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IMPACT OF THE VARIOUS MEASURES OF SIMILARITY ON THE STATISTIC HIERARCHICAL NEURAL RESPONSE METHOD

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ABSTRACT

The neural response is a critical semantic component in the hierarchical architecture proposed by Smale and known as similarity measures. One standard measurement used in Smale's framework is the inner product. However, there are many other measurements such as Dice similarity coefficient, Pearson correlation, cosine distance, and Euclidean. This work aims to compare and evaluate which of these similarity measures led to low computational operation with high accuracy results using Smale framework. The paper introduces a new approach for selecting an informative and practical template based on Coefficient of Variation (CV) statistical concept. In this technique, the developed method reduces the redundancy of the training images and extract compact template sets with better discrimination ability. The study has shown that comparing the effectiveness of the Square Pearson correlation (SPCC), Dice similarity coefficient (DSC), inner product (IP), Euclidean distance (ED), and chord distance (CD) enables a change in similarity method along with MNIST database.

Keywords: *Chord Distance, Coefficient of Variation, Cosine Distance, Dice Similarity Coefficient, Euclidean Distance, Square Pearson Correlation Coefficient, Statistic Hierarchical Neural Response.*

1 INTRODUCTION

Measuring similarities is a critical step in many applications of artificial intelligence. It can be used with a range of image processes and tasks, such as data mining, object recognition, feature extraction and classification. One fundamental question in neuroscience concerns measures of similarity that the mind uses to do similarity computations and how it differs across brain tasks and regions. Whereas researches in neuroscience usually consider Pearson correlation and inner product that capture the similarity between different brain states [1-5], psychologists have used a dizzying array of competing accounts of similarity [6, 7]. In general, the task of finding the similarity between images is still a problem of computing a distance measure. Thus, various distance measures have been suggested for computing the similarity between images such as, the Euclidean distance [8], taken from Euclidean geometry field. Another similarity metrics is Manhattan distance [9], known as the taxicab metrics. When images are represented as a vector, authors utilized the Canberra distance metrics [10], Cosine similarity [11] , Euclidean, Mahalanobis, and chord distances [12, 13]. More theoretical background concerning similarity measures can be found in [1].

A new idea of a natural image representation on the space of images in the neuroscience of visual cortex was introduced by Smale et al. [4]. The derived kernel and neural response are the main ingredients of interest in the framework that Smale et al. suggested. Indeed, the derived kernel is defined in terms of similarity measures. In [4], the author used inner product as a similarity measure (usually called a derived kernel). However, in the same framework, authors in [14-16] used sparse representation instead of similarity measure. Also, on the Smale's framework, Ramadhan et al. [3] suggested Squared Pearson correlation Coefficient (SPCC) method as a similarity measure. On the other hand, selection template plays an important part in neural response because the dimensionality of the neural response depends only on the number of templates at each layer.

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The present study aims to achieve two goals. The first goal is to compare the effects of the five similarity measurements: Cosine distance(inner product)[1, 4, 11, 17-19], SPCC[3, 20-26], dice distance[19, 21], Euclidean distance[1, 11, 12, 17-19, 27-31] and chord distance[32-36] on recognition accuracy. The second goal is to generate a technique to enhance a performance of feature extraction via extracting an effective and informative template. To date, several extraction template techniques have been suggested and investigated but they have been more of sheer theoretical speculations. Random extraction [4] was based on k-mean [16, 37] and arithmetic mean[5].

The following section presents a brief review on similarity measures used in this work. Section 3 contains a review of Smale's framework. Section 4 is dedicated to the discussion of the new method for selecting an effective template and essential operations analysis. Experimental analysis and results are dealt with in Section 5. Section 6 presents the findings of the study.

2 FAMILIES OF SIMILARITIES MEASURES USED IN THIS WORK

At date, there are three tools to decide the relationship between two vectors (or images): correlation coefficient, distance measure, and similarity measure. Distance measures commonly adopted for computing the similarity between two images will be presented in this section.

2.1 Cosine distance (Inner product) (CD)

In the inner product space, the inner product of two vectors (i.e. sub-images) are divided by the product of their lengths. This is defined as a cosine similarity measure which is used to compute the cosine of the angle between vectors. Motivated by this idea, we can determine whether the two sub-images are corresponding. Note that, the mathematical expression for cosine distance is:

$$D_{Cos}(s_1, s_2) = \frac{s_1 \cdot s_2}{\|s_1\| * \|s_2\|}$$

Where $\cdot$ denotes the inner product (i.e. scalar product), the Hadamard product is represented by *, and $\|.\|$ indicates the length of vector (i.e. the Euclidean norm). To the best of our knowledge, cosine distance range is between 0 and 1. Consequently, the higher of the $D_{Cos}(s_1, s_2)$ value represents the higher similarity between sub-images.

2.2 Square Pearson Correlation Coefficient (SPCC)

To detect and measure the linear relationship between two vectors, Pearson correlation coefficient is often used. It is represented by the equation:

$$PCC = \frac{\sum_{i=1}^{n}(x_i - \underline{x})(y_i - \underline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \underline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \underline{y})^2}}$$

where n indicates the vector size, the values of the two vectors of the ith data point is denoted by x_i and y_i, the average of x and y is represented by $\underline{x}$ and $\underline{y}$ respectively.

Subsequently, the squared Pearson correlation coefficient is the special case of the PCC which can be divided as:

$$SPCC = \left[\frac{\sum_{i=1}^{n}(x_i - \underline{x})(y_i - \underline{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \underline{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \underline{y})^2}}\right]^2$$

Note that, SPCC can take the value from 0 to 1. The value of 1 indicates a perfect similarity between x and y (i.e. the higher value of the similarity level depends on the higher value of the SPCC).

2.3 Dice similarity coefficient (DSC)

The DSC is a popular technique to compare the parts of the same image to each other. It can be defined as:

$$D(s_1, s_2) = \frac{2|s_1 \cap s_2|}{|s_1| + |s_2|}$$

where $|s_1| and\, |s_2|$ denote the cardinality of the vectors s_1 and s_2 respectively. $D(s_1, s_2) \in [0,1]$, such that if $D(s_1, s_2) = 0$, then the vectors s_1 and $s_2$are dissimilar. On the opposite side, the vectors s_1 and $s_2$are identical if $D(s_1, s_2) = 1$.

2.4 Euclidean Distance (ED)

The distance between two pixels in Euclidean space is called the Euclidean distance. Presently, among all the similarity measures, Euclidean distance is the most frequently used measure due to its simple computation. It can be seen as the straight-line distance between two points. Consequently, it is computed by utilizing the Euclidean norm in Euclidean space.

The mathematical expression for (ED) between two sub- images is:

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$$D_{Euc}(s_1, s_2) = \sqrt{\sum_{i=1}^{n} (s_{1i} - s_{2i})^2}$$

2.5 Chord distance (ChD)

The length of a chord between two pixels along the circumference of a unit circle is used to define the chord distance. It has the mathematical formula:

$$D_{Cho}(s_1, s_2) = \sqrt{\sum_{i=1}^{n} \left(\frac{s_{1i}}{\|s_1\|} - \frac{s_{2i}}{\|s_2\|}\right)^2}$$

Note that, this distance measure is simply the Euclidean measure computed but with a row normalized. In effect, this asymmetric distance measure has several advantages compared to Euclidean distance in that it is insensitive to double zeros (ignores double zeros) and has the maximum value (equal to $\sqrt{2}$), while Euclidean distance has no maximum value.

The brief mathematical properties of similarity measures used in this model can be found in the following table.

SIMILARITY MEASURES	MATHEMATICAL FORMULA	DOMAIN OF S	RANGE
CD	$\frac{s_1 \cdot s_2}{\|s_1\| * \|s_2\|}$	R	$0 < D < 1$
SQCC	$\left[\frac{\sum_{i=1}^{n} (x_i - \underline{x})(y_i - }{\sqrt{\sum_{i=1}^{n} (x_i - \underline{x})^2}\sqrt{\sum_{i=1}^{n}}}\right.$	R	0 < D< 1
DSC	$\frac{2\lvert s_1 \cap s_2\rvert}{\lvert s_1\rvert + \lvert s_2\rvert}$	R	0 < D< 1
ED	$\sqrt{\sum_{i=1}^{n} (s_{1i} - s_{2i})^2}$	R	D>0
ChD	$\sqrt{\sum_{i=1}^{n} \left(\frac{s_{1i}}{\|s_1\|} - \frac{s_{2i}}{\|s_2\|}\right)^2}$	R	$0 < D < 2$ FOR FULL HYPERSPHERE. $0 < D < \sqrt{2}$ FOR QUARTER HYPERSPHERE

3 A REVIEW OF SMALE'S FRAMEWORK

In this subsection, a brief summary of the Smale's model is provided. For more details about this model, the reader is encouraged to read [4, 14-16, 38-40].

To begin with, the derived kernel consists of four components.

The first one is a finite-layers hierarchical architecture which is defined by nested layers (i.e. this work considers three nested patches:$u, v, and\ sq\ \epsilon R^2, such\ that\ u \subset v \subset sq$).

The second element is transformation function which is mapped between two adjacent patches such as:

$$v\ H_v \rightarrow\ sq$$
$$u\ H_u \rightarrow\ v$$

The third component is function space which is defined on three layers as

$$F_{sq} = \{f: sq \longrightarrow [0,1]\}$$
$$F_v = \{f: v \longrightarrow [0,1]\}$$
$$F_u = \{f: u \longrightarrow [0,1]$$

The last but the very important key ingredient is template which is utilized as a connection between the mathematical architecture and a real-world distribution

$$T_{sq} \supset T_v \supset T_u$$

3.1 Template extraction

As mentioned above, template is a very important part of Smale's framework. It is randomly sampled. Consequently, only some templates fully reflect the structural characteristics of the image to be recognized. For the previous reason, there are many attempts to develop a method for template selection problem such as [37] using the k-mean technique to obtain an informative template. Hong Li et al. in [16] suggested a new criterion to achieve compact template sets. A new template selection approach based on the entropy notion was introduced in [41]. Recently, Ramadhan et al. have presented a statistical criterion based on an arithmetic mean for the selection of informative template[5].

This work proposed a new critical measure to extract an effective template based on the coefficient of variation (CV) concept.

3.2 Coefficient of variation

Coefficient of Variation (CV) is defined as the proportion of standard deviation of the mean. Hence, it is a factual measure of distribution of pixel information around the mean. The Coefficient of Variation (CV) is expressed in the following formula:

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$$Coefficient\ of\ Variation = \left(\frac{Standard\ Deviation}{Mean}\right) \times 100.$$

$$In\ symbols: CV = \frac{\sigma}{\mu} \times 100$$

This paper utilizes the CV for comparing the degree of variation from sub-image series to full image via CV values. Subsequently, the value of CV for sub-image is close to the value of CV for full image. This indicates that the sub-image belongs to template selection sets.

4 ESSENTIAL OPERATIONS IN PROPOSED METHOD

The key semantic idea of the developed method is to extract the feature of the input object by an alternation between similarity measure (i.e. derived kernel) and maximum pooling operation (i.e. neural response) within a hierarchical structure as in Smale's mathematical framework.

The first operation used in constructing the template set is given through the following section:

4.1 TEMPLATE SELECTION

The developed model of template selection is presented below. However, there are some remarks concerning the creation of template that readers must pay attention to:

1. This work considers the system with three layers architecture. So, only two template arrays (i.e.T_u and T_v) demand to be built.
2. The first layer template which is denoted byT_u, is rationally small since it covers only the basic parts of the objects (see fig1).
3. The second layer template which is denoted byT_v, is practically large enough to include approximately a full image with more discriminative structure (see fig2).
4. It is worth noting that the number and best size of the template is determined experimentally.
5. The objective of this technique is to construct the informative template sets P_u and P_v through eliminating the uninformative parts of initial template sets T_u and T_v such as

$$P_v \subset T_v \ and\ P_u \subset T_u$$

Now, the template extraction procedure can be given as follows:

a) Calculate the CV of an image patch, i.e., $CV = getCV(I)$

b) Randomly generate sub-image (i.e. initial template(T_1^u)) from image patch

c) Compute the CV1 of an initial template, i.e., $CV1 = getCV(T_1^u)$

d) Test the threshold. If the CV1 is greater than a given threshold T, return to step b), else, add T_1^u to S_u and then return to step b.

e) Carry on these repetitions until a certain tolerance arrives.

f) The repetitions are terminated.

Algorithm1 shows the detailed implementation of template selection procedures, where d is the number of the result templates (i.e. a certain tolerance) and count is the loop variable.

Algorithm 1 extracting of P_u
Input: Full image (I) and number of template d
Output: P_u

1. count=1
2. $P_u = \emptyset, m = \emptyset$
3. CV= $getCV$ (I)
4. While count < d
5. Constructing sub-image of size u(i.e. t_u^1) (randomly)
6. CV1= $getCV$ (t_u^1)
7. if CV1≤ TH
8. P_u=P_u∪t_u^1, count=count+1.
9. End if.
10. go to 4
11. end while
12. P_u =P_u∪∅

Moreover, the threshold (TH) can be decided experimentally. The next step is concatenating sub-images into a single matrixP_u; as given:
$P_u = [t_u^1, t_u^2 \ldots t_u^d]$ such that $P_u \subset T_u \subset Im(u)$

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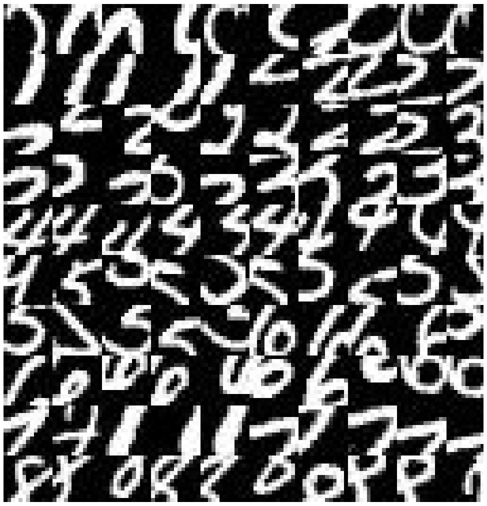

Figure 1 Samples templates of the size 13x13 in $\boldsymbol{T_u}$

In the same way yet with different size, the second layer template S_v is constructed as $P_v = [t_v^1, t_v^2, ..., t_v^L]$, such that $P_v \subset T_v \subset Im(v)$.

Figure 2 Samples templates of the size 21x21 in $\boldsymbol{T_v}$

4.2 Similarity measures

The second operation defines the similarity between the sub-image (i.e. $t_v^i \in P_v \subset Im(v)$) and the template P_u. The similarity steps are given below:

1) Taking sub-image (i.e. $t_v^i \in P_v \subset Im(v)$).
2) Performing associating transformations such that ($t_v^j \circ H_u = t_u^v, j = 1,2, \dots, L$).
3) Executing the similarity measures as $f_u(t_u^v, P_u)$, note that function f_u is a similarity measure implementation (such as inner product, ED, CD…) to estimate the similarity between a sub-image and a template set P_u. The similarity matrix result for the given sub-images $t_v^j \in Im(v)$ can be represented as:

$$S_v = [s_{11} \ \dots \ s_{1d} \ \vdots \ \dots \ \vdots \ s_L \ \dots \ s_{Ld}]$$

where each row consists of the value of similarity measure between a patch $t_v^j \in Im(v)$ and templates in P_u. It should be noted that the process of similarity measures starts with some normalized initial vector to avoid distortions traveling up the hierarchy, and provides a more interpretable as well as comparable quantity[4].

4.3 Pooling operation

After checking the application of similarity, we come to the last operation which is the pooling function. This is required to operate row-wise on the matrix of similarity S_v(i.e. $N_v(t_v, t) = max\{S_v : t_v \in P_v, t \in P_u\}$).

The developed method is illustrated in Fig. 1. Note that $N_{sq}(t_{sq}, t)$ represents the feature vector of input image extracted by the suggested model.

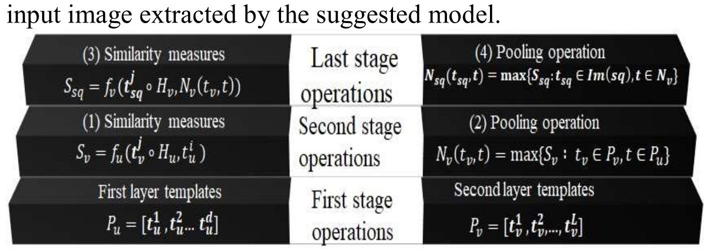

Figure 3 Architecture of the proposed model

5 EXPERIMENTAL RESULTS AND ANALYSIS

This section describes the experimental performance of the developed technique for feature extraction task on the MNIST handwriting digit dataset.

5.1 MNIST dataset

MNIST dataset is a handwritten digit recognition benchmark. It consists of 70,000 grayscale images, each size is 28×28. These data were written by different people with several styles, so they include a scaling, small amount of translations, rotations and other differences which, as one may expect, can be found in a corpus containing human handwritten objects. Note that 10,000 images were selected for

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testing, and 60,000 images were used as the training set.

Figure 4 Example images of the MNIST dataset

5.2 Procedures of the Experiments

When applying this system to the MNIST dataset for training, 5903 images from the whole training data were utilized, and1032 examples were exploited per class as a testing set. The template sets T_u and T_v were created randomly by choosing 500 image patches (of size u and/or v) from images, with the size u=13 and v=21.

For the experiments, the training and testing sets were fixed. To obtain the average recognition accuracy, the introduced model was instigated to over 10 random template sets selected for every similarity measure. The above process was repeated using the suggested technique to extract informative template. After the features were extracted by the developed method, SVM algorithm was taken to obtain classification accuracy. It is worth pointing out here that the experiments were implemented on a computer with 3.70 GHz Intel Core i3 processor and 4 GB RAM. The code was applied in MATLAB R2014a version.

5.3 Discussion

By utilizing the same Smalean framework, the developed algorithm largely outperforms Smale's model (i.e randomly selecting template sets) itself. This highlights the effectiveness of the proposed methods of informative templates. On the other hand, the experiments made some comparisons between the several similarity measures to demonstrate, compare and evaluate which of the several similarity measures can led to low computationally with high accuracy results. Detailed comparison results are mentioned in Table 2.

To sum up, the experiments demonstrate that the Square Pearson Correlation Coefficient (SPCC) and Cosine distance (CD) lead to more accurate similarity between objects. The reason is that SPCC deals with the statistical relationship between pixel intensities while CD is based on the geometric point of view. ED, DSC and ChD are based on the different forms of algebra distance measures between pixel intensities which are not appropriate in this model because the assumption of affine relationship between image intensity values does not always hold here (where different scaling, amount of translations, rotations of the same object, result in a complex relationship between pixel intensities).

Table 2 Accuracy and Running time comparison on the MNIST database

Similarity measures	Accuracy of developed method (%)	Accuracy of random extraction (%)	Time (s)
Cosine distance (CD)	83.66011	**81.68928**	0.083082
Square Pearson Correlation Coefficient (SPCC)	**89.47368**	80.66709	0.288061
Dice similarity coefficient (DSC)	81.17185	78.49207	0.027563
Euclidean Distance(ED)	63.06151	59.48763	**0.001594**
Chord distance (ChD)	79.87064	72.46417	0.233342

6 CONCLUSION

After a scanning overview of the commonly used similarity measures and methods, this paper has proposed an effective template selection method based on coefficient of variation (CV). The paper has been designed to develop a new method for selection informative template sets with large power of discrimination. To achieve this, experimental computations were performed. 5903 images from the whole training data were utilized, and1032 examples were exploited per class as a testing set. The template sets T_u and T_v were created randomly by choosing 500 image patches (of size u and/or v) from images, with the size u=13 and v=21. To obtain the average recognition accuracy, the introduced model was applied to over 10 random template sets selected for every similarity measure. The above process was repeated using the suggested technique to extract informative template. After the features were extracted by the developed method, SVM algorithm was taken to obtain classification accuracy. It has been found that the derived kernel based on the developed technique is more appropriate for object recognition and leads to a greater recognition performance. Additionally, this work has done some fundamental comparisons focusing on the similarity and distant measures for MNIST database based on different forms of algebra distance measures, such as Euclidean distance measure(ED), Dice similarity coefficient (DSC) and Chord distance (ChD), Cosine distance (CD), and the statistical relationship between pixel intensities such as Square Pearson Correlation Coefficient (SPCC). The experimental results have shown that the Square Pearson Correlation Coefficient (SPCC) leads to more accurate similarity between MNIST dataset. In future work, this avenue of research will be benefited immensely to a large and several dataset of real life objects. On the other hand, since the selection of template is still an open problem, other machine learning methods such as non-negative matrix factorization and manifold learning can be utilized thoroughly in future research works.

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