

ASSESSMENT OF CREDIT LOSSES BASED ON ARIMA-WAVELET METHOD

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ABSTRACT

The aim of this paper is to estimate and forecast the loss-given defaults (LGD) using a sample data of credit portfolio loan collected from a bank in Jordan for the period up from January 2010 to December 2014. We use a wavelet-inspired analysis to convert the original observations into a time-scale domain. Then, we combine the wavelet transform with the ARIMA (Auto-Regressive Integrated Moving Average) model to get an ARIMA-WT new model to forecast the LGD data time series. We evaluate four wavelet functions, which are Haar (Haar), Daubechies (d4), least Asymmetric (La8), and Coiflet (C6). The numerical results show that the ARIMA-WT is more accurate than the pure ARIMA and the other considered ARIMA-Wavelet transform based models. We consider several metrics (MAPE, MASE, RMSE, AIC, AICs and BIC) to measure the performance of our proposed model. The combination between ARIMA-WT and La8 function improves highly the forecasting accuracy. According to our findings, we can say that the resulting forecast model is able to produce a high quality result.

Keywords: CREDIT RISK, LGD, WT, ARIMA, FORECASTING.

1. INTRODUCTION

Forecasting financial time series is an important issue in finance and insurance. It has received a considerable attention by several researchers [1]. Forecasting is a way employed in credit risk management for protection from financial loss. Credit risk management refers to the set of practices and techniques used by financial and monetary organizations to manage the financial risks. It is the mitigation of the probability of the losses of a company when a borrower defaults in making payments on any type of debt. According to the Basel Committee, the credit risk can be assessed by using three main approaches, which are the standardized approach (with low accuracy, low complexity, and high capital charge), the Internal Ratings-Based (IRB) approach (with medium

accuracy, medium complexity and medium capital charge) and the advanced IRB approach (with high accuracy, high complexity and low capital charge). The two IRB based approaches are sensitive to risk; they depend on the bank's interior risks. The IRB based approach is widely used compared to the standardized approach because of its high accuracy and low capital charge [2].

The evaluation of credit risk is crucially important for banks because credit portfolio has the lion's share of bank assets. The Basel II Capital Structure announced by the Basel Committee supervision in June 2006, requires that banks should hold a minimum capital to cover the exposures of credit, market, and operational risks. For this reason, all banks are required to assess their portfolio risk, including credit risk [2].

Several credit risk models have been suggested since the introduction of the classical Z-score model for confirming the grant of credit of counterparties [3]. The ZETA discriminate examination model is developed through the linear function of market variables and accounting, and can be used to separate the repayment and non-repayment of a credit borrower. The logistic regression model is applied to foresee the likelihood of a borrower's default. Also, the parametric and non-parametric model, which assumed that the aggregate likelihood has a parametric and non-parametric functional form, is used to foresee the likelihood of a borrower's default [2, 4].

Wavelet Transform is a mathematical model used to convert the original observations into a time-scale domain [5, 6]. The wavelet transform model is suitable for financial data that is utmost non-stationary [7, 8].

The main contribution of our current study is to estimate and forecast the LGD using the data of credit portfolio loan from Jordan in the period from January 2010 to December 2014. The estimation uses both the ARIMA and the ARIMA-Wavelet transform. The ARIMA-WT is a combination of the wavelet transform and the ARIMA model. It has received great attention in the economic and finance areas. The ARIMA model can be defined as the following: ARIMA (p, d, q) where p is the order of autoregressive part (AR), d is the degree of first differentiation (I) and q is the order of the first moving part (MA).

We consider four wavelet functions namely: Haar (Haar), Daubechies (d4), least Asymmetric (La8), and Coiflet (C6). In addition, we use the accuracy criteria to compare the different studied models. We use several metrics (the MAPE, MASE, RMSE, the AIC, AICs and the BIC metrics) to evaluate the performance of our new model. The numerical results show that the ARIMA-WT with La8 function has more precision than the ARIMA and other ARIMA-WT based models.

The rest of the paper is organized as follows. Section 2 presents some related works. Section 3 gives a background on the mathematical formulation used in this study. Section 4 details the research design and the data descriptions. Section 5

reports the results of the empirical analysis. Finally, Section 6 concludes and gives some future works.

2. LITERATURE REVIEW

The aim of this section is to give an overview of some important related works.

The loss given default (LGD) is commonly used in credit risk management. The most common approach for estimating the LGD is LossCals. The latter is presented by Mody's 2005; it depends on the multivariate linear regression model that includes certain risk factors, industry and macroeconomic components. In [9], authors suggested a beta distributed LGD and applied different kinds of beta regressions for modeling the LGD. In [10], authors used the Japanese bank loan information to analyze the factors affecting the LGD. They build a multistage model for predicting the LGD and the expected loss (EL) values. They found that the collateral, the guarantees, and the loan size affect the LGD value. In addition, they found that the multistage LGD model has superior predictive accuracy than the corresponding debit model, OLS model, and inflated beta regression model. Furthermore, in [11], the author investigated a new model for bank loan loss given default by leveraging time to recovery. In [12], authors estimated the downturn LGD modeling by using quintile regression. Finally, in [13] authors constructed a model to predict the risk of a cardholder for the lifetime of the account where the survival analysis methodologies was applied to a case study from capital card services.

The LGD is nonlinear and nonstationary data. Therefore, the filtering method such as kalman [14] and spectrum filter models are suitable. The spectral analysis is a tool for extracting embedded structures in a time series. In particular, the Fourier analysis was used extensively for extracting deterministic structures from time series. However, it is unable to detect nonstationary features often present in geophysical time series [15]. The wavelet analysis can extract transient features embedded in time series, with a wavelet power spectrum representing variance (power) of a time series as a function of time and period.

Wavelet Transform is a mathematical model used to convert the original observations into

a time-scale domain [5, 6]. Wavelet is an interesting method that can be used in credit risk. Several works have been studied the effectiveness of wavelets in finance and credit risk. Among these works, we give the following ones: authors in [16] designed and implemented a new numerical method for inverting the Laplace transform based on Haar wavelets approach. This model was used to estimate expected shortfall for the individual loan portfolios under the one-factor Merton model with constant loss given default. The result showed that Wavelet Approximation method is accurate, fast, robust, and able to deal with concentrated or small portfolios at high loss levels. In [17], authors investigated the continuous wavelet coherency for the "time-varying" correlation for three different loans categories; loans to non-financial corporations, loans to private households (without mortgages) and mortgages. The study was done on German data from 1971 to 2010.

To investigate the lead/lag relationship, the wavelet phase difference is computed for various frequency bands. The results showed that the coherence between real GDP (gross domestic product) and loans to non-financial corporations changes over time. In [18], authors proposed a new methodology to estimate the Value at Risk [19] for quantifying losses in credit portfolios. They approximated the cumulative distribution of the loss function by using a finite combination of Haar wavelet basis functions. They calculate the coefficients of the approximation by inverting its Laplace transform. The loss function assumed to be exposure at default for creditors. The results showed that Wavelet model is an accurate, robust and fast method, allowing estimating VaR much more quickly than a Monte Carlo (MC) method at the same level of accuracy and reliability.

Authors in [20] worked on 100 companies according to industry types to construct wavelet structural model to improve predictive ability of corporate defaults. They applied wavelet decomposition; built different models separately for low frequency part and high frequency part and then reconstruct the predictive return. They found that the wavelet structural model has more sensibility and more precision than time series model. In [21], authors investigated a Discrete Wavelet Transformation process to achieve a better characterization of the loan applicants on the basis of the information previously gathered by the credit scoring system. The performed experiments

demonstrated how such approach outperforms the state-of-the-art solutions.

3. BACKGROUND AND MATHEMATICAL FORMULATIONS

This section gives a background of the main concepts used in our study.

3.1. Loss-Given Defaults (LGD)

The internal rating based (IRB) approach is a well-known technique for measuring credit risk. It permits to banks to model the probability of default. Further, the advanced IRB approach can model the loss given default (LGD). LGD can be defined as the amount of money lost when a borrower defaults.

The capital necessities for credit risk in Basel II Internal Rating Based (IRB) can be defined by using the Risk-Weighted Assets [8] equation where the LGD is considered as the main parameter to estimate RWA.

The monthly value of LGD is computed as shown in Equation (1):

$$\text{Monthly LGD} = \frac{\sum_{i=1}^n \text{LGD for each Borrower}_i}{n} \quad (1)$$

Where *Borrower_i* represents the borrower number *i* in the same month and *n* is the number total of borrowers for the given month. In addition, the LGD of each borrower *i* is estimated by using the proxy given in Equation (2).

$$\text{LGD for each borrower} = \frac{\text{outstanding amount}}{\text{Credit amount}} \quad (2)$$

LGD is then the proportion of exposure that is lost in the event of a default [22].

3.2. Wavelet Transform Formula

As already said, the wavelet transform [23] is a mathematical model for converting the original observations into a time-scale domain. The WT based model is an appropriate model for analyzing financial data because most of the financial data are non-stationary. We distinguish between two WT models, which are the discrete wavelet transform (DWT), and the continuous wavelet transform

(CWT). DWT consists of many functions such as Haar, Daubechies, and Maximum overlapping Wavelet transform (MODWT) and others. All of these functions have the same properties with different applications.

In this section, we present the main concept of WT. For more details, the readers can refer to [6, 7,24].

Wavelets theory is based on Fourier analysis, which represents any function as the sum of the sine and cosine functions. A wavelet is simply a function of time t that obeys a basic rule, known as the wavelet admissibility condition[7]:

$$C_\varphi = \int_0^\infty \frac{|\varphi(f)|}{f} df < \infty \tag{3}$$

where $\varphi(f)$ is the Fourier transform and a function of frequency f , of $\varphi(t)$. The WT is a mathematical tool that can be applied to numerous applications, such as image analysis and signal processing. It was introduced to solve problems associated with the Fourier transform, when dealing with non-stationary signals, or signals that are localized in time, space, or frequency.

There are two types of wavelets within a given function/family. Father wavelets describe the smooth and low-frequency parts of a signal, and mother wavelets describe the detailed and high-frequency components. Equation (4) represents the father wavelet and mother wavelet respectively, with $j=1,2,3,\dots, J$ in the J -level wavelet decomposition:[25].

$$\phi_{j,k} = 2^{\frac{-j}{2}} \phi \left(t - \frac{2^j k}{2^j} \right) \tag{4}$$

$$\varphi_{j,k} = 2^{\frac{-j}{2}} \varphi \left(t - \frac{2^j k}{2^j} \right)$$

Where J denotes the maximum scale sustainable by the number of data points and the two types of wavelets stated above, namely father wavelets and mother wavelets and satisfies:

$$\int \phi(t)dt = 1 \text{ and } \int \varphi(t)dt = 0 \tag{5}$$

time series data, i.e., function $f(t)$, is an input represented by wavelet analysis, and can be built up as a sequence of projections onto father and mother wavelets indexed by both $\{k\}$, $k = \{0, 1, 2,\dots\}$ and by $\{S\}=2^j$, $\{j=1,2,3,\dots J\}$.

When analyzing real discretely sampled data, we need to create a lattice for making calculations. Mathematically, it is convenient to use a dyadic expansion, as shown in Equation (5). The expansion coefficients are given by the projections:

$$S_{j,k} = \int \phi_{j,k} f(t) dt, d_{j,k} = \int \varphi_{j,k} f(t) dt, \tag{6}$$

The orthogonal wavelet series approximation to $f(t)$ is defined by:

$$F(t) = \sum S_{j,k} \phi_{j,k}(t) + \sum d_{j,k} \varphi_{j,k}(t) + \sum d_{j-1,k} \varphi_{j-1,k}(t) + \dots + \sum d_{1,k} \varphi_{1,k}(t) \tag{7}$$

$$S_j(t) = \sum S_{j,k} \phi_{j,k}(t) \text{ and } D_j(t) = \sum d_{j,k} \varphi_{j,k}(t) \tag{8}$$

The WT is used to calculate the coefficient of the wavelet series approximation in Equation (7) for a discrete signal, where $S_j(t)$ and $D_j(t)$ are introducing the smooth and details coefficients respectively. The smooth coefficients gives the most important features of the data set and the details coefficients are used to detect the main features in the dataset.

2.3. Comparison Between the Four Main Wavelet Transform Functions

As already said, two terms are used when describing a wavelet. The first is non-orthogonal wavelet (e.g. Morlet wavelet). The second is orthogonal wavelet, which refers to an orthogonal set of functions. Non-orthogonal wavelet used with the DWT (Discrete Wavelet Transform) or CWT (Continues Wavelet Transform) while orthogonal wavelet is used with dWT only a variety of

different WT algorithms can be applied over the data for analyzing depending on the nature of the data. Each transform has its own characteristics and advantages over other algorithms based on the individual cases. WT has four popular transform functions[6], namely Haar, Daubechies, Symlet and coiflet. Table (1) compares the main properties of Haar, Daubechies, coiflet, and symlet wavelets.

These functions are summarized as follows:

- 1) Haar Wavelet Transform (Haar) is the first algorithm with the simplest and oldest Wavelet function in the WT family. It was developed to overcome problems in the Fourier transform (FT). Haar is considered the simplest wavelet function[26].
- 2) Daubechies wavelet transform (d4) is widely used in statistics and finance

application and represents a development and an improvement of the HWT in terms of frequency-domain characteristics and arbitrary regularity.[26].

- 3) Coiflets Wavelet Transform (C6) was also proposed by Daubechies in 1992. However, Coiflets wavelet transform has unusual properties in the zero moments of the scaling function, because the wavelet function associated with Coiflets wavelet transform has 2h zero moments. [6, 26].
- 4) Least Asymmetric (La8) : wavelet is considered a good general-purpose wavelet whose “width” (8) strikes a balance between providing smooth approximations with few artifacts, and minimal edge effects at the boundaries of the data[27].

TABLE 1: Comparison Between Haar, Daubechies, Coiflet, And Symlet Wavelet Functions

Property	Haar	Daubechies (d4)	Least Asymmetric (La8)	Coiflet (C6)
Arbitrary regular	No	Yes	Yes	Yes
Orthogonal and compact support	Yes	Yes	Yes	Yes
Symmetry	Symmetry	A Symmetry	A Symmetry	Near Symmetry
Arbitrary number of zero moments	Yes	Yes	Yes	Yes
Existing of the scale function	Yes	Yes	Yes	Yes
Orthogonal analysis	Yes	Yes	Yes	Yes
Bio-orthogonal analysis	Yes	Yes	Yes	Yes
Continuous transformation	Yes	Yes	Yes	Yes
Discrete transformation	Yes	Yes	Yes	Yes
Exact reconstruction	Yes	Yes	Yes	Yes
Fast algorithm	Yes	Yes	Yes	Yes
Explicit expression	Yes	No	No	No
Real or complex wavelet	Real	Real	Real	Real

2.4 Autoregressive Integrated Moving-Average Model (ARIMA)

The auto-regressive moving average (ARMA) models are used in time series analysis to describe stationary time series. The ARMA model is a combination of a moving average (MA) model and an autoregressive (AR) model. A time series $\{e_t\}$ is called a white noise [28] process, $\{Y_t\}$ is called Gaussian process iff for all t , e_t is $iidN(0, \sigma^2)$. A time series $\{Y_t\}$ is said to follow the ARMA(p, q) model if:

$$Y_t = \mu + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} \dots + \phi_p Y_{t-p} + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2} \dots - \theta_q e_{t-q} \quad (9)$$

where q and p are non-negative integers, p represents order of autoregressive part (AR), q is defined as order of the first moving part (MA) and $\{e_t\}$ is the white noise [28] process. An extension of the ordinary ARMA model is the auto-regressive integrated moving-average model (ARIMA(p, d, q)) given by :

$$\phi_p(B)(1 - B)^d Y_t = \theta_0 + \theta_q(B)e_t \quad (10)$$

where p , d and q denote orders of auto-regression, integration (differencing) and moving average, respectively. When $d=0$, the ARIMA model reduces to the ordinary ARMA model.

3. RESEARCH DESIGN

The aim of this work is to forecast the LGD based on data of credit portfolio loan issued from Jordan bank in the period from January 2010 to December 2014. To do this, we use both the ARIMA and a combination of ARIMA and WT where the three parameters (p , d , and q) are estimated. Also, we consider the four WT functions which are: Haar(Haar), Daubechies (d4), least Asymmetric (La8), and Coiflet (C6). Further, we make use of the accuracy criteria to compare our models. The proposed framework is detailed in following.

3.1. Proposed Framework for an Accurate WTForecasting.

We propose several models to forecast the LGD data. We use a wavelet transform to convert the original data into a time-scale domain. Then we combine the WT with the ARIMA model to forecast the LGD data time series. We study various functions of WT to evaluate our models. Figure 1 draws the different steps of the WT forecasting process.

We note that when the data pattern is very rough, the wavelet process is repeatedly applied. The aim of the pre-processing step is to minimize the Root Mean Squared Error (RMSE) between the signal before and after transformation. The noise in the original data can thus be removed. Importantly, the adaptive noise in the training pattern may reduce the risk of over fitting in training phase. Thus, we adopt WT twice for the pre-processing of training data in this study.

As shown in Figure1, the Maximum Overlapping Wavelet transform (MODWT) converts the data into two sets: details series (DA1 (n)) and approximation series (CA1 (n)). These two series give a good behavior for the data set especially with the financial data since it is significantly fluctuated. Then, the transformed data is anticipated more precisely. The purpose behind the good behavior of these two series is the filtering effect of the MODWT. Additionally, we use the approximation series since the series behave as the main component of the transform.

Our methodology can be summarized as follows: To handle our data, first, we break down through the MODWT the available historical return data. Second, we use an improved ARIMA model fitted to the approximation series to make the forecasting. As already said, we propose a hybrid method combining the ARIMA model with WT decomposition. The decomposition process is done by using four WT functions which are: HaarWT, WTd4, WT La8 and WT C6. The overall method starts by applying WT in order to decompose time series data and reconstructed it in two parts: details series (DA1 (n)) and approximation series (CA1 (n)). The ARIMA model is applied on approximation series. The aim of the ARIMA-WT model is to forecast the LGD data time series. The

new technique is compared with a pure ARIMA model used directly to forecast the LGD data series by utilizing the specified criteria. Further, the four ARIMA-WT based proposed models are compared based on several metrics. The best model is the one given the best accuracy.

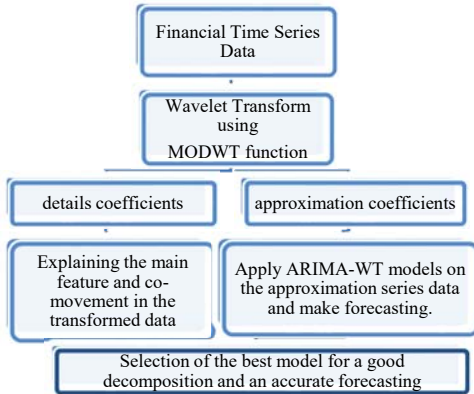


Figure 1. The Flowchart Of The WT Forecasting.

3.2. Accuracy Criteria

We consider several types of accuracy criteria: The Mean absolute percentage error (MAPE), the Mean absolute scaled error (MASE), the Root means squared error (RMSE), the Akaike information criterion(AIC, AICs), the Bayesian information criterion (BIC), and the Log likelihood test. The MAPE criterion is also known as mean absolute percentage deviation (MAPD) that is a measure of prediction accuracy of a forecasting method in statistics. It usually expresses accuracy as a percentage, and it is defined as:

$MAPE = \frac{\%100}{n} \sum_{t=1}^n \left| \frac{X_t - F_t}{X_t} \right|$, where X_t is the actual value and F_t is the forecast value. The absolute value in this calculation is summed for every forecasted point in time and divided by the number of fitted points n .

Also, the MASE is recommended for determining comparative accuracy of forecasts. It is estimated by

$$MASE = \frac{\sum_{t=1}^n |X_t - F_t|}{\frac{n}{n-1} \sum_{t=2}^n |X_t - X_{t-1}|}$$

where the numerator is the forecast error for a given period, defined as the actual value (X_t) minus the forecast value (F_t) for that period, and the denominator is the mean absolute error which uses the actual value from the prior period as the forecast: $F_t = X_{t-1}$.

Next, The RMSE is known also as root-mean-square deviation (RMSD) that is a frequently used as a measure of the differences between estimators. It measures the average error performed by the model in predicting the outcome for an observation. It is defined as the square root of the mean square errors: $RMSE = \sqrt{MSE} = \sqrt{\frac{\sum_{i=1}^N (\text{actualvalue} - \text{predictedvalue})^2}{N}}$, where N represents the number of observations.

Another accuracy criterion is AIC which is defined as: $AIC = -2 * \log - \text{likelihood} + k * npar$, where $npar$ represents the number of parameters in the fitted model, $k = 2$, and n being the number of observations, where AICc is a version of AIC corrected for small sample sizes.

Finally, the BIC is $BIC = -2 * \log - \text{likelihood} + k * npar$, where $npar$ represents the number of parameters in the fitted model, $k = \log(n)$, and n being the number of observations.

4. EMPIRICAL RESULTS

In this section, we present the data used in our study. Then we give the numerical results obtained when applied our models to the considered dataset.

4.1. Data Description

The sample data of credit portfolio are collected from a bank in Jordan and contain confidential information on credit of loans. The month-to-month information of credit portfolio was collected from January 2010 until December 2014. The size of portfolio is 4393, while the aggregate number of defaults throughout the 5-year time span is 495. A borrower is declared default when his/her cash installment is not paid within 3 months or more. The total number of observations for mathematical convenience in orthogonal wavelet transform is suggested to be divisible by 2^j ($j=1,2,3,\dots$). However, in this study, the dataset does not satisfy this condition. Therefore we adapt the Maximum overlapping discrete wavelet transform (MODWT)

since the MODWT is a wavelet function does not consider the number of observations[6, 7].

Table 2 shows the risk exposures (number of loans at risk) and the number of defaults in each year. The highest number of defaults occurred in the second year, and the highest number of defaults per exposure occurred in the same year (168 defaults from 1125 exposures).

Table 2. Number Of Exposures And Defaults In Each Year

Year	Exposure	# of defaults	% (# of defaults per exposure)
2010	1265	137	10.83
2011	1125	168	14.93
2012	783	67	8.56
2013	652	41	6.29
2014	568	82	14.44
Total	4393	495	-

4.2. Results and Discussion

Table 3 gives the results found by the four studied models. The ARIMA- Direct is the model obtained when we have applied the pure ARIMA model directly on the original data. ARIMA-WT (haar) is the combination of the ARIMA model with WT with the Haar function. ARIMA-WT (d4) is the combination of the ARIMA model with WT with the Daubechies function. ARIMA-WT (La8) is the combination of the ARIMA model with WT with the least Asymmetric function. ARIMA-WT (C6) is the combination of the ARIMA model with WT with the Coiflet function. We consider several metrics to evaluate the five studied models

Table 3 Appendix 1.

For a fair comparison, the same sample data has been chosen. The selected suitable ARIMA model for forecasting the sample data is

ARIMA (1,1,1) with RSME equal to 0.083 as shown in Table 3. However, the forecasting accuracy is improved when using HWT combined with the suitable ARIMA model which is ARIMA (0,1,0) with RMSE equal to 0.0615 as given in Table 3.

Similarly, an enhancement in the forecasting accuracy is noted when DWT with the d4 function is combined with a suitable ARIMA model (that is, (0,1,0)) with RMSE equal to 0.0606. The same remark is noted, that is the forecasting accuracy is also improved when DWT using C6 is combined with a suitable ARIMA model (that is, (0,2,0)) with RMSE equal to 0.010.

Also, the best forecasting accuracy is noted when the WT using the function La8 with the ARIMA (3,2,0) with RMSE is 0.0033. We should note that the difference in terms of RMSE between the methods used is relatively small. Nevertheless, it is possible to draw two important conclusions. First, the forecasting accuracy is improved when we use WT combined with the suitable ARIMA model since the forecasting using WT with ARIMA produces less RMSE than the forecasting carried out directly using only the ARIMA model. Second, the difference in RMSE between WT's functions contradicts existing opinion regarding WT (WT (Haar) WT (d4), WT (La8) and WT(C6)), since it is well known that the best WT in the literature is Daubechies WT.

In order to further corroborate our findings and hence conclusions, we used another statistical criterion to analyze the small difference in RMSE. We use both MAPE and MASE measures. Table 3 shows the obtained results using MAPE. When we take the results of both RMSE and MAPE collectively, we can see that the forecasting accuracy is improved using WT functions combined with a suitable ARIMA model compared to the forecasting accuracy using ARIMA model directly.

Thus, we can say that the findings in this study are significant and novel contribution to the area of WT and forecasting time series. Thus far, the results obtained in this paper indicate that WT is an effective model to decompose LGD data for forecasting since it is able to remove outliers, noise and residuals. In addition, it is able to produce smooth data.

4.3. Monthly Forecast of LGD Based on ARIMA-WT

Figure 2 shows the decomposition of LGD and default losses based on ARIMA-WT. The decomposition consists of a_1 , which is the approximated coefficients used for the proper forecasting, and d_1 , d_2 , d_3 , d_4 and d_5 , which show the fluctuations of data. The forecasting results for the ARIMA with MODWT are depicted in Figure 3.

As shown in Figure 2, the LGD data is decomposed using the four different functions; Haar WT, WTd4, WT La8 and WT C6. We can say that the WT La8 function is the suitable method to reveal the fluctuations, magnitudes and phases for the closing price data from the data used. The application of the WT La8 to the historical data decomposes them into a variety of resolution levels that expose their essential structure and it generates detailed coefficients at each one of the three decomposition levels.

According to WT mechanism, the three levels of decomposition can be carried out by the WT using the following equation: $X=TV1+TW1+TW2+TW3$ where X refers to the original signal which is represented in the topmost part of Figure 2. Then the next part consists of one approximation level (TV1) which shows the plot of the approximation coefficients for the transformed data using WTL8. The following parts of TW1, TW2 and TW3 represent the details levels, whereby, TW1 is the plot of the first level of the details coefficients, TW2 is the plot of the second level of the details coefficients and TW3 is the plot of the third level of the coefficients details. Any of these three levels (TW1, TW2, TW3) can be adopted for explaining the data.

Figure 2 Appendix 1

The estimated point of volatility at 95% and 99% confidence interval for LGD are given in Table 4. The forecast LGD value is 0.6841 in the first month, which is the highest point in the first year. The lowest point is in the last month at the same year. In the losses, the estimators always interested in the upper limit in confidence interval more than lower limit. Indeed, confidence interval for losses is frequently used as the maximum (0, upper limit).

Table 4 Appendix 1

Figure 3 shows the diagrams of past and forecasting values of LGD using WT functions combined with ARIMA model. According to the numerical results, we can say that the ARIMA-WT method succeeds in finding good results compared to the pure ARIMA. Moreover, when using ARIMA-wavelet with the La8 function the results are promising which leads us to conclude that ARIMA-WT with the La8 function based model is suitable for forecasting loss-given-defaults and losses.

Figure 3 Appendix 1

5. CONCLUSION

The probability of default (PD) and loss given default (LGD) are two important parameters in credit risk management. In this study, we proposed an ARIMA-WT new model to forecast the loss-given defaults (LGD) data time series. We evaluated four functions of WT, which are Haar (Haar), Daubechies (d4), least Asymmetric (La8), and Coiflet (C6). The method was validated on a sample data of credit portfolio loan collected from a bank in Jordan for the period up from January 2010 to December 2014. In addition, the wavelet models are compared with the ARIMA based on several metrics: the MAPE, MASE, RMSE, the AIC, AICs and the BIC.

The numerical results are interesting and show the significance of our work. The ARIMA-WT (La8) method is more accurate than the ARIMA and other ARIMA-Wavelet transform models. The new method is able to remove outliers, noise and residuals and to produce smooth data. However, this study can be improved by considering further factors in particular macroeconomic factors such as inflation, unemployment, Gross domestic product (GDP). This would be the main point to be explored in a near future work. In addition, we plan to use more sophisticated machine learning technique for default forecasting. Further, it would be nice to use some meta-heuristics techniques such as genetic algorithms and local search method for optimization purposes and appropriate factors selection.

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Appendix 1

TABLE 3. Comparing Between ARIMA And WT's Models With Different Accuracy Criteria

Models	ARIMA	Level	MAPE	MASE	RMSE	AIC	AICc	BIC
ARIMA-WT (haar)	(0,1,0)	1	5.6200	0.9836	0.0616	-158.5300	-158.4600	-156.4600
ARIMA-WT (d4)	(0,1,0)	1	5.5808	0.9836	0.0607	-160.2512	-160.1810	-158.1736
ARIMA-WT (La8)	(3,2,0)	3	0.3378	0.1621	0.0033	-487.2200	-486.4653	-478.9783
ARIMA-WT (C6)	(0,2,0)	3	0.9842	0.4408	0.0102	-362.7947	-362.7232	-360.7342

TABLE 4. Monthly Forecasting Of WT-La8 Of LGDF For One Year

Months	Estimation	Confidence Interval 95%		Confidence Interval 99%	
		Lower	Upper	Lower	Upper
1	0.6841	0.6775	0.6907	0.6755	0.6927
2	0.6617	0.6411	0.6822	0.6346	0.6887
3	0.6386	0.5985	0.6787	0.5859	0.6913
4	0.6146	0.5508	0.6784	0.5308	0.6984
5	0.5897	0.4983	0.6812	0.4695	0.7100
6	0.5645	0.4410	0.6880	0.4022	0.7268
7	0.5391	0.3793	0.6988	0.3291	0.7490
8	0.5134	0.3137	0.7131	0.2510	0.7759
9	0.4876	0.2446	0.7306	0.1682	0.8070
10	0.4617	0.1720	0.7513	0.0810	0.8423
11	0.4356	0.0964	0.7749	0.0000	0.8815
12	0.4096	0.0177	0.8014	0.0000	0.9245

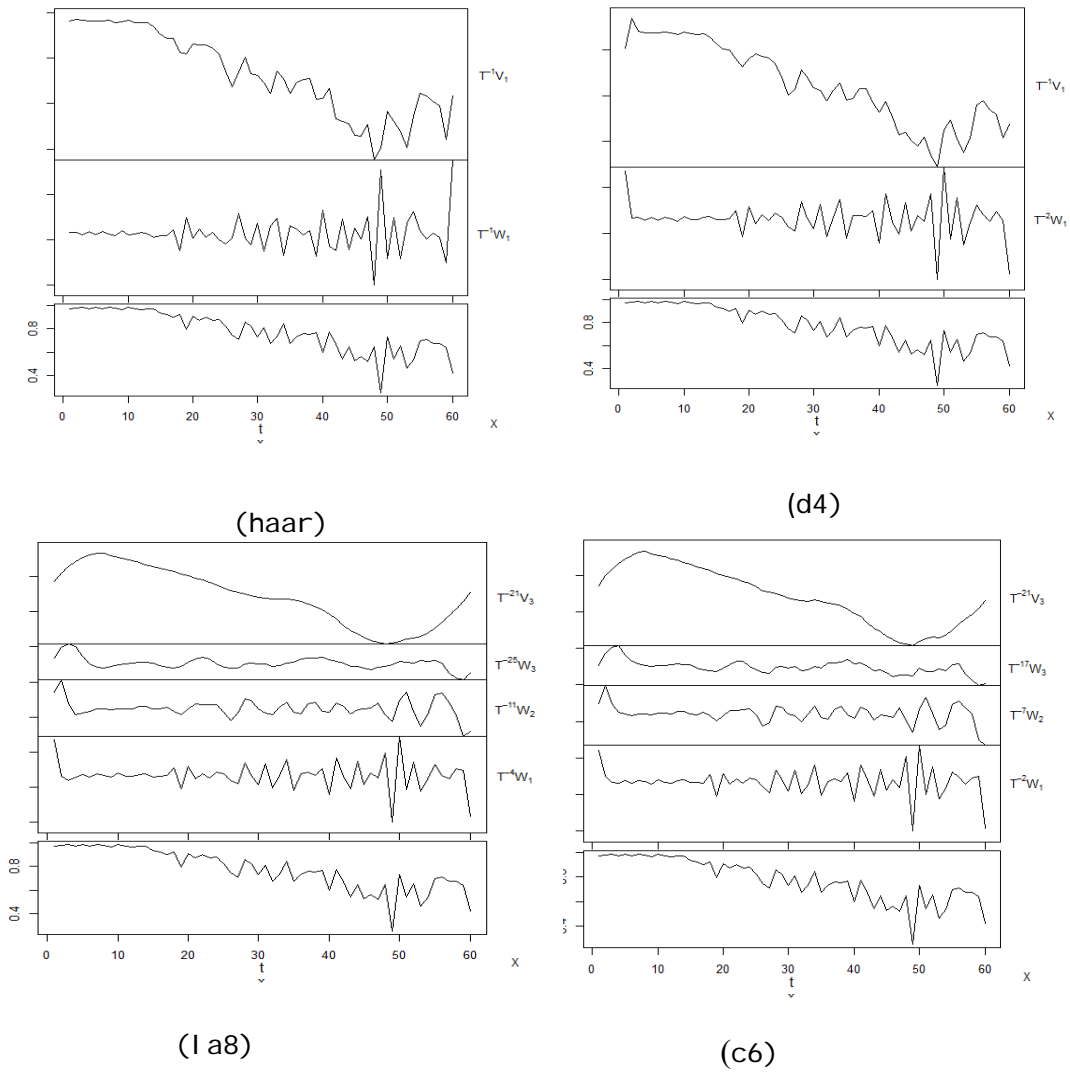
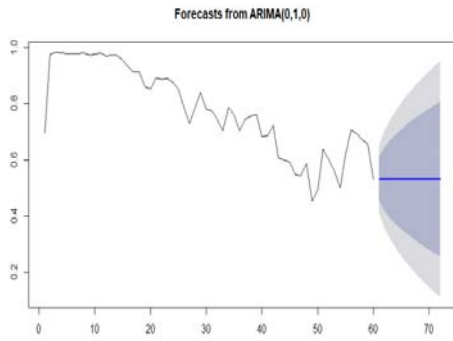
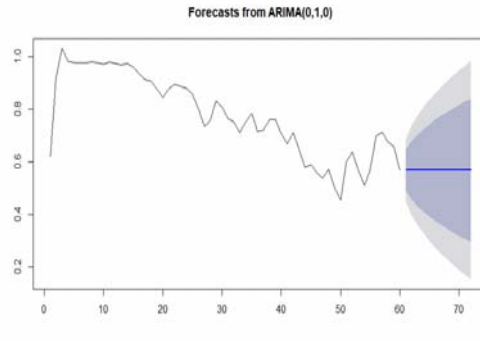


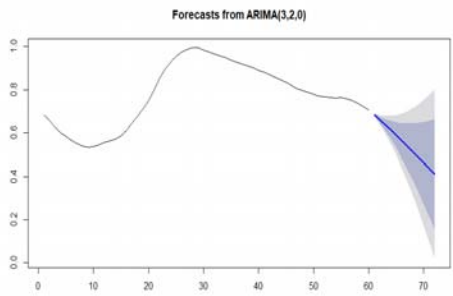
Figure 2: Data Decomposition Using WT's Functions.



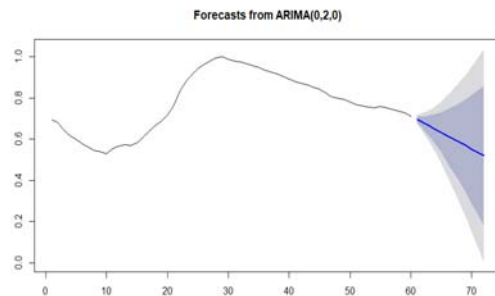
(haar)



(d4)



(1a8)



(c6)

Figure 3. Forecasting Diagram For WT's Functions.