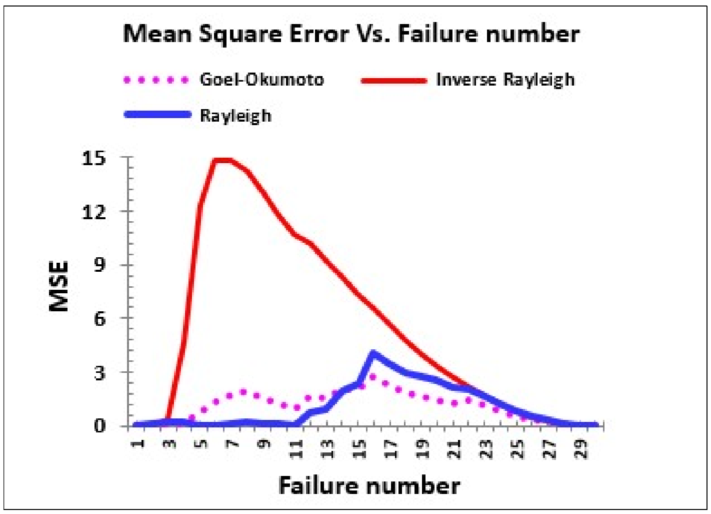

Figure 2: Transition of Mean Square Error(MSE)

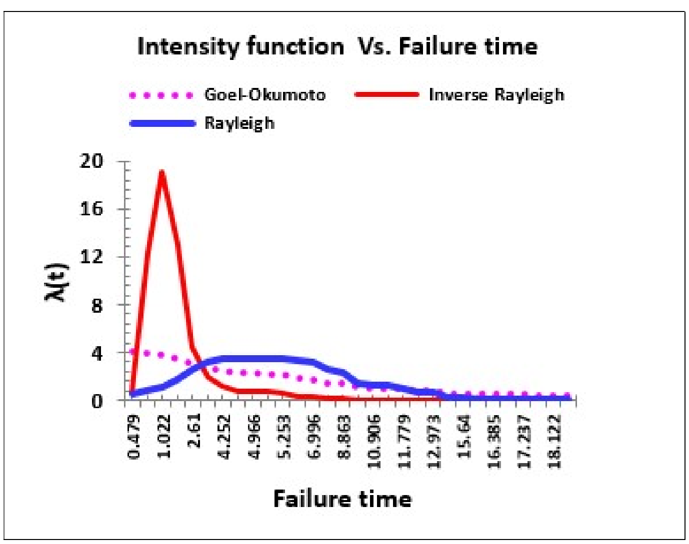

Figure 3: Transition of Intensity Function $\lambda(t)$

Table 3: Intensity Function Estimate of The Proposed Models.

Failure Number	Failure Time(hours) $\times 10^{-1}$	Basic Distribution	Rayleigh-type Distribution	
		Goel-Okumoto Model	Inverse-Rayleigh Model	Rayleigh Model
1	0.479	4.013277217	0.673491752	0.538725223
2	0.745	3.877179548	12.22319278	0.832778993
3	1.022	3.740357027	19.10094504	1.131952182
4	1.576	3.481026664	13.02509917	1.698955324
5	2.61	3.04413707	4.375864933	2.593741131
6	3.559	2.691590193	1.930545743	3.168214905
7	4.252	2.460218258	1.177180233	3.418881085
8	4.849	2.276909791	0.810655111	3.520314681
9	4.966	2.242618752	0.757169997	3.528255297
10	5.136	2.1937124	0.687436406	3.533106053
11	5.253	2.160674343	0.64429307	3.53195138
12	6.527	1.831586569	0.343023777	3.309686456
13	6.996	1.723493385	0.27996127	3.148762759
14	8.17	1.480064205	0.17737455	2.631116111
15	8.863	1.352836323	0.139453808	2.286247426
16	10.771	1.05626148	0.078226477	1.373832653
17	10.906	1.037927845	0.075383713	1.316588503
18	11.183	1.001300213	0.069967145	1.203332654
19	11.779	0.926814509	0.059952836	0.979937797
20	12.536	0.840141683	0.049803941	0.737845139
21	12.973	0.793847715	0.044969887	0.619199796
22	15.203	0.59446429	0.028016511	0.222694276
23	15.64	0.561707779	0.025743212	0.177815248
24	15.98	0.537475806	0.024141605	0.148433381
25	16.385	0.509971707	0.022402387	0.118954073
26	16.96	0.473322866	0.020208507	0.085864406
27	17.237	0.456619686	0.019253347	0.073031122
28	17.6	0.435619758	0.018090554	0.058790762
29	18.122	0.40710305	0.016576888	0.042633958
30	18.735	0.375989135	0.015007243	0.028822693

Fig 4 shows the pattern trend for the mean value function, which is the failure occurring expected value. In this figure, all models show overestimated from the difference between the true values, but the Rayleigh model has the smallest overestimated pattern. That is, the Rayleigh model is more efficient than the inverse Rayleigh model because the error width is the smallest.

We will analyze the reliability performance of the proposed models for future mission time.

Here, reliability is the probability that a software failure will occur when testing at $x_n = 187.35 \times 10^{-1}$, and no software failure will occur between confidence intervals $[x_n, x_n + \tau]$ where τ is the future mission time. Therefore, the reliability of future mission time is as follows [17].

$$\hat{R}(\tau|x_n) = e^{-\int_{x_n}^{x_n+\tau} \lambda(\tau)d\tau} \quad (31)$$
$$= \exp[-\{m(x_n+\tau) - m(x_n)\}]$$
$$= \exp[-\{m(18.735+\tau) - m(18.735)\}]$$

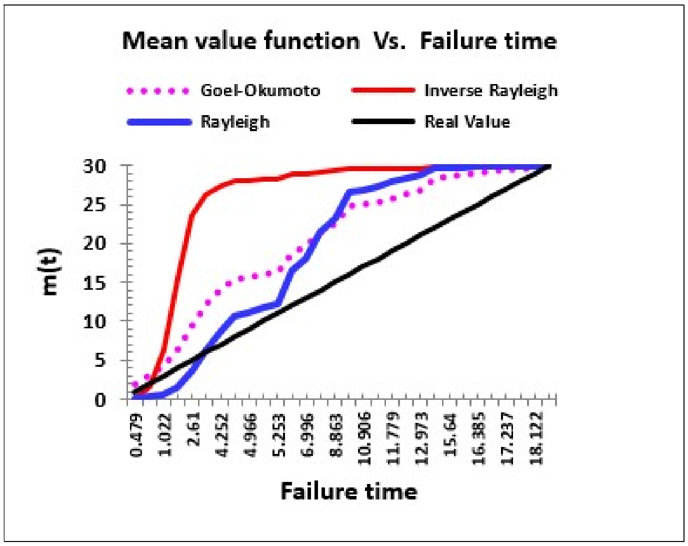

Figure 4: Pattern of Mean Value Function $m(t)$

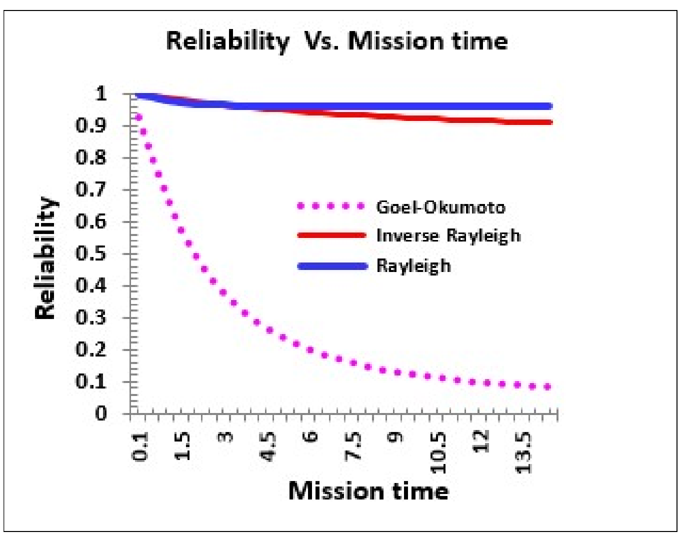

Figure 5: Transition of Reliability $\hat{R}(t)$

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Table 4: Reliability Estimate of The Proposed Models.

Failure Number	Mission Time(hours)	Basic Distribution	Rayleigh-type Distribution	
		Goel-Okumoto Model	Inverse-Rayleigh Model	Rayleigh Model
1	0.1	0.927164451	0.998512277	0.997213628
2	0.5	0.802280729	0.992811511	0.987793357
3	1	0.676450995	0.986206928	0.979155924
4	1.5	0.576499026	0.980123792	0.973086809
5	2	0.496273344	0.974507874	0.968855487
6	2.5	0.43125056	0.969311894	0.965930106
7	3	0.378066466	0.964494471	0.963925365
8	3.5	0.334191906	0.960019252	0.962564017
9	4	0.29770601	0.955854192	0.961648174
10	4.5	0.267135716	0.951970948	0.961037868
11	5	0.241340956	0.948344372	0.960635053
12	5.5	0.219431657	0.944952091	0.960371744
13	6	0.200707148	0.941774137	0.96020129
14	6.5	0.184611493	0.938792645	0.960092013
15	7	0.17070028	0.935991589	0.960022635
16	7.5	0.15861571	0.93335656	0.959979017
17	8	0.14806775	0.930874571	0.959951858
18	8.5	0.138819776	0.928533889	0.959935113
19	9	0.13067754	0.926323895	0.959924888
20	9.5	0.12348064	0.924234956	0.959918704
21	10	0.117095878	0.922258316	0.959915001
22	10.5	0.111412048	0.920386006	0.959912805
23	11	0.106335825	0.918610752	0.959911515
24	11.5	0.101788508	0.91692591	0.959910764
25	12	0.097703414	0.915325399	0.959910332
26	12.5	0.094023786	0.913803644	0.959910085
27	13	0.09070111	0.912355524	0.959909946
28	13.5	0.087693741	0.910976334	0.959909868
29	14	0.084965794	0.909661737	0.959909825
30	14.5	0.082486226	0.908407738	0.959909801

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As shown in Fig 5, the Rayleigh model shows a higher reliability trend than the other models. That is, in terms of reliability, the Rayleigh model is further reliable than the inverse Rayleigh model because the reliability is the highest.

Also, in the analysis of the reliability, the reliability estimation ($\hat{R}(t)$) of the proposed models are shown in Table 4 [18]. As shown in Table 4, the larger the reliability estimate, the better the reliability performance.

That is, the Rayleigh model showed the best performance among the proposed models because of its high reliability.

4. CONCLUSION

It is possible to efficiently improve the reliability performance by analyzing the attributes after quantitatively estimating the occurrence rate of the failure in the software development process. In this study, we compared and analyzed the software reliability performance of the Inverse Rayleigh model and the Rayleigh model together with Goel-Okumoto basic model. Also, we presented the optimal software reliability model through the analysis results.

The results of this study can be summarized as follows.

First, in terms of intensity function, the Rayleigh model and the inverse Rayleigh model were effective because the occurrence rate of the failure increased and decreased as the failure time passed same as the actual failure phenomenon. But, the Goel-Okumoto basic model decreased inversely.

Second, in the performance analysis of the mean value function, all the proposed models showed overestimation patterns in the error estimation for true values, but the Rayleigh model was efficient because it had the smallest error value than the other models.

Third, in the performance analysis of mission reliability, the Rayleigh model showed a higher reliability trend than the other models. That is, the Rayleigh model is further reliable than the inverse Rayleigh model because the reliability is the highest. In other words, a comprehensive analysis of these simulation results showed that the Rayleigh model has the best reliability performance among the proposed models.

As a result, through this study, together with a new analysis on the reliability performance of the proposed model without existing research examples, we were able to provide the research information that software developers can use the basic design guideline.

Also, future research will be needed to find the optimal model through the reliability performance analysis after applying the same type of software failure time data to various distribution models.

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