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NEW APPROACH TO CONSTRUCT A NEW ISLAMIC GEOMETRIC PATTERNS USING THE HASBA METHOD

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ABSTRACT

By its beauty and the symmetry of its patterns, Islamic geometric Pattern (IGP) art has always attracted the interest of scientists. These methods to make geometric patterns were analyzed and proposed by several author scientists. In this paper, we present a new method called the ‘Âark method’ to construct new geometric patterns by using the method called “Hasba” (measurement). That is widely adopted by Moroccan artisans. This proposed method, based on the concept of symmetry, allows building numerous patterns by systematic and dynamic processes. Symmetrical patterns are constructed from an asymmetric element called the ‘fundamental region’ by applying reflections and rotations. Compared to the classical ‘Ribbon method’ which will be explained later on, the proposed method is tolerant to construct more than 200 patterns with different visuals. This demonstrates the ability of the proposed method to construct various motifs and can easily be used in a softwere program, for an automatic generation of a large number of patterns without the intervention of the human being, to overcome the problem of limitation of the patterns posed by the craftsmen of Islamic geometric art.

Keywords: *Islamic Patterns, Symmetry, Islamic Art, Geometric Art, Âark Method.*

1. INTRODUCTION

Over centuries, the Islamic world has had great artistic and decorative traditions, we find artistic treasures of unrivaled beauty. The Islamic ornamental art that is found on diverse materials such as tiles, bricks, wood, brass, and plaster may be classified into two types: the tri-dimensional ornamental art called ‘Muqarnas’ (Stalactites) and the plane ornamental ‘Arabesque‘, which may also be divided into two categories: floral and “Tastir” or Geometric Patterns which are the subject of the present paper as illustrated in Figure.1.

Method “Hasba” or (measurement) [1], of construction of the geometric patterns encountered in western Islamic art, especially in Morocco. Is widely adopted by the Moroccan craftsmen. It's based on geometric rules and the concept of symmetry. It can be adapted to various materials: plaster, metal, marble.... A large variety of simple patterns, as well as sophisticated patterns, are obtained by tracing grids with precise criteria of measurement within a polygonal frame.

In this study, our work is focused only on square patterns. Patterns with octagonal, triangular, and other shapes are generally deduced in their design from the square.

This paper aims to describe a new approach by proposing a new method called ‘Âark method’ to construct new Islamic geometric patterns (IGP) that respect some precise criteria of measurement used in the "Hasba" method.

The rest of the paper is organized as follows, in section 3: we describe the method Hasba with ‘Ribbon or Qtib’. In section 4: we apply the recent proposed approach. Section 5: gives the results of our proposed method. Section 6 contains the conclusion and future works.

Figure 1: The motif of Moroccan geometric art produced according to the ‘Hasba’ method.

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2. RELATED WORK

Islamic geometric art has always attracted the interest of scientists. Several authors focused their work on analysis and proposing many methods of construction of these Islamic geometric patterns, Castera [2], a Spanish author, presented a practical method of constructing Moroccan zellijs (ceramic tiles).

Dewdney's method [3], Arabesque software allows the users to construct star patterns. Lu and Steinhardt [4] found out a wide usage of subdivisions in traditional Islamic design systems e.g. overlaid Penrose kites and darts on Iranian designs. Cromwell [5] has described a tiling-based method to construct Islamic geometric designs.

Izadi et al [6] proposed a computerized and heuristic algorithm to make geometrics used in traditional building patterns and tiling. They determine "fundamental area" repeated in the entire sketch so they create the entire sketch by simply drawing the fundamental area, using the geometric technique. Finding the fundamental area or the "fundamental region" is not a new problem. Lalvani [7] has also developed a method to construct complex geometric patterns based on infinite classes of subdivisions of 'the fundamental region'.

Aboufadil et al [8] has shown that it is possible to obtain the 17 plane groups of symmetry generated from n-fold rosettes by the Hasba method. By repeating the rosettes, considered as the basic motif, they make up patterns having various symmetry groups.

Lee [9] shows that a pattern should be extended by drawing pieces of lines from the middle of tiling edges. He illustrates the polygons-in-contact technique.

In this article, we present for the first time the method 'Âark' a simplified process of the 'Ribbon' method or 'Qtib' method used in the construction of Islamic geometric patterns (IGPs).

3. CONSTRUCTION OF THE RIBBON PATTERN RESPECTING THE HASBA METHOD

In this section, we give a brief reminder of the ribbon method, commonly used by the artisans working with rulers and compasses for the construction of Islamic geometric patterns (IGP).

The Hasba (i.e. measurement) is the method of construction of Islamic geometric patterns (IGP) that is mostly used by Moroccan craftsmen.

The artisans start their work by drawing the general frame of the pattern, which is often the square; rectangular, octagonal and other polygonal designs are not uncommon. On the sides of the frame, they define an empirical unit division q. The sides of the square have thus a width L equal to a multiple of q: L = h q. The ratio h is the specific measure or 'Hasba' of the pattern. The type of pattern achieved depends strongly on h, which may be an integer or rational number greater than eight. Figure. 2.

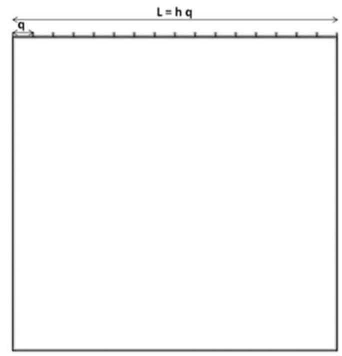

*Figure 2: Assessment of the dimensions of the square frame with the side L=h*q.*

The construction of an Islamic geometric pattern with ribbon method is done according to the following four steps which will be explained after wards:

- Construction of the underlying grid
- Tracing of the segments inside the fundamental region.
- Application of the elements of symmetry (4-fold axis and mirrors) to the structural unit.
- Construction of several patterns.

3.1 Construction of The Underling Grid

We draw 8 pairs of concentric circles with diameter respectively 4q and 2q at the corner and in the middle of the sides of the frame Figure. 3.

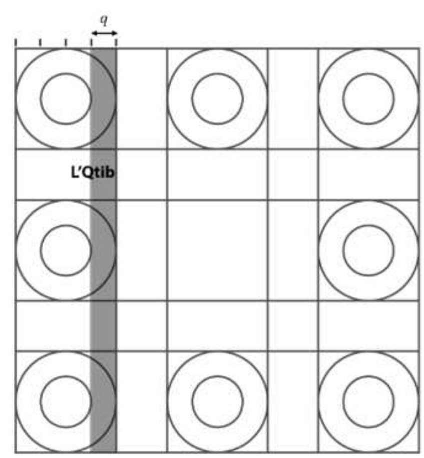

Figure 3: The eight concentric circles and Unit of measure (qassma) q.

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3.2 Tracing of the Segments inside the Fundamental Region.

In this section, we draw the tangents to all circles. We obtain an underlying grid constituted by four sets of crossed parallel lines. Two and two sets are related by the 4-fold axis rotations located at the center of the square, mirror reflections in lines joining the mid of its sides, and reflections in its diagonal. The strip delimited by the vertical (respectively horizontal) tangents to the small circles and the vertical (respectively horizontal) tangents to the large circle is called Ribbon ('Qtib') whose width is q. Figure. 4.

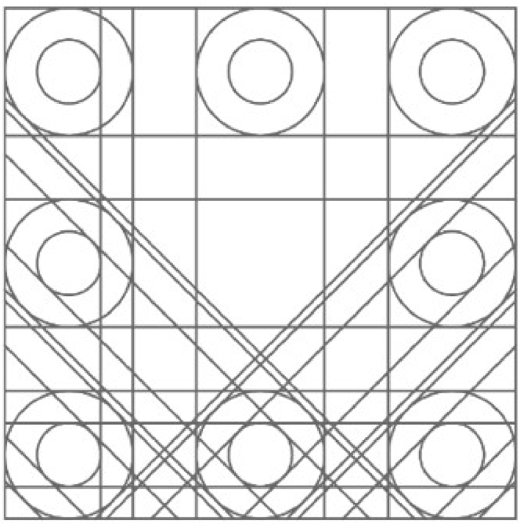

Figure 4: The underling grid

A small region called fundamental region is constructed, in which we define a structural unit to which we apply the elements of symmetry mirrors m and axis of order 4, Figure. 5.

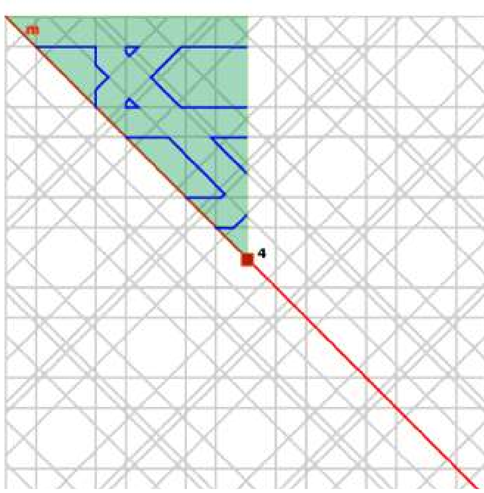

Figure 5: The fundamental region.

3.3 Application of the elements of symmetry (4mm).

By application of the element of symmetry, the first part of the geometric pattern is the image of the second part and we will make a successive rotation of 90°, 180°, and 270° around a center point of our pattern. For each point of our segment plot in the asymmetric zone, we will apply one of the equations Eq. 1, 2, and 3 depending on the angle of rotation.

For rotation $\theta = 90°$ the general formula is as follows:

$$\begin{cases} x_r = -(y - c_y) + c_x \\ y_r = x - c_x + c_y \end{cases} \quad (1)$$

With:

P(x, y): indicates the starting point
Pr (x_r, y_r): image of P by rotation of 90°
C (c_x, c_y): Center of rotation.

For rotation $\theta = 180°$ the general formula is as follows:

$$\begin{cases} x_r = 2 * c_x - x \\ y_r = 2 * c_x - y \end{cases} \quad (2)$$

For rotation $\theta = 270°$ the general formula is as follows:

$$\begin{cases} x_r = y - c_x + c_x \\ y_r = c_x - x + c_y \end{cases} \quad (3)$$

3.4 Islamic geometric pattern (IGP)

The fundamental region is considered the heart of the motif. Indeed a wide variety of patterns can be obtained from the same underlying grid by simply changing the structural unit Figure. 6.

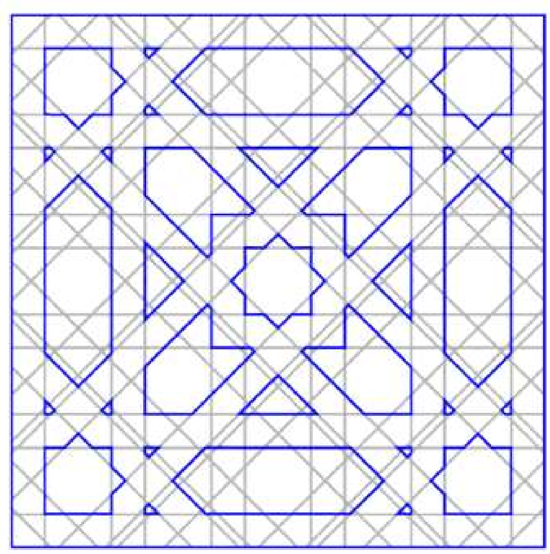

Figure 6: The Islamic geometric pattern (IGP)

4. PROPOSED APPROACH

In this paragraph, we will present our new method called 'Âark method', to construct new Islamic geometric patterns that respect the rules of the 'Hasba' method.
The main difficulty encountered by using the ribbon method to construct the pattern is the complexity of the underlying grid. The fundamental region, which contains 165 segments complicates the process to construct Islamic geometric patterns, Figure. 7.

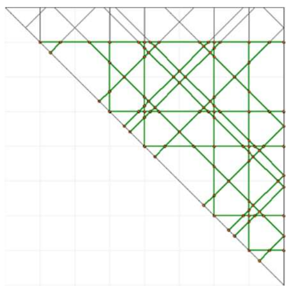

Figure 7: The 165 segments of the fundamental region funded by Ribbon method.

The objective of this work is to give a solution to the artisans to construct new Islamic patterns simply by using the proposed method called 'Âark', which is very simple and gives more results if compared with the ribbon method, moreover, we move with the ribbon method from 165 segments in the fundamental region to 32 segments which make our new underlying grid simpler.

4.1. Transition From Ribbon to Âark Patterns

To simplify the construction of the Islamic geometric patterns by artisans, we have transformed the ribbon to a strand by tending the ribbon's thickness q towards zero. We thus obtain a stranded pattern having a width L = (h-1) q.
We define h '= h -1 as the new hasba of the strand pattern as showing in the Figure. 8.

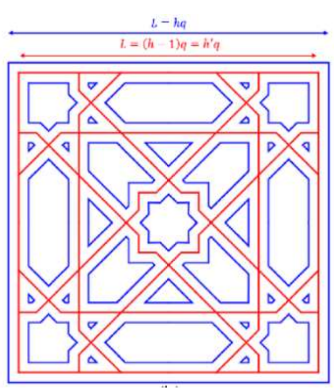

Figure 8: Pattern made with the Ribbon (Qtib)

4.2. Construction of The Âark Patterns

The strand patterns are built in a square frame with side L = (h-1) * q, in this work h =16 and q=30 according to the following process:

4.2.1. Drawing of circles

We draw eight circles, Figure.9, of radius $^3/_2\, q$ with center located at : $C_{ij}\big(O_{ij}(x_{ij}, y_{ij});\ R\big)$; where i,j=1,3 and (i, j) ≠ (2,2).

$$x_{ij} = \left(\frac{L}{2} - \frac{3}{2}q\right)\ (j-2);$$
$$y_{ij} = \left(\frac{L}{2} - \frac{3}{2}q\right)(2-i).$$

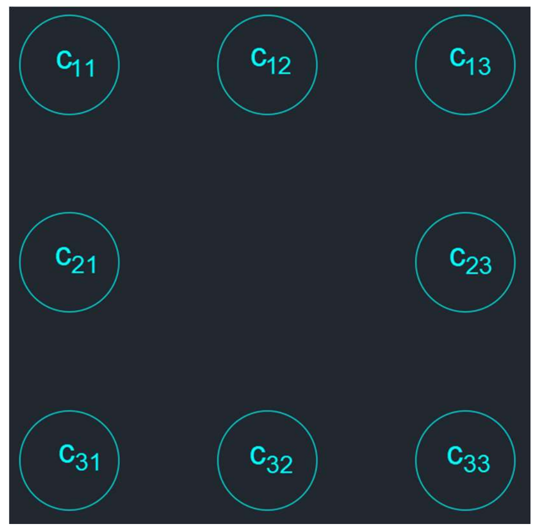

Figure 9: Circles (Cij, R=3/2 q).

4.2.2. Tangents to all circles

In this step, we will trace the different tangents for the different circles. We need to draw four sets of parallel lines: horizontal, vertical, and diagonal lines whose slope of the tangent line α = 1 and α = -1.

4.2.2.1. Horizontal and vertical tangents

The circles C_{i1} for i=1, 3 allow drawing all horizontal and vertical tangents defined by the equations Eq. (4) and in the same way for the circles C_{1j}, Eq. (5).we draw all vertical tangents, Figure. 10.

$$y = y_{i1} \pm \frac{3}{2}q \ ;\ i = 1{,}3 \qquad (4)$$
$$y = y_{1j} \pm \frac{3}{2}q \ ;\ j = 1{,}3 \qquad (5)$$

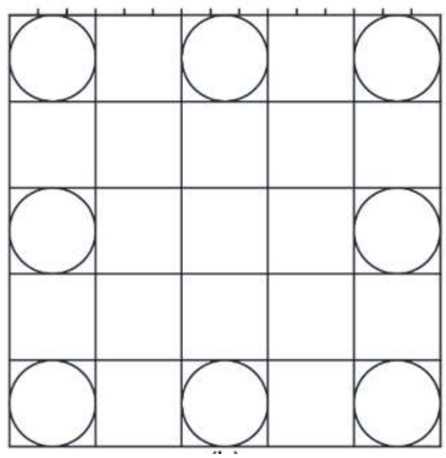

Figure 10. Horizontal and Vertical tangents

4.2.2.2. Oblique tangents

The oblique tangents are drawing according to the following steps:

- Step 1: Determination of the coordinates of the intersection points:

For each circle, we have four points of intersection, Figure. 11. We begin by finding the coordinates of these points and then find the equation of each tangent line at this point.

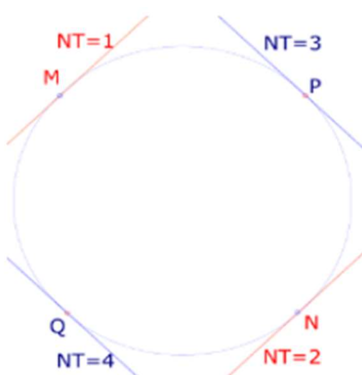

Figure 11. The points of intersection of the oblique tangents.

The coordinates of $M(x_0 - d_1, y_0 + d_2)$, with $d_1 = r * \sin(\beta)$, $d_2 = \cos(\alpha)$ or $\alpha = \beta = 45$. We deduce $\cos(\alpha) = \sin(\alpha) = \sin(\beta) = \cos(\beta) = \frac{\sqrt{2}}{2}$ and $d_1 = d_2$.Figure. 12.

$M(x_0 - r * \sin(\beta), y_0 - \cos(\alpha))$ Then be written, Eq. (6).

$$M\left(x_0 - q\frac{3\sqrt{2}}{4}, y_0 + q\frac{3\sqrt{2}}{4}\right) \quad (6)$$

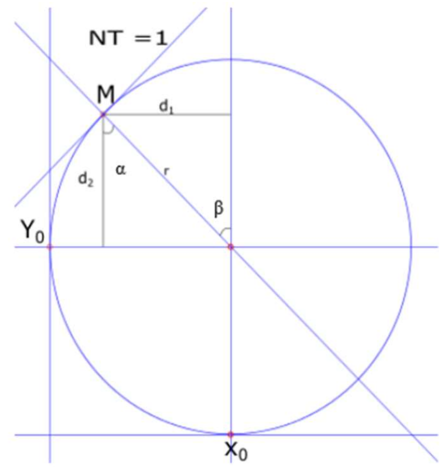

Figure 12. Coordinates of the point M

With the same way, the points N, P and Q can be written, Eqs. (7), (8) and (9), as showing in Figures. 13, 14 and 15.

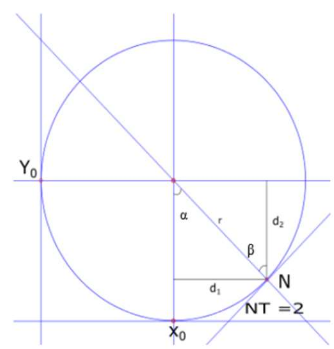

Figure 13. Coordinates of the point N

$$N\left(x_0 + q\frac{3\sqrt{2}}{4}, y_0 - q\frac{3\sqrt{2}}{4}\right) \quad (7)$$

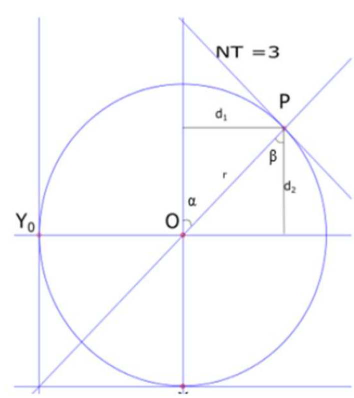

Figure 14. Coordinates of the point P.

$$P\left(x_0 + q\frac{3\sqrt{2}}{4}, y_0 + q\frac{3\sqrt{2}}{4}\right) \quad (8)$$

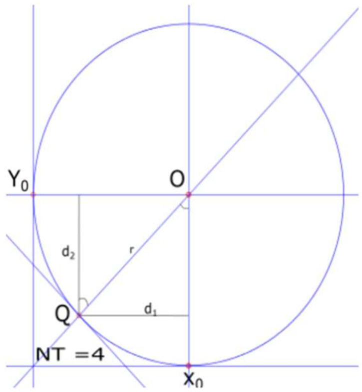

Figure 15. Coordinates of the point Q

$$Q\left(x_0 - q\frac{3\sqrt{2}}{4}, y_0 - q\frac{3\sqrt{2}}{4}\right) \quad (9)$$

- Step 2: Define the lines tangents of the circles:

The coefficient α of tangents NT = 1, NT = 2 equals 1 and the coefficient α of tangents NT = 3 and NT= 4 equals -1.For the tangents NT=1 and NT=2.

$y = ax + b$ We deduce $b = y - x$.

Point $M\ (x_m, y_m)$ is the intersection of the tangent. X_m and Y_m check the equation of the tangent NT=1 Eq. (10):

1. $b = y_m - x_m$
2. $b = y_0 + q\frac{3\sqrt{2}}{4} - x_0 + q\frac{3\sqrt{2}}{4}$
3. $b = y_0 - x_0 + q\frac{3\sqrt{2}}{2}$.

$$y = x + y_0 - x_0 + q\frac{3\sqrt{2}}{2} \quad (10)$$

With the same way, for the points $N(x_n, y_n)$, $P(x_p, y_p)$ and $Q(x_q, y_q)$, the equations of tangents can be written, Eqs. (11), (12) and (13).

$$y = x + y_0 - x_0 - q\frac{3\sqrt{2}}{2} \quad (11)$$

$$y = -x + y_0 + x_0 + q\frac{3\sqrt{2}}{2} \quad (12)$$

$$y = -x + y_0 + x_0 + q\frac{3\sqrt{2}}{2} \quad (13)$$

- Step 3: Define the segments tangents to the circles :

After having determined all the equations of the lines tangent to the circles, for the patterns we need only the segments and not the lines, then we proceeded as follows to determine the ends of these lines' tangent.

The ends are calculated from the intersection between the lines and the edges of the grid, Defined by the following equations, Eqs. (14), (15), (16) and (17).Figure. 16.

$$y = 0 \quad (14)$$

$$x = 0 \quad (15)$$

$$y = 15q \quad (16)$$

$$x = 15q \quad (17)$$

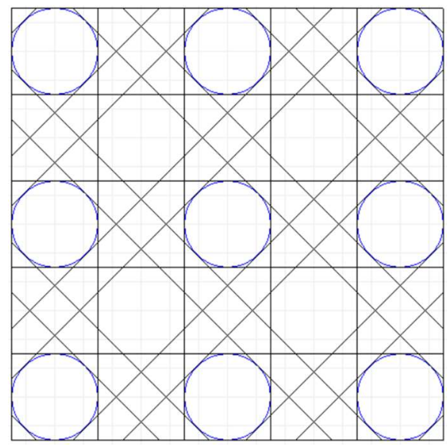

Figure 16: The new underlying Grid

4.2.3. Determination of the segments in the fundamental region

After drawing the underlying grid, we will determine a fundamental region Figure. 17, which will be drawn our pattern according to the new Âark method.

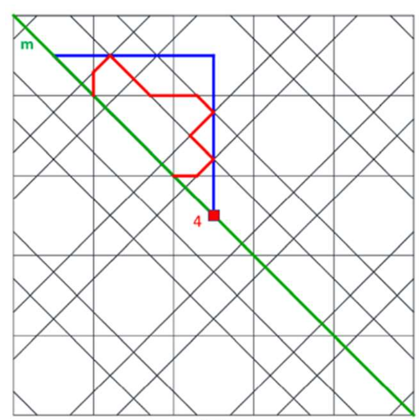

Figure 17. Fundamental region of the IGP.

To do this we will proceed to find all the segments inside our structural unit, through the intersection of all the tangents that we have already drawn from our new underlying grid with the edges of the fundamental region, Figure. 18, which have to use the following equations, Eqs. (18), (19), and (20).

$$y = -x + 15q \quad (18)$$
$$x = {}^{15}/_{2}\, q \quad (19)$$
$$y = 15q \quad (20)$$

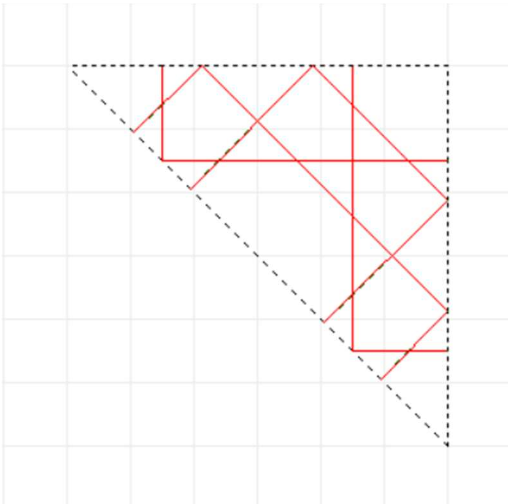

Figure 18: The 10 Segments of the fundamental region. Founded by 'Âark method'

To construct the new Islamic geometric patterns (IGPs) with our new Âark method, we have to find all the intersections points that will allow us to divide all segments in the structural unit, as showing in the Figure. 19.

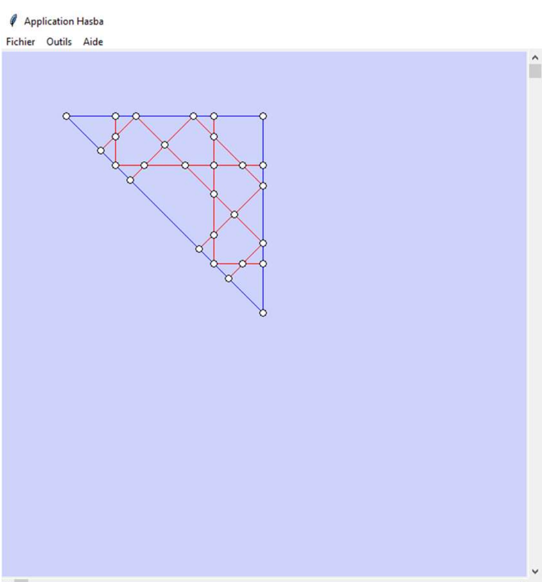

Figure 19: The structural unit with all intersections points

In Figure.20, for example, the segment with point number 0 to point number 5 is divided into two small segments. The first segment has starting point 0 and finishing point 2, and the other has starting point 2 and finishing point 5.

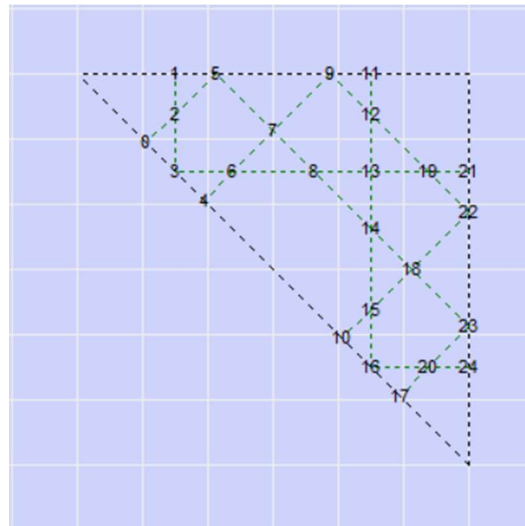

Figure 20: The 32 segments founded by method Âark.

Then we construct an array of all the segments, found, in which we store the type of segment (vertical: 0 horizontal: 1 oblique: 2), the numbers of the two vertices delimiting each segment, and the coordinates of each vertex Table 1:

Table 1: Segments with Limiting Point Coordinates

N° S_1	(x_1,y_1)	N° S_2	(x_2,y_2)	T
0	(91.81, 91.81)	2	(105, 78.63)	2
1	(105, 78.63)	1	(105, 60)	0
6	(131.36, 105)	8	(168.63, 105)	1
14	(195, 131.36)	13	(195, 105)	0
19	(221.36, 105)	22	(240, 123.63)	2
12	(195, 78.63)	19	(221.36, 105)	2
11	(195, 60)	12	(195, 78.63)	0

4.2.4. Drawing Islamic geometric pattern (IGP) in the fundamental region

We will apply the symmetry of 4mm, in the new fundamental region obtained according to our new method which contains only 32 segments instead of 165 segments, we trace the pattern on moving from one border of our region to the other. To obtain the other part of the pattern asymmetry is applied as showing in Figure. 21.

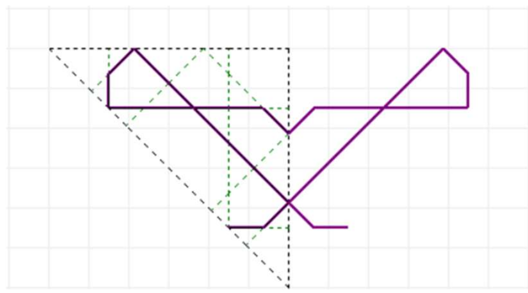

Figure 21: Islamic geometric pattern from point 16 to point 22.

By a succession rotation set of 90°, 180°, and 270°, with a point of coordinates (15/2q, 15/2q). We obtain our complete pattern according to our approach called the Âark method which fully respects the rules of Hasba, Figure. 22.

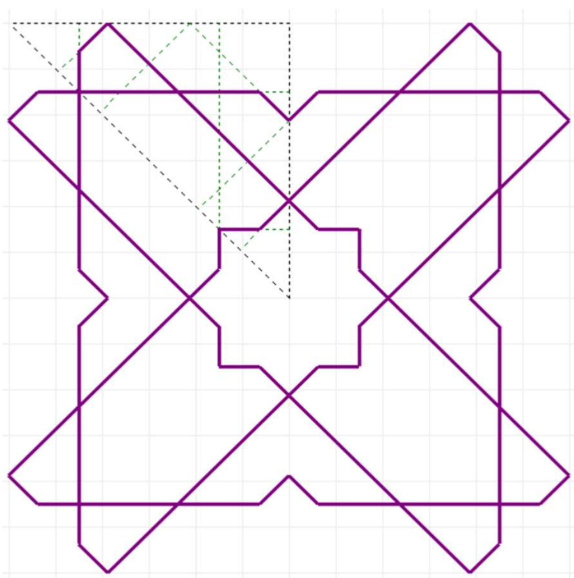

Figure 22: The final Islamic geometric Pattern

5. RESULTS AND DISCUSSION

In this subsection, we will present the results obtained by our new approach called the Âark method which respects the rules of construction of the 'Hasba' method. and which has the sole objective of reducing the complexity of the algorithm for constructing such patterns, instead of managing an area that contains 165 segments, with our method called Âark method we only manage 32 segments of our fundamental region or structural unit, and we get new geometric patterns that all respect the rules of the hasba method to help the artisans to create new Islamic geometric patterns (IGPs).

All the patterns obtained by the new 'Âark' method, are validated in a grid of dimension L=h*q with the Hasba h=15. What is makes our new method proposed in this paper very precise and fast as well as it gives patterns all valid.

The number of patterns obtained reaches 200 patterns, all of which comply with the 'Hasba' rules. Figure. 23 shows examples using the 'Âark' method.

6. CONCLUSION

The proposed method gives rise to new hasba patterns which are a very limited time given that the construction is done in most cases with a region that contains a fairly large number of segments, which makes the creation of new patterns quite difficult.

According to the method proposed in this article, up to 200 valid patterns were obtained that meet the measurement criteria of the 'Hasba' method.

The construction process consists of several steps described in detail. And reduce the complexity of the construction algorithm, instead of working on a region of 165 segments, our arrival by this method decreased the number to 32 segments and get a fairly large number of patterns.

This article presented a new method to simplify the creation of new patterns, including the aim to solve the problem of pattern limitation obtained so far by artisans.

The proposed method can be used to automatically generate a number very important of this Islamic geometric patterns (IGPs) without the intervention of the human being, to overcome the problem of limitation of the patterns posed by the artisans of Islamic geometric art.

In future work, we will adopt this method to make a process of generation automatically a large number of these Islamic geometric patterns.

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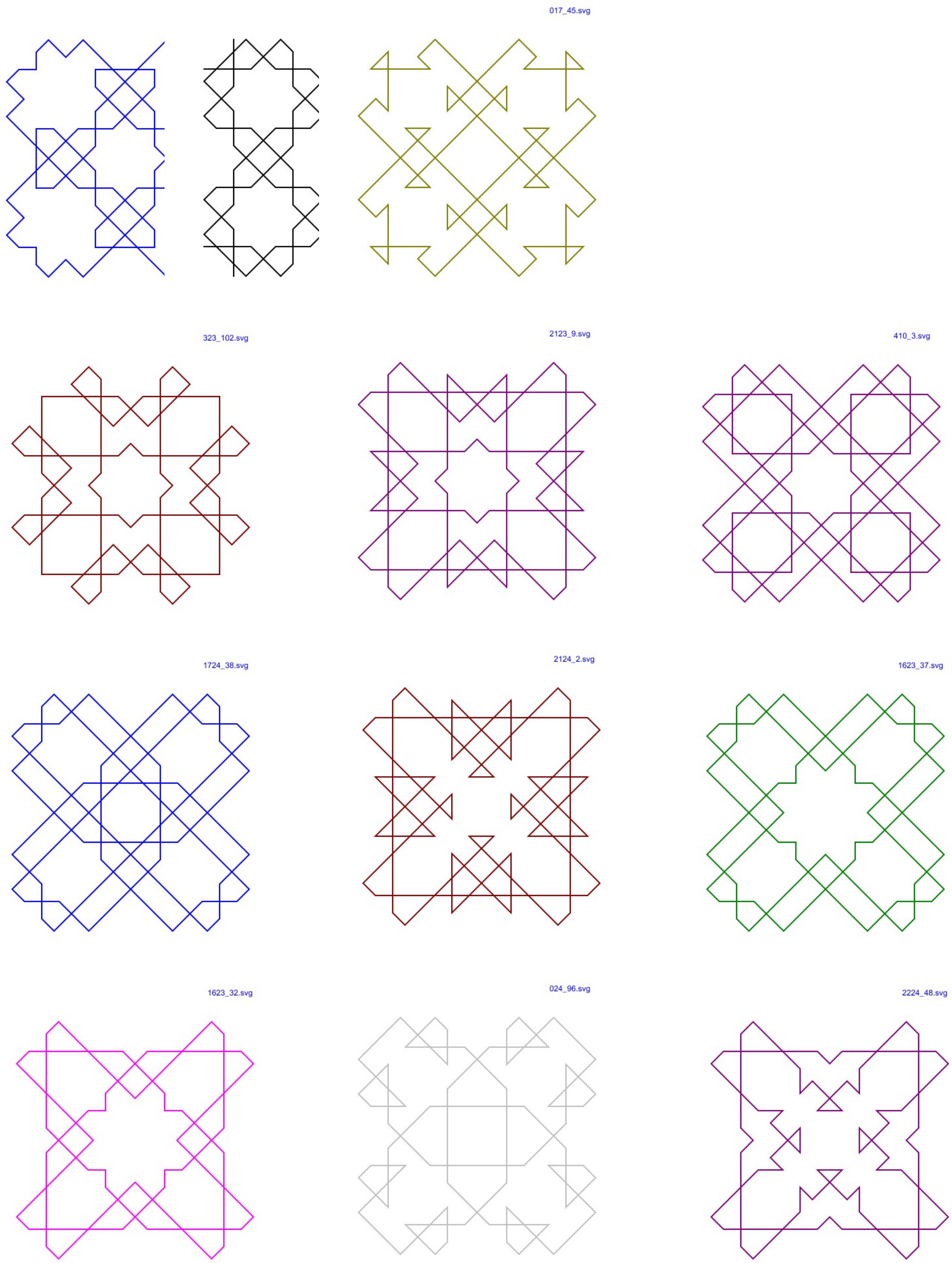

Figure 23: Examples of (IGPs) constructed by method Âark.