Journal of Theoretical and Applied Information Technology
31st August 2021. Vol.99. No 16

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

NUMERICAL SIMULATION OF WAVE PROPAGATION IN MIXED POROUS MEDIA USING FINITE ELEMENT METHOD

MARAT NURTAS[1], ZHARASBEK BAISHEMIROV[2,3,4], SULTAN ALPAR[1], FATIMA TOKMUKHAMEDOVA[1]

[1]International Information Technology University, Almaty, Kazakhstan
[2]Abai Kazakh National University, Almaty, Kazakhstan
[3]Institute of Information and Computational Technologies CS MES RK, Almaty, Kazakhstan
[4]Kazakh British Technical University, Almaty, Kazakhstan

E-mail: zbai.kz@gmail.com

ABSTRACT

In this paper we research the problem of acoustics in porous media in three separated subdomains. In each region assumes different physical properties: geometry of the pore, viscosity of fluid places in the middle of the two elastic domains. In this task, we first consider the solution of differential equations. A mathematical model of these physical phenomena described by the initial boundary-value problems for complex systems of differential equations in partial derivatives. Then these equations were solved using two numerical methods: finite element method (FEM) and the traditional finite difference method (FDM). Solutions allows to analyse wave propagation phenomena in porous media. The polynomial functions were used as the interpolation basis-test functions in order to get weak solution for the finite element method. The numerical results of our simulation illustrate the method is obviously effective, especially if we want research physical problems with complex domains in 2D and 3D spaces.
Keywords: *Numerical Simulation, Wave Propagation , Pours Media, Finite Method*

1. INTRODUCTION

Let a half-space $\Omega = \{x \in R | x > 0\}$ consist of three finite layers $\Omega_1 = \{x \in R | 0 < x < H_1\}$, $\Omega_2 = \{x \in R | H_1 < x < H_2\}$, $\Omega_3 = \{x \in R | H_2 < x < H_3\}$ and a semi-infinite layer $\Omega_4 = \{x \in R | x > H_3\}$. The domains Ω_1 and Ω_4 are elastic media without any pore structure, while the domains Ω_2 and Ω_3 are elastic porous media with porosity m_2 and m_3 respectively. The pores of the layer Ω_2 are filled with liquid 2 (oil) and the pores of the layer Ω_3 are filled with 3 (water). We assume that the solid skeleton of the domains Ω_2 and Ω_3 consists of the same material as the domains Ω_1 and Ω_4 [1, 2]. Of all the characteristics of a continuous medium, we will take into account only the density ρ_s and speed of sound c_s (dimensionless) of the elastic medium, the density ρ_2 and c_2 speed of sound of the first fluid, and the density ρ_3 and speed of sound c_3 of the second fluid [3]. According to [4, 5, 6, 7], the pressure of medium satisfies the acoustics equation in Ω if $t > 0$ [8]

$$\frac{1}{c^2(x)}\frac{\partial^2 p}{\partial t^2} = div\left(\frac{1}{\rho(x)}\nabla p\right) \quad (1)$$

Where

$$\frac{1}{c^2(x)} = \frac{h_1(x)}{{c_s}^2} + h_2(x)\left(\frac{1-m_2}{{c_s}^2} + \frac{m_2}{{c_2}^2}\right) + h_3(x)\left(\frac{1-m_3}{{c_s}^2} + \frac{m_3}{{c_3}^2}\right) + h_4(x)\frac{1}{{c_s}^2}$$

$$\rho(x) = \rho_s h_1(x) + h_2(x)(\rho_s(1-m_2) + \rho_2 m_2) + h_3(x)(\rho_s(1-m_3) + \rho_3 m_3) + \rho_s h_4(x)$$

$h_i(x), i = 1,2,3,4$ - the characteristic functions of the domains Ω_i, i.e.,$h_i(x) = 1$, $x \in \Omega_i$ and $h_i(x) = 0$ if $x \notin \Omega_i$.

At the boundary $\Gamma = \{x | x = 0\}$ we set the normal displacement of the medium, which, by virtue of the motion equation [4]

$$\rho \frac{\partial^2 \vec{W}^2}{\partial t^2} = -\nabla p$$

means to set the derivative

$$\frac{1}{\rho} \nabla p \vec{n} = u_1(t), x = 0, t > 0 \quad (2)$$

Γ boundary additional condition:

$$p = u_0(t), x = 0, t > 0 \quad (3)$$

Problem (1) - (3) is closed by setting the initial conditions [9, 10, 11]

$$p(x,0) = 0, \frac{\partial p}{\partial t}(x,0) = 0 \quad x > 0, t > 0 \quad (4)$$

First, changing the variables we write the problem in a more convenient form. Namely, we introduce a new spatial variable by the formula

$$y = \frac{x}{H_1} \quad 0 < x < H_1 \quad (\text{in } \Omega_1)$$
$$y = 1 + \frac{x - H_1}{H_2 - H_1} \quad H_1 < x < H_2 \quad (\text{in } \Omega_2)$$
$$y = 2 + \frac{x - H_2}{H_3 - H_2} \quad H_2 < x < H_3 \quad (\text{in } \Omega_3)$$
$$y = 3 + (x - H_3) \quad x > H_3 \quad (\text{in } \Omega_4)$$

Thus, the Ω_i domains will transform to G_i, $i = 1,2,3,4$ domains where

$$G_1 = \{y \in R | 0 < y < 1\}$$
$$G_2 = \{y \in R | 1 < y < 2\}$$
$$G_3 = \{y \in R | 2 < y < 3\}$$
$$G_4 = \{y \in R | y > 3\}$$

and equation (1) for the function $u(y,t) = p(x,t)$

$$\frac{1}{\tilde{c}^2(y)} \frac{\partial^2 u}{\partial^2 t} = \frac{\partial}{\partial y}\left[\frac{1}{\tilde{\rho}(y)} \frac{\partial u}{\partial y}\right] \quad (5)$$

where

$$\frac{1}{\tilde{c}^2(y)} = \frac{H_1 X_1(y)}{{c_s}^2} + H_2 X_2(y)\left(\frac{1 - m_2}{{c_s}^2} + \frac{m_2}{{c_2}^2}\right) + H_3 X_3(y)\left(\frac{1 - m_3}{{c_s}^2} + \frac{m_3}{{c_3}^2}\right) + \frac{X_4(y)}{{c_s}^2}$$

$$\tilde{\rho}(x) = \rho_s H_1 \chi_1(y) + H_2 \chi_2(y)(\rho_s(1 - m_2) + \rho_2 m_2) + H_3 \chi_3(y)(\rho_s(1 - m_3) + \rho_3 m_3) + \rho_s \chi_4(y)$$

$\chi_i(y)$- characteristic functions of domains G_i, $i = 1,2,3,4$.

When $y = 0,\ t > 0$

$$u(0,t) = u_0(t) \quad (6)$$

At the

$$\rho_s \frac{\partial u}{\partial y}(0,t) = u_1(t) \quad (7)$$

$$u(0,t) = \frac{\partial u}{\partial y}(0,t) = 0 \quad (8)$$

Basic assumptions

1. $u_0(t)$ and $u_1(t)$ - smooth bounded functions
2. $0 \le H_1 \le H_2 \le H_3 \le H_*$- specified values
3. $0 \le c_1, c_2, c_3 \le c_*$ - specified values
4. $0 < \rho_* \le \rho_1, \rho_2, \rho_3 \le \rho^*$- specified values.

$u_0(t) = 101325,\ u_1(t) = 0,\ H_1 = 1000, H_2 = 2000,\ H_3 = 3000.$

For the 1st layer (limestone):

$c_s = 3000,\ c_f = 0,\ \rho_s = 2700.$

For the 2nd layer (sand with oil):

$c_s = 2500,\ c_f = 1330,\ \rho_s = 2250,\ \rho_f = 850.$

For the 3rd layer (clay with water)

$c_s = 2000,\ c_f = 1400,\ \rho_s = 1600,\ \rho_f = 1000.$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

2. QUALITY FUNCTIONAL

In order to find the vector $\vec{V} = (H_1, H_2, H_3, \rho_s, \rho_1, \rho_2, m_2, m_3) \in R^{11}$ we consider problem (5), (7), (8) with given

$I(\vec{V}) = \int_0^T |u_0(t) - u(0,t)^2| dt$

Where $u(0,t) = A(\vec{V})$ - nonlinear operator defined by problem (5), (7), (8) for $\vec{V} \in K$. It is possible to achieve that functional $I(\vec{V})$ could reach its minimum value $I_* = I(\vec{V_*})$ for

3. MINIMIZATION OF FUNCTIONAL

Specify functional $I(\vec{V}) \geq 0$. The

$I(\vec{V}) \geq 0 \ \forall \vec{V} \in K$

According to the definition of the exact lower bound of a set, the sequence $\{\vec{V_n}\}$, $\vec{V_n} \in K$ is found, that is

$I(\vec{V_n}) \to I_*, \ n \to \infty$

(11)

But the set K is compact in $\boldsymbol{R^1}1$ (as a closed bounded set). Therefore we can assume (passing to the sequence if

4. GENERALIZED SOLUTION TO THE PROBLEM (5), (7),(10)

First of all, we should determine what is the solution of problem (5), (7), (8) for fixed $\vec{V} \in K$. Namely, we should find a generalized solution to problem (5),(7),(8) [6], which satisfies the integral identity

$\int_0^T \int_{-\infty}^0 \left\{\frac{1}{\tilde{c}^2(y,\tilde{v})} \frac{\partial u}{\partial t}\frac{\partial \varphi}{\partial t} - \frac{1}{\tilde{\rho}(y,\tilde{v})}\frac{\partial u}{\partial y}\frac{\partial \varphi}{\partial y}\right\}$

(14)

Proof. 1. Reduce the Cauchy problem to an initial-boundary value problem in the domain $\Omega_* = \{y \in R | 0 < y < H_*\}$ where H_*, taken $\sqrt{\alpha}T$ and

$\alpha = \max_{\tilde{v} \in k} \tilde{\rho}(y,\tilde{v}), \tilde{c}^2(y,\tilde{v})$

(16)

2. Solve the problem with the boundary condition $u(H_*, t) = 0$ using the Galerkin method in $\Omega_* \times (0,\mathrm{T})$.

5. FUNCTIONAL CONTINUITY $\boldsymbol{I}(\vec{V})$

Let $u_n(y,t) \equiv 0$ be the solution (generalized) of problem (5), (7), (8), corresponding to the set of parameters $v_n \in k$, $\widetilde{c_n}(y) = \widetilde{c_n}(y, \widetilde{v_n})$, $\widetilde{\rho_n}(y) =$

Recall that, we consider the initial-

$\tilde{c}(y,\tilde{V}), \tilde{\rho}(y,\tilde{V})$ functions where $\tilde{V} \in K$ and compact $K \subset R^{11}$ is determined by conditions (2) - (4). To meet a condition (6) we consider the functional

(9)

some $\tilde{V_*} \in K$ by varying $\tilde{V} \in K$. The value $\tilde{V_*}$ that determines the structure of the domain Ω will be called the solution to problem (1) - (4) [12, 13].

numerical set $M = \{z | z = I(\vec{V}), \ \vec{V} \in K\}$ is bounded below and has an exact lower bound I_* :

(10)

necessary) that

$\vec{V_n} \to \vec{V_*}, \ n \to \infty$

(12)

in the norm of $\boldsymbol{R^1}1$ space. We have to show that

$I(\vec{V_n}) \to I(\vec{V_*}), \ n \to \infty$

(13)

For an arbitrary smooth function $\varphi(y, \psi)$ finite in domain Ω.

Theorem 1. *For arbitrary $\vec{V} \in K$, there exists a unique generalized solution to problem (5), (7), (8) such that*

$\max_t \int_{-\infty}^0 \left\{\left(\frac{\partial u}{\partial t}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right\} dy \leq M$

(15)

where M depends only on the constants in conditions (2) - (4)

from conditions (2) –(4), may be considered more than

3. Show that in the domain.

$\{(y,t) \| y - y_0| \leq \sqrt{\alpha}(t - t_0)\} \ u(y,t) \equiv 0.$

for every $(t_0, y_0) \ 0 < t_0 < T, \ 0 < y_0 < H_*$.

The last proves the boundary condition $u(H_*, t) \equiv 0$.

$\widetilde{\rho_n}(y, \widetilde{v_n})$, $\widetilde{v_n} \to \tilde{v_*}$ for $n \to \infty$ easy to show that

$\widetilde{c_n}(y) \to$

$\tilde{c_*}(y) = \tilde{c}(y, \tilde{v_*})$,

$\widetilde{\rho_n}(y) \to$

$\tilde{\rho_*}(y) = \tilde{\rho}(y, \tilde{v_*})$ $n \to \infty$

boundary value problem equivalent to the

Cauchy problem in a domain $\Omega_* \times (0,\mathrm{T})$ with the additional condition
$u(H_*,t) \equiv 0$
(17)
The estimate (15) suggests that the sequence $\{u_n\}$ is weakly compact in space $\{u_n\},\left\{\frac{\partial u_n}{\partial t}\right\},\left\{\frac{\partial u_n}{\partial y}\right\}$.

$$\left.\begin{array}{l} u_{nk} \to u_* \\ \frac{\partial u_{nk}}{\partial t} \to \frac{\partial u_*}{\partial t} \\ \frac{\partial u_{nk}}{\partial y} \to \frac{\partial u_*}{\partial y} \end{array}\right\} \quad \text{weak in} \quad L_2(\Omega_* \times (0,\mathrm{T}))\ n_k \to \infty$$

or

where $n \to \infty$ for any $\varphi \in L_2(\Omega_* \times (0,\mathrm{T}))$ taking into account the convergence of the coefficients and passing to the limit where $n_k \to \infty$ in identity (14), we obtain the

$$\int_0^T \int_{\Omega_*} \frac{1}{\widetilde{c_*}^2} \frac{\partial u_*}{\partial y}\frac{\partial \varphi}{\partial t} - \widetilde{\rho_*}\frac{\partial u_*}{\partial y}\frac{\partial \varphi}{\partial t}\,dydt = -\int_0^T \varphi(0,t)\,u_1(t)dydt \qquad (18)$$

Further we use the theorem from [8] $W_2^1(\Omega_* \times (0,\mathrm{T})) \to L_2(0,\mathrm{T})$ (on the boundary $y = 0$), which states that every weakly convergent sequence $\{u_n\}$ in converges strongly in [14] $W_2^1(\Omega_* \times (0,\mathrm{T})) \to L_2(0,\mathrm{T})$ (on the boundary $y =$

$\omega_2^1(\Omega_* \times (0,\mathrm{T}))$, which means that subsequences $\{u_{nk}\},\left\{\frac{\partial u_{nk}}{\partial t}\right\},\left\{\frac{\partial u_{nk}}{\partial y}\right\}$ weakly convergent in $L_2(\Omega_* \times (0,\mathrm{T}))$ could be distinguished from sequences

$$\int_0^T \int_{\Omega_*} u_{nk} \cdot \varphi dydt \to \int_0^T \int_{\Omega_*} u_* \cdot \varphi dydt$$
$$\int_0^T \int_{\Omega_*} \frac{\partial u_{nk}}{\partial t} \cdot \varphi dydt \to \int_0^T \int_{\Omega_*} \frac{\partial u_*}{\partial t} \cdot \varphi dydt$$
$$\int_0^T \int_{\Omega_*} \frac{\partial u_{nk}}{\partial y} \cdot \varphi dydt \to \int_0^T \int_{\Omega_*} \frac{\partial u_*}{\partial y} \cdot \varphi dydt$$

identity

0). That is, the sequence $\{L_2(0,\mathrm{T})\}$
$$\int_0^T |u_{nk}(0,t) - u_*(0,t)|^2\,dt \to 0, \qquad n_k \to \infty$$
This means functional continuity $(\vec{V})$:
$$I(\overrightarrow{V_{nk}}) = \int_0^T |u_{nk}(0,t) - u_*(t)|^2\,dt \to \int_0^T |u_*(0,t) - u_*(t)|^2 dt, \qquad n_k \to \infty$$
$$I(\vec{V_*}) = I_*$$
which completes the proof of the theorem.

6. MATHEMATICAL MODEL DISCRETIZATION BY THE FINITE ELEMENT METHOD

6.1 Weak formulation

We derive an equivalent formulation for the one-dimensional wave equation,using [15, 16].

Let the function $v(x)$ be differentiable and $v(0) = v(1) = 0$, therefore it is equal to zero at the boundaries of our domain [17]. We multiply our wave equation by this function, integrate over the interval, and use integration by parts to obtain the following

$$0 = \int_0^1 \left(\frac{\partial^2 u}{\partial t^2}(x,t) - \frac{c^2}{\rho}\frac{\partial^2 u}{\partial x^2}(x,t)\right) v(x)dx.$$

$$0 = \int_0^1 \frac{\partial^2 u}{\partial t^2}(x,t)v(x)dx - \frac{c^2}{\rho}\left(\left[\frac{\partial u}{\partial x}(x,t)v(x)\right]_0^1 - \int_0^1 \frac{\partial u}{\partial x}(x,t)\frac{\partial v}{\partial x}(x,t)dx\right) = \int_0^1 \frac{\partial^2 u}{\partial t^2}(x,t)v(x)dx + \frac{c^2}{\rho}\int_0^1 \frac{\partial u}{\partial x}(x,t)\frac{\partial v}{\partial x}(x,t)dx = a(\mathrm{u}(\mathrm{x},\mathrm{t}),\mathrm{v}(\mathrm{x}) \qquad (19)$$

where

$$a(u(x,t),v(x)) = \int_0^1 \frac{\partial^2 u}{\partial t^2}(x,t)v(x)dx + \frac{c^2}{\rho}\int_0^1 \frac{\partial u}{\partial x}(x,t)\frac{\partial v}{\partial x}(x,t)dx \qquad (20)$$

$$a(u(x,t),v(x)) = 0 \qquad (21)$$

This is called a weak formulation, since the function $u(x,t)$ should keepcondition (21) towards some functions $v(x)$, called test functions.

6.2 Nodes and Elements

We divide our interval [0,1] into N elements and distribute it into an equal number of nodes. The

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

size of each element $h = \frac{1}{N}$, thus the nodes are given by

$$x_i = (i - 1)h \quad i = 1,\ldots,N + 1. \quad (22)$$

The elements as an interval

$$E_i = [x_i, x_{i+1}] \quad i = 1,\ldots,N. \quad (23)$$

Nodes and elements are shown in Figure 1.

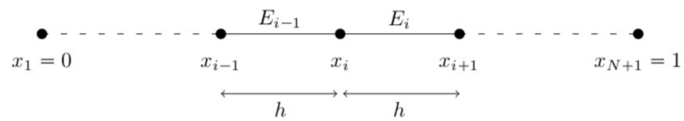

Fig 1. Partitioning our interval [0,1] into elements and nodes [13].

6.3 Basis functions

Let the set be given where the functions $v(x)$ are defined. These functions are represented as a linear combination of basis functions and are equal to zero at the boundaries [18]. Let's take $N + 1$ of basis functions, one for each node, and denote them $\varphi_i(x), i = 1,\ldots,N + 1$.

Then

$$v(x) = \sum_{i=1}^{N+1} v_i \varphi(x), \text{ with } v_i \epsilon R$$
$$v(0) = v(1) = 0. \quad (24)$$

The basis functions for receiving piecewise smooth linear functions for ele-ments are defined as follows: for $j = 2,\ldots,N$ we define basis functions that are equal to zero everywhere, except for the elements E_{j-1} and E_j, for which they are linear. Basic functions can be written as

$$\varphi_j(x) = \begin{cases} \frac{x - x_{j-1}}{h} & x_{j-1} \le x \le x_j \\ \frac{x_{j+1} - x}{h} & x_j \le x \le x_{j+1} \\ 0 \end{cases}, \quad \text{for } j = 2,\ldots,N. \quad (25)$$

We define the basis functions for $j = 1, N + 1$ which lie on the boundaries of the interval so that they are not equal to zero for the elements E_1 and E_N accordingly and are equal to zero for the rest. These functions can berepresented as

$$\varphi_1(x) = \begin{cases} \frac{x_2 - x}{h} & x_1 \le x \le x_2 \\ 0 \end{cases}, \quad \varphi_{N+1}(x) = \begin{cases} \frac{x - x_N}{h} & x_N \le x \le x_{N+1} \\ 0 \end{cases} \quad (26)$$

The basis functions are shown in Figure 2.

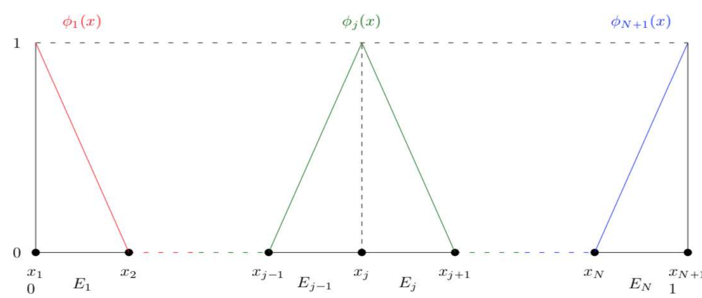

Fig 2. Piecewise smooth linear basis functions $\varphi_1, \varphi_j, \varphi_{n+1}$ [13].

An important property of these basis functions is that the value of the function $\varphi_i(x)$ must be equal to 1 on the node x_i and to zero on the other nodes. Therefore

$$\varphi_i(x_j) = \begin{cases} 1, & i = j \\ 0, & i \ne j \end{cases} \quad (27)$$

Then the property is preserved that $v(x_i) = v_i, i = 1,\ldots,N + 1$.

Journal of Theoretical and Applied Information Technology
31[st] August 2021. Vol.99. No 16

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

6.4 Solution by finite elements

Using the just derived basis functions, we determine the solution by finite elements and test functions of finite elements:

$U(x,t)=\sum_{i=1}^{N+1} U_i(x)\varphi(x)$,

$V(x)=\sum_{i=1}^{N+1} V_i\varphi_i(x)$.

(28)

The statement of finite elements can be written as follows

$a(U\ (x,t),V\ (x)) = 0$,

(29)

Thus, if (30) is preserved for all basis functions φ_i, then it is preserved for all

For each basis function $\varphi_i(x)$ that is not on the boundary for $=2,\dots,N$, we can write as

$$0=a\left(U\ (x,t),\varphi_j\ (x)\right)=$$

where

$$T_{i,j}=\int_0^1 \varphi_i\ (x)\varphi_j\ (x)dx$$

As a result, we get $N-1$ linear ordinary differential equations. For two basis functions that remain unaffected, we use additional equations (32) to preserve the boundary conditions.

$\ddot{U}_1(t)=0 \quad \ddot{U}_{N+1}(t)=0$ (32)

For these $N+1$ linear differential equations, we define a vector $U\ (t)=(U_1(t),\dots,U_{N+1}(t))$ and write differential

$\int_0^1 f(x)dx=\sum_{i=1}^{N+1} f(x)dx$.

(34)

Since the basis functions have two elements maximally that are not equal to zero, the values $T_{i,j}$ and $S_{i,j}$ could be calculated by several elements integration where both basis functions are not equal to zero. This allows to calculate the following coefficients:

as a scalar product of a solution by finite elements and test functions keeping the initial and boundary conditions. To find a solution by finite elements, it is enough to test only with basis functions. This follows from the linearity of the second variable $a(u,v)$ and therefore follows

$$0=a\left(U\ (x,t),V\ (x)\right)= a(U\ (x,t),\textstyle\sum_{i=1}^{N+1} V_i\varphi_i(x))= \sum_{i=1}^{N+1} V_i a\left(U\ (x,t),\varphi_i(x)\right). \quad (30)$$

functions $V(x)$.

$$=\int_0^1\left(\sum_{i=1}^{N+1}\frac{\partial^2}{\partial t^2}U_i(t)\varphi_i\ (x)\varphi_j\ (x)\right)dx + \frac{c^2}{\rho}\int_0^1\left(\sum_{i=1}^{N+1}U_i(t)\frac{\partial}{\partial x}\varphi_i\ (x)\frac{\partial}{\partial x}\varphi_j\ (x)\right)dx =$$

$$=\sum_{i=1}^{N+1}\frac{\partial^2}{\partial t^2}U_i(t)\int_0^1\varphi_i\ (x)\varphi_j\ (x)dx + \frac{c^2}{\rho}\sum_{i=1}^{N+1}U_i(t)\int_0^1\frac{\partial}{\partial x}\varphi_i\ (x)\frac{\partial}{\partial x}\varphi_j\ (x)dx =$$

$$=\sum_{i=1}^{N+1}\frac{\partial^2}{\partial t^2}\ddot{U}_\iota(t)T_{i,j}+\frac{c^2}{\rho}\sum_{i=1}^{N+1}U_i(t)\,S_{i,j}$$

(31)

$$S_{i,j}=\int_0^1\frac{\partial}{\partial x}\varphi_i\ (x)\frac{\partial}{\partial x}\varphi_j\ (x)dx$$

equation (31) with coefficients T and S, except the first and last rows derived in (32).

$T\ddot{U}(t)+\frac{c^2}{\rho}\ SU(t)=0$

(33)

The values $T_{i,j}$ and $S_{i,j}$ could be easily calculated by dividing their integrals into all elements.

$$T_{i,j}=\int_{x_{i-1}}^{x_i}\varphi_i\ (x)^2dx+\int_{x_i}^{x_{i+1}}\varphi_i\ (x)^2dx=\frac{2}{3}h,$$

$$T_{i,j-1}=\int_{x_{i-1}}^{x_i}\varphi_i(x)\ \varphi_{i-1}(x)dx=\frac{1}{6}h,$$

$$S_{i,j}=\int_{x_{i-1}}^{x_i}\left(\frac{\partial\varphi_i(x)}{\partial x}\right)^2dx\ +\int_{x_i}^{x_{i+1}}\left(\frac{\partial\varphi_i(x)}{\partial x}\right)^2dx\ =\frac{2}{h},$$

$S_{i,j-1} = \int_{x_{i-1}}^{x_i} \frac{\partial \varphi_i(x)}{\partial x} \frac{\partial \varphi_{i-1}(x)}{\partial x} dx = -\frac{1}{h}.$

(35)

For all other values of the basis functions, in order not to overlap nonzero elements, their multiplication is zero.

The result is a second-order linear differential equation with constant coef-

ficients (33). The solution of this equation can be used as a numerical solution of the one-dimensional wave equation with finite elements.

6.5 Time approximation

Let consider two numerical methods for solving differential equation (33). For the first method, we use the second-order approximation for the second time derivative

$$\ddot{U}(t) \approx \frac{U(t+dt)-2U(t)+U(t-dt)}{dt^2}$$

(36)

substituting (36) into the differential equation (33), we obtain

$$T\left(\frac{U(t+dt)-2U(t)+U(t-dt)}{dt^2}\right) + \frac{c^2}{\rho} SU(t) = 0$$

(37)

After some algebraic transformations, this equation can be rewritten as

$$U(t+dt) = -\frac{c^2}{\rho} dt^2 T^{-1} SU(t) +$$

$$2U(t) - U(t-dt)$$

(38)

The initial values $U(0)$ and $\dot{U}(0)$ can be used for the initial step of the first order approximation

$$U(dt) = U(0) + dt\dot{U}(0),$$

(39)

and use equation (38) for subsequent iterations.

For (38), an inverse sparse matrix T is required, but the inverse matrix generally is not sparse. For a more efficient calculation, we use

$$T\left(\frac{U(t+dt)-2U(t)+U(t-dt)}{dt^2}\right) = -\frac{c^2}{\rho} SU(t)$$

(40)

First, we calculate the right-hand side and $\frac{U(t+dt)-2U(t)+U(t-dt)}{dt^2}$ from which we could calculate $U(t+dt)$. Using this method, the inverse matrix calculation is not required,

get a linear system to solve

but the system of linear algebraic equations at each iteration is required, which is more accurate than the inverse matrix calculation.

The second method for solving equation (33) is to rewrite it as a first-order linear differential equation. Then we solve

(33) in two steps.

$$\begin{cases} T\left(\frac{\dot{U}(t+dt)-\dot{U}(t)}{dt}\right) = -\frac{c^2}{\rho} SU(t), \\ U(t+dt) = U(t) + dt\dot{U}(t) \end{cases}$$

(41)

In the first equation, we solve a system of linear equations to find $\dot{U}(t+dt)$ and the second equation can be solved explicitly.

7. NUMERICAL RESULTS OBTAINING BY THE FINITE DIFFERENCE METHOD

7.1 Mathematical Model Discretization

7.2 Scheme

Using Taylor series we expand our mathematical model by [19, 18, 20].

$$\frac{U_i^{j+1}-2U_i^j+U_i^{j-1}}{\Delta t^2} = \frac{c^2}{\rho} \frac{U_{i+1}^j-2U_i^j+U_{i-1}^j}{\Delta y^2}$$

(42)

$$U_i^{j+1} = \frac{c^2 \Delta t^2}{\rho} \frac{U_{i+1}^j-2U_i^j+U_{i-1}^j}{\Delta y^2} + 2U_i^j - U_i^{j-1}$$

(43)

Initial conditions:

$$U_i^0 = 0 \qquad i = 0, \dots, N_y$$

(44)

$$U_i^1 = U_i^0 \qquad i = 0, \dots, N_y$$

(45)

Boundary conditions:

$$U_0^j = u_0(t_j) \qquad j = 0, \dots, N_t$$

(44)

$$\rho \frac{U_1^j - U_0^j}{\Delta y} = u_1(t_j) \qquad j = 0, \dots, N_t$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

(45)

8. NUMERICAL RESULTS

Solution of our problem is shown down below, check other experiments in [21,22, 23, 24].

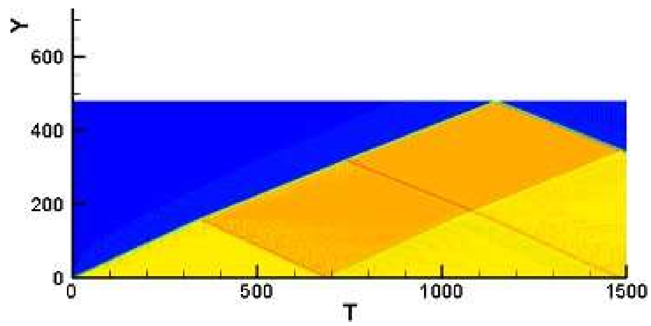

Fig 3. Acoustic wave propagation into three domains using the finite difference method (T - time (s), Y - depth (m)).

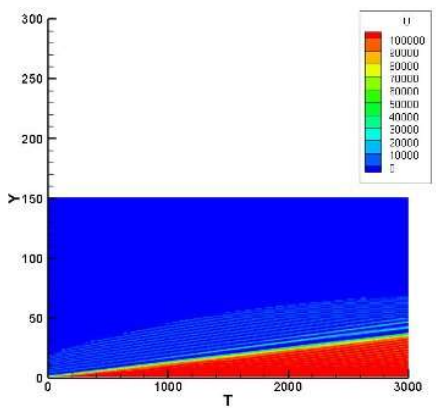

Fig 4. Acoustic wave propagation graph in one domain using the finite element method (T - time (s), Y - depth (m)).

9. CONCLUSION

Using both the finite difference method and the finite element method, a numerical solution of the one-dimensional poroelastic wave equation in a mixed domain was found. Their graphical propagations are shown also.

When comparing the numerical results, it can be noted that the finite element method with increasing time becomes unstable.

In this regard, to obtain further propagation of the wave and its reflection in other domains becomes impossible.

Also, the graph of finite differences did not show the condition of a semi- infinite layer.

This work was done in order to further complicate the domain and to transit to two- and three-dimensional space.

For this reason, the finite-element method in the one-dimensional domain is not considered at the given moment. Further research is required.

ACKNOWLEDGEMENTS

The work was performed under the grant №AP09261179 of the Committee of Science of the Ministry of Education and Science of the Republic of Kazakhstan (2021-2023).

REFERENCES:

[1] Poesio P., Ooms G., Schraven A., and van der Bas F. Theoretical and experimental investigation of acoustic streaming in a porous material. // Phys. Rev., 2010, E **66**.
[2] Hamzehpour H., Asgari M., and Sahimi M. Acoustic wave propagation in heterogeneous two-dimensional fractured porous media. // Phys. Rev., 2016, E **93**.
[3] Allard J. F., Noureddine A. Propagation of Sound in Porous Media. // Wiley, 2009.
[4] Meirmanov A., Nurtas M. Mathematical models of seismics in com- posite media: elastic and poro-elastic components. // Electronic Journal of Differential Equations, **2016**, pp. 1–22.
[5] Meirmanov A. A description of seismic acoustic wave propagation in porous media via homogenization. // SIAM J. Math. Anal., 2008, **40**(3), pp. 1272–1289.
[6] Biot M. Theory of propagation of elastic waves in a fluid-saturated porous solid. I. Low-frequency range. // Journal of the Acoustical Society of America, 1955, pp. 168–178.
[7] Ovsyannikov L. Introduction into continuum mechanics. // Novosibirsk, russian edition, 1977.
[8] Meirmanov A. Mathematical models for poroelastic flows. // Atlantis Press, 2013.
[9] Meirmanov A. Homogenized models for filtration and for acoustic wave propagation in thermo-elastic porous media. // European Journal of Applied Mathematics, 2008, pp. 259–284.
[10] Meirmanov A. Double porosity models in incompressible poroelastic me- dia. // Mathematical Models and Methods in Applied Sciences, 2010, pp. 635–659.
[11] Meirmanov A. Derivation of equations of seismic and acoustic wave propa- gation and equations of filtration via homogenization of periodic structures. // Journal of Mathematical Sciences, 2009, pp. 11–172.
[12] Meirmanov A., Nurtas M. Direct and inverse acoustic problems in porous media. Part I: formation of approximation solutions. // Herald of Kazakh-British Technical University, 2016.
[13] Nurtas M. Direct and inverse problems of the equation of acoustics ina porous elastic medium. // Herald of Kazakh-British Technical University, 2015.
[14] Meirmanov A. Nguetseng's two-scale convergence method for filtration and seismic acoustic problems in elastic porous media. // Siberin Mathematical Journal, 2007, pp. 519–538.
[15] Larson M.G., Fredrik B. The Finite Element Method: Theory, Implementation, and Applications. // Springer-Verlag Berlin Heidelberg, 2010.
[16] Larson M., The Finite Element Method: Theory, Implementation, and Applications. // Springer, 2013.
[17] Mazumder S. Numerical Methods for Partial Differential Equations. // Aca- demic Press, 2016.
[18] Chung T. Computational Fluid Dynamics. // Cambridge University Press, 2010.
[19] Samarskii A. Introduction into numerical methods. //Lan׳, 2005.
[20] Lehtinen J. Time-domain numerical solution of the wave equation. // (2003).
[21] Santillán A., Bozhevolnyi S.I. Demonstration of slow sound propaga- tion and acoustic transparency with a series of detuned resonators. // Phys. Rev., 2014.
[22] Malinouskaya I., Mourzenko V.V., Thovert J.F., Adler P.M. Wave propagation through saturated porous media. // Phys. Rev., 2008.
[23] Hamzehpour H., Kasani F.H., Sahimi M., Sepehrinia R. Wave prop- agation in disordered fractured porous media. // Phys. Rev., 2014.
[24] Tournat V., Pagneux V., Lafarge D., Jaouen L. Multiple scattering of acoustic waves and porous absorbing media. // Phys. Rev., 2004.
[25] Baishemirov Zh., Tang J.-G., Imomnazarov K., Mamatqulov M. Solving the problem of two viscous incompressible fluid media in the case of constant phase saturations. // Open Engineering, 2016, 6(1), pp. 742-745.
[26] Nurtas M., Baishemirov Zh., Ydyrys A.,

Altaibek A. 2-D finite element method using escript for acoustic wave propagation // ACM International Conference Proceeding Series, 2020, 3410774

[27] Panfilov M, Baishemirov Zh., Berdyshev A. Macroscopic model of two-phase compressible flow in double porosity Media // Fluid Dynamics, 2020, 55(7), pp. 936-951.

[28] Bekbauov B., Berdyshev A., Baishemirov Zh. Numerical simulation of chemical enchanced oil recovery processes // CEUR Workshop Proceeding, 2016, 1623, pp. 664-676.