OPTIMIZATION OF TOURIST TRANSPORTATION

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ABSTRACT

Transport is one of the most important components of the material base of any country's economy. Since ancient times, transport has been the engine of progress. The person used any available means to transport people and goods. With the invention of the wheel, and a little later of various types of engines, man began to develop vehicles accordingly: carts, coaches, steamers, locomotives, planes, etc. This made it possible to travel long distances and for various purposes.

In this research, we explore the optimization of tourist transportation problem. In our research we use branch and bound method, and dynamic programming techniques to ensure optimization in tourist transportation problem.

Keywords: Transportation, Optimization, Suicidal Ideation Detection, Machine Learning, Social Media

1. INTRODUCTION

Currently, transport is one of the most important components of the state's economy, both developing and with a highly developed economic and social base [1-3]. Transport ensures the normal functioning of the economy, increases the efficiency of public production, creates conditions for the rational placement of production forces on the territory of the country, taking into account the most appropriate approach of enterprises in various sectors of the economy to the sources of raw materials and areas of consumption of products, specialization and cooperation of production, allows the development of such industries as trade, agriculture and others [4-7]. Transport is a leading factor in the development of tourism

Transport plays a great role in solving social problems, ensuring business, cultural and tourist trips of the population, and developing cultural exchange within the country and abroad

Transport provides for the development of international economic relations, contributing to the implementation of mutually beneficial exchange between different countries [8-10]

Historically, the formation of transport systems in various countries has been influenced by their geographical location, natural potential, and often climate and landscape characteristics [11]. This has led to the formation of those transport and technical bases that are most efficiently used in the conditions of a particular region and state [12].

Transport services are one of the main types of services in tourism. They also account for the main share in the structure of the tour price. Depending on the length and distance of the trip, this percentage (in most cases) ranges from 20 to 60 % [13].

Tourism is considered to be one of the most dynamic sectors of the national economy as a whole and in individual regions [14-17]. A feature of the tourism industry, in contrast to other service industries, is its direct connection to recreational and tourist resources [18]. The capacity of the tourist market is strictly determined by the capacity of the tourist object (resource) and the degree of development of the corresponding infrastructure. Tourism and recreation activities are currently increasingly stimulating the development of other sectors of the economy, among which logistics occupies an important place [19-20].

The complex of components of the tourist service covers the means of accommodation and food for tourists, their transportation and software and tour support, information and financial services. In the functional aspect, both a separate tourist service and its component components necessarily involve logistics functions of supply, production and sales [21]. Therefore, each component of the tour service should be considered as a separate tourist
This is the delivery of tourists to the place of rest, ensuring sustainable tourism development; to solve a number of problems in the industry: infrastructure, recreational and other resources [23-25]. These circumstances lead to the need to solve one of the most pressing problems of the industry – ensuring its sustainable development. This is especially true for those places and regions that are very attractive for tourists, which can, with the right approach to organizing tourist flows, ensure the development of the industry. In addition, the most attractive objects for tourists significantly increase the load on the existing resource base, not only in the places where they are located, but also on the entire economy of the region, the city, its infrastructure, recreational and other resources [23-25].

Effective organization of tourism logistics helps to solve a number of problems in the industry: ensuring sustainable tourism development; preserving and restoring the resource base of the industry; reducing (and ideally excluding) the risks of environmental degradation; reducing the quality of tourist services, threats to the health and safety of tourists; can serve as a basis for determining a strategy for sustainable tourism development in the country and its regions.

There is one problem for all the tourist companies. It is the optimization of tourist travel. In this paper, we consider this problem and suggest our solution to optimization of tourist transportation by applying branch and bound method, dynamic programming techniques [26-28].

2. MATERIALS AND METHODS

One of the most important services that all travel companies provide is the transportation of tourists [29]. Consider the main types of transportation that implements the tourism industry. Depending on how the tour is formed, one or another transportation may be included in the package of services.

For the effective implementation of such services it is proposed to use a complex of mathematical methods [30]. For this purpose, a statement was made and an attempt was made to classify typical problems of transportation of groups of tourists.

There are three main types of transportation. This is the delivery of tourists to the place of rest, their movement from the place of arrival to the place of residence and the movement of tourists along the tour route or objects of excursion programs. The first type of transportation includes the delivery of tourists to a vacation destination by various means of transport—by plane, by train, by bus [31].

The second type of transportation includes the movement of tourists from the place of arrival (airport, train station, etc.) to the place of residence and back [32]. Such an auxiliary transport service is called transfer.

The third type of transportation consists in moving tourists along the tour route or through objects of excursion programs. In tourist practice, as a rule, the package of services includes the first two types of transportation [33]. Moreover, the use of the third type of transportation depends on the structure of the formed tour. Consider the first type of transportation of tourists. The delivery of tourists to the place of rest is one of the main services of travel companies, which is included in the main package of services for the tour offered to customers [34]. There is an international classification of all vehicles that implement such transportation. Among them, the main modes of transport are airplanes, trains, buses. However, depending on the location of the resting place, several modes of transport can be used. Among all modes of transport, air transportation is currently the most demanded in the whole world.

Transportation of Russian tourists is carried out on international and domestic airlines. If transportation is carried out on international airlines, then their provision from the point of departure in Russia to the first service point in the host country is assigned entirely to travel agencies.

For this reason, travel agencies must plan transportation, reserve seats, provide insurance, provide passport and visa support, issue travel documents to tourists. Based on the above tasks, the travel agency must choose the appropriate airline. The main criteria for this are:
- Tariff and benefits;
- Delivery speed to destination;
- Flight comfort;
- Reliability and reputation of the airline.

Depending on the type of transportation, which can be an individual, group, business tour, etc. the availability of tourists and available seats on this route, a specific carrier is selected. For the effective implementation of such tasks, a mathematical approach should be used. These tasks can be attributed to multi-criteria optimization tasks. Moreover, the optimization criteria are both
quantitative and qualitative. Quantitative criteria include tariffs, delivery speed, etc. Qualitative criteria are the types of benefits provided, flight comfort, level of service, reputation of the air company, etc. Analysis of the types of transportation (scheduled, charter, business and congress tours according to the system business office, individual) allows you to determine the restrictions imposed on the resources used. In this case, this is the number of seats on the plane, the regulated costs of transportation, etc [35]. The use of certain resources is determine not only by the state of the system certain point in time (the required number of seats for this flight), but also by other reasons. For example, in the high season, in addition to scheduled flights, it makes sense to use charter flights. Special flights must also be used if a tour is developed to a country to which there are no direct flights.

The most commonly used travel transportation of tourists. Large travel companies that are accredited by an international airline organization (IATA) are entitled to sell and book airline tickets. Depending on which air company the tickets are booked for, how much air transportation costs, the cost of this type of service is determined and therefore the cost of the tour [36]. Thus, when deciding which company to choose as a carrier, the travel agency manager must solve a multi-criteria problem, the main criteria of which are:

- Tour cost;
- Type of transportation (flight, charter);
- Type of aircraft used;
- Flight range;
- Basic tariffs and benefits, etc.

Consider charter flights that are used for mass transportation of tourists. A travel company can rent one flight per month, season, etc. from the airline for a certain period of time. In this case, the following tasks arise:

- Choosing the airline in which the plane is rented;
- Forecasting the flow of tourists;
- Organization of transportation to the place of rest and back without performing empty flights.

The last task requires a clear timetable for the delivery of tourists, taking into account the duration of rest of individual groups. Second place in popularity after air travel is occupied by road transportation of tourists. The most popular tourist transport is a bus [37].

Such transportation can be scheduled or take place on a specific route. When considering scheduled flights, tasks arise similar to scheduled flights. These are, as a rule, optimization multi-criteria problems. They consist in minimizing costs, while it does not matter if the travel agency has its own fleet or a bus must be ordered at a trucking company. Here, as well as during air transportation, it is necessary to formulate a generalized optimization criterion which is determined by the following quantitative and qualitative criteria:

- tariffs;
- delivery time to the destination;
- travel comfort, etc.

Restrictions on resources are determined by the number of seats in buses of various types, the possibilities of representing buses of a certain type for transportation, etc. The main modes of transport that are used in the implementation of the second type of transportation, i.e. transfer, are automobile or water. This raises a number of tasks that can be attributed to the so-called transport problems solved by linear programming methods. In the simplest case, these optimization tasks are single-criteria and require minimizing costs or maximizing income. However, in real conditions, a number of qualitative goals arise that require considering such tasks as multi-criteria [38]. All the tasks formulated above for the optimal organization of passenger transportation will be attributed to the first class of tasks for optimizing such transportation. These class includes optimization tasks that implement the third type of transportation of tourists along the tour router on objects of excursion programs [39]. When developing various tours, it may be necessary to design some route for the movement of tourists. In this case, the problem arises of choosing the optimal route in accordance with a given quality criterion. At present, mathematical methods are not used to find the best route for transporting tourists. However, in applied mathematics such problems are known as combinatorial problems of non-differentiable optimization.

Using modeling and optimization will improve the quality of the created routes and reduce their cost. In addition, it is very important to emphasize that the problems of the second class, as will be shown below, are also problems of multi-criteria optimization. We will be interested in designing at the stage when cities or points that tourists need to visit have already been selected. There are various types of routes-roundabout, radial, linear [40]. Depending on this, different tasks arise for constructing a route. If a ring route is being designed the ninth is case a problem arises which is known in applied mathematics as the traveling salesman problem. It consists in finding some closed route, including a number of cities and having a minimum length or cost. At the same time,
it is stipulated that none of the cities should be visited twice. For tourist practice, minimizing the length or cost of the route is not enough. Here it is necessary to introduce a number of important criteria in our opinion, which will be discussed in detail in the next section. If it is necessary to develop a linear or radial route, then the rather stringent restrictions of the previous problem are replaced by weaker ones. For example, for a linear route, moving from one city to another can be done in various ways.

Based on the behavioral analysis in picture 1, the classification and scheme of the problems of managing transport transportation of tourists are presented and methods for solving them are proposed, which will be described in the next section. Thus, the analysis of transport transportation of tourists allows us to draw the following conclusions.

- To effectively manage the transport of tourists, it is necessary to use mathematical modeling.
- The tasks of organizing tourist transportation and designing tour routes are tasks of multi-criteria optimization.
- The selection of optimization criteria is carried out by each travel agency, depending on what kind of transportation will be implemented.
- Restrictions on resources are usually the number of available seats in the mode of transport in question.

Currently, in the rapidly developing tourist industry, there are a number of well-established tour operators in the tourism market who develop both tours to various countries and domestic tours. One of the main tasks of developing a new tour is to design a tour route. The main stages of route design are:

1. Analysis of the source in formation about the route and semantic (verbal) statement of the problem.
2. Formalized statement of the problem and the creation of an adequate mathematical model.
3. The choice of method for solving the problem.
4. Determination of the possibilities of implementing the task of finding the optimal route.

Consider the first stage, which always precedes the creation of a mathematical model and has a phenomenological basis. In this case, it is necessary to analyze the technology of route design.

The task of designing a route can be considered from two interconnected positions, from economic and recreational. In economic terms, it is necessary to create such a route so that it has the lowest costs and at the same time satisfies the wishes of customers, that is, consumers of the developed tourist product. For this reason, an important point when choosing a route is conducting market research to identify the most popular object. To do this, it is necessary to carry out the appropriate statistical observation by collecting information about the expected design object. Then, based on the processed statistical information, a decision is made to create a new tour route. The analysis of the object allows you to determine the recreational resources. These are the most interesting cultural-historical points of visit to the route or display objects when developing excursions.

We single out the main tasks of route design:

- Route design when developing a new tour
- Adjustment of the existing tour route
- Designing the route of the excursion program
- Planning the excursion route in the museum

All of the above tasks have common features and are of an optimization nature since they require the achievement of certain goals. The statement of the optimization problem is as follows. There is a set of points or display facilities to which you need to deliver tourists at the lowest cost. As costs, you can take the length of the route, travel time, the cost of transporting tourists, etc. In addition, tourists should be satisfied with comfortable transportation, convenient accommodation and food, cultural and historical informational content of the route, etc.
Thus, the optimization problems under consideration require setting clear goals when developing a specific route. Moreover, the goals can be both quantitative and qualitative. These stage in the design of the tour route is to formalize the optimization problem statement and develop a mathematical model. As you know, optimization models include objective functions and limitations. The objective function may be different depending on what goal or goals the route model developer sets for himself in each particular case. Therefore, in the end, not one but many models will be obtained, allowing to solve the problem of optimizing the route being developed. Let us consider some criteria and the corresponding mathematical models of this problem. The main goal of any tour operator is to reduce costs in the implementation of the designed route, as this leads to a reduction in the cost of the tour. The concept of costs can include such indicators (factors) as the length of the route, the time it takes, the amount of fuel used by vehicles. In addition, when developing a route, it is necessary to take into account the recreational potential of visiting points. This is not only cultural and historical sites, but also the availability of places of residence (hotels, campsites, etc.) catering facilities (restaurants, cafes).
When drawing up various routes, it may be necessary to set the priority of visiting certain points according to its logical structure. In this case, it is necessary to take into account factors such as the relief of the route (descents, ascents), the condition of the road surface, hazardous sections (turns, narrowing the road, etc.) Thus, in the general case, the tasks of designing tour routes are multi-purpose. Therefore, these tasks are multi-criteria and they are usually called vector optimization problems. An important feature of the task of designing a route is the availability of both quantitative and qualitative goals. Quantitative criteria are determined by some numerical indicators (cost of transportation, length of the route, time it took to travel, speed along the route, the amount of fuel used by vehicles, etc.) Qualitative goals can be evaluated as discrete logical variables that take values “0” or “1” or more complex expressions. In some cases, when uncertain factors are taken into account, for example, weather conditions, one or more criteria can be probabilistic.

Thus, not one, but several matrices can be used as a model. A certain route for transporting tourists, consisting of a number of points, can be represented in the form of a directed graph, for example, circular, radial, or linear. In practice, routes can be of mixed type and have a more complex topology. Using the apparatus of graph theory, they usually present these to possible routes in the form of a directed graph, (X,D), where the set of its vertices X={X1,X2,...,Xn}defines cities, and the set of directed segments, which are arcs of the graph (i,j) and indicate the distance dij, between cities Xi and Xj, D determines the total length of the route in accordance with the sequence of city walks. In our statement, dij it determines a certain indicator, which is a convolution of the selected factors in the area between the cities xi and xj. We call this indicator conditionally costs.

Table 1. Cost Matrix for Transportation Between Cities.

<table>
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<tr>
<th>City number</th>
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<td>d1i</td>
<td>...</td>
<td>dij</td>
<td>...</td>
<td>d1n</td>
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<td>2</td>
<td>d21</td>
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<td>...</td>
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As a rule, a certain indicator called a long arc is given on the graph d(Xi,Xj)=dij. Let Dkls be the path length between the peaks k and l. It is equal to the sum of the arcs of elementary paths:

Dkl = \sum_{i,j} d_{ij}

As you know, the Hamiltonian path is such a path that means the elementary path passing through all the vertices of the graph. The traveling salesman problem is to determine the minimum Hamiltonian path in accordance with the chosen criterion. Based on the constructed route graph, we compose a cost matrix whose elements will be d_{ij} (table1)

If equality d_{ij} = d_{ji}, holds, then the cost matrix will be symmetric. If you need to return to the city from which the route began, then the task is considered closed. Otherwise, the task is open. Elements of the main diagonal d_{ii} are equal to infinity since at each step under consideration, it is forbidden to return to the city from which tourists left.

Consider the closed problem of designing tour routes as an optimization problem. Let it be necessary to find some closed route, including a number of cities and having minimal costs. Moreover, none of the cities on this route can be visited twice. In applied mathematics, this problem is known as the traveling salesman problem. The general method for solving such combinatorial problems is only a complete enumeration of options. However, in practice, the number of possible options is so great that is often impossible to do this. If for example, the number of cities is equal n, then the number of routes is equal (n-1)!. Thus, at n=10, it is necessary to consider approximately 3*10^6 options, which is practically impossible. For this reason, it is necessary to develop and use effective algorithms to solve this problem.
We formalize the statement of the traveling salesman problem. Let the transportation costs between the cities that you need to visit be known. We number the cities and denote the costs of transportation between cities \( I \) and \( j \) through \( d_{ij} \).

Picture 2 shows, for example, the many possible routes between the five cities in the form of a graph.

\[ d_{ij} \]

Figure 2: Graph Of Possible Routes

The cost matrix for the traveling salesman problem is presented in table1.

On the graph, as a rule, the simplest methods for solving such problems are heuristic methods based on intuition, between the vertices that allow you to find a fairly good, but not always optimal solution. However, the use of these methods in practical problems usually leads to solutions that are much better than randomly selected solutions. One of the heuristic approaches to solving the transportation optimization problem is to introduce the concept of inversion and calculate the change in the length of the Hamiltonian path by replacing arcs \((i, i+1)\) and arcs \((j, j+1)\). Picture 3 shows an example of applying inversion.

\[ \Delta_{ij} = d_{i,i+1} + d_{j,j+1} - d_{ij} - d_{i+1,j+1} \]

Denote by \( \Delta_{ij} \) change of the Hamiltonian path. Then

\[ \Delta_{ij} = d_{i,j+1} + d_{j,i+1} - d_{ij} - d_{i+1,j+1} \]

If \( \Delta_{ij} > 0 \), this means that the new path is shorter, then the inverse operation improves the solution. If \( \Delta_{ij} < 0 \), this means that there is no improvement.

We compose a matrix of changes in the Hamiltonian path. \( \Delta_{ij} \) (table2). Elements of the matrix determine the change in distances \( \Delta_{ij} \).

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Table2. The Matrix Of Changes In The Hamiltonian Paths

In the matrix constructed in this way, in each row, the element with the maximum value is selected, and the inversion corresponding to this element is entered on the graph. At each step, you can improve the resulting solution. The termination condition for this algorithm will be all negative elements of the matrix. This suggests that any changes to the solution will not improve, that is, a local minimum is found. However, the solution found may not be optimal. To solve the problem of optimizing transportation, you can use statistical modeling methods that give satisfactory results. When applying the Monte Carlo method, a random sequence of \( n \) city walks is randomly generated. Then for the obtained sequence, the length of the Hamiltonian path is determined. Next a new sequence is generated and a new path length is calculated. If the obtained path length is less than the previous one, then it is remembered, otherwise the previous value is left and so on. To stop such an algorithm, a certain number of steps of non-improved solutions are specified and then the sequence obtained in this way is taken as optimal. The paper proposes a more efficient algorithm, which is determined by the joint use of the Monte Carlo method and heuristic methods. At the beginning, a random sequence is generated that determines the order of visiting the cities included in the route. Then, the path is improved by one of the heuristic algorithms, for
example, by the inversion method. The obtained value of the path length is stored, then a new sequence is generated, it is improved and a new path length is compared with the previous one.

Application of the developed algorithm gives good practical results. To find the optimal Hamiltonian path, there are a number of deterministic algorithms that are however much more complicated than heuristic methods.

3. BRANCH AND BOUND METHOD

Consider the branch and bound method. The idea of this method is that the entire set of Hamiltonian path is divided into somewhat more often into two subsets, for which some estimate is calculated, which allows one to judge which of the subsets is most likely to find the optimal solution. The study is conducted with the subset in which the optimal Hamiltonian path is most likely to be found. In most cases, the lower bound of the set of feasible solutions is taken as such an estimate. As you know, the lower boundary of a set is a certain number that is always less than the value of a function given on the set in question.

The number $N$ will be the lower boundary of the set $X$, if for any $x \in X$ there exists $N \leq \phi (x)$.

For the branch and bound method, the most difficult is to choose a method for determining the lower boundary of the set. In the task of optimizing transportation, the lower boundary of the set is the lowest cost for a given set of options.

The idea of a method for determining the lower boundary of a set is based on the following. Let a cost matrix be given between cities (table.1). From linear algebra it is known that if we solve the optimization problem for a given and reduced matrix, then the solution will differ by the amount of reduction.

The solution to the traveling salesman problem is the sequence of visits to cities. The costs for the original and reduced matrices will differ by the number of casts. Thus, if you use the cast operation for all rows and columns of the original matrix, then the optimal costs will be determined as follows:

\[ L = L_0 + h, \]

Where

\[ h = \sum_{i=1}^{n} \sum_{j=1}^{n} h_{ij}, \]

$L_0$ – is the Hamiltonian path for the reduced matrix, $h$ – is the sum of the numbers of the reduction of the cost matrix in rows and columns.

The value of $h$ can serve as the lower boundary of the set of costs in solving the problem of choosing the optimal route.

To find $h$ it is necessary:

1. In each row of the original matrix, select the minimum element, which is taken as the cast number of or a given row $h_{ai}(a=1,...,n)$. Then subtract this number from all elements of the string.

2. In each column of the original matrix, select the minimum element, which is taken as the number of casts for this column $h_{aj}(a=1,...,n)$. Then subtract this number from all elements of the column. As a result of performing the cast operation in each row and in each column, we get at least one zero element.

Summing up all the reduction numbers we get the lower boundary of the set. This means that the costs cannot be less than $h$.

Let a cost matrix be given between cities $d_{ij}$. We denote $G(\theta)$–by the set of all possible routes. Then $G(ij)$ is as of the set $G(\theta)$, including an arc $(ij)$, which means the possibility of moving from city $i$ to city $j$. A subset $G'(ij)$–is part of a set $G(\theta)$, which means $(i,j)$, the impossibility of moving from city $i$ to city $j$. Dividing the set into two such subsets, we find the lower boundary of the original set of routes.

Division of the set of feasible solutions is carried out by element $(ij)$. This element is selected from the following conditions:

1. $d_{ij}$ in the matrix should be equal to zero. Such elements after the cast are in each line.
2. For each zero element, a penalty for “non-use” is determined $\theta$. This means that if the arc $(i,j)$ is not included in the route, then some element of the row $i$ and column $j$ will enter it. Therefore, the cost of “non-use” $(i,j)$ in any case will be no less than the sum of the minimum elements of the row $i$ and column $j$. Of all the zero elements, the maximum “non-use” penalty is selected for which $\theta$.
3. Select the subset with the lowest score, which is defined as the sum of the number of casts $h$ and the penalty for “non-use” $\theta$. Using the algorithm described above for this subset, we again obtain two subsets from which we select the best one in accordance with the given criterion. Thus, as a result, we obtain a tree of routes from which you can choose the best.
4. DYNAMIC PROGRAMMING METHOD

Consider the application of the dynamic programming method to determine the optimal Hamiltonian path.

The task of optimizing transportation can be represented as a multi-step decision-making process, that is, choosing a point to which you should move at every step. Based on the principle of Bellman’s optimality it is necessary to determine the benefit function at the k step, which in general terms can be written as follows:

\[ f_k(x) = \max \{ g_k(x,u) + f_{k-1}(x) \}. \]

\[ 0 \leq u \leq x \]

Here \( k \) is the number of steps in to which the whole process is divided, \( x \) – is a variable characterizing the state of the system, and is a \( u \) -control variable, the value \( f_k(x) \) of the optimization criterion for \( k \) steps \( g_k(x,u) \) determines the gain at the \( k \)-step.

Let \( j_0, j_1, j_2, \ldots, j_n \) – it be the numbers, of the cities to be visited. It is necessary to find the Hamiltonian path of minimum length at which it is possible to bypass all the remaining points \( j_0 \) from the point with minimal cost. We denote by the \( f_0(j_0, j_1, j_2, \ldots, j_n) \) function the benefits for this path. Arc length \( j_0, j_1 \) denote by \( d_{j_0,j_1} \). Then the costs provided that the movement comes from a point \( j_0 \) at any point on the route at each stage will be determined as follows:

\[ d_{j_0,j_1}+f_{j_1}(j_2, j_3, \ldots, j_n), \]
\[ d_{j_0,j_2}+f_{j_2}(j_1, j_3, \ldots, j_n), \]
\[ \ldots \ldots \ldots \ldots \ldots \]
\[ d_{j_0,j_m}+f_{j_m}(j_1, j_2, j_3, \ldots, j_m-1, j_{m+1}, \ldots, j_n). \]

The cost function, provided that the movement occurs from point \( j_0 \) to point \( j_m \) is determined as follows:

\[ f_{j_0,j_m} = \min \{ d_{j_0,j_m} + f_{j_m}(j_1, j_2, \ldots, j_n) \}, \]

\[ 1 \leq j_m \leq n. \]

\( j_0 \)– this is the start and end point of the route. As any point \( j_0 \) of the planned route can be selected.

Next, you need to find the numerical values of the function \( f_{j_m}(j_1, j_2, j_3, \ldots, j_m, j_{m+1}, \ldots, j_n) \).

The calculations are carried out from the first element to the last according to the following formulas

\[ f_{j_0}(j_1) = d_{j_0,j_1} + d_{j_1,j_n} \]

\[ f_{j_0,j_1}(j_2, j_3) = \min_{j_1,j_2} \left\{ d_{j_0,j_1} + f_{j_1}(j_2) \right\} \]
\[ f_{j_0,j_1,j_2}(j_3) = \min_{j_1,j_2,j_3} \left\{ d_{j_0,j_1} + f_{j_1}(j_2) + f_{j_2}(j_3) \right\} \]

three point gain function

The complexity of this solution is that it is necessary to remember the optimality function at each optimization step. If the number of steps into which the decision-making process is divided is large then the calculations are quite complicated. We examined the problem of choosing the optimal route on which severe restrictions are imposed. However, in tourist practice, there are often problems of choosing a route for which restrictions allow some concessions. For example, it is allowed to return to some cities of the route, as in some cities there are no hotels for tourists. In this case, a generalization of the travelling salesman problem is required. In general, the considered traveling can be posed as a multi-criteria optimization problem.

Such a problem must be reduced to a single-criterion optimization problem by convolution of criteria. In the case of transportation it makes sense to rank all the selected criteria and using the method of expert estimates to determine the weight of each criterion. Then it is necessary to carry out an economic convolution of criteria. The matrix elements \( d_{ij} \) for the problem under consideration determine the total costs at each step and its structure will be asymmetric. Further, you can use any algorithm that implements the traveling salesman problem.

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