Journal of Theoretical and Applied Information Technology
31st January 2021. Vol.99. No 2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

A NEW APPROACH FOR SYNTHESIS OF THE CONTROL SYSTEM BY GRADIENT-VELOCITY METHOD OF LYAPUNOV VECTOR FUNCTIONS

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ABSTRACT

We suggest technique for the synthesis of control systems with the state vector by means of gradient-velocity method of Lyapunov vector functions. Gradient-velocity method of Lyapunov vector functions is based on the presence of a gradient in a control system with a potential function in the form of Lyapunov function. Total derivative of the Lyapunov vector function with respect to time always exists as a negative definite function representing scalar product of the gradient vector and components of the velocity vector expansion in coordinates. In this paper, the synthesis of control systems is provided with the state vector directly from the matrix components of the closed loop system. Meanwhile, the controller synthesis is considered as a method for determining the allowed range of controller parameters to provide desired performance in a closed loop system.

Keywords: *Control system, controller synthesis, system with m inputs and n outputs, gradient-velocity method, Lyapunov vector function*

1. INTRODUCTION

The development of methods for studying robust stability and the synthesis of multidimensional control systems relate to the challenging tasks in the dynamic control principles and practices.

The known methods of control with a state vector [1-3] are based on the preliminary block diagonalization of the state matrix. In this case, the columns of nonsingular matrix used for canonical transformation are determined by eigenvectors of the control matrix, whereas corresponding rules and complex and ambiguous computing algorithms are described in [4], [5]. Moreover, it is desirable to calculate the roots of the system characteristic polynomial with a high-degree modal controller.

Recent studies have also shown that the methods based on Lyapunov vector function that derive from the geometric representation of the theorem on asymptotic stability in the phase space can be effectively used to study the robustness of linear or nonlinear control systems. The use of the method of Lyapunov functions for addressing various practical linear or nonlinear problems is limited by the lack of universal approach in defining a Lyapunov function [6], [7]. Lyapunov functions are generally defined as the functions of entropy and total energy of the system. It should be noted that any failure in choosing or defining a desirable Lyapunov function does not denote the instability of a system, it indicates only a failure in defining a Lyapunov function.

In this paper, we suggest technique for the synthesis of control systems having m inputs and n outputs with the state vector by means of gradient-velocity method of Lyapunov vector functions [8-10]. The study of the robust stability of automatic control systems is based on defining Lyapunov vector functions and the presence of a gradient in a dynamic control system [11], [12]. For these, we used the basic principles of the Lyapunov theorem on asymptotic stability and the concept of stability in dynamical systems.

There are different methods to evaluate stability. For the complex systems it is common to use Lyapunov function. But identifying the Lyapunov function is not trivial task, and there is no general technique for constructing such a function. The vast majority of existing methodologies fall in one of the following two categories:1) methods which construct or search for Lyapunov function and 2) methods which try to approximate it [13], [14], [15]. Prokhorov in [13] suggests a Lyapunov Machine, which is a special-design artificial neural network, for approximating Lyapunov function. In [14] author suggests an algorithm for approximating Lyapunov function when system is asymptotically stable. In the other works, Lyapunov functions are constructed by neural networks [16], [17]. Based on this ideas, two different approaches were used in order to calculate the appropriate network's weights. Another recently reported studies are in [15],[18],[19].

The task of the controller synthesis is addressed with the state vector directly from the matrix components of the closed loop system, whereas with Lyapunov vector functions approach it may be considered a method for determining the allowed range of controller parameters to provide the desired performance in a closed loop system.

2. ANALYSIS OF THE CONTROL SYSTEM PARAMETERS

Consider the control system having m inputs and n outputs. The state vector is fully measurable, and the system is fully controllable.

The dynamics of time-invariant linear control system is defined by the equation of fully controllable target as:

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad (2.1)$$

Assume the state vector x(t) of the system (2.1) is measurable. Consider control law of the type

$$u = -Kx \qquad (2.2)$$

where K is the n×m matrix of the controller coefficients to be evaluated. Closed-loop system target-controller is defined by the following equation as

$$\dot{x} = (A - BK)x, \qquad (2.3)$$

where $x(t) \in R^n, A \in R^{n*n}, B \in R^{n*m}, K \in R^{m*n}$

$$BK = \left\| \begin{matrix} \sum_{i=1}^{m} b_{1i}k_{i1} \sum_{i=1}^{m} b_{1i}k_{i2} \sum_{i=1}^{m} b_{1i}k_{i3}, \dots, \sum_{i=1}^{m} b_{1i}k_{in} \\ \sum_{i=1}^{m} b_{2i}k_{i1} \sum_{i=1}^{m} b_{2i}k_{i2} \sum_{i=1}^{m} b_{2i}k_{i3}, \dots, \sum_{i=1}^{m} b_{2i}k_{in} \\ \dots \\ \sum_{i=1}^{m} b_{ni}k_{i1} \sum_{i=1}^{m} b_{ni}k_{i2} \sum_{i=1}^{m} b_{ni}k_{i3}, \dots, \sum_{i=1}^{m} b_{ni}k_{in} \end{matrix} \right\|$$

Expanded forms of equations (2.3) are given as:

$$\begin{cases} \dot{x}_1 = (a_{11} - \sum_{i=1}^{m} b_{1i}k_{i1})x_1 + \dots + (a_{1n} - \sum_{i=1}^{m} b_{1i}k_{in})x_n \\ \dot{x}_2 = (a_{21} - \sum_{i=1}^{m} b_{2i}k_{i1})x_1 + \dots + (a_{2n} - \sum_{i=1}^{m} b_{2i}k_{in})x_n \\ \dots \qquad \dots \qquad \dots \\ \dot{x}_1 = (a_{n1} - \sum_{i=1}^{m} b_{ni}k_{i1})x_1 + \dots + (a_{nn} - \sum_{i=1}^{m} b_{ni}k_{in})x_n \end{cases} \qquad (2.4)$$

The condition for robust asymptotic stability of the system (2.4) can be found by means of gradient-velocity method of Lyapunov vector functions [20]. To do this, the components of a gradient vector of Lyapunov vector function $V(x)=(V_1(x),\ V_2(x),\dots,\ V_n(x))$ can be found from (2.4) as

$$\frac{\partial V_k(x)}{\partial x_j} = -(a_{kj} - \sum_{i=1}^{m} b_{ki}k_{ij})x_j,$$
$$k = 1,\dots,n,\ j = 1,\dots,n \qquad (2.5)$$

The components of the velocity vector expansion in coordinates $(x_1,\dots,x_n)$ can be found from (2.4) [10]:

$$\left(\frac{dx_k}{dt}\right)_{x_j} = (a_{kj} - \sum_{i=1}^{m} b_{ki}k_{ij})x_j, \qquad (2.6)$$
$$k = 1,\dots,n,\ j = 1,\dots,n$$

Total derivative of the Lyapunov vector function with respect to time is defined as a scalar product of the gradient vector (2.5) and the velocity vector (2.6) as:

$$\frac{dV(x)}{dt} = \sum_{k=1}^{n} \sum_{j=1}^{n} \frac{\partial V_k(x)}{\partial x_j} \left(\frac{dx_k}{dt}\right)_{x_j} =$$
$$= \sum_{k=1}^{n} \sum_{j=1}^{n} -(a_{kj} - \sum_{j=1}^{m} b_{ki}k_{ij})^2 x_j^2, \qquad (2.7)$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Total derivative of the Lyapunov vector functions (2.7) is a negative definite function.

Lyapunov function can be rewritten from (2.5) as:

$$V(x) = \frac{1}{2}\sum_{j=1}^{n}\left(\sum_{i=1}^{m} b_{1i}k_{ij} - a_{1j} + \ldots + \sum_{i=1}^{m} b_{ni}k_{ij} - a_{nj}\right)x_j^2 \quad (2.8)$$

Necessary conditions for Lyapunov vector functions, i.e. positive definiteness of Lyapunov functions are defined from criterion

$$\sum_{i=1}^{m} b_{1i}k_{ij} - a_{1j} + \ldots + \sum_{i=1}^{m} b_{ni}k_{ij} - a_{nj} > 0, \quad j = 1,\ldots,n \quad (2.9)$$

Criterion (2.9) defines the robust superstability of the control system directly from the state vector.

3. ANALYSIS OF A SYSTEM WITH THE DESIRABLE PARAMETERS

Consider a system with desired performance defined by a matrix

$$G_3 = \begin{Vmatrix} -d_{11} & -d_{12} & -d_{13}, & \ldots, & -d_{1n} \\ -d_{21} & -d_{22} & -d_{23}, & \ldots, & -d_{2n} \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ -d_{n1} & -d_{n2} & -d_{n3}, & \ldots, & -d_{nn} \end{Vmatrix}$$

The task is to determine the controller coefficients (components of the matrix K) so that the matrix components of the closed loop system have given values $d_{ij}(i=1,\ldots,n; j=1,\ldots,n)$

The stability of the system having given values of the coefficients $d_{ij}(i=1,\ldots,n; j=1,\ldots,n)$ is analyzed by gradient-velocity method of Lyapunov vector functions [22].

For the system with given parameters, the expanded form of the state equation is defined as:

$$\begin{cases} \dot{x}_1 = -d_{11}x_1 - d_{12}x_2 - d_{13}x_3 -,\ldots,-d_{1n}x_n \\ \dot{x}_2 = -d_{21}x_1 - d_{22}x_2 - d_{23}x_3 -,\ldots,-d_{2n}x_n \\ \ldots \quad \ldots \quad \ldots \\ \dot{x}_n = -d_{n1}x_1 - d_{n2}x_2 - d_{n3}x_3 -,\ldots,-d_{nn}x_n \end{cases} \quad (3.1)$$

The components of a gradient vector of Lyapunov vector function $V(x)=(V_1(x),\ldots, V_n(x))$ are found from (2.1) as:

$$\frac{\partial V_k(x)}{\partial x_j} = d_{kj}x_j, k = 1,\ldots,n, j = 1,\ldots,n \quad (3.2)$$

The components of the velocity vector expansion in coordinates are also found from (3.1):

$$\left(\frac{dx_k}{dt}\right)_{x_j} = -d_{kj}x_j, \quad k = 1,\ldots,n, j = 1,\ldots,n \quad (3.3)$$

According to (3.2) and (3.3) total derivative of the Lyapunov vector function with respect to time is defined as a scalar product of the gradient vector of Lyapunov vector functions (3.2) and the velocity vector (3.3):

$$\frac{dV(x)}{dt} = -\sum_{k=1}^{n}\sum_{j=1}^{n}\frac{\partial V_k(x)}{\partial x_j}\left(\frac{dx_k}{dt}\right)_{x_j} = \sum_{k=1}^{n}\sum_{j=1}^{n} - d_{kj}^2 x_j^2 \quad (3.4)$$

From (3.4), it follows that the total derivative of the Lyapunov vector function with respect to time is a negative definite function.

From (3.2), Lyapunov vector functions can be presented in a scalar form:

$$V(x) = \frac{1}{2}\sum_{j=1}^{n}(d_{1j} + d_{2j} + d_{3j},\ldots,+d_{nj})x_j^2, \quad (3.5)$$

Necessary conditions for Lyapunov vector functions of the system (3.1) are defined by a system of inequalities:

$$d_{1j} + d_{2j} + d_{3j} +,\ldots,+d_{nj} > 0, j = 1,\ldots,n, \quad (3.6)$$

It is necessary to equalize values of inequalities (2.9) and (3.6) to make a system (2.4) having desired performance. Then, the desirable values of the coefficients in the matrix K are expressed following way:

$$\sum_{i=1}^{n} b_{1i}k_{ij} - a_{1j} + \ldots + \sum_{i=1}^{n} b_{ni}k_{ij} - a_{nj} = d_{1j} + d_{2j} + d_{3j} +,\ldots,+d_{nj}, \quad j = 1,\ldots,n, \quad (3.7)$$

Journal of Theoretical and Applied Information Technology
31st January 2021. Vol.99. No 2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

When comparing coefficients in the equation (3.7) for each component, the following expression is defined:

$$\begin{cases} \sum_{i=1}^{n} b_{1i}k_{ij} - a_{1j} - d_{1j} = 0 \\ \sum_{i=1}^{n} b_{2i}k_{ij} - a_{2j} - d_{2j} = 0 \quad j=1,\dots,n, \\ \dots\dots\dots\dots\dots\dots\dots \\ \sum_{i=1}^{n} b_{ni}k_{ij} - a_{nj} - d_{nj} = 0 \end{cases} \quad (3.8)$$

From this system of n algebraic equations one can find the value n^2 for the components of matrix K.

4. RESULTS

Consider controller synthesis by implementing principle of the gradient-velocity method of Lyapunov vector functions. Assume a control system is defined by equation:

$$\frac{dx}{dt} = Ax + bu, x \in R^n, u \in R^4 \quad (4.1)$$

where

$$A = \begin{bmatrix} 1 & -2 & -5 & 2 \\ 2 & -1 & 7 & 2 \\ 0 & 2 & -8 & 2 \\ 1 & 0 & -7 & 1 \end{bmatrix} B = \begin{bmatrix} 4 \\ 2 \\ 0 \\ 1 \end{bmatrix} x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

whereas $u(t)=-k^T x(t)$, $k^T=||k_1\ k_2\ k_3\ k_4||$. One shows the results of a numerical experiment which are the graphs of transient process in a control system (4.1), i.e. time response of the system (a) and transient behavior of the state variables (b) in a control system are shown in Figure 1. The system is stable and its stability margin is 1 (according to eigenvalues). Controller should bring the system to desired performance (with bigger stability margin).

Then, the expanded form of a control system (4.1) is defined as:

$$\begin{cases} \dot{x}_1 = (1-4k_1)x_1 - (2+4k_2)x_2 - \\ \quad -(5+4k_3)x_3 + (2-4k_4)x_4 \\ \dot{x}_2 = (2k_1-2)x_1 - (1+2k_2)x_2 - \\ \quad -(7+2k_1)x_3 + (2k_4-2)x_4 \\ \dot{x}_3 = 0 + 2x_2 - 8x_3 + 2x_4 \\ \dot{x}_4 = (1-k_1)x_1 - k_2x_2 + \\ \quad +(7+k_3)x_3 + (1-k_4)x_4 \end{cases} \quad (4.2)$$

The components of a gradient vector of the Lyapunov vector function are determined from the state equation (4.2) with account for a system gradient as:

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_1} = (4k_1-1)x_1, \quad \frac{\partial V_1(x)}{\partial x_2} = (2+4k_2)x_2, \\ \frac{\partial V_1(x)}{\partial x_3} = (5+4k_3)x_3, \quad \frac{\partial V_1(x)}{\partial x_4} = (4k_4-2)x_4, \\ \frac{\partial V_2(x)}{\partial x_1} = (2k_1-2)x_1, \quad \frac{\partial V_2(x)}{\partial x_2} = (1+2k_2)x_2, \\ \frac{\partial V_2(x)}{\partial x_3} = (7+2k_3)x_3, \quad \frac{\partial V_2(x)}{\partial x_4} = (2k_4-2)x_4, \\ \frac{\partial V_3(x)}{\partial x_2} = -2x_2, \quad \frac{\partial V_3(x)}{\partial x_3} = 8x_3, \quad \frac{\partial V_3(x)}{\partial x_4} = -2x_4, \\ \frac{\partial V_4(x)}{\partial x_1} = (k_1-1)x_1, \quad \frac{\partial V_4(x)}{\partial x_2} = k_2x_2, \\ \frac{\partial V_4(x)}{\partial x_3} = (7+k_3)x_3, \quad \frac{\partial V_4(x)}{\partial x_4} = (k_4-1)x_4 \end{cases} \quad (4.3)$$

By expanding the components of the velocity vector in coordinates in the state equation (4.2), the velocity vector is rewritten as:

$$\begin{cases} \left(\frac{dx_1}{dt}\right)_{x_1} = (1-4k_1)x_1, \quad \left(\frac{dx_1}{dt}\right)_{x_2} = -(2+4k_2)x_2, \\ \left(\frac{dx_1}{dt}\right)_{x_3} = -(5+4k_3)x_3, \quad \left(\frac{dx_1}{dt}\right)_{x_4} = (2-4k_4)x_4, \\ \left(\frac{dx_2}{dt}\right)_{x_1} = (2-2k_1)x_1, \quad \left(\frac{dx_2}{dt}\right)_{x_2} = -(1+2k_2)x_2, \\ \left(\frac{dx_2}{dt}\right)_{x_3} = -(7+2k_3)x_3, \quad \left(\frac{dx_2}{dt}\right)_{x_4} = (2-2k_4)x_4, \\ \left(\frac{dx_3}{dt}\right)_{x_2} = 2x_2, \quad \left(\frac{dx_3}{dt}\right)_{x_3} = -8x_3, \quad \left(\frac{dx_3}{dt}\right)_{x_4} = 2x_4, \\ \left(\frac{dx_4}{dt}\right)_{x_1} = (1-k_1)x_1, \quad \left(\frac{dx_4}{dt}\right)_{x_2} = -k_2x_2, \\ \left(\frac{dx_4}{dt}\right)_{x_3} = -(7+k_3)x_3, \quad \left(\frac{dx_4}{dt}\right)_{x_4} = (1-k_4)x_4 \end{cases}$$

(4.4)

Let represent the total derivative of the Lyapunov vector function $V(x)=(V_1(x), V_2(x), V_3(x), V_4(x))$ with respect to time as a scalar product of gradient vector (4.3) and velocity vector (4.4):

$$\frac{dV(x)}{dt} = -(1-4k_1)^2x_1^2 - (2+4k_2)^2x_2^2 -$$
$$-(5+4k_3)^2x_3^2 - (2-4k_4)^2x_4^2 - (2-2k_1)^2x_1^2 -$$
$$-(1+2k_2)^2x_2^2 - (7+2k_3)^2x_3^2 - (2-2k_4)^2x_4^2 -$$
$$-4x_2^2 - 64x_3^2 - 4x_4^2 - (1-k_1)^2x_1^2 - 4k_2^2x_2^2 -$$
$$-(7+k_3)^2x_3^2 - (1-k_4)^2x_4^2$$

(4.5)

From (4.5), it is evident that the total derivative of the Lyapunov vector function is a negative definite function. Following the Lyapunov vector function (4.3), the scalar form of Lyapunov functions is given as:

$$V(x) = \frac{1}{2}(7k_1 - 4)x_1^2 + \frac{1}{2}(1 + 7k_2)x_2^2 + \\ + \frac{1}{2}(27 + 7k_3)x_3^2 - \frac{1}{2}(k_4 - 7)x_4^2 \quad (4.6)$$

Necessary conditions for Lyapunov vector functions, i.e. positive definiteness of a quadratic equation (4.6) are defined by a system of inequalities:

$$7k_1 - 4 > 0, \quad 1 + 7k_2 > 0, \\ 27 + 7k_3 > 0, \quad 7k_4 - 7 > 0 \quad (4.7)$$

Let some system with desired performance be given as a matrix

$$G = \begin{bmatrix} 1 & -40 & 3 & 26 \\ 2 & -20 & -3 & 14 \\ 0 & 2 & -8 & 2 \\ 1 & -9{,}5 & -5 & 7 \end{bmatrix}$$

Assume the expanded form of a system with desired performance as:

$$\begin{cases} \dot{x}_1 = x_1 - 40x_2 + 3x_3 + 26x_4 \\ \dot{x}_2 = 2x_1 - 20x_2 - 3x_3 + 14x_4 \\ \dot{x}_3 = 2x_2 - 8x_3 + 2x_4 \\ \dot{x}_4 = x_1 - 9{,}5x_2 - 5x_3 + 7x_4 \end{cases} \quad (4.8)$$

The desirable time response of the system (a) and transient behavior of the state variables (b) in a control system are shown in Figure 2. As seen in the figure the system is aperiodic, stable and its stability margin is 3.

Correspondingly, the components of a gradient vector of the Lyapunov vector function are given as:

$$\begin{cases} \frac{\partial V_1(x)}{\partial x_1} = -x_1, & \frac{\partial V_1(x)}{\partial x_2} = 40x_2, \\ \frac{\partial V_1(x)}{\partial x_3} = -3x_3, & \frac{\partial V_1(x)}{\partial x_4} = -26x_4, \\ \frac{\partial V_2(x)}{\partial x_1} = -2x_1, & \frac{\partial V_2(x)}{\partial x_2} = 20x_2, \\ \frac{\partial V_2(x)}{\partial x_3} = 3x_3, & \frac{\partial V_2(x)}{\partial x_4} = 14x_4, \\ \frac{\partial V_3(x)}{\partial x_1} = 0, & \frac{\partial V_3(x)}{\partial x_2} = -2x_2 \\ \frac{\partial V_3(x)}{\partial x_3} = 8x_3, & \frac{\partial V_3(x)}{\partial x_4} = -2x_4, \\ \frac{\partial V_4(x)}{\partial x_1} = -x_1, & \frac{\partial V_4(x)}{\partial x_2} = 9{,}5x_2, \\ \frac{\partial V_4(x)}{\partial x_3} = 5x_3, & \frac{\partial V_4(x)}{\partial x_4} = -7x_4 \end{cases} \quad (4.9)$$

The velocity vector expansion in coordinates (x_1, x_2, x_3, x_4) is found from (4.8):

$$\begin{cases} \left(\frac{dx_1}{dt}\right)_{x_1} = x_1, & \left(\frac{dx_1}{dt}\right)_{x_2} = -40x_2, \\ \left(\frac{dx_1}{dt}\right)_{x_3} = 3x_3, & \left(\frac{dx_1}{dt}\right)_{x_4} = 26x_4, \\ \left(\frac{dx_2}{dt}\right)_{x_1} = 2x_1, & \left(\frac{dx_2}{dt}\right)_{x_2} = -20x_2, \\ \left(\frac{dx_2}{dt}\right)_{x_3} = -3x_3, & \left(\frac{dx_2}{dt}\right)_{x_4} = 14x_4, \\ \left(\frac{dx_3}{dt}\right)_{x_1} = 0, & \left(\frac{dx_3}{dt}\right)_{x_2} = 2x_2, \\ \left(\frac{dx_3}{dt}\right)_{x_3} = -8x_3, & \left(\frac{dx_3}{dt}\right)_{x_4} = 2x_4, \\ \left(\frac{dx_4}{dt}\right)_{x_1} = x_1, & \left(\frac{dx_4}{dt}\right)_{x_2} = -9{,}5x_2, \\ \left(\frac{dx_4}{dt}\right)_{x_3} = -5x_3, & \left(\frac{dx_4}{dt}\right)_{x_4} = 7x_4 \end{cases} \quad (4.10)$$

The total derivative of the Lyapunov vector function with respect to time is found as:

$$\frac{dV(x)}{dt} = -\frac{1}{2}x_1^2 - \frac{1}{2}x_1^2 - \frac{1}{2}x_1^2 + 40\frac{1}{2}x_2^2 + \\ + \frac{1}{2}20x_2^2 - 2\frac{1}{2}x_2^2 + 9{,}5x_2^2 - 3\frac{1}{2}x_3^2 + \\ + \frac{1}{2}x_3^2 + 8\frac{1}{2}x_3^2 + 5\frac{1}{2}x_3^2 - 26\frac{1}{2}x_4^2 - \\ - 14\frac{1}{2}x_4^2 - 2\frac{1}{2}x_4^2 - 7\frac{1}{2}x_4^2 \quad (4.11)$$

From (4.11), it is evident that the total derivative of the Lyapunov vector function is a negative definite function. Following (4.9), Lyapunov functions are rewritten as:

$$V(x) = -4x_1^2 + 67{,}5x_2^2 + 13x_3^2 - 49x_4^2 \quad (4.12)$$

To determine the controller parameters, it is necessary to equalize corresponding components of Lyapunov function defined for the control system and those defined for the system with desired performance [23]. Then, the controller parameters k are determined from the above system of equations.

Let equalize the left-hand portions of inequalities (4.7) and (4.12) to make the system (4.1) having desirable properties. Then, the necessary values of coefficients k_1, k_2, k_3, k_4 can be found.

$$\begin{matrix} 7k_1 - 4 = -4 \\ 1+7k_2 = 67{,}5 \\ 27+7k_3 = 13 \\ 7k_4 - 7 = -49 \end{matrix} \rightarrow \begin{matrix} k_1 = 0 \\ k_2 = 9{,}5 \\ k_3 = -2 \\ k_4 = -6 \end{matrix} \quad (4.13)$$

The time response of the system with the controller (a) and transient behavior of its state variables (b) are shown in Figure 3. As seen in the figure the system is also aperiodic and shows the same dynamic as desirable system. This proofs that coefficients defined using presented method are correct. Present study denotes system stability at different values of the coefficients k_1, k_2, k_3 and k_4, whereas the control system demonstrates superstability behavior allowing high-quality control process to run with no evidence of over-control or repeated variations. Again, the control system provides desirable transient behavior confirmed by the results of experimental simulations of the system model. Moreover this method needs less computations and it is applicable for high degree systems.

The given gradient-velocity method of Lyapunov vector functions allows to study of the robust stability, moreover if there is given state matrix of desirable performance, then this method allows to define a controller, that assures required performance.

5. CONCLUSIONS

Recent studies have shown that the method of the Lyapunov vector functions can be efficiently implemented and applied for the analysis of robust stability in both linear and non-linear control systems. Wide-range applications of this method are limited by the lack of universal approach in choosing and defining a Lyapunov function as well as by their difficult algorithmic representation. Any inappropriate choosing or defining a desirable Lyapunov function does not denote the instability of a system, it indicates only a failure in finding a Lyapunov function.

Suggested approach in defining Lyapunov functions allows the evaluation of the area of robust stability as the simple inequality for a control system with undefined parameters. The given approach named gradient-velocity method of Lyapunov vector functions. In this study, this methodology has been developed for synthesis of the control system, which ensures desired stability margin of the system by equalizing robust stability inequalities to the corresponding inequalities of “desired” system. This gave us to define controller ensuring desired performance. This is clearly visible on figures. In comparison with methods of modal control given approach in this study has less iteration. Both the efficiency and applicability of the suggested technique are evident.

ACKNOWLEDGMENT

This research has been funded by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan (Grant No. AP08857114).

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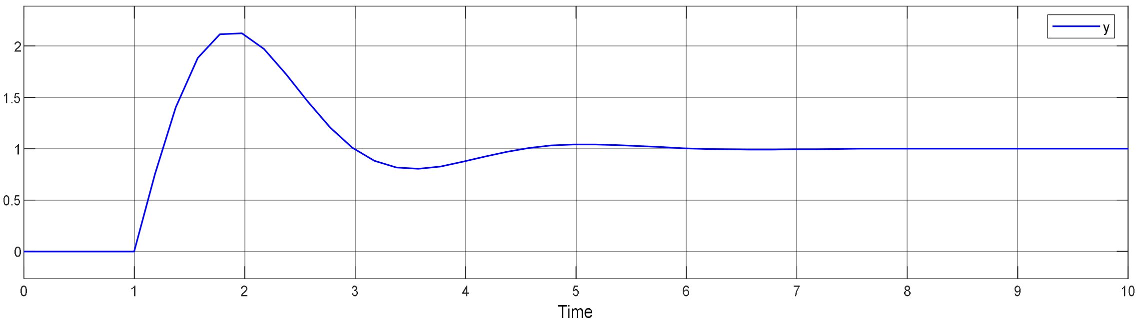
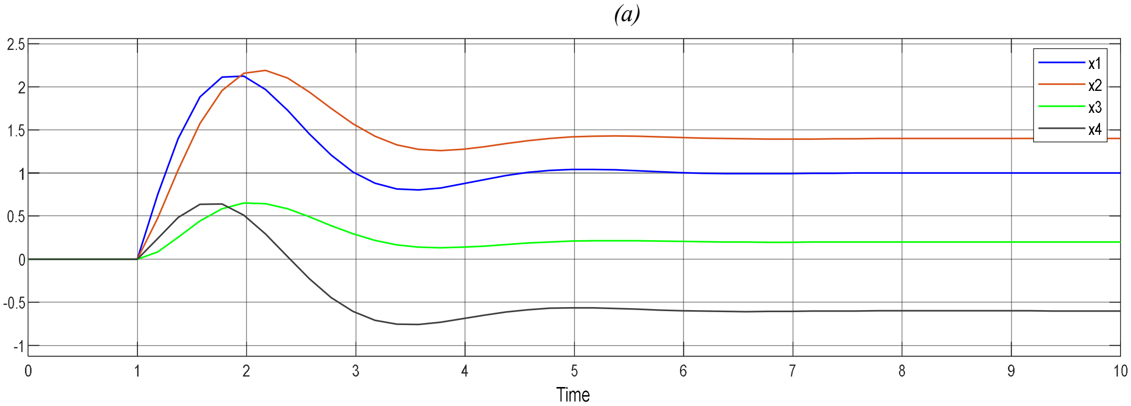
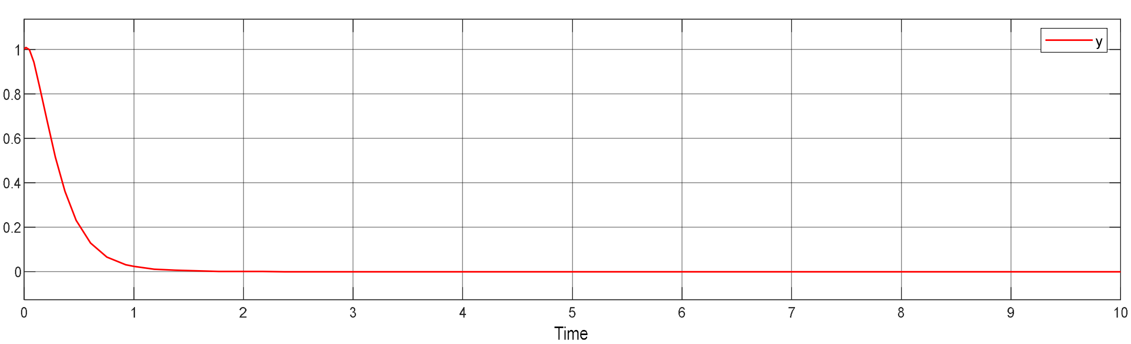


(a)

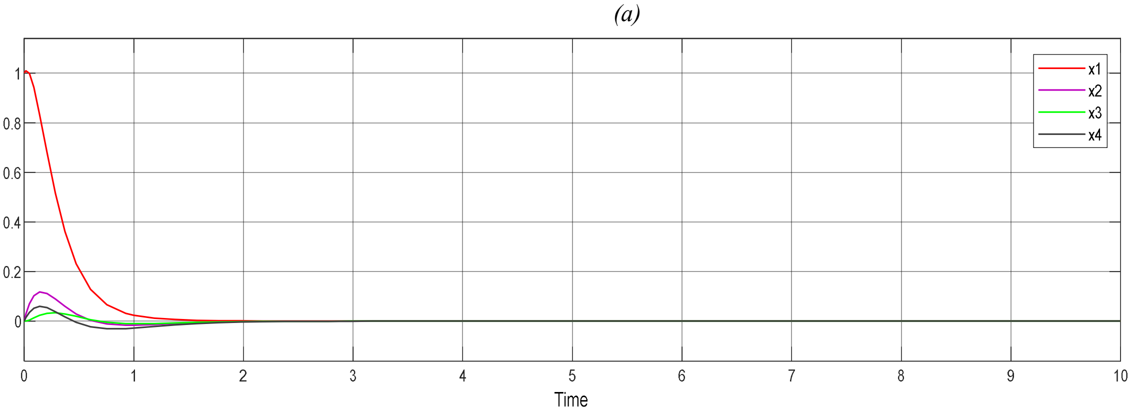


(b)

Figure 1. Time response of the system (a) and transient behavior of the state variables (b)



(a)

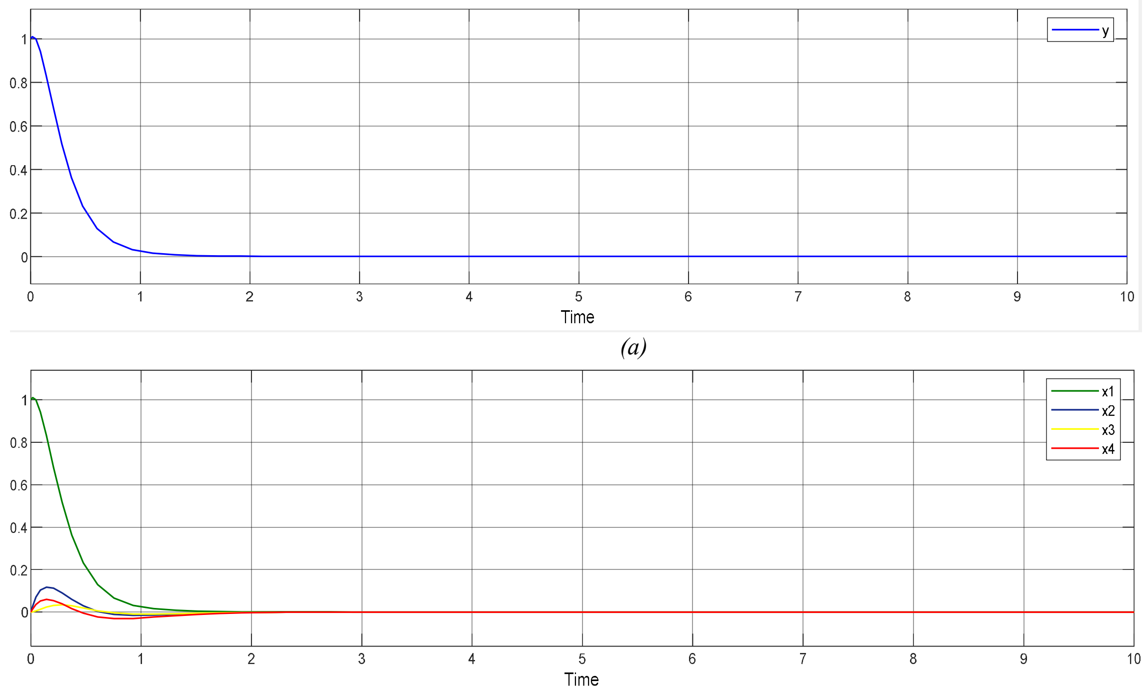


(b)

Figure 2. Desired performance:
time response of the system (a) and transient behavior of the state variables (b)

Journal of Theoretical and Applied Information Technology
31st January 2021. Vol.99. No 2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195



(b)

Figure 3. The system with the controller: time response of the system (a) and transient behavior of the state variables (b)