ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

IMPLEMENTATION FINITE VOLUME METHOD OF SEA WAVES IN SUNDA STRAIT

[1] NURISSAIDAH ULINNUHA, [2] USWATUN KHASANAH

[1,2]Department of Mathematics, Islamic State University of Sunan Ampel Surabaya, Indonesia

E-mail: [1]nuris.ulinnuha@uinsby.ac.id , [2]uswatun.khasanah741@gmail.com

ABSTRACT

The Sunda Strait is directly linked to the Java Sea and the Indian Ocean. Geographically, the Sunda Strait is situated between Sumatra Island and Java Island, which plays an important role in inter-island transport routes. Sea transport depends very much on their own waves, if the waves of the sea are higher than usual, they will be dangerous for the transport of the sea. Sea waves are one of the problems that can be solved mathematically. This research is useful for obtaining a wave model that is close to the actual situation. This research contributes to obtain the numerical solution of a tidal wave of sea water using finite volume method and the terms stability of shallow water wave equation. Based on the mathematical model obtained by using the finite volume method, that is Godunov scheme, a stable sea waves model is obtained with the stability conditions that is $\frac{\Delta t}{\Delta x}\sqrt{2gb} \leq 1$.

Keywords: *Sunda Strait, Wave Modelling, Finite Volume Method, Godunov Scheme*

1. INTRODUCTION

Indonesia is a state of the archipelago adjacent to the Pasific Ocean and the Hindi Ocean. Indonesia has three modes of transport to connect one island to another, namely land transport, sea transport and air transport. However the cost of air transport has increased recently [1]. That's why many people in Indonesia are switching to a different kind of transport. According to data from Badan Pusat Statistik (BPS) on January-April 2019, domestic passengers decreased by 20,5% to 23,98 million passengers compared to the previous period [2].

One of the transport routes is going through the straits. There are many straits in Indonesia, such as the Madura Strait, the Bangka Strait, the Sunda Strait, the Malaka Strait, etc. Sunda Strait is the transport route to Java Island and Sumatra Island [3]. Geographically, the Sunda Strait is in equator, with irregular wind and tidal waves causing an accident at sea [4]. A sea accident also happens in Indonesia on 13 December 2016, when a ship sinks into the Sunda Strait due to bad weather conditions and an unpredictable sea wave height [5].

Sea waves is one of the physical problems that can be solved by mathematics, that is wave simulation with differential equations [6]. One wave model is the use of a shallow water equation [7]. By building a shallow water equation model, a solution will be found to predict where water flows, the speed of water flow, the area of the impact of incoming water and the rescue route to run to safe areas. [8]. One of the studies on numerical simulation of shallow water is carried out by Farouq using a finite-volume approach. The result of the research is the largest MSE of 0.002193 and the smallest MSE of 0.0003817 [7].

Generally, the shallow water equation is difficult to know, so a numerical approach is used. Some numerical methods are finite element method [9], finite difference method [10], finite volume method [11][12], etc. One of them can be used for numerical simulations from many type of conservation law, especially mechanical and fluid which is finite volume method.

There are two schemes in the finite volume method, namely the Lax scheme and the Godunov scheme. Calculation of fluid dynamics is used in this process by solving equations. The solution will be found by discretion, which is used to separate the results into new equations [13].

A research by Bobby M. Ginting [12] on 2011 about the 2-dimensional flood flow propagation models caused by dam ruins using the finite volume method. The result is finite volume method that give a good and accurate solution with error value for the depth is 1.33% and error value for the velocity is 1.85%.

Previous study of the finite volume method held by Zulbahrum Caniago in 2015 as a non-linear equation with the island expansion case. It was induced by deposition, which revealed that the findings of the volume approach were reasonable and suitable for use as a numerical solution. The finite volume method shows good and clean simulation result, even though it applied in hydrodynamics of fluid case which have high flexibility characteristic [11].

The distinction with previous research is the research object and the process scheme used. The entity being modeled is the waves of sea water in the Sunda Strait and the scheme used is Godunov because this method has the advantage of producing smooth and stable shock profiles.

Therefore, this research contributes to obtain the numerical solution of a tidal wave of sea water using finite volume method and the terms stability of shallow water wave equation. In this study the application of the finite volume method that is Godunov scheme as a model of sea waves during the Sunda Strait movement will be carried out. This study aims to learn and make a mathematical models of the sea height in the Sunda Strait.

2. RELEVANT STUDIES

2.1 Wave

Wave is the movement of each fluid particle that moves longitudinally and orbital simultaneously caused by the flow of mass, energy and momentum through various forms of material Another example of waves requiring a medium is sea water waves. Sea water waves that occur at sea and coast are important factors in determining the layout of sea transportation bases, shipping lane schemes, building planning in coastal areas, etc [14].

Sea water waves can be divided into several according to the factors that influence it, namely the wind waves that occur as a result of wind blowing on the surface of the sea, the tidal waves that occur as a result of the attractive force of the moon with the sun against the ocean, the tsunami waves that occur as a result of volcanic eruptions or sea-centered earthquakes, and the waves produced by moving ships, etc. [14].

2.2 Shallow Water Equation

The shallow water equation (SWE) is a hyperbolic system equation that can be used to describe a variety of fluid-related events. In addition, this equation is commonly used in the simulation of fluid movements that can take place both in one-dimensional space and in all directions in two-dimensional space [7]. This equation consists of two equations, the law of conservation of masses and the law of conservation of momentum. Generally, the SWE model is used for wave modeling on the surface.

In the SWE, the movement of the water surface is influenced by the depth and the length of the wavelength (λ) is greater than the depth of the water (d_0) [7].

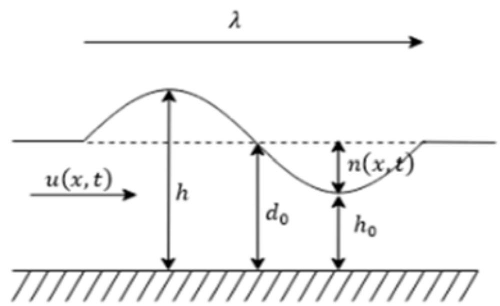

Figure 1: Shallow Water Equation Phenomenon

Figure 1 is an example of one-dimensional shallow water equation system where the variable x is the distance variable, t is the time variable, λ is the wavelength of wave, $h = h\,(x,t)$ is the depth of water obtained from the highest wave, $u = u\,(x,t)$ is the velocity in $x, n\,(x,t)$ is the change in wave height, d_0 is the sea height, and $g = 9.81\ m/s^2$ is the acceleration due to gravitation.

One dimensional SWE can be showed in Equation (1) [7].

$$h_t + (uh)_x = 0 \quad (1)$$

$$(hu)_t + \left(hu^2 + \frac{1}{2}gh^2\right)_x = 0 \quad (2)$$

The two-dimensional SWE can be seen in Equation (1) [15].

$$\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} + \frac{\partial (vh)}{\partial y} = 0 \quad (3)$$

$$\frac{\partial (uh)}{\partial t} + \frac{\partial (hu^2 + \frac{g}{2}h^2)}{\partial x} + \frac{\partial (huv)}{\partial y} = -gh\,\frac{\partial}{\partial x}b \quad (4)$$

Journal of Theoretical and Applied Information Technology
30th April 2020. Vol.98. No 08

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\frac{\partial(vh)}{\partial t} + \frac{\partial(hv^2 + \frac{g}{2}h^2)}{\partial y} + \frac{\partial(huv)}{\partial x} = -gh\frac{\partial}{\partial y}b$$

h representing the height of the surface, u as the symbol of air velocity in the x direction, v is the symbol of velocity in the y direction, g is the working force acting on the fluid, and b is the deserved sea height. Equation (4) consists of momentum conservation and Equation (3) shown continuity equation [15].

2.2.1 Continuity Equation

Continuity equation were obtained based on law conservation of mass which passed by fluid. Law conservation of mass is the mass of a system that will be constant even though there are so many processes in the system [16].

2.2.2 Momentum Equation

Momentum equation were obtained physically concept that related with momentum. One of physically concept is Newton II law. Newton II law is stated the sum of forces ($\sum F$) which is result of acceleration (α) and mass (m). Newton II law can be showed in Equation (5).

$$\sum F = m.a \qquad (5)$$

According to K. Anastasiou and C.T. Chan, SWE has some superiorities such as [8]:

1. It can able to handle problem with high complexity domain and fickle.
2. It can able to simulate stable flow, unstable flow, subcritical flow or supercritical flow.
3. It can simulate discontinue wave.

2.3 Finite Volume Method

The finite volume method is a computational method used to describe the results of numerical simulations of different types of conservation law, i.e. mass conservation and momentum conservation transitions from differential equations to algebraic equation [7].

From Equation (1) and Equation (2), both have a conservation mass which can be expressed as $q(x,t)t + f\big(q(x,t)\big)x = 0$ [14]. As an example, one-dimensional domain discretized as many as a finite-volume control is shown in Figure 2.

Figure 2: Discretization of Finite Volume Method

With $\Delta x = x_i - x_{(i+1)}$ and the boundary conditions exist at the specified points A and B and the discrete time domain becomes Equation (6) with $n = 0, 1, 2, 3, \ldots..n$.

$$t^n = n.\Delta t \qquad (6)$$

The value of Q_i^n is the quantity of average volume $q(x,t)$ approach in interval i-th space in the t^n on the time domain [17]:

$$Q_t^n \approx \frac{1}{\Delta x}\int_{x_{t-\frac{1}{2}}}^{x_{t+\frac{1}{2}}} q(x,t^n)dx$$

The value of $F_{t+\frac{1}{2}}^n$ is approach to the flux function $f(q(x,t))_x$ is:

$$F_{t+\frac{1}{2}}^n \approx \frac{1}{\Delta t}\int_{t^n}^{t^{n+1}} f(q(x,t^n))dt$$

The discrete form for the finite volume method is:

$$Q_t^{n+1} = Q_t^n - \frac{\Delta t}{\Delta x}(F_{t+\frac{1}{2}}^n - F_{t-\frac{1}{2}}^n)$$

Or it can be shown as:

$$\frac{Q_t^{n+1} - Q_t^n}{\Delta t} + \frac{F_{t+\frac{1}{2}}^n - F_{t-\frac{1}{2}}^n)}{\Delta x} = 0$$

The average value in cell q is updated in time units. In general, the time integral of the right side of the equation can not be calculated because the value of q varies with time along each edge of the cell and has no exact solution. Thus, it obtained a discreet form of the law on the conservation of mass using the volume method.

SWE is a hyperbolic system equation that can be used to define a variation of scheme. The definition of the finite-volume method is used as a discretization of the acceptable number obtained by the actual results. One of the scheme in the volume system is the Godunov scheme. [13]. The Godunov scheme is a conservative finite-volume method which solves exact, or approximate Riemann problems at each inter-cell boundary.

The form of Godunov scheme is on the Equation (7) [14].

$$F_{i-\frac{1}{2}}^n = \begin{cases} \min(f(Q_{i-1}^n),(Q_i^n)), & Q_{i-1}^n < Q_i^n \\ \max(f(Q_{i-1}^n),(Q_i^n)), & Q_{i-1}^n > Q_i^n \end{cases} \qquad (7)$$

2.4 Stability

According to Zulbahrum [11], instability in the finite volume method produces numerical errors from the exact solution so that the numerical solution does not have the exact value desired. One method that can be used to analyze stability is von Neumann or commonly known as Fourier. To analyze the stability of von Neumann, Equation (8) is replaced by the equation shown below.

$$u_j^n = \rho^n e^{iaj} \quad (8)$$

where n is the time, superscript $i = \sqrt{-1}$, j is vector, and a is in interval $[0, 2\pi]$. Based on the terms and conditions for von Neumann's stability in is Equation (9).

$$|\rho| \le 1 \quad (9)$$

With ρ is the norm of the eigenvalue of the matrix.

2.5 Perturbation Method

The perturbation method is a method that can be used to line up the fluid equations by using an equilibrium solution, which the solution is not depend on t parameter. The equation used in the perturbation method can be seen in Equation (10) [18].

$$\eta = 0 + \eta' \Rightarrow \eta(x,y,t) = \eta_0(x,y,t) + \epsilon\eta_1(x,y,t) + \cdots + \epsilon^n\eta_n(x,y,t)$$
$$u = 0 + u' \Rightarrow u(x,y,t) = u_0(x,y,t) + \epsilon u_1(x,y,t) + \cdots + \epsilon^n u_n(x,y,t) \quad (10)$$
$$v = 0 + v' \Rightarrow v(x,y,t) = v_0(x,y,t) + \epsilon v_1(x,y,t) + \cdots + \epsilon^n v_n(x,y,t)$$

With approximating $\eta_0(x,y,t) \approx 0, u_0(x,y,t) \approx 0, v_0(x,y,t) \approx 0$

3. METHOD

In this study, the mathematical modeling of sea waves in the Sunda Strait uses the finite-volume method using the Godunov scheme. The data used are sample data in the Sunda Strait. The purpose of this study is to obtain a mathematical model and simulation of air height results in the Sunda Strait. The flow of this research is as follows:

a. Study of Literature

A study of literature was carried out in order to obtain theories as well as studies of models of sea waves in the Sunda Strait.

b. Model Regulatory Equations

The basic fluid system is mathematically modeled to get the regulatory equation by integrating the Navier-Stokes 3D equation which is a formula derivation of mass conservation and three-dimensional momentum equilibrium to the volume element which then produces a shallow water flow equation in Equation (1) and (2).

c. Discretization Model with Finite Volume Method

Equations obtained later using partial differential equations. Next, the result was a model to be used as a numerical comparison using the Godunov scheme for higher finite volume, as can be seen in Equation (7).

d. Convergence Analysis

Convergence analysis is carried out by evaluating the stability of the equation obtained in order to obtain a successful simulation result. The stability analysis shall be carried out using the von Neumann condition in Equation (8). Von Neumann's sufficient stability conditions are shown in Equation (9).

e. Numerical Equation Simulation Model

The numerical equation simulation is done by testing the model through initializing the conditions to be modeled using Sunda Strait sample data.

f. Analysis of The Result

Next, the results of the numerical simulation model were analyzed to find out the mathematical model of sea waves in the Sunda Strait.

4. RESULT AND DISCUSSION

4.1 Regulatory Equations In Fluid Construction

The two-dimensional shallow water equation is related to the space variable x, y and time variable t. The three-dimensional model of fluid flow is related to the shallow equation of two-dimensional water using the space variable x, y and the time variable t. The construction of the two-dimensional shallow water wave model starts from a derivation using the continuity equation and the momentum equation, the kinematic boundary conditions of the sea wave, then begins to derive the hydrostatic pressure equation for the two-dimensional shallow water wave equation from the z direction momentum equation.

It can be seen in Figure 3, the shallow water equation derived from two equations, namely continuity with three dimensions and also the momentum equation, with x, y, z being the dimensions of space and t as dimensions of time. The equation of shallow water equation as in Equation (1).

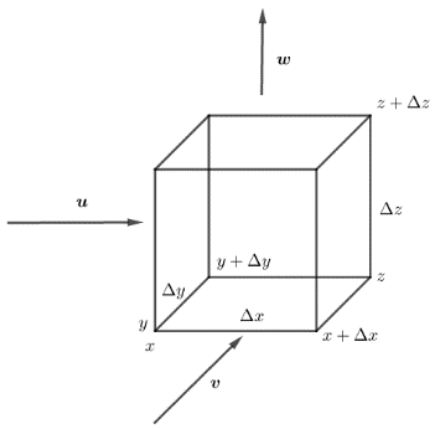

Figure 3: Ilustration of Element Volume

4.1.1 Continuity Equation

Based on the law of mass transfer to an element, the volume is assumed to be in the form of a cube. These sides are x, y, and z which is passed by the fluid, then the formula can be reduced in the continuity equation. For example, ρ is the density of a fluid, then u, v, and w or can be symbolized by q. It can be said that $q\ (u, v, w)$ is velocity of fluid particles that move in the direction of the coordinates of space the fluid, those are x, y, and z.

The velocity and cross-sectional area of the fluid affects the amount of fluid volume entering or leaving the volume element. Table 1 shows the total mass of fluid entering and leaving, both from the x, y and z

Table 1: Mass of Fluid

Amount Volume Entering	Amount Volume leaving
$(\rho u)\|_x \Delta y \Delta z$	$(\rho u)\|_{x+\Delta x} \Delta y \Delta z$
$(\rho v)\|_y \Delta x \Delta z$	$(\rho v)\|_{y+\Delta y} \Delta x \Delta z$
$(\rho w)\|_z \Delta x \Delta y$	$(\rho w)\|_{z+\Delta z} \Delta x \Delta y$

In chemical reactions, it is assumed that there is no loss of mass in the experiment, nor is there any change in average mass. The change is the difference between the average in-field fluid and the average out-of-field fluid. Then the equation of mass balance is obtained:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (11)$$

4.1.2 Momentum Equation

In a cube-shaped volume element with the sides x, y and z through which the fluid passes, the momentum equation can be derived based on the equilibrium of the three-dimensional momentum and the physical principle of Newton's Law II. Newton's Law II states that the total force (F) is the product of the mass of the fluid which is symbolized by m and its acceleration which is represented by a. Newton's Law Formula can be seen in Equation (5). By using the definition of acceleration that acceleration is the result of a derivation in velocity (v) with respect to the time unit (t), the Newton II Law equation can be rewritten as:

$$\begin{aligned} \sum F &= m\frac{dv}{dt} \\ &= \int m\,dv \\ &= mv \end{aligned}$$

mv is physic definition of momentum (Pm) so that it can be written as:

$$\begin{aligned} Pm &= mv \\ Pm &= (\rho V)v \\ Pm &= \rho v(\Delta x \Delta y \Delta z) \end{aligned}$$

From the results above, it appears that the momentum change in the fluid occurs in three directions namely x, y and z. In order to evaluate the average change in the momentum of the volume element, the difference between the average momentum entering the system and the average output of the system is calculated, and then the results are added to the amount of forces working on the system. Illustration of the momentum balance is shown in Figure 4. Momentum balance is a vector equation, where each variable working on a fluid states the direction of fluid motion that corresponds to the coordinates of the finite element.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

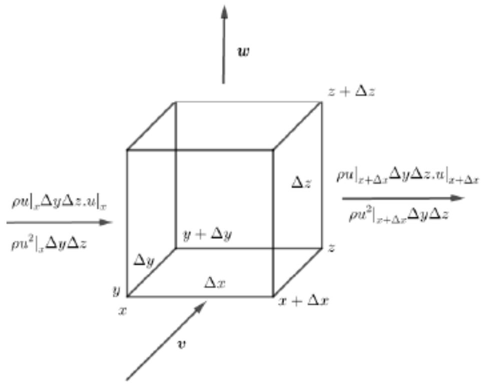

Figure 4: Ilustration of the Balance Momentum

a. x Direction

The momentum balance in x direction can be seen in Table 2.

Table 2: The Momentum Balance in x Direction

Momentum Entering	Momentum leaving
$(\rho u^2)\|_x \Delta y \Delta z$	$(\rho u^2)\|_{x+\Delta x} \Delta y \Delta z$
$(\rho uv)\|_y \Delta x \Delta z$	$(\rho uv)\|_{y+\Delta y} \Delta x \Delta z$
$(\rho uw)\|_z \Delta x \Delta y$	$(\rho uw)\|_{z+\Delta z} \Delta x \Delta y$

There are other factors involved in the average change in momentum, i.e. the forces which occur in the volume dimension, namely the gravity or symbolized W, which is influenced by the gravitational force as well as the force which is influenced by the pressure (P). The results of the calculation can be written below:

$$W = m.g$$
$$= (\rho V)g$$
$$= (\rho g(\Delta x \Delta y \Delta z))$$

Pressure (P) can be shown as:

$$P = \frac{F}{A}$$
$$F = P.A$$

Where F is the difference between the input force and output force which is influenced by pressure and areas:

$$F = F_{in} - F_{out}$$

The forces acting in the x direction and the resultant of other forces acting in the x direction can be written as follows:

$$\frac{\partial \rho u}{\partial t}\Delta x \Delta y \Delta z = Momentum\ Entering - Momentum\ leaving + Resultan\ of\ the\ force$$

The result of the momentum on x direction is:

$$\frac{\partial u}{\partial t} + \frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = g - \frac{1}{\rho}\frac{\partial P}{\partial x}$$

b. y Direction

The momentum balance in y direction can be seen in Table 3.

Table 3: The Momentum Balance in y Direction

Momentum Entering	Momentum leaving
$(\rho uv)\|_x \Delta y \Delta z$	$(\rho uv)\|_{x+\Delta x} \Delta y \Delta z$
$(\rho v^2)\|_y \Delta x \Delta z$	$(\rho v^2)\|_{y+\Delta y} \Delta x \Delta z$
$(\rho vw)\|_z \Delta x \Delta y$	$(\rho vw)\|_{z+\Delta z} \Delta x \Delta y$

Another factor that affects the momentum changes direction x, applies also to the y direction. The forces acting in the y direction as well the resultant other forces acting in the y direction be written as follows:

$$\frac{\partial \rho u}{\partial t}\Delta x \Delta y \Delta z = Momentum\ Entering - Momentum\ leaving + Resultan\ of\ the\ force$$

The result of the momentum in y direction is:

$$\frac{\partial v}{\partial t} + \frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} + \frac{\partial vw}{\partial z} = g - \frac{1}{\rho}\frac{\partial P}{\partial y}$$

c. z Direction

The momentum balance in z direction can be seen in Table 4.

Table 4: The Momentum Balance with The z Direction

Momentum Entering	Momentum leaving
$(\rho uw)\|_x \Delta y \Delta z$	$(\rho uw)\|_{x+\Delta x} \Delta y \Delta z$
$(\rho vw)\|_y \Delta x \Delta z$	$(\rho vw)\|_{y+\Delta y} \Delta x \Delta z$
$(\rho w^2)\|_z \Delta x \Delta y$	$(\rho w^2)\|_{z+\Delta z} \Delta x \Delta y$

Another factor that affects the momentum changes direction x and y, applies also to the z direction. The forces acting in the z direction as well

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

the resultant other forces acting in the z direction be written as follows:

$$\frac{\partial \rho u}{\partial t}\Delta x \Delta y \Delta z = Momentum\ Entering - Momentum\ leaving + Resultan\ of\ the\ force$$

The result of the momentum on z direction is:

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}=g-\frac{1}{\rho}\frac{\partial P}{\partial x}$$

The gravitational force only works in the direction or z plane only, so that the gravitational force on x and y is 0. The momentum equation in the z direction can be shown as follows:

$$\frac{\partial u}{\partial t}+\frac{\partial u^2}{\partial x}+\frac{\partial uv}{\partial y}+\frac{\partial uw}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\frac{\partial v}{\partial t}+\frac{\partial uv}{\partial x}+\frac{\partial v^2}{\partial y}+\frac{\partial vw}{\partial z}=-\frac{1}{\rho}\frac{\partial P}{\partial y}$$

$$\frac{\partial w}{\partial t}+\frac{\partial uw}{\partial x}+\frac{\partial vw}{\partial y}+\frac{\partial w^2}{\partial z}=g-\frac{1}{\rho}\frac{\partial P}{\partial z}$$

The momentum equation in the z direction is elaborated to determine the hydrostatic pressure for shallow water waves two dimensions. Because the fluid does not rotate, the total yield for w is 0. The result of substitution is shown below:

$$\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}+\frac{\partial (vh)}{\partial y}=0$$

$$\frac{\partial (uh)}{\partial t}+\frac{\partial (hu^2+\frac{g}{2}h^2)}{\partial x}+\frac{\partial (huv)}{\partial y}=-gh\frac{\partial}{\partial x}b \quad (13)$$

$$\frac{\partial (vh)}{\partial t}+\frac{\partial (hv^2+\frac{g}{2}h^2)}{\partial y}+\frac{\partial (huv)}{\partial x}=-gh\frac{\partial}{\partial y}b$$

4.2 Linearization Of Shallow Water Equation System

Equation (3) and (4) will be lineared so that the shallow water equation system is more easily resolved. Linearization of shallow water equation is using perturbation method, namely at Equation (10).

The two-dimensional shallow water equation model is related to the space variable x, y and time variable t. The three-dimensional model of fluid flow in Equation (1) is substituted with Equation (10) then the results are completed only up to the order $\in$.

The equation is obtained in a linear two-dimensional sea wave, which is in Equation (14).

$$\frac{\partial h}{\partial t}+\frac{\partial (uh)}{\partial x}+\frac{\partial (vh)}{\partial y}=0$$

$$\frac{\partial (uh)}{\partial t}+\frac{\partial (hu^2+\frac{g}{2}h^2)}{\partial x}+\frac{\partial (huv)}{\partial y}=0 \quad (14)$$

$$\frac{\partial (vh)}{\partial t}+\frac{\partial (hv^2+\frac{g}{2}h^2)}{\partial y}+\frac{\partial (huv)}{\partial x}=0$$

Or it can be shown as:

$$\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}[u(\eta+b)]+\frac{\partial}{\partial y}[v(\eta+b)]=0 \quad (a)$$

$$\frac{\partial}{\partial t}[u(\eta+b)]+\frac{\partial}{\partial x}[u^2(\eta+b)+\tfrac{1}{2}g(\eta+b)^2]+\frac{\partial}{\partial y}[uv(\eta+b)]=0 \quad (b) \qquad (15)$$

$$\frac{\partial}{\partial t}[v(\eta+b)]+\frac{\partial}{\partial x}[uv(\eta+b)]+\frac{\partial}{\partial y}\left[v^2(\eta+b)+\tfrac{1}{2}g(\eta+b)^2\right]=0 \quad (c)$$

The result of Equation (15a) is:

$$\frac{\partial \eta}{\partial t}+\frac{\partial}{\partial x}[u(\eta+b)]+\frac{\partial}{\partial y}[v(\eta+b)]$$

$$=\frac{\partial}{\partial t}\eta_0(x,y,t)+\in\left(\frac{\partial}{\partial t}\eta_1(x,y,t)\right)+$$
$$\left(\frac{\partial}{\partial x}u_0(x,y,t)+\in\left(\frac{\partial}{\partial x}u_1(x,y,t)\right)\right)(\eta_0(x,y,t)+$$
$$\in\eta_1(x,y,t)+b)+(u_0(x,y,t)+\in u_1(x,y,t))$$
$$\left(\frac{\partial}{\partial x}\eta_0(x,y,t)+\in\left(\frac{\partial}{\partial x}\eta_1(x,y,t)\right)\right)+\left(\frac{\partial}{\partial y}v_0(x,y,t)+\in\right.$$
$$\left.\left(\frac{\partial}{\partial y}v_1(x,y,t)\right)\right)(\eta_0(x,y,t)+$$
$$\in\eta_1(x,y,t)+b)+(v_0(x,y,t)+\in v_1(x,y,t))$$
$$\left(\frac{\partial}{\partial y}\eta_0(x,y,t)+\in\left(\frac{\partial}{\partial y}\eta_1(x,y,t)\right)\right)$$

$$=\left(\frac{\partial}{\partial x}u_0(x,y,t)\right)\eta_1(x,y,t)+\eta_0(x,y,t)$$
$$\left(\frac{\partial}{\partial x}u_1(x,y,t)\right)+\eta_0(x,y,t)\left(\frac{\partial}{\partial y}v_1(x,y,t)\right)+$$
$$\eta_1(x,y,t)\left(\frac{\partial}{\partial y}v_0(x,y,t)\right)+\left(\frac{\partial}{\partial x}u_1(x,y,t)\right)b+$$
$$u_0(x,y,t)\left(\frac{\partial}{\partial x}\eta_1(x,y,t)\right)+\left(\frac{\partial}{\partial x}\eta_0(x,y,t)\right)$$
$$u_1(x,y,t)+\left(\frac{\partial}{\partial y}v_1(x,y,t)\right)b+v_0(x,y,t)$$
$$\left(\frac{\partial}{\partial y}\eta_1(x,y,t)\right)+\left(\frac{\partial}{\partial y}\eta_0(x,y,t)\right)v_1(x,y,t)$$
$$+\frac{\partial}{\partial t}\eta_1(x,y,t)$$

$=\left(\frac{d}{dx}0\right)\eta_1(x,y,t)+\eta_1(x,y,t)\left(\frac{d}{dx}0\right)+$

$\left(\frac{\partial}{\partial x}u_1(x,y,t)\right)b+\left(\frac{d}{dx}0\right)u_1(x,y,t)+\left(\frac{d}{dy}0\right)v_1(x,y,t)+$
$\frac{\partial}{\partial t}\eta_1(x,y,t)$

$0=\left(\frac{\partial}{\partial x}u_1(x,y,t)\right)b+\left(\frac{\partial}{\partial y}v_1(x,y,t)\right)b+\frac{\partial}{\partial t}\eta_1$

The result of Equation (15b) is:

$\frac{\partial}{\partial t}[u(\eta+b)]+\frac{\partial}{\partial x}[u^2(\eta+b)+\frac{1}{2}g(\eta+b)^2]+$
$\frac{\partial}{\partial y}[uv(\eta+b)]$

$=\left(\frac{\partial}{\partial t}u_0(x,y,t)+\in\left(\frac{\partial}{\partial t}u_1(x,y,t)\right)\right)(\eta_0(x,y,t)+$

$\in\eta_1(x,y,t)+b)+(u_0(x,y,t)+\in u_1(x,y,t))$

$\left(\frac{\partial}{\partial t}\eta_0(x,y,t)+\in\left(\frac{\partial}{\partial t}\eta_1(x,y,t)\right)\right)+$

$\left(\frac{\partial}{\partial x}\eta_0(x,y,t)+\in\left(\frac{\partial}{\partial x}\eta_1(x,y,t)\right)\right)+(u_0(x,y,t)+$

$\in u_1(x,y,t)^2+2(\eta_0(x,y,t)+\in\eta_1(x,y,t)+$
$b)(u_0(x,y,t)+\in u_1(x,y,t))$

$\left(\frac{\partial}{\partial x}u_0(x,y,t)+\in\left(\frac{\partial}{\partial x}u_1(x,y,t)\right)\right)+$
$g(\eta_0(x,y,t)+\in\eta_1(x,y,t)+b)+$

$\left(\frac{\partial}{\partial x}\eta_0(x,y,t)+\in\left(\frac{\partial}{\partial x}\eta_1(x,y,t)\right)\right)+$
$\left(\frac{\partial}{\partial y}v_0(x,y,t)+\in\left(\frac{\partial}{\partial y}v_1(x,y,t)\right)\right)(u_0(x,y,t)+$
$\in u_1(x,y,t)+\in v_1(x,y,t))+(\eta_0(x,y,t)+$

$\in\eta_1(x,y,t)+b)\left(\frac{\partial}{\partial y}u_0(x,y,t)+\in\right.$

$\left.\left(\frac{\partial}{\partial y}u_1(x,y,t)\right)\right)(v_0(x,y,t)+\in v_1(x,y,t))+$

$(\eta_0(x,y,t)+\in\eta_1(x,y,t)+b)(u_0(x,y,t)+$
$\in u_1(x,y,t))\left(\frac{\partial}{\partial y}v_0(x,y,t)+\in\left(\frac{\partial}{\partial y}v_1(x,y,t)\right)\right)$

$=2\left(\frac{d}{dx}0\right)u_1(x,y,t)+\eta_1(x,y,t)\left(\frac{d}{dx}0\right)g+$

$\left(\frac{\partial}{\partial x}\eta_1(x,y,t)\right)gb+u_1(x,y,t)\left(\frac{d}{dx}0\right)v_1(x,y,t)b$

$+\left(\frac{\partial}{\partial t}u_1(x,y,t)\right)b+\left(\frac{d}{dt}0\right)\eta_1(x,y,t)+$
$\left(\frac{d}{dt}0\right)u_1(x,y,t)$

$0=b\left(\left(\frac{\partial}{\partial x}\eta_1(x,y,t)\right)g+\left(\frac{\partial}{\partial t}u_1(x,y,t)\right)\right)$

The result of Equation (15c) is:

$\frac{\partial}{\partial t}[v(\eta+b)]+\frac{\partial}{\partial y}[v^2(\eta+b)+\frac{1}{2}g(\eta+b)^2]+$
$\frac{\partial}{\partial x}[uv(\eta+b)]$

$=\left(\frac{\partial}{\partial t}v_0(x,y,t)+\in\left(\frac{\partial}{\partial t}v_1(x,y,t)\right)\right)(\eta_0(x,y,t)+$

$\in\eta_1(x,y,t)+b)+(v_0(x,y,t)+\in v_1(x,y,t))$

$\left(\frac{\partial}{\partial t}\eta_0(x,y,t)+\in\left(\frac{\partial}{\partial t}\eta_1(x,y,t)\right)\right)+$

$\left(\frac{\partial}{\partial x}\eta_0(x,y,t)+\in\left(\frac{\partial}{\partial x}\eta_1(x,y,t)\right)\right)+(v_0(x,y,t)+$

$\in v_1(x,y,t)^2+2(\eta_0(x,y,t)+\in\eta_1(x,y,t)+$
$b)(u_0(x,y,t)+\in u_1(x,y,t))$

$\left(\frac{\partial}{\partial x}u_0(x,y,t)+\in\left(\frac{\partial}{\partial x}u_1(x,y,t)\right)\right)+$
$g(\eta_0(x,y,t)+\in\eta_1(x,y,t)+b)+$

$\left(\frac{\partial}{\partial x}\eta_0(x,y,t)+\in\left(\frac{\partial}{\partial x}\eta_1(x,y,t)\right)\right)+$
$\left(\frac{\partial}{\partial y}v_0(x,y,t)+\in\left(\frac{\partial}{\partial y}v_1(x,y,t)\right)\right)(u_0(x,y,t)+$
$\in u_1(x,y,t)+\in v_1(x,y,t))+(\eta_0(x,y,t)+$

$\in\eta_1(x,y,t)+b)\left(\frac{\partial}{\partial y}u_0(x,y,t)+\in\right.$

$\left.\left(\frac{\partial}{\partial y}u_1(x,y,t)\right)\right)(v_0(x,y,t)+\in v_1(x,y,t))+$

$(\eta_0(x,y,t)+\in\eta_1(x,y,t)+b)(u_0(x,y,t)+$
$\in u_1(x,y,t))\left(\frac{\partial}{\partial y}v_0(x,y,t)+\in\left(\frac{\partial}{\partial y}v_1(x,y,t)\right)\right)$

$$= \left(\frac{d}{dx}0\right) u_1(x,y,t)b + v_1(x,y,t)\left(\frac{d}{dx}0\right) b + \eta_1(x,y,t)\left(\frac{d}{dy}0\right) g + 2\left(\frac{d}{dy}0\right) v_1(x,y,t)b + \left(\frac{\partial}{\partial y}\eta_1(x,y,t)\right) gb + \left(\frac{\partial}{\partial t}v_1(x,y,t)\right) b + \left(\frac{d}{dx}0\right)\eta_1(x,y,t) + \left(\frac{d}{dx}0\right) v_1(x,y,t)$$

$$0 = b\left(\left(\frac{\partial}{\partial y}\eta_1(x,y,t)\right) g + \left(\frac{\partial}{\partial t}v_1(x,y,t)\right)\right)$$

Effects of the linearization of the shallow water wave equation, i.e. a set of shallow water equation system that is linear can be seen in Equations (13).

$$\frac{\partial h}{\partial t} + b\frac{\partial u}{\partial x} + b\frac{\partial v}{\partial y} = 0$$
$$\frac{\partial u}{\partial t} + g\frac{\partial h}{\partial x} = 0 \qquad (16)$$
$$\frac{\partial v}{\partial t} + \frac{\partial h}{\partial y} = 0$$

In shallow water 2-dimensional linear equation, the wave is a function of the space variable x and y and also the t variable. On Equations (16), the variable h is the surface deviation water from the equilibrium condition z = b, the variable u is the fluid velocity at surface in x direction, variable v is the velocity of the fluid at the surface in y direction, and g is the acceleration due to gravity in the fluid.

4.3 Godunov Scheme

The domain used is $[0, M_x] \times [0, M_y]$ with each partitioned by $\frac{1}{2}\Delta x$ and $\frac{1}{2}\Delta y$ interval, that can be seen in Figure 5.

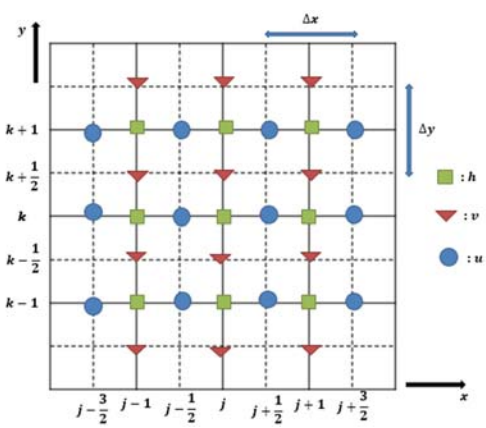

Figure 5: Arakawa C-Grid

The 2-dimensional shallow water equation in Equation 13 can be described as discreet by Godunov scheme:

$$h^{n+1}_{j,k} = h^n_{j,k} - b\frac{\Delta t}{\Delta x}\left(u^n_{j+\frac{1}{2},k} - h^n_{j-\frac{1}{2},k}\right) - b\frac{\Delta t}{\Delta y}\left(u^n_{j,k+\frac{1}{2}} - u^n_{j,k-\frac{1}{2}}\right)$$
$$u^{n+1}_{j+\frac{1}{2},k} = u^n_{j+\frac{1}{2},k} - b\frac{\Delta t}{\Delta x}\left(h^{n+1}_{j+1,k} - h^{n+1}_{j,k}\right) \qquad (17)$$
$$v^{n+1}_{j,k+\frac{1}{2}} = u^n_{j,k+\frac{1}{2}} - b\frac{\Delta t}{\Delta x}\left(h^{n+1}_{j,k+1} - h^{n+1}_{j,k+1}\right)$$

4.4 Analysis of Stability

Stability analysis is using the von Neumann stability requirements to find out the system produced by convergence, by substituting $h^n_{j,k} = \rho^n e^{ia(j+k)}$, $u^n_{j+\frac{1}{2},k} = r^n e^{ia(j+\frac{1}{2}+k)}$, $v^n_{j,k+\frac{1}{2}} = s^n e^{ia(j+k+\frac{1}{2})}$ and the result is:

$$\begin{bmatrix}\rho^{n+1}\\ r^{n+1}\\ s^{n+1}\end{bmatrix} = \begin{bmatrix} 1 & -2ib\frac{\Delta t}{\Delta x}sin\left(\frac{a}{2}\right) & -2ib\frac{\Delta t}{\Delta y}sin\left(\frac{a}{2}\right)\\ -2ig\frac{\Delta t}{\Delta x}sin\left(\frac{a}{2}\right) & 1-4gb\left(\frac{\Delta t}{\Delta x}\right)^2 sin^2\left(\frac{a}{2}\right) & -4gb\frac{\Delta t}{\Delta x\Delta y}sin^2\left(\frac{a}{2}\right)\\ -2ig\frac{\Delta t}{\Delta x}sin\left(\frac{a}{2}\right) & -4gb\frac{\Delta t}{\Delta x\Delta y}sin^2\left(\frac{a}{2}\right) & 1-4gb\left(\frac{\Delta t}{\Delta x}\right)^2 sin^2\left(\frac{a}{2}\right)\end{bmatrix}\begin{bmatrix}\rho^n\\ r^n\\ s^n\end{bmatrix}$$

With B

$$= \begin{bmatrix} 1 & -2ib\frac{\Delta t}{\Delta x}sin\left(\frac{a}{2}\right) & -2ib\frac{\Delta t}{\Delta y}sin\left(\frac{a}{2}\right)\\ -2ig\frac{\Delta t}{\Delta x}sin\left(\frac{a}{2}\right) & 1-4gb\left(\frac{\Delta t}{\Delta x}\right)^2 sin^2\left(\frac{a}{2}\right) & -4gb\frac{\Delta t}{\Delta x\Delta y}sin^2\left(\frac{a}{2}\right)\\ -2ig\frac{\Delta t}{\Delta x}sin\left(\frac{a}{2}\right) & -4gb\frac{\Delta t}{\Delta x\Delta y}sin^2\left(\frac{a}{2}\right) & 1-4gb\left(\frac{\Delta t}{\Delta x}\right)^2 sin^2\left(\frac{a}{2}\right)\end{bmatrix}$$

Based on von Neuman's requirements, the eigenvalue of the matrix must be less than or equal to 1. The next step is to simplify by assuming $\Delta y = \Delta x$, then the eigenvalue of the matrix B obtained is:

$$(\lambda - 1)\left(\lambda^2 - 2\left(1 - 4gb\left(\frac{\Delta t}{\Delta x}\right)^2 sin^2\left(\frac{a}{2}\right)\right)\lambda + 1\right) = 0$$

The stability requirements from the volume method to the Godunov scheme for the two-dimensional water wave equation are obtained as:

$$\frac{\Delta t}{\Delta x}\sqrt{2gb} \leq 1$$

4.5 Simulation and Interpretation

After the result of the stability requirements are obtained from Godunov scheme, the values from Δt and Δx can be selected that meet the stability

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

requirements which are used in the simulation of sea waves in the Sunda Strait. The finite volume method requirements for two-dimensional sea waves in this study is:

$$\frac{\Delta t}{\Delta x}\sqrt{2gb} \leq 1$$

In this simulation, initial values $h(x, y, 0) = b \times e^{-5(x^2+y^2)}$ and $u(x, y, 0) = 0$ with Δy = Δx = 0.85, Δt = 0.05, and Sunda Strait depth = 100m. Figure (5), (6) and (7) show the solution of linier shallow water equation 2-dimension by using Godunov scheme to fulfill the stability requirements.

From the Figure 5, we can see that the waves at $t = 0$ is an initial wave occurs.

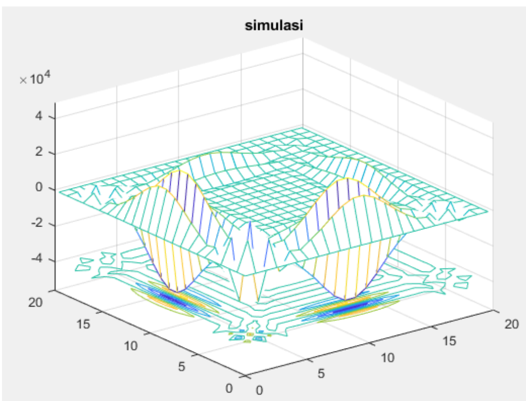

Figure 5: $t = 0$

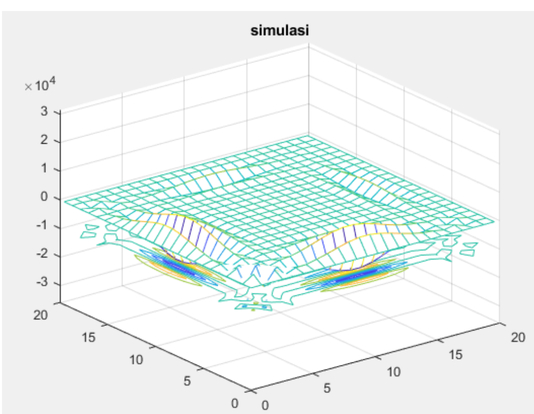

Figure 6: $t = 6$

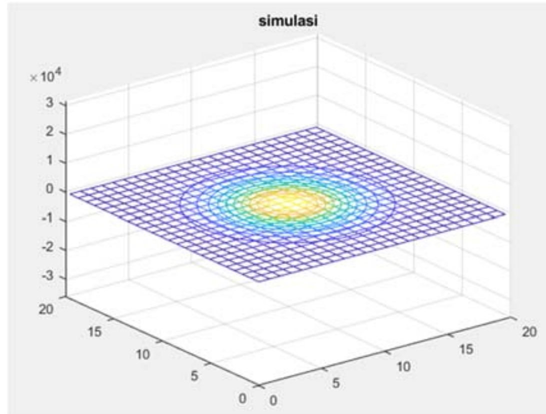

Figure 7: $t = 12$

In Figure 6, when t = 6 sea water waves are still forming, while at t = 12 (Figure 7) the results show that the surface conditions of the waves are stable and not repeatable.

5. CONCLUSION

Based on the results of the study, it was found that the numerical resolution of sea waves of the Sunda Strait, using the finite volume method with the Godunov system, has the stability requirement of the system that is $\frac{\Delta t}{\Delta x}\sqrt{2gb} \leq 1$. It means that the stability requirements depend on the value of the interval of direction x (Δx), time interval (Δt), the force of gravity (g) and the lower limit of the wave base (b). If it does not meet the stability requirements, there will be a blow up or infinity solutions.

The limitation of this research is that the samples used in the simulation model is only taken from the Sunda Strait. This study also uses only one scheme, the Godunov scheme. In addition, this research took a long time because the reduction in the model formula was quite complex.

REFERENCES:

[1] L. Hanafi and D. Indrajaya, “Simulasi Model Gelombang Pasang Surut dengan Metode Beda Hingga,” vol. 2, no. 2, pp. 93–101, 2005.

[2] B. P. Statistik, *Data Aktual Pengguna Moda Transportasi*, no. April. 2019.

[3] S. Prawiradisastra, “Penyebab Timbulnya Bencana Gelombang Tsunami di Wilayah Selat Sunda dan Upaya Penanggulangan,” *Alami: Jurnal Teknologi Reduksi Risiko Bencana*, vol. 10, no. 2. 2005.

[4] R. B. A. Nugraha, H. Surbakti, J. Risandi, and L. O. N. Mbay, “Simulasi Pola Arus Laut Dua

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Dimensi Di Perairan Sekitar Nusa Penida, Bali," *J. Kelaut. Nas.*, vol. 9, no. 1, p. 37, 2014.

[5] Komite Nasional Keselamatan Transportasi, "Data Investigasi Kecelakaan Pelayaran," *Media Release KNKT*, vol. 2016, no. November, pp. 16–17, 2016.

[6] Hasan, T. Yulianto, R. Amalia, and Faisol, "Penerapan Metode Beda Hingga pada Model Matematika Aliran Banjir dari Persamaan Saint Venant," *Math J.*, vol. 2 No. 1, 2016.

[7] R. Farouq and D. Adytia, "Simulasi Numerik Shallow Water Equations Menggunakan Skema Eksplisit Collocated Lax-Friedrichs Finite Volume Method," vol. 5, no. 2, pp. 3767–3773, 2018.

[8] C. F. Ai and S. Jin, "Solution of the 2D shallow water equations using the high-resolution finite-volume method on unstructured meshes," *Jisuan Lixue Xuebao/Chinese J. Comput. Mech.*, vol. 26, no. 6, pp. 900–905, 2009.

[9] I. G. P. Budiana and Y. A. Pranata, "Pemodelan metode elemen hingga nonlinier dinding panel gewang laminasi 2D terhadap beban lateral (192S)," *Konf. Nas. Tek. Sipil 7 (KoNTekS 7)*, vol. 7, no. KoNTekS 7, pp. 209–217, 2013.

[10] C. A. Hapsoro and W. Srigutomo, "Method Pemodelan Aliran Fluida 2-D Pada Kasus Aliran Permukaan Menggunakan Metode Beda Hingga," no. August 2013, 2018.

[11] Z. Caniago, "METODA VOLUME HINGGA DENGAN JALA TAK-TERSTRUKTUR UNTUK MENSIMULASIKAN PERLUASAN PULAU AKIBAT DEPOSISI SEDIMEN," vol. IV, pp. 11–16, 2015.

[12] B. M. Ginting, D. K. Natakusumah, M. S. B. Kusuma, and D. Harian, "Model 2 Dimensi Propagansi Aliran Banjir Akibat Keruntuhan Bendungan dengan Metode Volume Hingga," *Pros. Konf. Nas. Pascasarj. Tek. Sipil*, 2011.

[13] S. Mungkasi, U. S. Dharma, and S. G. Roberts, "ANUGA software for numerical simulations of shallow water flows," no. May 2014, 2012.

[14] P. Higuera, M. Jesus, J. L. Lara, I. J. Losada, Y. Guanche, and G. Barajas, "Numerical simulation of three-dimensional breaking waves on a gravel slope using a two-phase flow Navier – Stokes model," *J. Comput. Appl. Math.*, vol. 246, pp. 144–152, 2013.

[15] A. Jameson, W. Schmidt, and E. Turkel, "AIAA 1981 – 1259 Numerical Solution of the Euler Equations by Finite Volume Methods Schemes AIAA 14th Fluid and Plasma Dynamic Conference Palo Alto , California," *Convergence*, vol. M, no. AIAA 81-1259, pp. 1–19, 1981.

[16] H. M. Marbun and M. Hazwi, "SIMULASI ALIRAN FLUIDA PADA POMPA HIDRAM DENGAN TINGGI AIR JATUH 2.3 M DENGAN MENGGUNAKAN PERANGKAT LUNAK CFD," *J. e-Dinamis*, vol. 7, no. 3, pp. 136–145, 2013.

[17] M. Shearer, *Partial Differential Equations An Introduction to Theory and Applications*. 2015.

[18] H. J. Wilson and J. M. Rallison, *Non-Newtonian Fluid Mech*. 1997.