INTELLIGENT SYSTEM AND OPTIMAL MINIMISATION OF ENERGY CONSUMPTION ON THE ATTITUDE CONTROL OF A VTOL UAV HEXACOPTER BASED ON FRACTIONAL CONTROL LAWS

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ABSTRACT

The objective of this research study based on numerical results is to validate the feasibility and effectiveness of a new method for controlling hexacopter type UAVs, which belong to the family of multirotors established on laws fractional control (FOC), taking into consideration an intelligence system for smart cities that will have economic impacts in the future. The control method asserts advantages regarding of response time and stabilisation at the desired altitude and attitude. This control is intended to be used to control and maintain the desired trajectory during several manoeuvres while minimising energy consumption. The system's performance and stability are analysed with several tests, from simple hovering flight. This new solution applied to multi-rotor UAVs will totally revolutionise this technology in terms of stability and solve the problem for many industries. All the simulations discussed in this article were performed in the MATLAB/Simulink environment.

Keywords: Drone, Unmanned Aerial Vehicle (UAV), Hexacopter, Smart Cities, Intelligent Systems, Fractional Order Control (FOC).

1. INTRODUCTION

One of the characteristics of rotary wing UAVs is that they maintain a stable attitude in all directions while moving through space and also while hovering in the presence of any disturbances and environmental stresses. In order to guarantee a good mission accomplishment, it is necessary to have a robust flight controller to counteract the attitude drifts caused by these disturbances, in particular atmospheric disturbances such as torques due to gravitational and aerodynamic forces. Figure 1 shows the schematic of the attitude control system for a 6-motor UAV [6]:

This approach deals with the stabilization of an optimized hexacopter system [16] by the principle of initial state feedback, this law based on fractional control offers a good compromise between the different aerodynamic criteria and the stability performances such as the temporal responses of the system, the accuracy of the displacement of the UAV and the energy consumed by the UAV. In order to succeed in this, it is imperative to take into
account in the tests [7] [8] the strength provided or the degree of stability as well as the dimensional parameters internal to the design of the system.

In the beginning, the 4-motor UAV was an interesting platform to carry out several aerial missions without human intervention. At the beginning of this technology, the 4-motor UAV was an interesting platform to perform several aerial missions without the intervention of the human being. However, with time, the users require several conditions concerning the UAV’s nacelle, the battery autonomy which has a direct link with the UAV’s [14] flight time. Adding more motors to the system can also cause disadvantages, such as the weight of the drone which will be higher than the 4 motor configurations, the cost and the external dimensions.

For many industrialists and researchers consider that this configuration remains a good choice to solve some problems but it is also necessary to take into account the dynamics of the drone at the stability level because of the added weight and the consumed energy. This is why it is necessary to conceive a robust controller which manages well the energy consumption of the drone.

There is a lot of research work already translated into reality for different control methods such as PID [9] [10], LQR [11] [13], etc. However, there is less work on flight test verification due to the complexity of real-time implementation.

In this study, we aim to design a controller never before used for hexacopter type UAVs in order to see how reliable and efficient it is to have a stable UAV. If this stability condition is automatically validated, we can deduce that the management of the electrical power consumption of our system will be optimised and regulated. The so-called FOC controller is based on fractional control laws. We will study this controller in terms of response time and stabilisation at the desired altitude and attitude. All simulations and tests are generated in MATLAB/Simulink.

The organization of this article is as follows, In the first part, presents the nonlinear equations of the general model of the UAV with 6 motors, then the representation of the design of the controller based on fractional laws under the software matlab, discussion of the results of the simulations made and at the end a conclusion to see the feasibility of this method.

2. MATERIALS AND METHODS

1. Dynamic modelling of the 6-motor rotary wing UAV

Using the Newton-euler principle we can obtain the dynamic equations of the 6-motor rotary wing UAV. The equations of the hexacopter with 6 degrees of freedom are almost the same as those of the quadcopter UAV except for a change in the boundary conditions of the system control inputs. In the figure below, the geometric representation of the 6-motor UAV with the reference marks:

![Reference Frames, Force & Torque Directions](image)

2. Definition of bearings and parameters of the 6-motor UAV:

Figure 2 shows the design and configuration of the 6-motor UAV used in the study. There are two reference points for the derivation: the first one is the fixed reference point of the Earth base $F_E$ which is the non-accelerated inertial reference. The second one $F_B$ is the fixed reference but this time linked to the structure of the UAV located at the origin of the center of gravity $C_g$, which is permanent displacement with the UAV whatever its movement (translation, rotation). The two references and unit vectors are represented in figure 2. For the simulations, the origins of the two references are considered as a single reference at $t=0$. 
3. System inputs and established strategy:

Concerning the hexacopter drone we have 6 motors in it there are 4 parameters: the angles of Euler \((\theta,\phi,\psi)\) and the altitude in \((z)\), which are the direct responsible of the controls and followed. For a good interpretation of the results the command \(U\) is established in the design of the controller:

\[
U = PU_r;
\]

\[U_r = 7U\]

\[
P = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 \\
0 & -0.87d & 0 & 0.87d & 0 & 0.87d \\
-\theta & -\theta/2 & \theta/2 & \theta/2 & -\theta/2 & -\theta/2 \\
-\phi & -\phi & \phi & -\phi & -\phi & -\phi \\
-\psi & -\psi & \psi & \psi & -\psi & -\psi \\
-\psi & -\psi & -\psi & -\psi & \psi & \psi
\end{bmatrix}
\]

\[; \quad c = d \sqrt{3}/2 = 0.87d\]

The given result of the matrix \(P\) is found using the above mentioned configuration of the 6-motor UAV in figure 2.

\(U\) which is the virtual control input composed of \(M_x c, M_y c, M_z c, F_z c\) which represents the moments in the \(x\), \(y\), \(z\) axes by the rotation of the propellers and the forces generated in the \(z\) axes of the UAV. These moments and forces are responsible for the movements of the drone and will be the object of control. This virtual control proposes a physical interpretation of the inputs related to the control and the addition of the integral effect of the controller. This has advantages in the design part of the controller then it is necessary to make a control of the (rpm) angular velocity for each engine of the drone. But we correspond the virtual control input \(U\) to the real input control \(U_r\) which has a relationship with the angular velocity of the 6 engines.

4. 6-DOF Nonlinear Dynamic Model:

Once the control inputs are defined, the dynamics of the hexacopter can be generated by considering the external forces and moments acting on the cg of the hexacopter. The equations of motion in 6 degrees of freedom are given as follows:

\[
\Sigma F_{ext} = m \ddot{P} + w \times (m \dot{V}_z)
\]

\[\Sigma M_{ext} = m \ddot{P} + w \times (J \dot{w})\]

Note that the above equations are expressed in relation to \(F_{\text{in}}\), the fixed reference frame of the UAV.

\[
\sum F_{\text{ext}} \quad \text{and} \quad \sum M_{\text{ext}}\]

are the external moments and forces acting on the 6-motor UAV and the calculation of the \(\sum M_{\text{ext}}\) is with respect to the center of gravity \(C_g\).

\[
\sum F_{\text{ext}} = F_{\text{grav}} + F_{\text{prop}} + F_{\text{drag}}
\]

\[
F_{\text{grav}} = -L_{\text{ext}} \begin{bmatrix} 0 & 0 & mg \end{bmatrix}^T
\]

\[
F_{\text{prop}} = \begin{bmatrix} 0 & 0 & F_{\text{ext}} \end{bmatrix}^T
\]

\[
F_{\text{drag}} = -K_d \begin{bmatrix} u & v & w \end{bmatrix}^T
\]

\[
\sum M_{\text{ext}} = M_{\text{prop}} - \begin{bmatrix} M_{\text{ext}} & F_{\text{ext}} \end{bmatrix}^T
\]

For the hexacopter drone \(F_{\text{drag}}\) characterizes the aerodynamic force of the drag, \(F_{\text{grav}}\) characterizes the force given by the effect of the gravity, concerning the dynamics of the drone is given by the actuators the force and the moment \(F_{\text{prop}}, M_{\text{prop}}\) which are generated by the system engine plus propeller, With:

\[
L_{\text{ext}} = \begin{bmatrix}
\cos(\theta) & \sin(\theta) & \cos(\theta) & -\sin(\theta) & \cos(\theta) & -\sin(\theta) \\
\sin(\theta) & \cos(\theta) & \sin(\theta) & -\cos(\theta) & \sin(\theta) & -\cos(\theta) \\
\cos(\theta) & \sin(\theta) & \cos(\theta) & -\sin(\theta) & \cos(\theta) & -\sin(\theta) \\
\end{bmatrix}
\]

\[L_2 = \begin{bmatrix}
1 & 0 & -s(\theta) \\
0 & c(\theta) & s(\theta) \cos(\theta) - c(\theta) s(\theta) c(\theta) \\
0 & -s(\theta) & c(\theta) \cos(\theta)
\end{bmatrix}
\]

With the following kinematics the rotational and translational velocities can be developed:
By combining motion equations, translational and rotational dynamics of hexacopter is obtained [3] [4] as follows:

\[
\begin{align*}
\dot{z} &= \left[ z, \dot{y}, \dot{z} \right]^T = \left[ \frac{m}{I_{xx}} \left( \frac{k_2 u}{z} - \frac{k_2 u}{z} \right), \frac{m}{I_{yy}} \left( \frac{k_2 u}{z} - \frac{k_2 u}{z} \right), \frac{m}{I_{zz}} \left( \frac{k_2 u}{z} - \frac{k_2 u}{z} \right) \right]^T \\
\end{align*}
\]

Equation (15), expresses the nonlinear attitude dynamics which is achieved by replacing the two equations (13) and (14) in equation (11) and eliminating the impacts of the actuator dynamics in the linearization.

\[
\begin{align*}
\dot{\phi} &= \phi \left( \psi - \hat{\psi} \right) + M_{\phi \psi} \\
\dot{\theta} &= \theta \left( \psi - \hat{\psi} \right) + M_{\theta \psi} \\
\dot{\psi} &= \psi \left( \psi - \hat{\psi} \right) + M_{\psi \psi} \\
\end{align*}
\]

B. The inclusion of the state integral in the state space representation:

The objective is to study the behavior of a fractional controller that tracks desired attitude and altitude commands, \( \psi \) state control and \( z, \phi, \theta \) states. It is noted that the primary purpose for the design of fractional commands are the inputs and state matrices. For simplification the state space representation is given for the states \( z, \phi, \theta, \psi \), their derivatives \( z', \phi', \theta', \psi' \)and their integrals \( \int z dt, \int \phi dt, \int \theta dt, \int \psi dt \) all exist in \( X \) the state vector \( X \) as represented by equation (16) and equation (17) below:

\[
X = \left[ x_1, x_2, ..., x_n \right]^T = \left[ x_1, \dot{x}_1, x_2, \dot{x}_2, ..., x_n, \dot{x}_n \right]^T
\]

\[
U = \left[ u_1, u_2, ..., u_m \right]^T = \left[ u_1, \dot{u}_1, u_2, \dot{u}_2, ..., u_m, \dot{u}_m \right]^T
\]

The following first order differential equations are found from equation (12) and equation (15).

To obtain the linear equations of the state space, the equations (18) are linearized with respect to a control condition. The matrix A and the matrix B are respectively the equations of the state space. This calculation was done with the Matlab software.

\[
\left[ \begin{array}{c}
\dot{z} \\
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{array} \right] = \left[ \begin{array}{c}
\frac{m}{I_{xx}} \left( \frac{k_2 u}{z} - \frac{k_2 u}{z} \right) \\
\frac{m}{I_{yy}} \left( \frac{k_2 u}{z} - \frac{k_2 u}{z} \right) \\
\frac{m}{I_{zz}} \left( \frac{k_2 u}{z} - \frac{k_2 u}{z} \right) \\
\phi \left( \psi - \hat{\psi} \right) + M_{\phi \psi} \end{array} \right]
\]

\[
\left[ \begin{array}{c}
[\phi, \theta, \psi]^T \\
[\phi, \theta, \psi]^T
\end{array} \right]
\]
Whatever the value of the altitude for the dynamic model, the matrix A will always be equal to the matrix B. When the hexacopter drone in hover mode, several parameters will be zero, for the reason that equation (18), does not have a coupled nonlinear servo term. Except $F_2$ is the same as the weight of the UAV.

$$A_{ij} = \frac{\partial f_i}{\partial q_j}, \text{for } i = 1:12, j = 1:12$$  

(19)

$$B_{ij} = \frac{\partial f_i}{\partial u_{ie}}, \text{for } i = 1:12, j = 1:12$$  

(20)

$$X_0 = \text{zeros}(12,1), U_0 = [m_0, 0.0, 0]^T$$  

(21)

Equation (22) represents the state space dynamics of the 6-motor UAV according to the obtained time-invariant matrix A and matrix B.

$$\Delta X(t) = AX(t) + BUL(t) = CX(t)$$  

(22)

Where:

$$\Delta X(t) = X(t) - X_0, \Delta U(t) = U(t) - U_0$$

To move on to the calculation phase, the Matlab calculation software has a calculation platform based on predefined blocks to facilitate the implementation of the script, below is the Simulink Matlab representation of the Linear model of the Hexacopter:

Block diagram 1: Linear model of the Hexacopter in Simulink

6. Method of designing the fractional controller for the hexacopter uav

The control law for any servo system, for establishing a fractional controller, is:

$$u(t) = -x(t)^{\alpha}$$  

(23)

The function $x(t)$ of the derivation order is represented by the variable $\alpha$ which is a real number, the above equation then becomes:

$$x(t)^{\alpha} = \left[ \Gamma \left( \frac{1}{\alpha} \right) \right]^{\frac{1}{\alpha}} \left[ x(t)^{\alpha} \right]$$  

(24)

for a fractional derivative based on the Gamma function [1] [2] we have the following definition interpreted by Letnikov Grunwald:

$$\Delta x(t)^{\alpha} = \frac{1}{\Gamma(1+\alpha)} \sum_{k=0}^{n} \frac{\Gamma(n-k+1)}{\Gamma(k+1)} (t-k)^n \Delta x(t-k)$$  

(25)

Where:

$$\gamma(\alpha, h) = \frac{F(\alpha + h)}{F(\alpha) F(\alpha + h)}$$

Below is the Simulink matlab representation of the controller fractional, For the simulation the following system input values are defined in table 1:

<table>
<thead>
<tr>
<th>Inputs</th>
<th>values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>20</td>
</tr>
<tr>
<td>Roll</td>
<td>15</td>
</tr>
<tr>
<td>Yaw</td>
<td>30</td>
</tr>
<tr>
<td>Pitch</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 1: Input Values For The Hexapter System For Test Simulation In Simulink.

Block diagram 2: fractional controller (Simulink model).
3. RESULTS AND DISCUSSION

Table 2: Parameters And Variables Of The Hexacopter Design Needed For The Numerical Calculation [5]

<table>
<thead>
<tr>
<th>Par</th>
<th>Definition</th>
<th>value</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>hexacopter mass</td>
<td>15</td>
<td>kg</td>
</tr>
<tr>
<td>g</td>
<td>Acceleration of gravity</td>
<td>9.8</td>
<td>m/s²</td>
</tr>
<tr>
<td>k_f</td>
<td>Constant that relates F_i and ω_i</td>
<td>5.7 · 10⁻⁸</td>
<td>N / rpm²</td>
</tr>
<tr>
<td>k_t</td>
<td>Constant that relates F_i and T_i</td>
<td>0.016</td>
<td>m</td>
</tr>
<tr>
<td>J_xx</td>
<td>Hexacopter Inertie Matrix</td>
<td>1.220</td>
<td>kg m²</td>
</tr>
<tr>
<td>J_yy</td>
<td></td>
<td>1.226</td>
<td>kg m²</td>
</tr>
<tr>
<td>J_zz</td>
<td></td>
<td>2.065</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Distance of motor to center</td>
<td>0.71</td>
<td>m</td>
</tr>
<tr>
<td>L_d</td>
<td>Air-torque Coef.by torque dividing rotation-speed²</td>
<td>7.245</td>
<td>N m / (rad/s)</td>
</tr>
</tbody>
</table>

After linearization, time-invariant $A, B, C$ matrices are found as follows:

1. Fractional control law of the 6-engine rotary wing uav in state space:

We have

$$\dot{x}(t) = -k_x x(t)^{\alpha}$$

(26)

for different values of $\alpha$ and as a function of time we studied the behaviour of the Euler angles under Matlab and Simulink software, as shown in the following figures:
Negative values of $\alpha$ in the simulation introduce a robust fractional integrator which provides a faster response to the system, but has an impact on the stability i.e. it consumes more energy.

For positive values of $\alpha$, the system is more stable than for negative values of $\alpha$ and consumes less energy and the steady state error decreases. For this reason, the choice of $\alpha$ depends on the experiment.

To highlight the energy saving or rather consumption, 150sec for each $\alpha$ was plotted to see the control action that related to the reaction wheels.
Figure (6) shows the shape of the curves which indicates that the accuracy is good for negative values of α. In logarithmic scale, (SSE) the steady state error changes exponentially with $\alpha > 0$:

$$\text{SSE} = \exp(k_1 \alpha)$$

2. Proportional fractional derivative:

The last point revealed that control fractional is more exact. However, it consumes more energy unlike the values of $\alpha > 0$ there is a noticeable loss in the quality of accuracy, but on the other hand the energy consumption has been reduced.

To solve this problem and minimize the energy consumption without affecting the accuracy, the following fractional control law is given below:

$$u(t) = -K_1 x_1 - K_2 x_2^{(\alpha)}$$  \hspace{1cm} (27)

Where:

$$x_1 = (\phi, \theta, \psi)^T \text{ and } x_2 = (\dot{\phi}, \dot{\theta}, \dot{\psi})^T$$

The following tests in the figure below represent the angular velocities versus time with $t=150s$ of the Euler angles (Roll, Yaw and Pitch), for variations of the value of $\alpha$, from -0.2 to the value 0.3 with a step equal to 0.1. A variety of colors has been implemented for the good understanding and shown the quality of the curves. After all the calculation done, a remark has been made concerning this fractional derivative control law. That it gathers with the control law.
As the value of $\alpha$ increase from -0.2 to 0.3, there is an effect on the stability of the system which makes it more accurate, which can also be seen in the graphs showing the angular parameters of the velocity for Roll, Yaw and Pitch.

From the above figures, it should be mentioned that from the interpretation of the results in findings that the relative coefficient $a$ represents the role of a damping coefficient of the 6-motor UAV system. Figure (10) and figure (11) illustrate the response of the roll angle, which is part of the Euler angles, as a function of the time that the choice has been set to 150s. It can also be seen that the drone system is overdamped for the reason that the tolerance band is a ±0.05° and the lower limit is -0.05. For values of $\alpha$ which are in the vicinity of 0.4, so it can be concluded that the system behaves like a 1st order system. This means that it is only necessary to change the value of $\alpha$ for the UAV Hexacopter system [17] to be damped without having any change in the gain matrix.

### 3. Fractional derivative $D^{[\alpha]}d$:

The control law is given by:

$$u(t) = -K_1x_1^{[\alpha]} - K_2x_2$$

(27)

Where:

$$x_1 = (\phi, \theta, \psi)^T$$

and $n_2 = (\phi, \theta, \psi)^T$

The same behavior of the system is observed. Next figures give Euler angle [19] [20] versus time in addition to the angular velocities for different values of $\alpha$.

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**Fig 10:** As a function of $t=150$s, the angle of Roll, (the variation of $\alpha$ from -0.3 to 0.5 with a step of 0.1) according to the derived fractional control

**Fig 11:** As a function of $t=15$s, overshoot for $\alpha = 0.04$, (the variation of $\alpha$ from -0.025 to 0.045 with a step of 0.005)

**Fig 12:** As a function of $t=150$s, the angle of Roll, Yaw, Pitch (the variation of $\alpha$ from -0.3 to 0 with a step of 0.1) according to the derived fractional control law $D^{[\alpha]}d$. 

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From the above figures the fractional control law $P^{(1)}D$ is more accurate than the other fractional control laws studied, the interpretation of this accuracy is obtained by the integral term $I^{(1)}$ for $\alpha$ values $< 0$.

The interpretation of the previous control law in terms of the positive variation of the stabilisation as a function of time goes through discontinuities. this is generated at the time of the undershoots and overshoots that reach the lower and upper limit.

For the first fractional controller, there is a minimum of action of the reaction wheel control which has been achieved for the value of $\alpha = 0.1$. also for the tolerance band which is lower than that of the steady state error. for all the tests carried out in the steady state simulations the best error marked for $\alpha$ equal to $-0.3$.

Concerning the second fractional controller (PROPORTIONAL FRACTIONAL DERIVATIVE), the stabilisation time improved for minimum values of $\alpha = 0.4$ and above, also the control action of the reaction wheels increases slightly.

In the third controller $P^{(1)}D$ the integral part was introduced. For the purpose of further decreasing the stabilization response time, the minimum of the control reaction [15] was reached and the stabilization time of the hexacopter drone system was reduced, with a minimum obtained of $\alpha = -0.25$ in Roll with a slight increase of the power consumption.

4. CONCLUSIONS

In this article, the dynamic model of a Hexacopter UAV with 6 motors has been built and implemented to measure the feasibility of a controller based on the principle of fractional laws and then measured its importance for the control of the said system according to the response time, the time that the system will take to stabilize and the energy consumption caused by this type of controller.

Several tests have been carried out under different conditions, with the objective of obtaining reasonable and logical results. The linear model is generated using the MATLAB script environment. And all the controllers are tested using the same model also integrated in MATLAB which is the Simulink platform.

In the bibliography several different control methods for autonomous systems have proven to have great potential and have provided interesting results, but each one has advantages as well as disadvantages, for example the PID controller, which has a good characteristic regarding the response time which is fast, but trying to eliminate the error in steady state it allows the system to start oscillating. Another example of a controller known as LQR, among its characteristics is a robust design that produces a very low steady state error, but with a slow response time. For this reason, this paper is interested in the implementation of the method newly known as FOC, i.e. fractional order controller, with the objective of finding a solution that solves this compromise between the system's stabilization time and the energy consumed. The three fractional controllers presented take advantage of the LQR control method with the characteristic of having a fast settling time.

The results presented in this article show that the application of the fractional controller can provide satisfactory control in terms of the attitude
of the hexacopter system. If all the parameters are correctly defined and the choice of weighting matrices correctly established, the controller can be implemented on an on-board system with satisfactory results in most desired flight situations. The contribution of the adaptation of this fractional controller is to have reduced and optimized energy consumption. In fact, applying the FOC controller, results in implicit energy optimization. Furthermore, introducing a new criterion that forces the controller to alter performance in order to extend the mission duration is another level of optimization.

The results and simulation done in this paper as a whole prove that this fractional controller is a more flexible controller compared to the existing controller by giving better results in terms of response time and accuracy. In addition, other dynamic properties such as damping effects can be adjusted using this controller.

5. FUTURE WORK

In the future, this work can be improved by considering all the external constraints of the hexacopter drone system and see the results it will give and its impact on energy consumption and conservation.

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