

# METHODS OF MODEL SYNTHESIS AND MULTI-CRITERIA OPTIMIZATION OF CHEMICAL-ENGINEERING SYSTEMS IN THE FUZZY ENVIRONMENT

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## ABSTRACT

Methods of synthesis of mathematical models and multi-criteria optimization of parameters of chemical-technological systems in the conditions of initial information uncertainty are proposed. It is proposed a technique for constructing models of interconnected aggregates of technological systems using the example of a delayed coking unit (DCU). Methods for the synthesis of fuzzy and linguistic models based on the involvement of experts have been developed. Based on the proposed methods, the structure of coke oven models is determined and identified in the form of clear and fuzzy regression equations. Parametric identification is carried out on the basis of least squares methods and the theory of fuzzy sets. The problems of multicriteria optimization of the modes of operation of the ultrasonic system in a fuzzy environment are formulated and methods of their solution are proposed on the basis of various optimality principles. The novelty of these results lies in the fact that the tasks are posed and solved in a fuzzy environment without converting them to a system of clear tasks.

**Keywords:** *Mathematical Models, Multi-Criteria Optimization, Chemical-Engineering System, Fuzzy Information, Delayed Coking Unit.*

## 1. INTRODUCTION

When developing mathematical models of complex chemical-engineering systems (CES) that can be used to solve problems of optimizing their operating modes, there are often problems of uncertainty associated with the lack and fuzziness of the initial information. The most effective approach to solving these problems of uncertainty is the use of expert assessment methods and the theory of fuzzy sets based on the methodology of system analysis.

Problems of scarcity and uncertainty due to the vagueness of the initial information can be caused by the fact that many, including important parameters of technological systems of real production are poorly measured, their measurements are economically impractical or not measured at all. However, these non-quantifiable parameters can be adequately evaluated by a person, namely an experienced specialist, production personnel, and decision-maker (DM).

These specialists, based on their experience, knowledge and intuition, can fairly reliably evaluate them in natural language, i.e. with fuzzy information such as "below normal", "normal", "above normal", etc. The most effective methods for collecting, formalizing, processing, and using such additional fuzzy information when modeling and optimizing production facilities, i.e., methods for solving problems of source information scarcity are expert evaluation methods [1],[2] and fuzzy set theory [3],[4],[5], which are integrated based on the methodology of system analysis [6],[7].

Typically, in the conditions of uncertainty, it is proposed to apply the methods of probability theory and mathematical statistics to model and optimize the CES [8],[9],[10]. However, the application of these methods is wrongful if the uncertainty is related to the fuzziness of the initial information, which often happens under actual production conditions. Under these conditions, the reliable statistical information is missing or insufficient, and the axioms of probability theory

(statistical stability of the research object, repeatability of experiments under similar conditions) are not met. Sometimes, only the fuzzy information representing knowledge and experience (intuition, judgment, pronouncement) of an expert, a decision-maker (DM), is available, which can be formalized and applied in the development of models, decision making and control of complex CES [1], [11],[12],[13],[14],[15],[16].

With the competence of DM, experts and with the correct organization of their interview, collection and processing of fuzzy information on its basis, it is possible to construct models that take into account all the complex interrelations of various parameters and variables of complex CES. The resulting models can be more meaningful than the models developed by traditional methods, and most importantly, can adequately describe the actual CES and the problems.

Technological facilities of oil refining, for example, a delayed coking unit (DCU), refer to the complex CES, where technological processes for raw materials (tar, fuel oil) processing and products (petroleum coke and other petroleum products) production are carried out, which are evaluated by the vector of economic, environmental, technological criteria. For effective solving of such multi-criteria problems of the CES control, these criteria should be turned into an extremum, i.e. it is required to make decisions on the selection of optimal operating modes of the CES. Such problems, as a rule, are formalized as the problem of a multi-criteria selection in the fuzzy environment [15],[17], which are solved on the basis of knowledge of expert and mathematical models of the objects, constructed with regard to fuzziness of the initial information and preferences of decision-makers.

Successful solution of the problems of CES modeling and control under conditions of uncertainty requires the development of methods for the synthesis of fuzzy models, the further development of methods for formalizing and solving the problems of the CES operating modes control in the fuzzy environment that are the topical problems of chemical technology, technological systems and processes.

The aim of the work is to develop the methods for the synthesis of CES models in the fuzzy environment, formalize and state the multi-criteria problems of DCU parameters optimization in the fuzzy environment, and develop the heuristic algorithms for their solution.

## 2. STATEMENT OF THE PROBLEM

We formulate the general problem of multi-criteria optimization with the use of classical terms: it is necessary to maximize the objective function (criterion), taking into account various constraints  $\varphi(x) \geq 0$ , where  $x \in X$ , i.e.:

$$\min_x f(x),$$

$$\varphi(x) \geq 0, x \in X$$

Where  $f(x) = f_1(x), \dots, f_m(x)$  - criteria vector,  $\varphi(x)$  - functions of constraint,  $x = (x_1, \dots, x_n)$  - variables of the object state. The range of definition of  $f(x)$ ,  $\varphi(x)$  and  $x$  is as follows:  $0 \leq f(x) \leq f_{est}(x)$  ( $f_{est}(x)$  - set value;  $\varphi(x)^{\min} \leq \varphi(x) \leq \varphi(x)^{\max}$ ;

$x = [x^{\min}, x^{\max}]$ , where  $\varphi^{\min}, \varphi^{\max}$  - minimum and maximum values of constraint,  $x_i^{\min}, x_i^{\max}$  - the lower and upper limits of the variation of the variables  $x$ .

We will research the CES of production of petroleum coke, a delayed coking unit operating at the Atyrau oil refinery (refinery), as an optimization object. The problem is to construct mathematical models, formulate the problem of multi-criteria optimization of DCU in the fuzzy environment, and develop the heuristic methods for solving it.

## 3. METHODS FOR CONSTRUCTING FUZZY AND LINGUISTIC MODELS OF CES

Qualitative analysis of the CES which are actual complex and quantitatively difficult to describe, requires approaches for which the high accuracy and stringency of mathematical formalism is not absolutely necessary and in which a methodological scheme is used that accepts fuzziness and partial truths. There are the following approaches to the CES modeling with the fuzziness of the initial information [7],[11], which satisfy these requirements, which are based on the use of *the methods of the fuzzy set theory*.

1. The approach based on the construction of statistical models of CES with fuzzy coefficients on the basis of regression analysis methods. The models obtained by this approach are successfully used in the modeling and control of a number of processing facilities in the oil refining industry.

Assume that as a result of the research of CES or the experiment,  $L$  values of the input and standard parameters  $X_l$  ( $l = \overline{1, L}$ ,  $x_{il}$ ,  $i = \overline{1, n}$ ,  $l = \overline{1, L}$ ), and the corresponding fuzzy values of the output parameters  $\tilde{y}_j^o$  ( $y_{jl}$ ,  $j = \overline{1, m}$ ,  $l = \overline{1, L}$ ) were estimated by the experts.

To construct mathematical models of this CES, it is necessary to solve the following two stages of the identification problem:

a) Select the structure of the function (structural identification)

$$\tilde{y}_j^M = \tilde{f}_j(x_1, \dots, x_n, \tilde{a}_0, \dots, \tilde{a}_n), j = \overline{1, m}$$

(1)

Approximating function

$$\tilde{y}_j = \tilde{f}_j(x_1, \dots, x_n), j = \overline{1, m}.$$

At this stage, the qualitative analysis of CES is of decisive importance, as a result of which the main parameters influencing the functioning and their interrelations are identified, and the method is selected to identify the structure of the model.

b) Determine the ways of estimation of the parameters of the selected function (1) (parametric identification), for example, the values of fuzzy coefficients  $\tilde{a}_0, \dots, \tilde{a}_n$ . For such an estimation, one can use the criterion of minimizing the deviation of fuzzy values of the output parameter  $\tilde{y}_j^M$  obtained by model (1) from its selective fuzzy values obtained on the basis of expert evaluation  $\tilde{y}_j^o$ :

$$\tilde{J}_j = \sum_{l=1}^L (y_{jl}^o - \tilde{y}_{jl}^M)^2 = \sum_{l=1}^L (y_{jl}^o - \tilde{f}_j(x_1, \dots, x_n, \tilde{a}_0, \dots, \tilde{a}_n))^2, j = \overline{1, m}. \quad (2)$$

In the second stage, the main issue is the selection of the method of estimating unknown parameters that provides the necessary properties of the object under research.

Generally, fuzzy models obtained by this approach have the form of a fuzzy multiple regression equation.

$$\tilde{y}_j = \tilde{a}_{0j} + \sum_{i=1}^n \tilde{a}_{ij} x_{ij} + \sum_{i=1}^n \sum_{k=i}^n \tilde{a}_{ikj} x_{ij} x_{kj}, j = \overline{1, m} \quad (3)$$

$$\sum_{i=1}^n \sum_{k=i}^n \tilde{a}_{ikj} x_{ij} x_{kj}, j = \overline{1, m}$$

Using the concept of a set of level  $\alpha$  allows reducing the fuzzy regression equations to the system of ordinary regression equations. This

approach makes it possible to use classical regression methods to solve the problems above.

2. An approach based on the use of logical conditional output rules, for example, in the following form:

$$IF \tilde{x}_1 \in \tilde{A}_1 (\tilde{x}_2 \in \tilde{A}_2 (\dots, (\tilde{x}_n \in \tilde{A}_n), \dots), \quad (4)$$

$$THEN \tilde{y}_j \in \tilde{B}_j; j = \overline{1, m}$$

Where  $\tilde{x}_i$ ,  $i = \overline{1, m}$ ,  $\tilde{y}_j$ ,  $j = \overline{1, m}$  – input, output and linguistic variables, respectively, of the object,  $\tilde{A}_i, \tilde{B}_j$  – fuzzy subsets characterizing  $\tilde{x}_i, \tilde{y}_j$ .

The advantage of this approach is the possibility of its use in the CES modeling, for which the collection of statistical information is very expensive, difficult or impossible. In this case, the obtained linguistic models are the result of processing an expert survey of the experts who, as a rule, use qualitative information. Such information, provided sufficient competence of experts, allows considering all the range of complex internal interrelations of parameters of the object in the obtained models.

### Methods of models synthesis in the fuzzy environment.

We propose the following method for the synthesis of fuzzy models (FM) and linguistic models (LM), which implements the idea of the 1st approach to modeling quantitatively difficult-to-describe CTS, when the input parameters are clear (measurable), and the output parameters are fuzzy, and are evaluated by specialist experts.

FM Method.

1. Select the input  $x_i \in X_i$ ,  $i = \overline{1, n}$  and output  $\tilde{y}_j \in Y_j$ ,  $j = \overline{1, m}$  parameters of the CES element required to construct a model.

2. Collect information on the basis of an expert procedure, determine the term-set of fuzzy parameters describing the state of the CES element.

3. Determine the structure of fuzzy multiple regression equations

$$\tilde{y}_j = \tilde{f}_j(x_1, \dots, x_n, \tilde{a}_0, \tilde{a}_1, \dots, \tilde{a}_n), j = \overline{1, m}$$

4. (solution of the problem of structural identification).

5. Construct a membership function of fuzzy parameters of the CES element and model coefficients.

6. Estimate fuzzy coefficients ( $\tilde{a}_0, \tilde{a}_1, \dots, \tilde{a}_n$ ), of selected functions

$\tilde{y}_j$  (solution of the problem of parametric identification).

7. Check whether the model matches the actual data (adequacy of the model). In case of inadequacy of the model, find out the reason of it and return to the relevant item.

This method corresponds to the first approach of modeling the fuzzy CES. Let us explain some of the items of the above method.

In the first item, the most informative variables characterizing the quality of the CES performance are selected. For convenience, the ranges of variation of fuzzily described parameters are set in the form of segments, indicating the minimum ( $y^{\min}$ ) and the maximum value ( $y^{\max}$ ). These segments, depending on the judgment of experts, are broken down into several sampling intervals (quanta), for example:  $y_j^{\min} = y_j^1 < y_j^2 < \dots < y_j^l = y_j^{\max}$ .

Each quantum of the selected parameters is verbally characterized by the corresponding terms (fuzzy terms) to construct a term-set describing the state of the CES (item 2). For example, if  $\tilde{y}_j$  is the quality of produced products, they can be described by the following terms:

$\tilde{y}_j = \{low, below\ average, average, above\ average, high\}$

Thus, the adopted term-set is the values of linguistic variables describing the performance of the system under research. Each sampling interval obtained in item 1 is characterized by a certain term, and a fuzzy set which is described by the membership function at the corresponding grading level matches to this term.

The definition of the structure of fuzzy multiple regression equations (item 3) and identification of their fuzzy coefficients (paragraph 5) are carried out in accordance with stages 1 and 2 of the first approach above (items a and b). The problem of structural identification is solved on the basis of the results of system analysis and research of the CES, using, for example, the method of sequential inclusion of regressors [18], the essence of which consists in the sequential inclusion of the next regressors until the conditions of the model's correspondence with the actual data are met. A fuzzy analogue of the method of least squares can be used for parametric identification [19].

The construction of the membership function of fuzzy sets (parameters) (item 4) is one of the main stages of modeling the complex objects using

the methods of the fuzzy set theory. The main way to recover this function is a graphical construction of the curve of degree of membership of any parameter to the corresponding fuzzy set. On the basis of the obtained curve, the type of function that approximates it more accurately is selected. After that, the parameters of the selected function are identified [11], [15].

The problem of the final stage of the method (item 6) is to check whether the model matches to the object (the original). The model is considered to be adequate (identical) to the modeled CES if the parameters found with its help match, at any desired degree of precision, the actual data obtained experimentally on the CES itself.

As a rule, the error ratio of design (model)  $y^M$  and actual (experimental)  $y^E$  data are used as a criterion of adequacy, which is a measure of the model's correspondence with the object. In addition, the value of the permissible level of error  $R_D$  is selected. The model is considered adequate if  $R = |y^M - y^E| \leq R_D$ .

In case of inadequacy, the mathematical model is improved, sources of uncertainty are determined. This may be an underestimation of the significance of any important variable and its underestimation in the model, an incorrect or incomplete structure of fuzzy equations, an error in parametric identification, etc. After this, one should return to the corresponding item of the algorithm to complete the model.

*The method of synthesis of the CES models using linguistic input and output parameters.* The following proposed method realizes the idea of the second approach, which uses logical conditional output rules.

*LM Method*

Some items of this algorithm (1, 2, 6) are similar to the corresponding items of the FM-1 method, but the fuzziness of the input parameters  $\tilde{x}_i$ ,  $i = \overline{1, m}$  are taken into account.

1. Select the input  $\tilde{x}_i \in X_i$   $i = \overline{1, n}$  and output  $\tilde{y}_j \in Y_j$   $j = \overline{1, m}$  ( $X_i, Y_j$  – universal sets) parameters of CES, which are linguistic variables;

2. Based on expert evaluations, evaluate the values of the parameters  $\tilde{x}_i, \tilde{y}_j$  and construct a term-set  $T(X_i, Y_j)$ .

3. Construct the membership functions of fuzzy input  $\mu_{\tilde{A}_i}(\tilde{x}_i)$  and parameters  $\mu_{\tilde{B}_j}(\tilde{y}_j)$ , where  $\tilde{A}_i, \tilde{B}_j$  - fuzzy subsets, and  $\tilde{A}_i \subset X_i, \tilde{B}_j \subset Y_j$  (fuzzification).

4. Construct a linguistic model of CES and formalize fuzzy mappings  $R_{ij}$ , determining the relations between  $\tilde{x}_i$  and  $\tilde{y}_j$ .

5. Determine the fuzzy values of the output parameters of the CES, for example, on the basis of fuzzy rules and interference, and select their numerical values from the fuzzy set of solutions (defuzzification).

6. Check the adequacy of the model. If the condition of adequacy is met, the model is recommended for research and optimization of the CES, otherwise, return to the previous items to refine the model.

Let us describe some items of the proposed method in more detail.

The linguistic (qualitative) model of the CES is based on the results of expert information processing (item 4). For convenience, it can be presented in the form of a table where different values of input fuzzy parameters  $\tilde{x}_i, i = \overline{1, n}$  and the values of output fuzzy parameters corresponding to these variants  $\tilde{y}_j, j = \overline{1, m}$  are indicated. The table should be filled using the term-set selected in item 2. On the basis of the model obtained in such a way, fuzzy mappings are formalized  $R_{ij}$ , determining the relations between the input and output fuzzy parameters  $\tilde{x}_i$  and  $\tilde{y}_j$ . It is convenient to formalize such a fuzzy mapping by fuzzy logic or fuzzy interference. In this case, using expert information and term-sets  $T(X_i, Y_j)$  of input and output linguistic variables, a complete description of all possible situations is given. This description, which is called a linguistic model, consists of a set of nested logical rules of the form (4).

Fuzzy mappings for a quantum  $p$  are defined as  $R_{ij}^p = A_i^p \cdot B_j^p$ . For the convenience of using fuzzy mapping  $R_{ij}$  in the calculations, it is necessary to construct matrices of fuzzy relations:  $\mu_{R_{ij}}(x_i, y_j^M)$ , for example, for isolated quanta generally:

$$\mu_{R_{ij}}^p(x_i, y_j^M) = \min[\mu_{A_i}^p(x_i), \mu_{B_j}^p(y_j^M)],$$

$$i = \overline{1, n}, j = \overline{1, m}$$

The fifth item of the method of linguistic models synthesis is the application of the composite inference rule:

$$B_j = A_i \circ R_{ij},$$

Where  $A_i \subset X_i, B_j \subset Y_j, X_i, Y_j$  - universal sets. Using this rule, it is possible to calculate the output variables, for example, by the following expression (maximin):

$$\mu_{B_j}^p(y_j^*) = \max_{x_i \in X_i} \{ \min[ \mu_{A_i}(x_i^*), \mu_{R_{ij}}(x_i, y_j^M) ] \}$$

Let  $x_i^*$  be the measured (evaluated by experts) values of input variables, then the desired set, to which the current measured values of the input variables relate, is determined as the set for which the measured values have the highest degree of membership:

$$\mu_{A_i}(x_i^*) = \max_i \mu_{A_i}(x_i)$$

Specific numerical values of the output parameters  $y_j^*$  from the fuzzy set of solutions are determined from the following:

$$y_j^M = \arg \max_{y_j} \mu_{B_j}(y_j^*), j = \overline{1, m}$$

#### 4. THE METHOD OF CONSTRUCTING A SYSTEM OF MODELS FOR CES MULTI-CRITERIA OPTIMIZATION BY THE EXAMPLE OF DCU

We describe the methods of constructing a system of models of interrelated technological elements of CES by the example of DCU on the basis of system analysis [5],[20]. DCU is a complex CES, consisting of interrelated plants, which are simultaneously influenced by a large number of different parameters. The main units of DCU are coke chambers (CC), rectification columns (RC), tube furnaces (TF), and heat exchangers (HE). Since they are interrelated, changes in the regime parameters of one of them lead to the change in the parameters of others, which influence on the volume and quality of products produced by DCU (petroleum coke, gasoline, light and heavy gas oils).

In this regard, in order to optimize and control the coking process in the optimal (effective) mode, it is necessary to have associated mathematical models of the main equipment of the plant, constructed on the basis of the method of system analysis taking into account the influence of technological parameters on each unit, intermediate

and final products, and performance of the unit in the whole [1],[10],[21].

Models of each object in the system can be constructed using different approaches, i.e. it is possible to obtain a set of models for each DCU plant, for example, deterministic, statistical, fuzzy, and combined. To combine such different models of plants into a single system and for the purposes of system modeling and optimization, it is necessary to analyze the advantages and disadvantages of each model, to develop comparison and selection criteria for the system of modeling, optimization and decision making, to determine the principles of combining models into a single system [1],[12].

For this purpose, we analyzed various types of models of the main DCU plants. Based on the results of studies of the specifics of the process and plants of DCU, data of the experiments and expert demand and analysis of approaches to modeling of similar plants and CES, an evaluation of the possible types of models of each plant of the unit was carried out. The result of this analysis and model evaluation is presented in Table 1.

Table 1: System analysis and evaluation of model types of DCU plants

		DCU plants at the Atyrau oil refinery			
		CC	RC	TP	HE
1. Deterministic	1.Availability of necessary information	2	1.5	2,5	4
	2.Applicability in the SODM	3	3	3	4
	3. Accuracy	4.5	4.5	5	5
	4.Ability to construct	3	3	3.5	4
	5.Ability to integration	3	4	5	5
		15.5	16	19	22
2. Statistical	1.Availability of necessary information	2.5	2	3	3.5
	2.Applicability in the SODM	4.5	4,5	4.5	4.5
	3. Accuracy	3.5	3,5	4	4
	4.Ability to construct	4	4	4,5	4
	5. Ability to integration	3	3	5	4
		17.5	17	21	20

3. Fuzzy	1.Availability of necessary information	4	4	2.5	3.5
	2.Applicability in the SODM	4	4	4.5	4
	3. Accuracy	3.5	3.5	3	3
	4.Ability to construct	4.0	3.5	4	4
	5.Ability to integration	4	4	4	4
		19,5	19	18	18,5
4. Combined	1.Availability of necessary information	4	4	3.5	4
	2.Applicability in the SODM	4.5	4.5	4	4
	3. Accuracy	3.5	4.0	3.5	3.5
	4.Ability to construct	3	3	3.5	3
	5.Ability to integration	4	4	4	4
		19	19,5	18.5	18.5

Note: The expert evaluation was carried out on the basis of the five-point grading scale, where 1 is the lowest point and 5 is the highest point (estimates can be fractional and fuzzy).

Legend: TF - tube furnace; RC - rectification columns; CC - coke chambers; HE - heat exchangers.

To assess the possible model types the basic units of DCU, i.e. models CC, RC, TF and HE the expert assessment based on Delphi method is carried out. The expert evaluation involved 10 experts from the number of DCU operators, technologists, plant Manager and researchers of delayed coking processes. The 5th round of evaluation was conducted, after which the value of the concordance coefficient close to 1 was obtained, and more specifically, it turned out  $W = 0.97$ , i.e. the experts' opinions were agreed. Table 1 shows the results of the expert evaluation after the 5th round. Since the units of measurement of the values of all five model evaluation criteria are the same (points), the decision to choose the effective model type for aggregates is made on the basis of an integrated criterion, values of which are determined by summing all the particular criteria.

The following are selected as the main criteria for comparing different types of models for which they are evaluated: availability of the necessary information for building a model of the appropriate type, applicability of models in the optimization and decision support system (DSS) for optimal control of the installation, the possibility (or complexity) of its construction, accuracy, and

the possibility of combining them into a single system of models

Based on the information given in Table 1, it is possible to carry out multi-criteria selection of the type of models of DCU plants. If the estimations are fuzzy, then a procedure for selecting the best alternative based on fuzzy information can be developed using the methods of fuzzy set theory.

According to the results of the system analysis and evaluation by the selected criteria for heat exchangers (HE), the development of deterministic models for them is most efficient (by the sum of criteria, the maximum point is 22), and for tubular furnaces - statistical models (21 points) (refer to Table 1).

The results of the study of the performance of the DCU plant system and the possible set of their models show that it is inefficient or almost impossible to construct deterministic and statistical models for rectification columns (RC) and coke chambers (CC) due to the complexity of the plants, the difficulties in studying the processes in them, and the impossibility of obtaining reliable data.

Statistical (stochastic) models of the RC and CC of the DCU are convenient for combining them into a single system of models and are suitable for solving optimization problems and making decisions on plant control. However, the collection of statistical information for constructing regression models of these plants is complicated by the absence of special industrial instruments for measuring some important process parameters (for example, the phase composition of the raw materials, some product quality indices) and the low reliability of the available measuring means at the current DCU of the Atyrau oil refinery. In this regard, the methods of the fuzzy set theory are chosen as more effective means, supplementing the missing data on the basis of fuzzy information in the form of knowledge, experience, judgments and intuition of experts. The accuracy of the fuzzy models obtained is less than that of the previous models, but it is quite sufficient for modeling for the purposes of optimizing and making decisions on the coking process control. We note the applicability of fuzzy models to construct a computer system for optimization and decision support. As can be seen from the table for heat exchangers, the most effective type of mathematical model is deterministic models. This can be explained by the fact that these aggregates are quite simple in comparison with others and, most importantly, they are sufficiently studied from a theoretical point of view [24], the theoretical information and equations describing the processes

occurring in heat exchangers are known. Since for TF there is a sufficient amount of statistical data describing the combustion processes and thermal conditions of furnaces, it is effective for them to develop statistical models. This is confirmed by the result of the expert evaluation - 21 points (see table. 1)

In practice, one has to use any available information and consider the interconnection and integration of various collected data, for example, statistical and expert data, to construct models in case of insufficient information. Let us call the models of technological plants obtained on the basis of such data as *combined* [12],[21]. They can be obtained through various combinations of available data and are focused on taking into account the advantages of the above types of models. However, when constructing the combined models, it is necessary to pre-process and reconcile the collected data. Thus, the development of fuzzy models (19.5 points) for coke chambers and a combined model (19.5 points) for the rectification columns is effective in terms of the sum of the criteria evaluation.

## 5. FORMALIZATION AND STATEMENT OF THE PROBLEM OF MULTI-CRITERIA OPTIMIZATION IN THE FUZZY ENVIRONMENT, AND HEURISTIC METHOD FOR ITS SOLUTION

Let  $f(x) = f_1(x), \dots, f_m(x)$  be a vector of criteria of optimization and control of operating modes, assessing the quality of the CES performance. Each of the local criteria  $m$  depends on the vector  $n$  of variables (standard parameters - control actions)  $x = (x_1, \dots, x_n)$ , for example: temperature, pressure and other parameters of the CES. Assume that this dependence is described by the mathematical models considered above. In practice, there are always various constraints (economic, technological, and environmental) that can be described by some functions – constraints  $\varphi_q(x) \geq b_q, q = \overline{1, L}$ , and the range of parameters  $x = (x_1, \dots, x_n)$  is defined as  $x \in \Omega$ .

It is required to select such parameter values  $x = (x_1, \dots, x_n)$ , which provide extreme values of the criteria vector  $f(x)$ , when the given constraints are executed and some of the initial data is fuzzy, and also take into account the preference of the decision maker.

*Mathematical statement of the problem model:* The formalized optimization problem, under multi-criteria and fuzzy conditions, can be written in the form of fuzzy mathematical programming (FMP).

Find the optimal value of vector of the input and standard parameters  $x^* = (x_1^*, \dots, x_n^*)$ , providing such values of local optimization criteria that satisfy the decision maker:

$$\max_{x \in X} f_i(x), i = \overline{1, m} \quad (5)$$

$$X = \{x \in \Omega, \varphi_q(x) \lesssim \overline{b_q}, q = \overline{1, L}\} \quad (6)$$

Where  $f_i(x)$  – local criteria, the values of which are calculated by models;  $\varphi_q(x), q = \overline{1, L}$  – the constraint functions that determine the admissible range  $\Omega$  of the multi-criteria optimization problem (5) - (6);  $b_q$  – set numbers that can be fuzzy.

The solution of this problem is the value of the vector of the standard parameters  $x^* = (x_1^*, \dots, x_n^*)$ , providing extreme values of local criteria that satisfy the decision maker.

To obtain a stringent mathematical formulation of the initial problem, we first consider the situation when the problem of optimizing the operating modes of the CES is stated as a single-criterion problem (in the case of multi-criteria, it is possible to result local criteria to one integrated criterion) and several fuzzy constraints.

Let there be one normalized criterion –  $\mu_0(x)$  of the form (5) and  $L$  constraints of the form (6) with fuzzy instructions -  $\varphi_q(x) \lesssim \overline{b_q}, q = \overline{1, L}$ . Assume that the membership functions of the constraints execution is  $\mu_q(x), q = \overline{1, L}$  for each constraint are constructed as a result of a dialogue with the DM and experts.

Assume that the weight vector  $\beta = (\beta_1, \dots, \beta_L)$  is known that reflects the mutual importance of the constraints at the time of the selection problem statement. Then, in general, the initial problem:

$$\max_{x \in X} \mu_0(x)$$

under the conditions

$\varphi_q(x) \lesssim \overline{b_q}, q = \overline{1, L}$ , can be written as follows:

$$\max_{x \in X} \mu_0(x),$$

$$X = \left\{x : \arg \max_{x \in \Omega} \mu_q(x), q = \overline{1, L}\right\}$$

This formulation of the problem with non-fuzzy objective function and fuzzy constraints with fuzzy instruction reflects the desire to maximize the criterion, fully satisfying the requirements of the

constraints. If we assume that all the membership functions are normal, then the formulation of the initial fuzzy selection problem is as follows:

$$\max_{x \in X} \mu_0(x) \quad (7)$$

$$X = \left\{x : x \in \Omega \wedge \mu_q(x) = 1, q = \overline{1, L}\right\} \quad (8)$$

We obtained the ordinary (non-fuzzy) mathematical programming problem (7) - (8) with maximization of the objective function on non-fuzzy set  $X$ . Then we assume the concavity of the objective function  $\mu_0(x)$ , the constraints  $\mu_q(x), q = \overline{1, L}$ , and the convexity of the admissible set  $X$ . This problem can be solved by conventional methods and mathematical programming

In the well-known formulation of problems in the fuzzy environment and methods of their solving, one-criterion cases are mainly considered, there is no flexibility in taking into account the preferences of the decision maker. In this case, as a rule, the fuzzy problem at the stage of statement is replaced by an equivalent determinate one, which will lead to the loss of the main part of the initial fuzzy information [10],[23].

In many cases, qualitative factors (fuzzy statements and judgments) are the main types of initial information and are familiar to humans. The conversion of a fuzzy description into a quantitative one is not always possible or is not feasible. In this regard, the most promising approach based on the development of control methods adapted to the human language, qualitative factors of any kind, human decision-making and optimization procedures, is the approach when the problems are stated and solved in the fuzzy environment, without conversion into deterministic one, i.e. the available fuzzy information is not lost, but used in full volume.

The combined optimality principles modified for operation in the fuzzy environment are proposed in this work to solve the stated optimization problem.

Let us summarize the formulated problem (optimization to the problem of the FMP of a specific CES, for example, DCU).

Let  $\mu_0(x) = (\mu_0^1(x), \dots, \mu_0^m(x))$  be a normalized vector of criteria  $f_i(x), i = \overline{1, m}$ , assessing the effectiveness of DCU performance. Assume that the membership function for fuzzy constraint execution  $\mu_q(x), q = \overline{1, L}$  is constructed for each fuzzy constraint  $\varphi_q(x) \lesssim \overline{b_q}, q = \overline{1, L}$ . A number of priorities for local criteria  $I_k = \{1, \dots, m\}$  and constraints  $I_r =$



$\{1, \dots, L\}$ , or a weight vector reflecting the mutual importance of criteria  $\gamma = (\gamma_1, \dots, \gamma_m)$  and constraints  $\beta = (\beta_1, \dots, \beta_L)$  are known.

Then, modifying the idea of various *optimality principles* to work in fuzzy environment, one can get different statements of problems of DCU optimization as FMP problems and develop methods for their solutions.

In practice, when solving actual optimization problems, it is often enough if some principles are executed with a certain concession. For such problems with several constraints for the criteria, it is proposed to apply a new principle - the *principle of quasi-maximin*, and for constraints - the *idea of the ideal point method*:

$$\max_{x \in X} \mu_0^1(x), \quad (9)$$

$$X = \{x : \arg \max_{x \in \Omega} \min_{i \in I_0} (\gamma_i \mu_0^i(x) - \Delta_i) \wedge \arg(\mu_q(x) \geq \min \|\mu(x) - \mu^u\|_D), \quad (10)$$

$$I_0 = \{2, \dots, m\}, q = \overline{1, L}\}$$

Where  $\|\cdot\|_D$  – the used metrics  $D, \mu_0(x) =$

$(\mu_1(x), \dots, \mu_L(x)), \mu^u = (\max \mu_1(x), \dots, \max \mu_L(x))$ . It is possible to use  $\mu^u$  as the coordinates of the ideal point:  $\mu^u = (1, \dots, 1)$ ,  $\Omega$  – initial set of definition of variables  $x, I_0$  – set of indexes of criteria transferred to constraints.

In problem (9) - (10), the criterion with the number 1 is maximized, the other criteria are constrained by the principle of quasi-maximin (QMM), i.e. taking into account the concession  $\Delta_i, i = \overline{2, m}$ , and fuzzy constraints are considered on the basis of the modified ideal point (IP) method.

On the basis of methods of fuzzy set theories and modifying the principle of QMM and the IP method, we propose a heuristic method for solving the formulated optimization problem of CES (9) - (10), which consists of the following main points:

1. In the dialogue with the decision maker, the importance values of the local criteria are determined

$$\mu_0^i(x): \gamma = (\gamma_1, \dots, \gamma_m), \sum_{i=1}^m \gamma_i = 1, \gamma_i \geq 0, i = \overline{1, m}$$

Assign the value  $p_i, i = \overline{1, m}$  – a number of steps on each  $i^{th}$  coordinate.

$$2. \text{ Calculate } h_i = \frac{1}{p_i}, i = \overline{1, m} \text{ – the}$$

values of steps to change the coordinates of the weight factor  $\gamma_i$ .

3. Determine the set of weight vectors  $\gamma^1, \gamma^2, \dots, \gamma^N, N = (p_1 + 1)(p_2 + 1) \dots (p_m + 1)$  by varying the coordinates on segments  $[0,1]$  with a step  $h_i$ .

4. DM assigns the concession values for local criteria  $\Delta_i, i = \overline{2, m}$ .

5. If  $\mu_0^i(x), i = \overline{1, m}, \gamma = (\gamma_1, \dots, \gamma_m)$  are  $\Delta_i, i = \overline{2, m}$  fuzzy, then the term-sets are determined for them and the membership functions are constructed.

6. The term-set is determined and the membership functions of constraints execution are constructed  $\mu_q(x), q = \overline{1, L}$ .

7. The coordinates of the ideal point are determined. The maximum values of the membership function can be used as the coordinates of these points -  $\mu^u = (\max \mu_1(x), \dots, \max \mu_L(x))$  or units (if the MF are normal) -  $\mu^u = (1, \dots, 1)$ .

8. Select the type of metrics  $\|\mu(x) - \mu^u\|_D$ , which determines the distance of the solution  $x^*$  obtained from the ideal point -  $\mu^u$ .

10. The problem is solved  $\max \mu_0^1(x)$  on the set X, determined by the expression (10). The solutions are determined  $(x(\gamma, \Delta, \|\cdot\|_D)), \mu_0^1(x(\gamma, \Delta, \|\cdot\|_D)), \dots, \mu_0^m(x(\gamma, \Delta, \|\cdot\|_D)), \mu_1(x(\gamma, \Delta, \|\cdot\|_D)), \dots, \mu_L(x(\gamma, \Delta, \|\cdot\|_D))$

11. The decision is presented to the decision maker. If the current results do not satisfy the DM, the new values  $\gamma$ , and (or)  $\Delta$  and (or)  $\|\cdot\|_D$  are assigned, and one should return to item 2. Otherwise, it is necessary to return to item 12.

12. The search for the solution is stopped; the results of final choice of the DM are output: the values of the control vector  $x^*(\gamma, \Delta, \|\cdot\|_D)$ ; values of local criteria  $\mu_0^1(x^*(\gamma, \Delta, \|\cdot\|_D)), \dots, \mu_0^m(x^*(\gamma, \Delta, \|\cdot\|_D))$  and level of execution of fuzzy constraints  $\mu_1(x^*(\gamma, \Delta, \|\cdot\|_D)), \dots, \mu_L(x^*(\gamma, \Delta, \|\cdot\|_D))$ .

We present several variants of using the Euclidean metric ( $D = E$ ) for the implementation of item 9:

$$\begin{aligned} \|\mu(x) - \mu^u\|_D^2 &= \sum_{q=1}^L \beta_q \left( \frac{\max_{x \in \Omega} \mu_q(x) - \mu_q(x)}{\max_{x \in \Omega} \mu_q(x)} \right)^2, \\ \|\mu(x) - \mu^u\|_D^2 &= \sum_{q=1}^L \left( \frac{\max_{x \in \Omega} \mu_q(x) - \beta_q \mu_q(x)}{\max_{x \in \Omega} \mu_q(x)} \right)^2, \\ \|\mu(x) - \mu^u\|_D^2 &= \sum_{q=1}^L \beta_q \left( \max_{x \in \Omega} \mu_q(x) - \mu_q(x) \right)^2, \\ \|\mu(x) - \mu^u\|_D^2 &= \sum_{q=1}^L \left( \max_{x \in \Omega} \mu_q(x) - \beta_q \mu_q(x) \right)^2. \end{aligned}$$

Here are the statements of the problem of selecting the operating modes of CES of coke production with a constraint vector based on the maximin principles (for the criterion) and the Pareto optimality (for constraints):

$$\max_{x \in X} \mu_0^1(x), \quad (11)$$

$$X = \{x : \operatorname{argmax}_{x \in \Omega} \min_{i \in I_0} (\gamma_i \mu_0^i) \wedge$$

$$\operatorname{argmax}_{x \in \Omega} \sum_{q=1}^L \beta_q \mu_q(x) \wedge \sum_{q=1}^L \beta_q = 1 \wedge \beta_q \geq 0, \quad (12)$$

$$I_0 = \{2, \dots, m\}, q = \overline{1, L}\}$$

In problem (11) - (12), the main criterion with priority 1 is maximized, the other criteria are introduced into constraints under the maximin principle, and fuzzy constraints are taken into account on the basis of the Pareto optimality principle [24].

The structure of the method for solving this problem can be represented as follows:

MM+PO method:

1. The values of weight factors for local criteria are determined in the dialog mode with participation of DM

$$\mu_0^i(x), i = \overline{1, m}, \gamma = (\gamma_1, \dots, \gamma_m),$$

$$\sum_{i=1}^m \gamma_i = 1, \gamma_i \geq 0, i = \overline{1, m}$$

2. If the criteria  $\mu_0^i(x)$ , and the weight vector  $\gamma$  are fuzzy, then the term-sets are determined for them and the membership functions are constructed.

3. The values of weight factors for constraints are determined in the dialog mode with participation of DM  $\mu_q(x), q = \overline{1, L}$ :

$$\beta = (\beta_1, \dots, \beta_L), \sum_{q=1}^L \beta_q = 1, \beta_q \geq 0, q = \overline{1, L}.$$

4. Assign the value  $p_q, q = \overline{1, L}$  - a number of steps on each  $q^{th}$  coordinate.

5. Calculate  $h_q = \frac{1}{p_q}, q = \overline{1, L}$  - the values of steps to change the coordinates of the weight factor  $\beta_q$ .

6. Construction of a set of weight vectors  $\beta^1, \beta^2, \dots, \beta^N$ ,

$N = (p_1 + 1) \cdot (p_2 + 1) \cdot \dots \cdot (p_L + 1)$  by varying the coordinates on segments [0.1] with step  $h_q$ .

7. The term-set is determined and the membership functions of constraints execution are constructed  $\mu_q(x), q = \overline{1, L}$ .

8. On the basis of the CES model, the maximization problem  $\max_{x \in X} \mu_0^1(x)$  (11) on the

set X determined by expression (12) is solved. The current solutions

$$x(\gamma, \beta), \mu_0^1(x(\gamma, \beta)), \dots, \mu_0^m(x(\gamma, \beta)),$$

$$\mu_1(x(\gamma, \beta)), \dots, \mu_L(x(\gamma, \beta))$$
 are determined.

9. The obtained solution is presented to the DM. If the current results do not satisfy the decision maker, the new values are assigned or the values  $\gamma$  and (or)  $\beta$  are corrected, and one should return to item 2. Otherwise, it is necessary to return to item 10.

10. The search for the solution is stopped; the results of final choice of the DM are output: the variables providing optimal modes of the CES  $x^*(\gamma, \beta)$ ; optimal values of local criteria  $\mu_0^1(x^*(\gamma, \beta)), \dots, \mu_0^m(x^*(\gamma, \beta))$  and maximum values of fuzzy constraints execution  $\mu_1(x^*(\gamma, \beta)), \dots, \mu_L(x^*(\gamma, \beta))$ .

Thus, various statements of the problems of the CES operating modes by the example of DCU in the fuzzy environment were obtained. On the basis of various optimality principles and methods of FST, heuristic methods for their solution are proposed.

## 6. PRACTICAL APPLICATION OF THE PROPOSED METHOD FOR THE DEVELOPMENT OF MODELS OF DCU COKE CHAMBERS AND DISCUSSION OF THE RESULTS

For system modeling, optimization of parameters and DCU control, it is necessary to have a simple mathematical model of the main equipment. This is due to the fact that the costs of computer time for modeling should be minimized

as much as possible, since any algorithm of optimization and decision making repeatedly refers to the modeling of sub-program, and the response of the control system for issuing recommendations regarding control also should take little time.

Therefore, in constructing DCU models, the described approach as the most acceptable is proposed on the basis of the decomposition principle, according to which the model of this unit is first constructed according to the results of research of each plant and the data collected. Then these models are combined into a single system of models in order to describe the process in the whole.

When constructing multidimensional models of coke chambers, taking into account the simultaneous influence of several factors on the quantity and quality of coke, problems arose due to the lack of reliable data. The main reason of the lack of such data is the duration of the process cycle in the chamber (40-60 hours), the lack or shortage of industrial means for information collection, as well as the low accuracy of the available measuring means. In connection with this, we used the methods of synthesis of models based on fuzzy information proposed above for constructing the model of coke chambers. The expert evaluation was carried out using a fuzzy analog of the methods of the experimental design theory [2],[25]. DM and experts evaluated the variants of experiences in a qualitative manner (in the natural language with application of in advance accepted term-sets), i.e. simultaneous influence of various combinations of fuzzy input parameters on output and quality of coke.

Further, on the basis of the results of the analysis of Table 1 according to the proposed FM-1 method, let us study the development of mathematical models of coke chambers (CC) for the purposes of SODM development.

1. The following input and standard parameters that influence the process of delayed coking are selected as the necessary for the construction of the coke chamber model:  $x_1$  – consumption of secondary raw materials at the inlet of the CC (power supply);  $x_2$  - temperature at the inlet of the CC;  $x_3$  – pressure in the CC;  $x_4$  – coking ability of raw materials;  $x_5$  – recycle ratio, and the following parameters are selected as output parameters (criteria):  $y_1$  – coke production;  $\tilde{y}_2$  – volatility and  $\tilde{y}_3$  – ash content of coke.

2. The information was collected and, based on expert evaluation, a term-set of fuzzy

parameters describing the state of the CC was determined. To evaluate the output parameters the following was selected for the term-set:

$T(X) = \{\text{low, below average, average, above average, high}\}$ .

3. The structure of the model is determined, i.e. fuzzy multiple regression equations (solution of the problem of structural identification).

The structure of fuzzy models of the coke chamber is identified as an equation for multiple nonlinear regression (13) (to determine the volume of coke) and fuzzy multiple regression equations (14) (for assessing the quality of coke):

$$y_1 = a_{0j} + \sum_{i=1}^5 a_{ij} x_{ij} + \sum_{i=1}^5 \sum_{k=i}^5 a_{ijk} x_{ij} x_{kj} \quad (13)$$

$$\tilde{y}_j = \tilde{a}_{0j} + \sum_{i=1}^5 \tilde{a}_{ij} x_{ij} + \sum_{i=1}^5 \sum_{k=i}^5 \tilde{a}_{ijk} x_{ij} x_{kj}, \quad j = \overline{2,3} \quad (14)$$

Where  $y_1$  – coke volume (output);  $\tilde{y}_j$  – fuzzy output parameters (qualitative indices of coke: volatility and ash content),  $\sim$  means the fuzziness of the corresponding parameters and factors, other  $x_{ij}, x_{kj}$  – input, standard parameters, considered in item 1:  $\tilde{a}_{0j}, \tilde{a}_{ij}, \tilde{a}_{ijk}$  – the estimated fuzzy coefficients, respectively: the absolute term of the regression equation, the linear effects, and the effects of quadratic and pair interaction.

To solve such fuzzy equations on the basis of a set of level  $\alpha$ , we turn to the system of ordinary (non-fuzzy) multiple regression equations that, with varying accuracy (depending on the value  $\alpha$ ), describe the influence of input parameters on the output parameters:

$$y_j^{\alpha} = a_{0j}^{\alpha} + \sum_{i=1}^5 a_{ij}^{\alpha} x_{ij} + \sum_{i=1}^5 \sum_{k=i}^5 a_{ijk}^{\alpha} x_{ij} x_{kj}, \quad j = \overline{2,3} \quad (15)$$

The set of level  $\alpha$  in (8)  $L_\alpha = \{\alpha_l, l = \overline{1,5}, \alpha = (0.5, 0.75, 1, 0.75, 0.5)\}$ , determines the level of reliability of the obtained values for the model.

4. To construct the membership function (MF) of the fuzzy parameters of the CES elements and model coefficients, an analytical expression is used, which is determined on the basis of practical experience:

$$\mu_{B_j}^p(\tilde{y}_j) = \exp\left( Q_{B_j}^p \left| (y_j - y_{mdj}) \right|^{N_{B_j}^p} \right) \quad (16)$$

Where  $\mu_{B_j}^p(\tilde{y}_j)$  – MF of fuzzy output parameters  $\tilde{y}_j$  relating to a fuzzy set  $\tilde{B}_j$ ;  $p$  – the number of the quantum (of the interval describing the  $p$ -term);  $Q_{B_j}^p$  – parameter, which is determined when identifying the MF and characterizing the

level of fuzziness;  $N_{B_j}^p$  – a coefficient that changes the range of definition of terms and the shape of the MF graph of fuzzy parameters;  $y_{mdj}^p$  – a fuzzy variable most corresponding to a given term (in the quantum  $p$ ), for this value  $\mu_{B_j}^p(y_{mdi}) = \max_j \mu_{B_j}^p(y_j)$ .

5. For the purposes of evaluation, i.e. identification of fuzzy coefficients ( $\tilde{a}_{0j}, \tilde{a}_{ij}, \tilde{a}_{ijk}$ ) of fuzzy models (solution of the problem of parametric identification), it is necessary to determine such coefficients  $\alpha_{0j}^{\alpha_l}, \alpha_{ij}^{\alpha_l}, \alpha_{ijk}^{\alpha_l}$  of the models (7) at each level  $\alpha_l$  that satisfy the following condition:

$$J_j = \sum_{j=1}^m (y_j^{\alpha_l} - \hat{y}_j^{\alpha_l})^2 \rightarrow \min, \quad l = \overline{1,5}$$

Where  $\hat{y}_j^{\alpha_l}$  – observed, actual values,  $y_j^{\alpha_l}$  – obtained values after processing the expert information.

After the parametric identification of the model (5) on the basis of the least squares method using the REGRESS software package, a dependence of the coke volume  $y_1$  on the input parameters  $x_1, x_2, x_3, x_4$  and  $x_5$  is obtained:

$$\begin{aligned} \tilde{y}_1 = f_3(x_1, x_2, x_3, x_4, x_5) = & 2890.1725 + \\ & 11.5859 x_1 + 8.7110 x_2 - 71.0833 x_3 + 0.9789 x_1^2 \\ & + 0.0117 x_1 x_2 - 0.0125 x_1 x_3 + 0.1875 x_1 x_4 - \\ & - 0.0057 x_1 x_5 + 0.1610 x_2^2 + 1.5000 x_2 x_3 + \\ & + 0.0454 x_2 x_4 + 0.0454 x_2 x_5 - 1.2222 x_3^2 - \\ & 0.0297 x_3 x_4 \end{aligned}$$

The definitions of the fuzzy coefficients of the model (6) are reduced to the definition of a system of non-fuzzy coefficients on the set of  $\alpha$  levels that are determined using the known methods of parametric identification based on the method of least squares (using the Regress program and the MatLab system [26].

Thus, identification of fuzzy coefficients  $\tilde{a}_{0j}, \tilde{a}_{ij}, i = \overline{1,5}, j = \overline{1,4}$  and  $\tilde{a}_{ijk}, i = \overline{0,5}, k = \overline{i,5}; j = \overline{1,3}$  of the system of equations (14) is based on the application of sets of  $\alpha$  – level, for levels  $\alpha = 0.5, 0.75$  (left and right) and 1.

A fuzzy model describing the volatility of

coke  $\tilde{y}_2$  depending on the input parameters  $x_1, x_2, x_3, x_4$  and  $x_5$ :

$$\begin{aligned} \tilde{y}_1 = f_3(x_1, x_2, x_3, x_4, x_5) = & (0.5/5926 \\ & + 0.75/5933 + 1/5938 + 0.75/5943 + 0.5/5950) \\ & - (0.5//23.7045 + 0.75/23.9450 + 1/24.1345 + \\ & 0.75/24.3350 + 0.5/24.5455) x_1 - (0.5/20.0170 + \\ & 0.75/20.3265 + 1/20.5278 + 0.75/20.758 \\ & + 0.5/21.059) x_2 + (0.5/1.0085 + 0.75/1.1190 \\ & + (0.5/1.0085 + 0.75/1.1190 + 1/1.2395 \\ & + 0.75/1.3490 + 0.5/1.4595) x_3 + (0.5/3.2610 + \\ & + 0.75/3.4720 + 1/3.6833 + 0.75/3.8950 + \\ & + 0.5/4.1065) x_4 + (0.5/0.0035 + 0.75/0.0140 + \\ & + 1/0.0245 + 0.75/0.0350 + 0.5/0.0455) x_5 \\ & + (0.5/0.0007 + 0.75/0.0015 + 1/0.0333 + \\ & + 0.75/0.0633 + 0.5/0.0943) x_1^2 + (0.5/0.0135 \\ & + 0.75/0.0190 + 1/0.0243 + 0.75/0.0293 + \\ & + 0.5/0.0385) x_2^2 - (0.5/0.0150 + 0.75/0.0071 + \\ & 1/0.0101 + 0.75/0.0135 + 0.5/0.0155) x_3^2 + \\ & + (0.5/0.3205 + 0.75/0.4215 + 1/0.5218 + \\ & 0.75/0.6225 + 0.5/0.723) x_4^2 - (0.5/0.2805 + \\ & + 0.75/0.3810 + 1/0.4815 + 0.75/0.5820 \\ & + 0.5/0.6825) x_1 x_2 + (0.5/0.0205 + \\ & 0.75/0.0310 + 1/0.0417 + 0.75/0.0520 + \\ & + 0.5/0.0625) x_1 x_3 - (0.5/5.0640 + \\ & 0.75/5.0810 + 1/5.0926 + 0.75/5.1035 + \\ & + 0.5/5.1085) x_1 x_4 - (0.5/0.1730 + \\ & 0.75/0.1803 + 1/0.1944 + 0.75/0.1995 + \\ & + 0.5/0.2070) x_2 x_3 + (0.5/0.0635 + \\ & 0.75/0.0715 + 1/0.0755 + 0.75/0.0800 \\ & + 0.5/0.0850) x_2 x_4 \end{aligned}$$

The obtained values of the coefficients at various  $\alpha$  levels  $a_0^{\alpha_l}, a_i^{\alpha_l}, a_{ik}^{\alpha_l}, i = \overline{1,5}; k = \overline{i,5}; l = 0.5, 0.75, 1, 0.75, 0.5$  of model (6) are combined using the following expression:

$$\tilde{a}_i = \bigvee_{\alpha \in [0.5, 0.1]} \text{ or } \mu_{\tilde{a}_m}(a_i) = \text{SUP}_{\alpha \in [0.5, 0.75, 1]} \min\{\alpha, \mu_{a_i^{\alpha}}(a_i)\}$$

Where  $a_i^\alpha = \{a_i | \mu_{\tilde{a}_i}(a_i) \geq \alpha\}$ .

Similarly, the ash content of coke ( $\tilde{y}_3$ ) is determined.

6. To ensure the model's adequacy to actual data, i.e. for correct identification of coefficients  $\tilde{\alpha}_0, \tilde{\alpha}_i, \tilde{\alpha}_{ik}$  of fuzzy models, such coefficients as  $\alpha_0^{\alpha_i}, \alpha_i^{\alpha_i}, \alpha_{ik}^{\alpha_i}$  of fuzzy models (5) at each level  $\alpha_i$  are determined which satisfy the following condition:

$$J_j = \sum_{j=1}^m (y_j^{\alpha_i} - \hat{y}_j^{\alpha_i})^2 \rightarrow \min, l = \overline{1.5}$$

Where  $\hat{y}_j^{\alpha_i}$  - observed, actual values  $y_j^{\alpha_i}$  obtained after processing the expert information.

Thus, the problem of estimating the fuzzy coefficients of the fuzzy regression equations (13) with the introduction of the set of  $\alpha$  level reduces to the classical problems of estimating the multiple regression parameters (14).

*Evaluation of validity and comparison of modeling results*

The results of modeling the operation of coke chambers at the Atyrau oil refinery on the basis of the fuzzy models identified above are compared with the results of deterministic models of the coking process [22],[27],[28] and experimental production data of the plant. The main results of the comparison are given in the form of a table (refer to Table 2).

Table 2: Comparison of the results of the work of deterministic models, proposed fuzzy models of coke chambers and experimental data of the DCU of the Atyrau oil refinery

Determined parameters	Results of deterministic modeling	Results of fuzzy modeling	Experimental (actual) data
Output of petroleum coke, t/h	17.00	17.20	16.50
Volatility of coke, %	-	11.50	(12.00) <sup>1</sup>
Ash content of coke, %	-	0.27	(0.35) <sup>1</sup>

Note: the input and standard parameters of the process are taken to be the same for the experiments and the model, ()<sup>1</sup> means that these indices are determined in the laboratory - it means that these  $i$  indices are not determined by deterministic models.

The data shown in Table 2 shows that the modeling results match actual (experimental) data to high enough precision, and on the basis of fuzzy models it is possible to determine the qualitative indices of production in the fuzzy environment that are not determined by traditional deterministic modeling methods. As a result of the modeling, the criteria were improved; the volume of coke produced were increased by 4.24%, and the quality of coke was improved by 0.5% (volatility) and 0.08% (ash content)

**The difference between the proposed methods for developing mathematical models and multi-criteria optimization of CTS parameters in a fuzzy environment and the known methods for developing models and optimization.**

The main difference between the proposed method of selecting the type of mathematical models of elements of the CF and building a system of mathematical models from the known methods of model development is that the method based on the methodology of system analysis and expert assessments first allows you to determine what type of model should be built for each unit of the technological system. Then, based on the initial information of various types, including fuzzy information, it allows you to build effective models that are combined into a single package of mathematical models that allows you to systematically model the work of the CTS.

The proposed fuzzy approach to solving the problem of multicriteria optimization in a fuzzy environment, in contrast to known methods of fuzzy task, based on the comfort level  $\alpha$ , replaces a system of clear objectives, then solve with known methods, tasks are set and solved in a fuzzy environment, without converting them to deterministic problems. Thus, the proposed heuristic approach is based on the development of methods adapted to the human language, taking into account the human decision-making procedure. To solve the set fuzzy optimization problem, it is proposed to combine various optimality principles modified for working in a fuzzy environment, which allows for more complete use of available fuzzy information and to ensure the adequacy of solving production problems.

## 7. CONCLUSION

Methods of synthesis of CTS models in a fuzzy environment under various conditions of initial information indistinctness are developed. The proposed first method for developing models

based on fuzzy information (FI) is intended for constructing models with fuzzy coefficients under the conditions of measured (clear) input and fuzzy output parameters of the CTS. The developed second method (LM) is intended for constructing linguistic models of CTS in the conditions of fuzzy input and output parameters of the system.

The proposed method for constructing a system of models of interconnected units of the DCU is as follows: a criterion for comparing and choosing an effective type of model of elements (units) of the technological system is selected; based on system analysis, an expert assessment of types of aggregate models is carried out according to the selected criteria; Based on the available information of various nature, the corresponding models are constructed (deterministic, statistical, fuzzy, hybrid) and combined into a single system (package) of models.

On the basis of the results obtained, the structure of the models of coke chambers of the DCU was identified in the form of ordinary and fuzzy multiple regression equations, parametric identification was carried out on the basis of the least squares method and using the set level  $\alpha$  of the fuzzy set theory.

The advantages of the proposed methods for the synthesis of fuzzy, linguistic models of CTS include:

- with fuzzy output parameters of the object, it is possible to build fuzzy models in the form of fuzzy regression equations;
- if both the input and output parameters of an object are not clearly evaluated, i.e. they are linguistic variables, they allow us to build linguistic models of CTS.

Thus, these methods allow us to determine the type of model and build effective models in a fuzzy environment, when traditional approaches do not yield significant results.

In addition, models developed on the basis of the proposed methods take into account the internal relationships of the main parameters of the system, which can not be formalized.

The research contribution to the proposed methodology for developing a set of mathematical models of CTS is to conduct a systematic study, a comprehensive use of experimental statistical methods, expert evaluation methods, and fuzzy set theories. This makes it possible to obtain the emergence effect in the proposed system method, i.e., to obtain more efficient models that cannot be built by separate known methods.

New statements of the problems of multi-criteria optimization of the modes of operation of

the CTS DCU in a fuzzy environment in the form of a problem of fuzzy mathematical programming are formulated. For solving the above problems, the FMP based on the modification of the combined principles of optimality of the developed heuristic solution methods, which are based on the involvement of the decision maker to use his experience, knowledge and preferences in the process of solving the problem.

The novelty of these results is determined by the fact that the problems of optimizing the operating modes of the CTS are set and solved in a fuzzy environment without first converting them to deterministic equivalent problems, which allows more use of the original fuzzy information and adequately describe production situations in a fuzzy environment and obtain effective solutions to multi-criteria optimization problems in a fuzzy environment.

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