Journal of Theoretical and Applied Information Technology
15th December 2021. Vol.99. No 23

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

A COMPARATIVE STUDY ON THE ATTRIBUTES OF NHPP SOFTWARE RELIABILITY MODEL BASED ON EXPONENTIAL FAMILY AND NON-EXPONENTIAL FAMILY DISTRIBUTION

SEUNG KYU PARK

Department of Electronic Engineering, Namseoul University, 91 Daehak-ro, Seonghwan-eup, 31020

Seobuk-gu, Cheonan-city, Chungnam, South Korea

E-mail: skpark@nsu.ac.kr

ABSTRACT

In this study, after applying the exponential family (Goel-Okumoto, Erlang) and non-exponential family distributions (Pareto, Log-Logistic) which are used in the field of reliability to the finite failure NHPP software reliability model, we compared and analyzed the reliability attributes reflecting the shape parameters of the proposed distribution. For this, software failure time data was used, parametric estimation was applied to the maximum likelihood estimation method, and nonlinear equations were calculated using the bisection method. As a result, in the analysis of the intensity function, the Log-Logistic model of the non-exponential family was efficient because the failure occurring rate decreases with the failure time and the mean square error is small. In the analysis of the mean value function, all the proposed models showed a slightly underestimated value compared to the true value, but the Goel-Okumoto model of the exponential family had a smaller margin of error than other models. As a result of evaluating the software reliability after putting the mission time in the future, the Erlang model was high and stable, but the Log-Logistic and Pareto model showed a small decreasing tendency. In conclusion, the exponential family models showed more efficiency than the general non-exponential family model but were ineffective than the log type (log-logistic) model. In this study, we have newly analyzed the software reliability attributes of the exponential family and non-exponential family distributions, which have no previous research cases, and expect it to be used as a basic guideline for software developers to search for the optimal software reliability model.

Keywords: *Erlang Distribution, Exponential Family, Log-Logistic Distribution, NHPP Model, Non-exponential Family, Pareto Distribution*

1. INTRODUCTION

As software technology is widely applied in convergence industries, there is a growing need for high-quality software that can reliably process a variety of data without failure. To solve this problem, software developers are doing a lot of research and investment to explore ways to improve reliability. For this reason, software reliability models using the Non-Homogeneous Poisson Process (NHPP) have been extensively studied to improve software reliability. In particular, many NHPP software reliability models using the intensity function and the mean value function have been proposed to estimate the reliability attributes such as the number of residual failures and the failure rate [1]. In this process, Pham and Zhang [2] proposed a new model based on software reliability, while Gokhale and Trivedi [3] proposed an improved NHPP model. In particular, Kim [4] analyzed properties of the finite failure NHPP software model based on the modified Lindley type lifetime distribution, while Kim and Shin [5] analyzed the optimal software reliability attributes based on Gamma exponential family distribution models. Yang [6] proposed a new performance analysis results of the NHPP software reliability model based on Weibull lifetime distribution. Also, Yang [7] compared the property on the cost and release time of the software development model based on Lindley-type distribution. Therefore, in this study, after applying the exponential family and non-exponential family distributions which are used

Journal of Theoretical and Applied Information Technology
15th December 2021. Vol.99. No 23

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

in the field of reliability to the finite failure NHPP software reliability model, we were newly analyzed the reliability attributes of the proposed models and will present the optimal software reliability model.

2. RELATED RESEARCH

2.1 NHPP Software Reliability Model

The NHPP model contains property about mean value m(t) and intensity pattern λ(t). N(t) is the cumulative number of failures of the software detected up to time t, m(t) is a mean value function when λ(t) is expressed by an intensity function, the cumulative failure number N(t) follows a Poisson probability density function having a parameter m(t). The NHPP software reliability model is a model that measures the reliability using the mean failure rate function by the number of failures generated per unit time. That is,

Consi

$$P\{N(t) = n\} = \frac{[m(t)]^n \cdot e^{-m(t)}}{n!} \quad (1)$$

Note. $n = 0,1,2,\cdots \infty$.

The mean value function m(t) and the intensity function λ(t) of the NHPP model are as follows.

$$m(t) = \int_0^t \lambda(s)ds \quad (2)$$

$$\frac{dm(t)}{d(t)} = \lambda(t) \quad (3)$$

In terms of software reliability, the mean value function represents a software failure occurrence expected value, the intensity function is the failure rate function, and means the failure occurrence rate per defect. Also, the time domain NHPP models are classified into a finite failure that the failure does not occur at the time of repairing the failure, and an infinite failure that the failure occurs at the time of repairing failure.

In this study, we will analyze the software reliability performance based on finite failure cases. That is, in the finite-failure NHPP model, if the expected value of the failure that can be found up to time [0, t] is θ, then the mean value function and

the intensity function are as follows.

$$m(t|\theta, b) = \theta F(t) \quad (4)$$

$$\lambda(t|\theta, b) = \theta F(t)' = \theta f(t) \quad (5)$$

Considering Eq. 4 and Eq. 5, the likelihood function of the finite-failure NHPP model is derived as follows.

$$L_{NHPP}(\Theta|\underline{x}) = \left(\prod_{i=1}^{n} \lambda(x_i)\right) \exp[-m(x_n)] \quad (6)$$

Note. $\underline{x} = (x_1, x_2, x_3 \cdots x_n)$.

2.2 Exponential Family Distribution

2.2.1 Finite Failure NHPP: Goel-Okumoto Basic Model

The Goel-Okumoto model is a well-known basic model in the software reliability field. Let f(t) and F(t) for the Goel-Okumoto model be a probability density function and a cumulative density function, respectively.

Assuming that the failure expected value of the observation point [0, t] is θ, the finite failure strength function and the mean value function are as follows [8].

$$\lambda(t|\theta, b) = \theta f(t) = \theta b e^{-bt} \quad (7)$$

$$m(t|\theta, b) = \theta F(t) = \theta(1 - e^{-bt}) \quad (8)$$

Note that $\theta > 0,\ b > 0$.

If using Eq. 7 and Eq. 8, the likelihood function of the finite-failure NHPP model is derived as follows.

$$L_{NHPP}(\theta,\mathrm{b}|\underline{x}) = \left(\prod_{i=1}^{n} \theta b e^{-bx_i}\right) exp[-\theta(1-e^{-bx_n})] \quad (9)$$

Note that $\underline{x} = (0 \le x_1 \le x_2 \le \cdots \le x_n)$.

The log-likelihood function, using Eq. 9, is simplified to the following log conditional expression.

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln\theta + nlnb - b\sum_{k=1}^{n} x_k - \theta(1-e^{-bx_n}) \quad (10)$$

When Eq. 10 is partially differentiated concerning θ and b, the result is as follows.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n} = 0 \quad (11)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{n}{\hat{b}} - \sum_{i=1}^{n} x_n - \hat{\theta} x_n e^{-\hat{b}x_n} = 0 \quad (12)$$

Therefore, the maximum likelihood estimator $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ can be calculated using the bisection method.

2.2.2 Finite Failure NHPP: Erlang Distribution Model

The Erlang distribution is the lifetime distribution of the exponential family widely used in the reliability field. The probability density function and the cumulative density function considering the shape parameter (a) and the scale parameter (b) are as follows [9].

$$\mathrm{f(t)} = \frac{b^a}{\Gamma(a)} t^{a-1} e^{-bt} \quad (13)$$

$$F(\mathrm{t}) = \left(1 - e^{-bt}\sum_{i=0}^{a-1}\frac{(bt)^i}{i!}\right) \quad (14)$$

Note. $\mathrm{a,b} > 0$, $\mathrm{a} = 1,2,3\cdots$, $t \in [0,\infty]$.

Therefore, the strength function and the mean value function are as follows.

$$\lambda(\mathrm{t}|\theta,\mathrm{b}) = \theta\mathrm{f(t)} = \theta\left(\frac{b^a}{\Gamma(a)} t^{a-1} e^{-bt}\right) \quad (15)$$

$$\mathrm{m}(\mathrm{t}|\theta,\mathrm{b}) = \theta F(\mathrm{t}) = \theta\left(1 - e^{-bt}\sum_{i=0}^{a-1}\frac{(bt)^i}{i!}\right) \quad (16)$$

The Log-likelihood function to maximum likelihood estimation by using Eq. 15 and Eq.16 is derived as follows.

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln\theta - nln\Gamma(a) + nalnb + (a-1)\textstyle\sum_{i=1}^{n} lnx_i - b\sum_{i=1}^{n} x_i - \theta + \theta e^{-bx_n}\left(\displaystyle\sum_{i=0}^{a-1}\frac{(bx_n)^i}{i!}\right) \quad (17)$$

Note. $\underline{x} = (0 \le x_1 \le x_2 \le \cdots \le x_n$, Θ is parameter space.

When Eq. 17 is partially differentiated concerning θ and b, the result is as follows.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - 1 + e^{-\hat{b}x_n}\left(\sum_{i=0}^{a-1}\frac{(\hat{b}x_n)^i}{i!}\right) = 0 \quad (18)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{an}{\hat{b}} - \sum_{i=1}^{n} x_i$$

$$+\frac{\partial\left[\hat{\theta}e^{-\hat{b}x_n}\left(\sum_{i=0}^{a-1}\frac{(\hat{b}x_n)^i}{i!}\right)\right]}{\partial b} = 0 \qquad (19)$$

In this study, we will apply the case where the shape parameter(a) that determines the type of failure distribution is 2.

Therefore, Eq.18 and Eq. 19 are derived as follows.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - 1$$

$$+e^{-\hat{b}x_n}(1+\hat{b}x_n) = 0 \qquad (20)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = \frac{2n}{\hat{b}} - \sum_{i=1}^{n} x_i$$

$$-\hat{\theta}\hat{b}{x_n}^2 e^{-\hat{b}x_n} = 0 \qquad (21)$$

Therefore, the maximum likelihood estimator $\hat{\theta}_{MLE}$ $\hat{b}_{MLE}$ can be calculated using the bisection method.

2.3 Non-exponential Family Distribution

2.3.1 Finite Failure NHPP: Pareto Distribution Model

The Pareto distribution is a non-exponential family distribution widely used in the field of reliability that reflects mathematical and observable phenomena such as society, science, and physics. The probability density function and the cumulative density function considering the shape parameter (a) and the scale parameter (b) are as follows [10].

$$f(t) = \frac{a}{b}\left[1+\frac{t}{b}\right]^{-(a+1)} \qquad (22)$$

$$F(t) = 1 - \left[1+\left(\frac{t}{b}\right)\right]^{-a} \qquad (23)$$

Note. $a, b > 0,\ t \in [0, \infty]$

Assuming that the failure expected value of the observation point [0, t] is θ, the finite failure strength function and the mean value function are as follows.

$$\lambda(t|\theta, b) = \theta f(t) = \theta\left(\frac{a}{b}\left[1+\frac{t}{b}\right]^{-(a+1)}\right) \qquad (24)$$

$$m(t|\theta, b) = \theta F(t) = \theta\left(1-\left[1+\left(\frac{t}{b}\right)\right]^{-a}\right) \qquad (25)$$

Therefore, the log-likelihood function to maximum likelihood estimation by using Eq. 24 and Eq. 25 is derived as follows.

$$\ln L_{NHPP}(\Theta|\underline{x}) = nln\theta + nlna - nlnb$$

$$+\sum_{i=1}^{n} ln\left[1+\frac{x_i}{b}\right]^{-(a+1)} - \theta\left(1-\left[1+\left(\frac{x_n}{b}\right)\right]^{-a}\right)$$

$$(26)$$

In this study, we apply the case where the shape parameter(a) that determines the type of failure distribution is 2.

When Eq. 26 is partially differentiated concerning θ and b, the result is as follows.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial \theta} = \frac{n}{\hat{\theta}} - \left(1-\left[1+\left(\frac{x_n}{\hat{b}}\right)\right]^{-2}\right)$$

$$= 0 \qquad (27)$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial b} = -\frac{n}{\hat{b}} + \frac{2x_n\hat{\theta}}{\hat{b}^2}\left(1+\frac{x_n}{\hat{b}}\right)^{-3}$$

$$+\frac{3}{4}\sum_{i=1}^{n} ln\left(1+\frac{x_i}{2}\right) = 0 \qquad (28)$$

Note that $x = (x_1, x_2, x_3 \cdots x_n)$.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Therefore, the maximum likelihood estimator $\hat{\theta}_{MLE}$ and $\hat{b}_{MLE}$ can be calculated using the bisection method.

2.3.2 Finite Failure NHPP: Log-Logistic Distribution Model

The Log-Logistic distribution has a property that increases and decreases in the form of failure rate and thus is a non-exponential family distribution widely used in the reliability field.

The probability density function and the cumulative distribution function considering the shape parameter(k) are as follows [11].

$$\mathrm{f}(\mathrm{t}|\tau,\mathrm{k}) = \frac{\tau k(\tau t)^{k-1}}{[1+(\tau t)^k]^2} \qquad (29)$$

$$\mathrm{F}(\mathrm{t}|\tau,\mathrm{k}) = \frac{(\tau t)^k}{[1+(\tau t)^k]} \qquad (30)$$

Note that $\tau > 0,\ \mathrm{k} > 0$

Therefore, the intensity function and the mean value function of the finite fault NHPP Log-Logistic model are as follows.

$$\lambda(\mathrm{t}|\theta,\tau,\mathrm{k}) = \theta\mathrm{f}(\mathrm{t}) = \theta\,\frac{\tau k(\tau t)^{k-1}}{[1+(\tau t)^k]^2} \qquad (31)$$

$$\mathrm{m}(\mathrm{t}|\theta,\tau,\mathrm{k}) = \theta F(\mathrm{t}) = \theta\,\frac{(\tau t)^k}{[1+(\tau t)^k]} \qquad (32)$$

Therefore, the log-likelihood function to maximum likelihood estimation by using Eq. 31 and Eq. 32 are derived as follows.

$$\ln L_{NHPP}(\Theta|\underline{x}) = n\ln 2 + n\ln\theta + 2n\ln\tau + \sum_{i=1}^{n} x_i$$

$$-\sum^{n} \cdots \qquad (\tau x_n)^2$$

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\tau} = \frac{2n}{\hat{\tau}} - 2\hat{\tau}\sum_{i=1}^{n} {x_i}^2\,\frac{1}{ln[1+(\hat{\tau}x_i)^2]}$$

In this study, we apply the case where the shape parameter(k) that determines the shape of failure lifetime distribution is 2.
And, if Eq. 33 is partially differentiated concerning θ and τ, the maximum likelihood estimator $\hat{\theta}_{MLE}$ and $\hat{\tau}_{MLE}$ can be calculated using the bisection method.

$$\frac{\partial \ln L_{NHPP}(\Theta|\underline{x})}{\partial\theta} = \frac{n}{\hat{\theta}} - \frac{(\hat{\tau}x_n)^2}{[1+(\hat{\tau}x_n)^2]} = 0 \qquad (34)$$

$$-\hat{\theta}\left(\frac{2\hat{\tau}{x_n}^2(1+\hat{\tau}^2x_n - \hat{\tau}^2{x_n}^2)}{[1+(\hat{\tau}x_n)^2]^2}\right) = 0 \qquad (35)$$

Note that $x = (x_1, x_2, x_3 \cdots x_n)$.

4. RELIABILITY PROPERTY ANALYSIS USING SOFTWARE FAILURE TIME

We will compare and analyze the reliability property of the proposed models using the software failure time data as shown in Table 1 [12].

This software failure time is the data that was occurred 30 times in 738.68 unit time.

Therefore, these simulation results can be utilized for reliability analysis in various software convergence industries.

Table 1: Software Failure Time Data.

Failure Number	Failure Time (hours)	Failure Time (hours) $\times 10^{-2}$	Failure Number	Failure Time (hours)	Failure Time (hours) $\times 10^{-2}$
1	30.02	0.3	16	151.78	1.51
2	31.46	0.31	17	177.50	1.77
3	53.93	0.53	18	180.29	1.8
4	55.29	0.55	19	182.21	1.82
5	58.72	0.58	20	186.34	1.86
6	71.92	0.71	21	256.81	2.56
7	77.07	0.77	22	273.88	2.73
8	80.90	0.8	23	277.87	2.77
9	101.90	1.01	24	453.93	4.53
10	114.87	1.14	25	535	5.35
11	115.34	1.15	26	537.27	5.37
12	121.57	1.21	27	552.90	5.52
13	124.97	1.24	28	673.68	6.73
14	134.07	1.34	29	704.49	7.04
15	136.25	1.36	30	738.68	7.38

Laplace trend test was applied to confirm the software failure time data as shown in Figure 1.

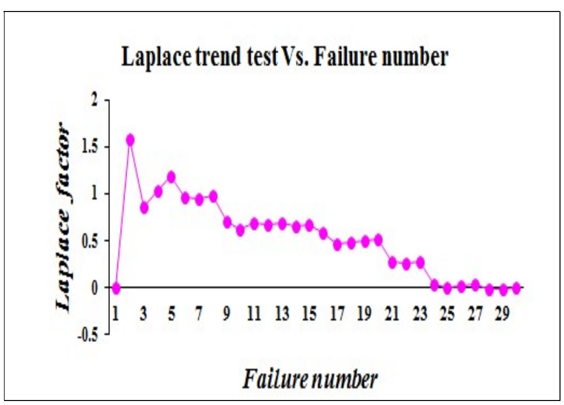

Figure 1: Estimation Results of Laplace Trend Test.

In general, if the Laplace factor estimates are distributed between -2 and 2, the data are reliable because the extreme values do not exist and are stable [13].

As a result of this test, the estimated value of the Laplace factor was distributed between 0 and 2, as shown in Figure 1. Thus, it is possible to use this data because there is no extreme value.

Also, the maximum likelihood estimation(MLE) method was used to calculate parameter estimation [14].

The calculation method of the nonlinear equations was solved by the bisection method, and the results are shown in Table 2.

As the basis for determining the efficient model, the mean square error(MSE) is defined as follows [15].

$$MSE = \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{n-k} \qquad (36)$$

Note that $m(x_i)$ is the total accumulated number of errors observed within time is $(0, x_i)$, $\hat{m}(x_i)$ is the cumulative number of errors at time x_i obtained from the fitting mean value function, n is the number of observations, and k is the number of parameters to be estimated.

When selecting an efficient model, the smaller the mean square error(MSE), the more efficient the model.

The coefficient of determination (R^2) is a measuring value to the explanatory power of the difference between the target value and the observed value.

When selecting an efficient model, the larger the value of the decision coefficient, the more efficient the model because the error is relatively small.

Thus, it can be derived as follows [16].

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(m(x_i) - \hat{m}(x_i)\right)^2}{\sum_{i=1}^{n}\left(m(x_i) - \sum_{j=1}^{n} m(x_j)/n\right)^2} \qquad (37)$$

As shown in Table 2, the Log-Logistic model is more efficient than the Erlang model.

But, the Goel-Okumoto basic model having the largest coefficient of determination and the smallest mean square error showed the best performance among the proposed models.

Table 2: Parameter Estimate of The Proposed Models.

Type	Model (Shape Parameter)	MLE (Maximum Likelihood Estimation)		Model Comparison	
				MSE	R^2
Exponential Family Distribution	Goel-Okumoto Basic	$\hat{\theta} = 33.409$	$\hat{b} = 0.309$	5.842	0.981
	Erlang (a=2)	$\hat{\theta} = 30.598$	$\hat{b} = 0.792$	14.191	0.955
Non-exponential Family Distribution	Pareto (a=2)	$\hat{\theta} = 31.209$	$\hat{b} = 1.810$	45.580	0.855
	Log-Logistic (k=2)	$\hat{\theta} = 32.241$	$\hat{\tau} = 0.495$	8.973	0.972

Journal of Theoretical and Applied Information Technology
15th December 2021. Vol.99. No 23

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Figure 2 shows the transition of mean square error (MSE) according to each failure number. Also, in this figure, the Log-Logistic model of the non-exponential family showed an efficient performance in terms of fitness because the mean square error tends to be smaller than the Erlang model of the exponential family.

Figure 3 shows trends in the strength function, which is the failure occurring rate per defect.

Also, in the analysis of the intensity function, the intensity function estimation ($\lambda(t)$) of the proposed models are shown in Table 3.

The Log-Logistic model shows the greatest increasing and decreasing tendency as the failure time passes, and the Erlang model also shows a similar pattern, and that is mean more efficient than the other models in terms of fitness [17].

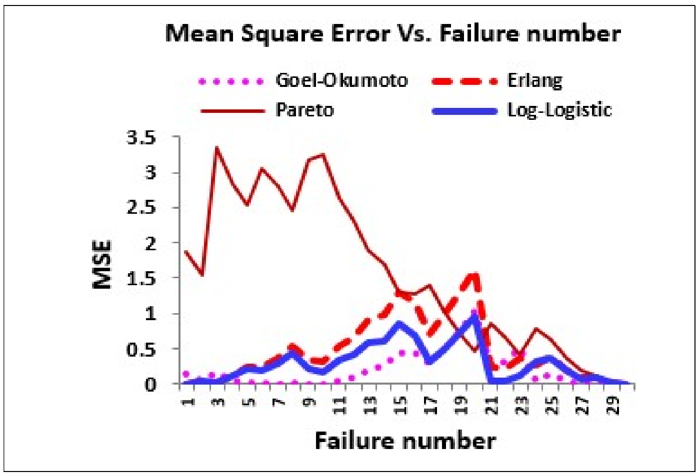

Figure 2: Transition of Mean Square Error.

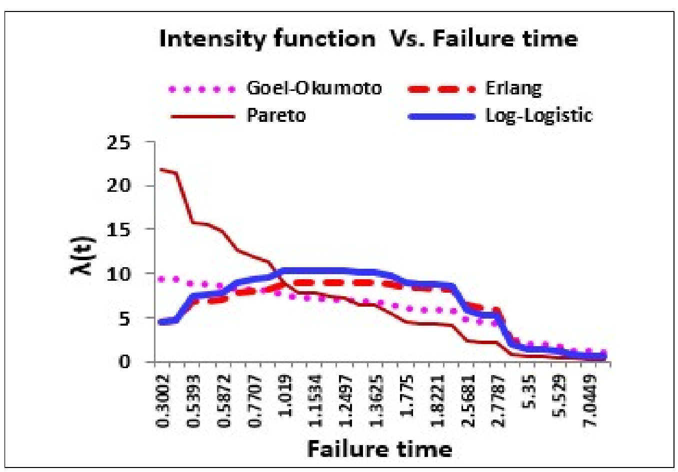

Figure 3: Transition of Intensity Function $\lambda(t)$.

Table 3: Intensity Function Estimate of The Proposed Models.

Failure Number	Failure Time(hours) $\times 10^{-2}$	Exponential Family Distribution		Non-exponential Family Distribution	
		Goel-Okumoto Model	Erlang Model	Pareto Model	Log-Logistic Model
1	0.3002	9.408894912	4.54451278	21.76048824	4.545635323
2	0.3146	9.367122098	4.70848365	21.32106023	4.743500224
3	0.5393	8.738806367	6.75530813	15.77017897	7.432673453
4	0.5529	8.702159464	6.85144663	15.49946363	7.568536258
5	0.5872	8.610414931	7.08142870	14.84367603	7.896493953
6	0.7192	8.266279338	7.81214276	12.63893346	8.959061908
7	0.7707	8.135775045	8.03687798	11.89732806	9.287722228
8	0.809	8.040058096	8.1841484	11.3830065	9.501500326
9	1.019	7.53490564	8.72868979	9.031752371	10.23880963
10	1.1487	7.238898175	8.87888558	7.89536308	10.37055053
11	1.1534	7.228392754	8.88208161	7.857858638	10.37134661
12	1.2157	7.090571963	8.91101209	7.38242861	10.3582857
13	1.2497	7.016468362	8.91679436	7.139063898	10.33371521
14	1.3407	6.821920354	8.90074204	6.538213671	10.21421964
15	1.3625	6.776120964	8.89059584	6.404363065	10.175112
16	1.5178	6.458629277	8.75746151	5.548978181	9.801194488
17	1.775	5.965198361	8.35358465	4.438362875	8.933033285
18	1.8029	5.91399283	8.29940995	4.336336986	8.827886249
19	1.8221	5.879010169	8.26117959	4.267935406	8.754777331
20	1.8634	5.804460705	8.17648718	4.125602398	8.595783626
21	2.5681	4.668670815	6.44795373	2.436948896	5.927516125
22	2.7388	4.428797529	6.00677399	2.172777021	5.370940504
23	2.7787	4.374529666	5.90466298	2.116592187	5.247713237
24	4.5393	2.539008395	2.39117452	0.798995599	1.958617726
25	5.35	1.976380831	1.48271057	0.557169725	1.31521086
26	5.3727	1.962566409	1.46246440	0.551903959	1.301345101
27	5.529	1.870033433	1.32973342	0.517388536	1.210715348
28	6.7368	1.287556336	0.62234380	0.327583379	0.723830421
29	7.0449	1.170631339	0.50985928	0.294565961	0.641976846
30	7.3868	1.053266312	0.40775735	0.262920476	0.564620756

Figure 4 shows the pattern trend for the mean value function, which is the failure occurring expected value.

In this figure, all models show underestimated the difference between the true values, but the Pareto model shows the biggest error estimation. Also, the Log-Logistic model is more efficient than the Erlang model because the error width is small.

Let analyze the reliability performance of the proposed models for future mission time.

Here, reliability is the probability that a software failure will occur when testing at $x_n = 738.68 \times 10^{-2}$, and no software failure will occur between confidence intervals $[x_n, x_n + \tau]$ where τ is the future mission time.
Therefore, the reliability of future mission time is as follows [14] [18].

$$\begin{aligned}\hat{R}(\tau|x_n) &= e^{-\int_{x_n}^{x_n+\tau}\lambda(\tau)d\tau} \\ &= \exp[-\{m(x_n+\tau) - m(x_n)\}] \\ &= \exp[-\{m(7.3868+\tau) - m(7.3868)\}]\end{aligned} \tag{38}$$

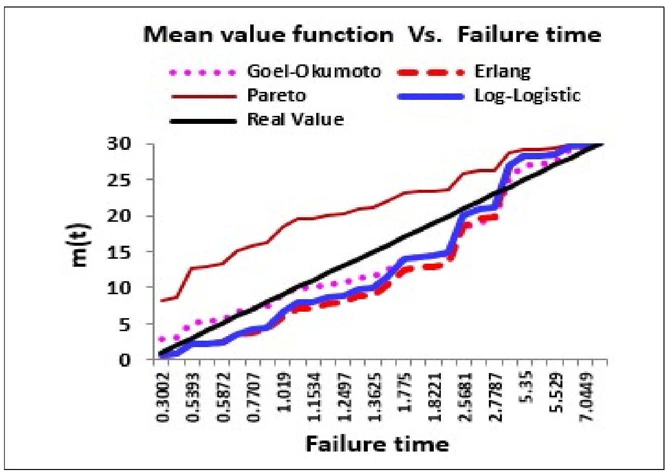

Figure 4: Pattern of Mean Value Function $m(t)$.

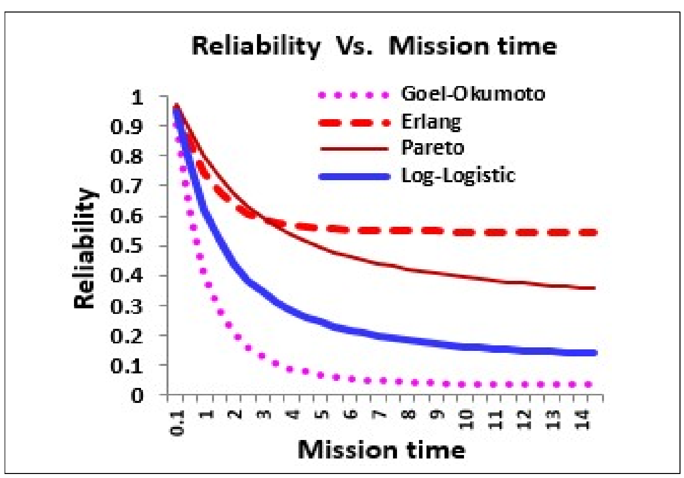

Figure 5: Transition of Reliability $\hat{R}(t)$.

Table 4: Reliability Estimate of The Proposed Models.

Failure Number	Mission Time(hours)	Exponential Family Distribution		Non-exponential Family Distribution	
		Goel-Okumoto Model	Erlang Model	Pareto Model	Log-Logistic Model
1	0.1	0.901481311	0.9613042	0.9744623	0.946070549
2	0.5	0.613872884	0.8405781	0.8856199	0.772413614
3	1	0.404105582	0.7421531	0.7981101	0.621850318
4	1.5	0.282426941	0.6789798	0.7295667	0.517513995
5	2	0.207773873	0.6373070	0.6747983	0.442393901
6	2.5	0.159720508	0.6092878	0.6302828	0.386557452
7	3	0.127492089	0.5902006	0.5935627	0.343925363
8	3.5	0.105103598	0.5770828	0.5628809	0.310623653
9	4	0.089075413	0.5680152	0.5369530	0.284096137
10	4.5	0.077301032	0.5617243	0.5148228	0.262604213
11	5	0.06845848	0.5573505	0.4957654	0.244933493
12	5.5	0.061691009	0.5543060	0.4792234	0.230215604
13	6	0.056427126	0.5521860	0.4647619	0.217816523
14	6.5	0.052275598	0.5507098	0.4520373	0.207264651
15	7	0.048962242	0.5496823	0.4407752	0.198203302
16	7.5	0.046290794	0.5489675	0.4307543	0.190358619
17	8	0.04411804	0.5484706	0.4217942	0.183517468
18	8.5	0.042337642	0.5481254	0.4137464	0.177511908
19	9	0.040869388	0.5478858	0.4064883	0.172208112
20	9.5	0.039651898	0.5477196	0.3999174	0.167498337
21	10	0.038637593	0.5476045	0.3939474	0.163295016
22	10.5	0.037789158	0.5475247	0.3885056	0.159526356
23	11	0.037077019	0.5474695	0.3835301	0.156133025
24	11.5	0.036477513	0.5474314	0.3789679	0.153065626
25	12	0.035971547	0.5474050	0.3747734	0.150282744
26	12.5	0.0355436	0.5473868	0.3709073	0.147749437
27	13	0.035180969	0.5473742	0.3673355	0.145436049
28	13.5	0.034873196	0.5473656	0.3640283	0.143317267
29	14	0.034611625	0.5473596	0.3609596	0.141371372
30	14.5	0.034389061	0.5473555	0.3581065	0.13957964

As shown in Figure 5, the Erlang model shows a higher reliability trend than the other models in which the reliability decreases with the mission time, but the Goel-Okumoto basic model showed the highest reliability trend.

Also, in the analysis of the reliability, the reliability estimate ($\hat{R}(t)$) of the proposed models are shown in Table 4. As shown in Table 4, the larger the reliability estimate, the better the reliability performance.

That is, the Erlang model showed the best performance among the proposed models because of its high reliability.

3. CONCLUSION

This study is based on the finite failure NHPP model that no new defects occur during the removal or correction process of software defects. Also, the software reliability attributes were solved by analyzing the reliability performance factor(m(t), λ(t), R(t)) using exponential family distribution widely applied in the field of reliability and non-exponential family distribution known as efficient distribution.

The results of this study can be summarized as follows.

First, in the analysis of the strength function, the Log-Logistic model of exponential family distribution was the most efficient in terms of reliability fitness. This is because the failure occurring rate of the Log-Logistic distribution shows the greatest increasing and decreasing tendency as the failure time passes, and the mean square error (MSE) is also small.

Second, in the analysis of the mean value function, all the proposed models showed underestimation patterns in the error estimation for true values, but the Pareto model showed the biggest error estimation. Also, the Log-Logistic model is more efficient than the Erlang model of non-exponential family distribution because of the small error.

Third, in the analysis of mission reliability, the Erlang model showed a higher reliability trend than the other models in which the reliability decreases with the mission time. That is, the Erlang model is more effective than other models because of its high reliability. In a comprehensive analysis, the exponential family distribution was found to be more efficient than the general non-exponential family distribution but was more ineffective than the Log-type (Log-Logistic) distribution.

As a result, we have newly analyzed the software reliability attributes of exponential family and non-exponential distributions, which have no previous research cases, and expect it to be used as a basic guideline for software developers to search for the optimal software reliability model.

Also, further research will be needed to find the optimal model through the reliability comparative analysis after applying the same type of software failure time data to various reliability models.

ACKNOWLEDGEMENTS

Funding for this paper was provided by Namseoul University.

REFERENCES:

[1] L. Kuo and T. Y. Yang., “Bayesian Computation of Software Reliability”, *Journal of the American Statistical Association*, Vol. 91, 1996, pp. 763-773.

[2] Pham H, Zhang X., “NHPP Software Reliability and Cost Models with testing coverage”, *Eur. J. Oper. Res,* Vol. 145, 2003, pp. 445-454.

[3] Gokhale, S. S., and Trivedi, K. S. A, “time/ structure-based software reliability model”, *Annals of Software Engineering,* Vol. 8, 1999, pp. 85-121.

[4] H. C. KIM, “A Comparative Study on the Finite Failure Software Reliability Model with Modified Lindley Type Lifetime Distribution”, *International Journal of Engineering Research and Technology,* Vol. 12, No. 6, 2019, pp. 760-764.

[5] H. C. KIM, H. C. Shin, “A Comparative Study of Software Optimal Release Time Based on Gamma Exponential and Non-Exponential Family Distribution Model”, *The Journal of Korea Society of Computer and Information*, Vol. 15, No. 5, 2010, pp. 125-132.

[6] Tae-Jin. Yang, “A Study on the Reliability Performance Analysis of Finite Failure NHPP Software Reliability Model Based on Weibull Life Distribution”, *International Journal of Engineering Research and Technology* Vol. 12, No. 11, 2019, pp. 1890-1896.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

[7] Tae-Jin. Yang, "Comparative study on the Attributes Analysis of Software Development Cost Model Based on Exponential-Type Lifetime Distribution", *International Journal of Emerging Technology and Advanced Engineering,* Vol. 11, No. 10, October 2021, pp. 166-176.

[8] Goel A L, Okumoto K, "Time-dependent fault detection rate model for software and other performance measures", *IEEE Transactions on Software Engineering,* Vol. 28, 1978, pp. 206-211.

[9] H. C. KIM, Y. S. Choi, J. G. Park, "An Approach for the NHPP Software Reliability Model Using Erlang Distribution", *The Journal of the Korea Institute of Maritime Information & Communication Sciences,* Vol. 10, No. 1, 2006, pp. 7-14.

[10] H. C. KIM, "A Study on the Utility of Finite-fault NHPP Software Reliability Model Applying Pareto Distribution Feature", *Asia-pacific Journal of Multimedia Service,* Vol. 7, No. 10, 2017, pp. 915 -923.

[11] Tae-Jin. Yang, "A Comparative Study on the Performance Attributes of Finite Failure NHPP Software Reliability Model with Logistic Distribution Property", *International Journal of Engineering Research and Technology,* Vol. 13, No. 3, 2020, pp. 438-443.

[12] R. Satya Prasad, K. R. H. Rao, and R.R.L Kantha, "Software Reliability Measuring using Modified Maximum Likelihood Estimation and SPC", *International Journal of Computer Applications (0975-8887)*, Vol. 21, No. 7, 2011, pp. 1-5.

[13] Kuei-Chen, C., Yu-Shiang, H. and Tzai-Zang, L., "A Study of Software reliability growth from the perspective of learning effects", *Reliability Engineering and System Safety 93*, 2008, pp. 1410-1421.

[14] Tae-Jin Yang, "Comparative Study on the Performance Attributes of NHPP Software Reliability Model based on Weibull Family Distribution", *International Journal of Performability Engineering,* Vol. 17, No.4, 2021, pp. 343-353.

[15] H. Pham, "Distribution Function and Its Application in Software Reliability", *International Journal of Performability Engineering,* Vol.15, No. 5, 2019, pp. 1306-1313.

[16] A. Alhaddad, B. Al-ibrahi, A. A bualkishik., "A Systematic Mapping Study on Software Effort Estimation", *Journal of Theoretical and Applied Information Technology,* Vol. 98, No. 17, 2020, pp. 3619-3643.

[17] E.E. Ismail, N.K. Utelieva, R.Z. Mulaev , T.S. Auelbekov, "Specific Features, Requirements, Attributes and Metrics of Safety of Software for Space Purposes", *Journal of Theoretical and Applied Information Technology,* Vol. 99, No. 2, 2021, pp. 411-419.

[18] A. Boranbayev, S. Boranbayev, M. A mirtaev, M. Bai,ukhamedov, A. N urbekov, "Development of a Real Time Human Face Recognition Software System", *Journal of Theoretical and Applied Information Technology,* Vol. 99, No. 7, 2021, pp. 1693-1705.