

A SCATTER SEARCH HYBRID APPROACH FOR TEAM ORIENTEERING PROBLEM

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ABSTRACT

The Team Orienteering Problem (TOP) is a particular vehicle routing problem in which the aim is to search a fixed number of paths that maximize the scores associated with a set of given locations within a limited time. Scatter Search explores a search space of solutions systematically by evolving a small set of reference solutions. It has strategies for diversification (in diversification generation and subset generation methods); and intensification (in the improvement and updating method). However, all these methods are very time consuming. This paper proposes a scatter search hybrid approach (SSHA) to deal with the TOP by reduce processing time and maintaining a good set of references solutions in terms of diversity and quality. It uses some new operators, called reference set queen bee-method to initializing and updating the *RefSet*, and greedy select parents to selecting pairs from a reference set for the combination method to generate a new solution. Furthermore, to improve the quality of the solution, a local search is employed, called steepest descent to explore neighborhood in a fully deterministic manner and then selects the best neighbour. Experiments conducted on the standard benchmark of TOP clearly show that proposed approach outperforms the solving methods in the scientific literature. Our algorithm detects all but one of the best known solutions. A statistical test was conducted to determine the algorithm that performed better compared with the others. The results revealed that SSHA outperformed all state-of-the-art algorithms and was comparable to one algorithm.

Keywords: *Optimization, Metaheuristic, Team Orienteering Problem, Scatter Search Algorithm, Local Search.*

1. INTRODUCTION

The Team Orienteering Problem (TOP) is a development of the orienteering problem (OP). It is a well-known, challenging combinatorial optimization problem that was first highlighted and heuristically tackled by Butt and Cavalier [1]. The TOP introduced by Chao, et al. [2]. The TOP can be defined as a team consisting of several players (e.g., two, three, or four players). Each player should start at the same point and must visit the subset of points, which have their own scores, before reaching the same end point in a specific time. Once the team player visits a point, no other player can visit that particular point. In other words, each point can be visited only once. The objective is to maximize the total team scores that have been collected before it reaches the end point within the allowed time [2, 3]. The TOP has been considered an NP-hard problem [4] because it goes beyond and is even more difficult than the OP.

The applications of TOP include athlete recruiting [2], technician routing [3] and tourist trip planning [5]. In this paper, we are interested in TOP as the core variant of OP for multiple vehicles. TOP have attracted a good deal of attention in operational research and artificial intelligent community. Apart from the difficulty of solving it, TOP has been selected for two reasons: it represents various real-world applications and its results are still possible to be improved. So far different exact method have been developed for TOP proposed by Boussier, et al. [6], Butt and Ryan [7], Dang, et al. [8] and Nicola, et al. [9]. However, unless $P = NP$, there is no algorithm which can find an optimal solution within time polynomial in the size of customers. An alternative approach to the TOP is metaheuristic, which aims to yield a satisfactory solution within reasonable time.

In contrast to exact solving approaches, a number of heuristics and metaheuristics have been developed for TOP. To date, ant colony optimization (ACO)

[10]that constructs the candidate solutions by employing four constructive heuristics: sequential, deterministic-concurrent, random-concurrent, and simultaneous methods. guided local search [11], large neighborhood search with three improvement methods: local search improvement, shift and insertion improvement, and replacement improvement [12], memetic algorithm with optimal split procedures for chromosome evaluation (MA) [13], particle swarm optimization [14], path relinking [15], Pareto mimic algorithm [16], tabu search with an adaptive memory procedure [3], and variable neighborhood search [5] have been applied to the TOP. The interested reader is referred to [17] for a survey. Among the current metaheuristics, the Pareto mimic algorithm in [16] ranked first when testing on the instances of [2]. Moreover, the results on larger new generated instances demonstrate that their algorithm is very effective though it consumes relatively long running time.

This research aims to investigate the SS algorithm that may improve the available search approaches for the TOP. The main goal is to adapt the SS algorithm, utilize its strengths, cover its weaknesses by hybridizing it with other metaheuristic algorithms, and attain a suitable balance between exploration and exploitation.

In this paper, a metaheuristic, called scatter search hybrid algorithm, is proposed. The main contribution of this paper are summarized as follows: (1) it uses different selective strategies in subset generation method called, greedy selecting parents (GSP), to generate new solution by selecting suitable parents for solution combination method, and different updating strategies in references set update method called, references set queen bee (RefSetQB), to update the references set solution with new solution. (2) To hybridize the SS algorithm with other local search algorithm called, steepest decent (SD), to enhance the exploitation search. Experiments conducted on the standard benchmark of TOP clearly show that SSHA outperforms most of the existing solution methods of the literature. It detects all but one of the best known solutions. Moreover, a strict improvement was found for one instance of the benchmark. The remainder of this paper is structured as follows. Section 2 provides a description of the TOP. SSHA are described in Section 3. In Section 4, the experimental study of the parameters is discussed. The computational results on the standard benchmark is described in Section 5. Finally, in Section 6, some conclusions and further developments are discussed.

2. DESCRIPTION OF THE TOP

The TOP can be represented as a complete graph $G = (V, E)$, where $V = [1, 2, \dots, n+1]$ is a set of points and $E = [(i, j) \mid i, j \in V]$ is a set of edges. The aim of the TOP is to find m routes that start at point 0 and end at point $n+1$, such that the total score of the visited points is maximized. Each point can be visited only once. For each route, the total time taken to visit the points cannot exceed the predetermined time limit T_{max} . Let c_{ij} be the travel time of edge $(i, j) \in E$, T_i is the service time for point i , and s_i is the score of point i . $K = [1, 2, \dots, m]$ is a set of routes, $V' = [1, 2, \dots, n]$ is a set of points visited by the k th route, and U is a subset of V that is not included in the solution. $y_{ik} = 1$ if the i th point is visited in the k th route. Otherwise, $y_{ik} = 0$. $x_{ijk} = 1$ if the edge (i, j) is visited in the k th route and point j is visited directly after point i . Otherwise, $x_{ijk} = 0$. The evaluation function, $f(x_p)$ for solution x , x_p can be formulated as Equation (1). The TOP can be formulated as follows [2, 10, 18]:

$$f(x_p) = \text{Max} \sum_{k=1}^m \sum_{i=1}^n s_i y_{ik}, \quad (1)$$

Subjected to:

$$\sum_{i=0}^n \sum_{j=0}^{n+1} c_{ij} x_{ijk} + \sum_{i=1}^n T_i y_{ik} \leq T_{max}, \quad \forall k \in K \quad (2)$$

$$x_{ijk} \in X \subseteq \{0, 1\} \quad (3)$$

Where constraint (2) is the time limit constraint, which indicates that the total traveling time of each route must not exceed the T_{max} . X is a decision variable used in constraints (4)–(9).

$$\sum_{j=0}^n x_{jik} = \sum_{j=1}^{n+1} x_{ijk}, \quad \forall i \in V'; k \in K \quad (4)$$

$$\sum_{j=1}^{n+1} x_{0jk} = 1, \quad \forall k \in K \quad (5)$$

$$\sum_{i=0}^n x_{ilk} = 1, \quad l = n+1; \forall k \in K \quad (6)$$

$$\sum_{i=1}^n x_{ijk} = y_{ik}, \quad \forall i \in V', \forall k \in K \quad (7)$$

$$\sum_{k=1}^m y_{ik} \leq 1, \quad \forall i \in V' \quad (8)$$

$$\sum_{i,j \in U} y_{ik} \leq |U| - 1, \quad U \text{ is an arbitrary subset of } V'; 2 \leq |U| \leq n; \forall k \in K \quad (9)$$

$$y_{ik}, x_{ijk} \in \{0, 1\} \quad (10)$$

Constraint (4) guarantees the connectivity of each route. Constraints (5) and (6) ensure that the route will start at point 0 and end at $n+1$. Constraint (7) describes the relation between x and y . Constraint (8) ensures that each vertex is visited only by one route. Constraint (9) imposes restrictions on the variables.

3. A SCATTER SEARCH HYBRID ALGORITHM

The SS is a population-based metaheuristic introduced by Glover [19]. It constructs the solutions by combining other elite solutions to exploit their positive attributes. The main aspect of the mechanisms within SS are not restricted to single uniform design allows the exploration of strategic possibilities that may prove effective in particular implementation [20, 21].

The search process of the SS algorithm starts with constructing a number of trial diverse solutions as an input population. Each solution in the population pool is enhanced using the improvement method. After initializing the population, a small set of good solutions is selected from the population to build a small population called a reference set, where the “good” criteria are based on the objective function value and diversity value of the solutions. After building a reference set, new subsets are generated using the subset generation method by selecting the solutions from the reference set pool in lexicographical order. Each subset of solutions is combined in a systematic way to produce one or more new solutions, each of which is mapped into an associated feasible solution. Each new solution is enhanced by the improvement method. The reference set pool is updated by a new trial solution using a steady-state replacement mechanism. This step is known as a dynamic update of the search experience [22, 23]. The pseudo-code of the SS is illustrated in Algorithm 1.

Algorithm 1. The main procedure pseudo-code of the SS

Set $P = \emptyset$, $|P| = PSize = 50$.

While (until $|P| = PSize$)

Step 1: Use the Diversification Generation Method to construct a population of solutions.

Step 2: Apply the Improvement Method. Let χ be the resulting solution. If $\chi \notin P$ then add χ to P , otherwise, discard χ .

End While

// employ nearest neighbour to construct the population of initial solutions.

// utilize steepest decent to improve all the solutions in the population.

Step 3: use the Reference Set Update method to build $RefSet$, $|RefSet| = 20$, $RefSet = [X^1, \dots, X^b], [Y^1, \dots, Y^d]$ with the “best” b_1 and “diverse” b_2 solutions in P . Order the solutions in b_1 according to their objective function value such that χ^1 is the best solution and χ^b the worst.

While (StoppingCriterion)

Step 4: Generate *NewSubsets* by the Subset Generation Method.

// apply greedy selecting parents to generate the subset.

While ($NewSubsets = \emptyset$) *// this is repeated until all NewSubsets are combined.*

Step 5: select the *Subset s* in lexicographical order to be combined.

Step 6: Apply the Solution Combination Method to *NewSubset s* to obtain one new solution χ' .

// share 25% of the divers solution Y^d attribute in the quality solution X^b to produce χ' .

// convert the infeasible solution to feasible using repair strategy.

Step 7: Apply the Improvement Method to the new solution χ' .

// utilize steepest decent to improve the produced new solution from_Solution Combination Method.

Step 8: apply the Reference Set Update method to update $RefSet$.

// utilize references set queen bee strategy.

Remove s from the *Subsets*

End While

If ($RefSet$ has not updated)

Apply Diversification Generation Method to initial new population of solutions

```
// performing nearest neighbour.
```

```
Apply the Improvement Method to the new solutions in the population.
```

```
// apply steepest decent.
```

```
Replace the diverse solutions in the RefSet by the new solutions in the population.
```

```
End If
```

```
End While
```

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Step 9: Return the best found solution
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The reference set pool is exploited to avoid the search from redundantly exploring the solutions that have been evaluated and prevent the duplication of solutions in the pool. The structure of reference set pool stimulates search diversification and intensification and may help the search to escape from the local optima because of the types of solutions in the pool and the updating strategy [24, 25].

Hence, the SS concentrates on maintaining the balance between intensification and diversification of the search. This balance is achieved by combining all possible pairs in the reference set pool to generate one or more new solutions, where the new solution may contain attributes that are not available in the original pairs. The SS exploits knowledge derived from the search space and is thus considered an information-derived algorithm [26].

The elements in the SS can be implemented in a variety of ways and degree of sophistication because of the flexibility of the SS framework [21, 22, 25]. For example, diversification generation method step might apply several constructive heuristic to build the population. Moreover, subset generation method can utilize different selection mechanism to select the parent for combination method [21].

A SS template consists of five procedures, as described by Glover, et al. [22], Glover, et al. [23], and Laguna and Martí [27]. Further details on the algorithm are given in the following subsections.

3.1 A Diversification Generation Method

Initializing the population using a diversification generation method plays an important role in developing the method that balances diversification and intensification [28]. The population generation method aims to generate good starting solutions to solve the TOP. The collection of initial solutions are constructed uniformly and distributed in the search space as input. This operation starts from scratch (empty solution) and constructs a solution by assigning points to one decision variable at a time until a complete solution is generated. As shown in

Algorithm 1, a large number of diverse solutions are constructed using the nearest-neighbor greedy algorithm (NNGA) to obtain a diverse region in the solution space [29]. All the generated solutions considered in this work are feasible.

3.2 An Improvement Method

In SS, an improvement method is introduced to transform a trial solution into one or more enhanced trial solutions. Thus, this component is used to enhance the generated solution quality via a local search procedure that drives the solution to the local optima [22]. The aim of this operation is to explore the neighborhood of the generated solutions by modifying it [21]. The solutions will be accepted if they are still feasible. We employed five common neighborhood structures (i.e. insert, swap, move, 2-opt, and replace), all of which were drawn from literature [11]. These neighborhood structures aims to shorten the total travel time and increase the total received score. The neighbors that improve the current solution are randomly selected at each iteration. We use a steepest descent heuristic as the improvement method to search for a better quality solution. We consecutively use a number of iterations as a termination criterion.

3.2.1 Steepest Descent (SD)

SD explores neighborhood N in a fully deterministic manner and then selects the best neighbor (solution) [30, 31]. Hence, *SD* exhaustively explores the neighborhood N of solution x , and all probable moves are tried for solution x to return the best neighboring (solution x'). In this work, *SD* is applied after solution x is generated (x is generated by the diversification generation or the combination method) to further improve its quality. Each of the five neighborhood N structures is selected randomly and explored for each solution x to return the one with the highest quality (x'). *SD* is terminated when it reaches the maximum number of iterations NI . The pseudo-code of the best improvement hill climbing (steepest descent) adopted in this work is presented in Algorithm 2.

Algorithm 2. Steepest Descent x = Initial solution; NI = number of iteration;**While** (stopping criterion is not met) Choose neighborhood $N()$ randomly; Select best neighbor x' where $x' \in N(x)$; x' new solution; **If** $f(x') > f(x)$ **then** $f(x)$ the objective function value of solution x
 $x = x'$ **End if** **End while****3.3 A reference set update method**

The reference set is the heart of a SS procedure, which may result in important modifications during the search process because of its initial composition [21]. The initial *RefSet* is built by selecting the elite solutions from the population and inserting them into the *RefSet*. These elite solutions are selected based on the objective value (solution quality) and diversity value (solution diversity). The diversity value is calculated by diversity measurement method, for more details see [32]. The *RefSet* is divided into two sets of solutions, namely, *RefSet*₁, which contains high-quality solutions; and *RefSet*₂, which contains the most diverse solutions. Moreover, the reference set update method is employed to update the *RefSet* after generating the new trial solution x through the combination and

improvement methods. In this work, a queen bee strategy is used to build and update the reference set.

3.3.1 Reference set queen bee-method (RefSet-QB)

The reference set queen bee-method idea came from the bee colony optimization algorithm [31]. This method consists of initializing and updating the *RefSet*. Initializing the *RefSet* starts with selecting the 10 best solutions from the population based on their objective function values. Then, the 10 most diverse solutions are selected from the population based on their diversity values. All the selected solutions (20 solutions) are placed and sorted in one pool (*RefSet*). The best solution in the initial *RefSet* is marked as the queen bee. Algorithm 3 illustrates the initialization step of the queen bee method.

Algorithm 3. Initial RefSetQBSet *RefSet* = \emptyset , $|RefSet| = RefSet_Size = 20$.*Step 1*: sort the population p of solutions based in the objective function $f(x)$.*Step 2*: chose the better 10 quality solutions and put them in the *RefSet*.*Step 3*: sort the population p of solutions based in the diversity vlaue $d(x)$.*Step 4*: chose the better 10 diverse solutions and put them in the *RefSet*.*Step 5*: sort the *RefSet* based on the objective function $f(x)$.Return *RefSet* = $[X^1, \dots, X^n]$

Meanwhile, the *RefSet* is updated by comparing the new trial solution (x') with the worst quality solution (x_b) in the *RefSet*. If the new trial solution (x') is better in terms of the objective function value, it will supersede the worst quality solution (x_b). Once the *RefSet* cannot be updated anymore, the new trial solution (x') will supersede the worst diverse

solution (x_y) if it is better in terms of the diversity value. When the updating strategy fails to update the *RefSet*, the solutions in the *RefSet*, except for the queen bee (best solution in the *RefSet*), are replaced by the other solutions generated by the diversification generation method. Algorithm 4 illustrates the updating step of the queen bee method.

Algorithm 4. Update RefSetQB*Now_Sol_List* = $[X^1, \dots, X^n]$; //new trial solutions generated by combination method and improved by improvement method.**While** (*Now_Sol_List* = \emptyset) x_b = worst quality solution in the *RefSet* based on the objective function value.

```

 $x_y$  = worst diversity solution in the RefSet based on the diversity value.
If ( $f(x') > f(x_b)$ ) //  $f(x')$  objective function of new trial solutions;
    replace the worst quality solution  $x_b$  in RefSet by  $x'$ .
    remove  $x'$  from Now_Sol_List.
Else If ( $d(x') > d(x_y)$ ) //  $d(x')$  diversity value of new solutions to RefSet;
    replace the worst diversity solution  $x_y$  in RefSet by  $x'$ .
    remove  $x'$  from Now_Sol_List.
Else
    remove  $x'$  from Now_Sol_List.
End If
End While
If (RefSet has not updated)
    Apply Diversification Generation Method to initial new population
    of solutions by performing nearest neighbour.
    Apply the Improvement Method to the new solutions in the
    population.
    // apply hill climbing.
    Replace the solutions except queen bee in the RefSet by the new trial
    solutions in the population.
End If

```

3.4 A subset generation method

Subset generation method (SGM) plays an important role in selecting pairs from a reference set for the combination method to generate a new solution [23]. The most common subset generation method is to generate the subset of pairs using a lexicographical order mechanism, namely, *Type-I* selection (i.e., the size of all subsets is 2). In the present work, greedy select parents (GSP) is employed as selection mechanism to generate a subset of pairs for the combination method.

3.4.1 Greedy select parents (GSP)

This selection strategy operates on two lists of quality and diverse solutions in the reference set to generate a subset of two solutions. This method generates three types of subsets: two quality solutions, two diversity solutions, and one quality and one diversity solution [33]. Each solution is elected randomly from each list once. The pseudo-code of this strategy is illustrated in Algorithm 5.

Algorithm 5. Greedy select parents

$TempListX = RefSet = [X^1, \dots, X^b]$

$TempListY = RefSet = [Y^1, \dots, Y^d]$

$SubsetList = \emptyset$.

Subset $S = \emptyset$, $|S| = SSize = 2$.

While (*TempList* is empty)

Generate empty subset S ;

While (until $|S| = SSize$)

Switch :

case 1

Step 1: select x randomly from *TempListX*; // x is solution quality

Step 2: add x to S ;

Step 3: remove x from *TempListX*;

case 2

Step 1: select y randomly from *TempListY*; // y is solution diversity

Step 2: add y to S ;

Step 3: remove y from *TempListY*;

case 3

apply case 1 & case 2;

Step 5: add S to SubsetList

End while

Step 6: return SubsetList

3.5 A solution combination method

The main goal of this component is to generate one or more trial solutions from various regions of the solution search space. These new trial solutions are subjected to the improvement method. In this study, we use the work of Martí, et al. [34] to design the combination method, which produces one trial solution in a systematic way. The solution in the subset with the highest quality is selected, and then 25% is taken from other solutions with the highest quality. The new solution has 75% of the quality solution attributes and 25% percent of the diverse solution attributes and is perhaps infeasible.

Repair strategy is applied on the infeasible solution to restore its feasibility by removing the repeated points and the lowest score so that the solution does not exceed the time limit. For instance, a strategy is applied in the case where the search operators used by the combination method may generate infeasible solutions.

The SS features the role of the reference set update method in maintaining elite solutions (in terms of solutions of high quality and high diversity) and the SGM in providing pairs for producing a new solution. A greedy select parents (GSP) is employed as selection mechanism to generate a subset of pairs in SGM which considers both quality and diversity solutions in the *RefSet* to be selected as a subset during the search process and produces new solutions that maintain the diversity of the *RefSet*. Steepest Descent is single-solution-based metaheuristics can improve the quality of the obtained results further by exploiting the promising area.

In this work, the repairing strategy first checks the repeated points. If the point is repeated in the solution, one of these points is randomly selected and then removed from the others. In the second step, the time budget for each route is examined. If the solution exceeds the available time, the edge that causes a long travel time between points is removed. In the next section, the performance of the SSHA on

the standard benchmark for TOP is discussed.

4. EXPERIMENTAL SETUP

We have conducted an experimental study to evaluate our proposals and to compare the algorithm, termed SSHA, with the state of the art. In this study, we have considered the standard benchmark dataset most widely used in the literature, namely, team orienteering problem [2], making a total of 387 instances with different sizes and flexibility (the number of points, time limit, and number of player).

The SS is coded using Java 1.7 and performed on a personal computer (Intel Pentium (R) Core i5 CPU at 3.40 GHz with 4 gigabyte RAM), running on Windows 8 operating system (64-bit). The proposed SS executed 31 independent runs with different random seeds for some instances of the TOP to assess the effectiveness of using different parameters. Talbi [31] recommended performing minimum 30 runs in the statistical analysis of the algorithm performance. Executing 31 runs can easily calculate the median value without the need for interpolation.

4.1 Parameter settings

There are three parameters in SSHA, that is, the population size (*Popsiz*), the number of iteration (*Iter_{max}*) based on the fix number (*k*), and references set size (*RefSet*). The parameter values are determined using statistical study. The tested values of these parameters are as follows: *Popsiz* = [30; 50; 100], *k* = [10; 20; 30; 40; 50], and *RefSet size* = [6; 10; 20]. Based on experimental results, we observed that when *Popsiz* = 20, *k* = 40, and *RefSet size* = 20, SSHA works best. To analyze the effect of each parameter, only one parameter is varied while others are fixed.

The settings of the parameters of the SS HA used for the TOP are presented in Table 1. We performed trial

and error methods called offline parameter tuning [31].

select parents (GSP) with the proposed algorithm. The second subsection compares the results of proposed algorithm against the other algorithms that have been introduced in the literature for TOP.

Table 1. Parameter Settings

#	Parameters	Values
1-	Population size (Pop_{size})	50 solutions
2-	References set size ($RefSet$)	20 solutions ($b_1 = 10, b_2 = 10$)
3-	References set quality ($RefSet_1$)	$b_1 = 10$
4-	References set diversity ($RefSet_2$)	$b_2 = 10$
5-	Number of Iteration ($Iter_{max}$)	$Iter_{max} = k \cdot n/m; k = 30$
6-	Subset generation size	2 solutions in each subset
7-	Solution Combination rate	1.0

We set the termination condition for the SS to solve the TOP according to the work of Dang, et al. [14]. The SSHA stops when the number of elapsed consecutive iterations reaches the obtained results of Equation (10).

$$Iter_{max} = k \cdot n/m \quad (10)$$

where k is the fixed number, n is the number of points, and m is the number of paths. The termination condition is changed based on the number of points and number of paths.

5. RESULTS AND COMPARISON

This section is divided into two subsections. The first subsection presents the assess the value of incorporating the Steepest Descent (SD), Reference set queen bee-method (RefSet-QB), and Greedy

5.1 Effectiveness Evaluation

This experiment examines the value of using of incorporating the Steepest Descent (SD), Reference set queen bee-method (RefSet-QB), and Greedy select parents (GSP) with SS. We have tested the proposed algorithm (denoted as SS) with RefSet-QB (denoted as SS-RefSetQB), GSP (denoted as SS-GSP), both together (denoted ISS) and incorporated ISS with SD (denoted as HISS-SD). To ensure a fair comparison between SS, SS-RefSetQB, SS-GSP, ISS, and HISS-SD the initial solution, number of runs, stopping condition and computer resources are the same for all instances. All algorithms have been tested 31 runs over all instances. Table 2 lists, for each instance, the maximum (Max), average (Avg), standard deviation ($Stdv$), and CPU average time ($CPU_{Avg} Time$) achieved by SS, SS-RefSetQB, SS-GSP, ISS, and HISS-SD. In the table, the maximum value is preferred (Maximization problem) and best results achieved by the compared algorithms are presented in boldfont.

From table 2, it can be notice that HISS-SD obtained the best results in all instance and outperformed the other tested algorithms, except ISS, across most instances. Moreover, both the average and standard deviation results gained by HISS-SD are better than other tested algorithms on all instances. The results revel that the cooperative SS with RefSet-QB, GSP, and SD contribute to HISS-SD performance and has an effect on the ability of HISS-SD in producing high quality and consistent results across all tested instances.

Table 2. The computational results of SS, SS-RefSetQB, SS-GSP, ISS, and HISS-SD

Algorithms	Ins. Name												
	p4.2.1	p4.3.p	p4.4.s	p5.2.y	p5.3.r	p5.4.v	p6.2.n	p6.3.n	p6.4.1	p7.2.o	p7.3.s	p7.4.1	
SS	Max	1071	1222	1254	1645	1125	1320	1260	1170	696	945	1079	590
	Avg	1064	1200.06	1225.84	1621.61	1124.03	1320	1245.1	1167.68	696	935.71	1073.35	589.52
	Stdv	7.19	16.32	18.5	12.27	2.01	0	11.26	4.82	0	7.51	5.39	1.98
	CPU _{Avg} Time	269.37	248.72	185.79	172.4	103.56	53.51	109.87	104.12	46.01	336.83	229.75	91.57
SS-GST	Max	1074	1222	1256	1645	1125	1320	1260	1170	696	945	1081	590
	Avg	1070.19	1208.77	1235.81	1635	1124.84	1320	1253.23	1168.84	696	941.23	1074.97	589.81
	Stdv	2.51	13.76	14.66	8.76	0.9	0	8.73	3.61	0	6.17	4.92	0.6
	CPU _{Avg} Time	199.12	105.26	69.75	138.45	90.3	42.34	86.73	98.85	38.15	219.1	160.71	78.13
SS-RefSetQB	Max	1073	1222	1256	1645	1125	1320	1260	1170	696	945	1081	590
	Avg	1067	1206.94	1225.77	1629.19	1122.9	1320	1246.26	1168.65	696	939.94	1070.68	589.48
	Stdv	4.95	18.44	15.71	9.58	5.59	0	9.32	3.7	0	4.63	3.91	2.53
	CPU _{Avg} Time	200.01	143.37	129.25	134.59	90.46	45.3	80.51	94.29	42.7	254.85	189.19	71.79
ISS	Max	1074	1222	1257	1645	1125	1320	1260	1170	696	945	1081	590
	Avg	1071.06	1215.16	1245.16	1635.65	1124.35	1320	1254.58	1168.89	696	942.61	1076.61	589.94
	Stdv	3.93	11.92	6.94	8.44	1.87	0	6.9	3.96	0	3.33	5.03	0.36
	CPU _{Avg} Time	173.47	93.84	66.8	137.04	90.29	46.04	89.3	91.5	41.2	257.05	186.82	78.32
HISS-SD	Max	1074	1222	1260	1645	1125	1320	1260	1170	696	945	1081	590
	Avg	1073.48	1221.68	1257	1645	1125	1320	1260	1170	696	945	1081	590
	Stdv	0.51	0.54	3.44	0	0	0	0	0	0	0	0	0
	CPU _{Avg} Time	195.49	107.07	86.32	152.8	99.64	50.12	95.11	96.61	42.62	285.95	197.45	83.3

To further support these positive results, the significant difference between the proposed algorithms were examined by performing a Wilcoxon rank test with the significance interval of 95% ($\alpha = 0.05$). Pair comparisons were executed for HISS-SD and the other proposed algorithms. Table 3 presents the *p*-value of the HISS-SD versus SS, SS-RefSetQB, SS-GSP, and ISS for the TOP instances. In Table 3, the symbol “+” denotes that HISS-SD is statistically better than the contending proposed algorithm (*p*-value < 0.05), “-” denotes that HISS-SD was outperformed by the contending proposed algorithm (*p*-value > 0.05), and “~” denotes that HISS-SD has the same performance as the contending proposed algorithm (*p*-value = 0.05).

Table 3. The *p*-value of HISS-SD versus SS, SS-RefSetQB, SS-GSP, and ISS.

HISS-SD vs.	SS	SS-GST	SS-RefSetQB	ISS
Ins. Name	<i>p</i> -value	<i>p</i> -value	<i>p</i> -value	<i>p</i> -value
p4.2.1	+	+	+	+
p4.3.p	+	+	+	+
p4.4.s	+	+	+	+
p5.2.y	+	+	+	+
p5.3.r	~	~	+	~
p5.4.v	~	~	~	~
p6.2.n	+	+	+	+
p6.3.n	+	~	~	+
p6.4.1	~	~	~	~
p7.2.o	+	+	+	+
p7.3.s	+	+	+	+
p7.4.1	~	~	~	~

The results in Table 3 show that HISS-SD is statistically better than the contending other proposed algorithms in most instances. These results support the fact that the SD algorithm improved the performance of the SS, SS-RefSetQB, SS-GSP, and ISS, while the HISS-SD obtained much better results than the other proposed algorithms. Indeed, the use the SD algorithm can effectively enhance the search performance to obtain very good results for all tested instances.

5.2 The comparison of HISS-SD with other algorithms for TOP.

This section compares the performance of the proposed algorithms with state-of-the-art of algorithms used to solve team orienteering problem (TOP). The results of the proposed algorithms for TOP are compared with state-of-the-art algorithms in terms of maximum obtained value, average CPU time, relative percentage error (RPE), and average RPE (ARPE). A statistical test is also performed to compare the proposed algorithms with other algorithms implemented to solve TOP.

The state-of-the-art algorithms used to compare the results are presented in Table 4. The symbols of these algorithms, including the descriptions and references are presented in Table 4.

The comparison of 157 instances with the standard benchmark for TOP, which are set as 4, 5, 6, and 7, is reported. Other instances (i.e., sets 1, 2, and 3) are eliminated from the comparison because all the

compared algorithms obtain the same best known results.

Tables 5 – 9 present the results obtained through state-of-the-art of algorithms for all compared instances and through the proposed algorithms. In

addition, a comparison is also performed with respect to the best known results for TOP. The best results are displayed in boldfont and with a dark background.

Table 4: Abbreviations of the compared methods for TOP.

#	Symbol	Descriptions	References
1-	ASe	Sequential Ant Colony Optimization	(Ke et al. 2008)
2-	ADC	Deterministic Concurrent Ant Colony Optimization	(Ke et al. 2008)
3-	ARC	Random Concurrent Ant Colony Optimization	(Ke et al. 2008).
4-	ASi	Simultaneous Ant Colony Optimization	(Ke et al. 2008)
5-	MA10	Memetic Algorithm	(Bouly et al. 2010)
6-	PSOMA	Particle Swarm Optimization Based Memetic Algorithm	(Dang et al. 2013)
7-	PSOiA	Particle Swarm Optimization Inspired Algorithm	(Dang et al. 2013)
8-	PMA	Pareto Mimic Algorithm	(Ke, Zhai et al. 2016)

According to the results in Tables 5 – 9, SS, ISS, and HISS-SD generate excellent results for most instances of TOP. HISS-SD obtains the best results compared with the proposed algorithms in 153 out of

157 instances. This result indicates that this algorithm generalizes well among all TOP instances instead of producing excellent results for few instances only.

Table 5. Best results of SS, ISS, and HISS-SD compared with other algorithms for test set 4

Ins.	HISS-SD	ISS	SS	ASe	ADC	ARC	ASi	MA10	PSOMA	PSOiA	PMA	Best Known
p4.2.a	206	206	206	206	206	206	206	206	206	206	206	206
p4.2.b	341	341	341	341	341	341	341	341	341	341	341	341
p4.2.c	452	452	452	452	452	452	452	452	452	452	452	452
p4.2.d	531	531	531	531	530	531	531	531	531	531	531	531
p4.2.e	618	618	618	618	600	600	613	618	618	618	618	618
p4.2.f	687	687	687	687	672	672	672	687	687	687	687	687
p4.2.g	757	756	756	757	756	756	756	757	757	757	757	757
p4.2.h	835	834	835	827	819	819	820	835	835	835	835	835
p4.2.i	918	918	918	918	900	918	918	918	918	918	918	918
p4.2.j	965	962	965	965	962	962	962	965	965	965	965	965
p4.2.k	1022	1022	1022	1022	1016	1016	1016	1022	1022	1022	1022	1022
p4.2.l	1074	1074	1074	1071	1070	1071	1069	1071	1071	1074	1074	1074
p4.2.m	1132	1132	1132	1130	1115	1119	1113	1132	1132	1132	1132	1132
p4.2.n	1174	1173	1172	1168	1149	1158	1169	1174	1174	1174	1174	1174
p4.2.o	1218	1216	1206	1215	1209	1198	1210	1218	1218	1218	1218	1218
p4.2.p	1242	1241	1239	1242	1229	1233	1239	1242	1241	1242	1242	1242
p4.2.q	1267	1264	1258	1263	1253	1252	1260	1267	1267	1268	1268	1268
p4.2.r	1292	1288	1276	1288	1278	1278	1279	1292	1292	1292	1292	1292
p4.2.s	1304	1304	1290	1304	1304	1303	1304	1304	1304	1304	1304	1304
p4.2.t	1306	1306	1306	1306	1306	1306	1306	1306	1306	1306	1306	1306
p4.3.c	193	193	193	193	193	193	193	193	193	193	193	193
p4.3.d	335	335	335	335	333	333	335	335	335	335	335	335
p4.3.e	468	468	468	468	468	468	468	468	468	468	468	468
p4.3.f	579	579	579	579	579	579	579	579	579	579	579	579
p4.3.g	653	653	653	653	652	653	652	653	653	653	653	653
p4.3.h	729	729	729	720	713	713	713	728	729	729	729	729
p4.3.i	809	809	809	796	793	793	786	809	809	809	809	809
p4.3.j	861	861	860	861	857	855	858	861	861	861	861	861
p4.3.k	919	919	919	918	913	910	910	919	919	919	919	919
p4.3.l	979	978	979	979	958	976	966	979	979	979	979	979



p4.3.m	1063	1056	1061	1053	1039	1028	1046	1063	1063	1063	1063	1063
p4.3.n	1121	1121	1121	1121	1109	1112	1103	1121	1121	1121	1121	1121
p4.3.o	1172	1172	1170	1170	1163	1167	1165	1172	1172	1172	1172	1172
p4.3.p	1222	1222	1222	1221	1202	1207	1207	1222	1222	1222	1222	1222
p4.3.q	1253	1246	1251	1252	123	1239	1238	1253	1253	1253	1253	1253
p4.3.r	1273	1272	1262	1267	1263	1263	1263	1273	1273	1273	1273	1273
p4.3.s	1295	1293	1282	1293	1291	1289	1291	1295	1295	1295	1295	1295
p4.3.t	1305	1305	1299	1305	1304	1303	1304	1305	1304	1305	1305	1305
p4.4.e	183	183	183	183	183	183	183	183	183	183	183	183
p4.4.f	324	324	324	324	324	324	324	324	324	324	324	324
p4.4.g	461	461	461	461	461	460	460	461	461	461	461	461
p4.4.h	571	571	571	571	556	556	556	571	571	571	571	571
p4.4.i	657	657	657	657	653	653	653	657	657	657	657	657
p4.4.j	732	732	732	732	731	731	731	732	732	732	732	732
p4.4.k	821	820	821	821	820	818	818	821	821	821	821	821
p4.4.l	880	878	878	880	877	875	875	880	880	880	880	880
p4.4.m	919	916	919	918	911	911	911	916	919	919	919	919
p4.4.n	976	976	967	961	956	956	956	969	969	976	976	977
p4.4.o	1061	1061	1061	1036	1030	1029	1029	1061	1061	1061	1061	1061
p4.4.p	1124	1120	1124	1111	1108	1110	1110	1124	1124	1124	1124	1124
p4.4.q	1161	1160	1157	1145	1150	1148	1148	1161	1161	1161	1161	1161
p4.4.r	1216	1213	1211	1200	1195	1194	1194	1216	1216	1216	1216	1216
p4.4.s	1260	1257	1256	1249	1256	1252	1252	1260	1259	1260	1260	1260
p4.4.t	1285	1284	1282	1281	1281	1281	1281	1285	1285	1285	1285	1285

Table 6. Best results of SS, ISS, and HISS-SD compared with other algorithms for test set 5.

Ins.	HISS-SD	ISS	SS	ASe	ADC	ARC	ASi	MA10	PSOMA	PSOiA	PMA	Best Known
p5.2.h	410	410	410	410	410	410	410	410	410	410	410	410
p5.2.j	580	580	580	580	580	580	580	580	580	580	580	580
p5.2.k	670	670	670	670	670	670	670	670	670	670	670	670
p5.2.l	800	800	800	800	800	800	800	800	800	800	800	800
p5.2.m	860	860	860	860	860	860	860	860	860	860	860	860
p5.2.n	925	925	925	925	920	920	925	925	925	925	925	925
p5.2.o	1020	1020	1020	1020	1020	1010	1010	1020	1020	1020	1020	1020
p5.2.p	1150	1150	1150	1150	1150	1150	1150	1150	1150	1150	1150	1150
p5.2.q	1195	1195	1195	1195	1195	1195	1195	1195	1195	1195	1195	1195
p5.2.r	1260	1260	1260	1260	1260	1260	1260	1260	1260	1260	1260	1260
p5.2.s	1340	1340	1330	1340	1330	1330	1330	1340	1340	1340	1340	1340
p5.2.t	1400	1400	1400	1400	1400	1400	1400	1400	1400	1400	1400	1400
p5.2.u	1460	1460	1430	1460	1460	1460	1460	1460	1460	1460	1460	1460
p5.2.v	1505	1505	1490	1505	1495	1500	1495	1505	1505	1505	1505	1505
p5.2.w	1565	1560	1550	1560	1555	1555	1555	1560	1560	1565	1565	1565
p5.2.x	1610	1610	1595	1610	1610	1610	1610	1610	1610	1610	1610	1610
p5.2.y	1645	1645	1645	1645	1645	1645	1645	1645	1645	1645	1645	1645
p5.2.z	1680	1680	1670	1680	1680	1680	1680	1680	1680	1680	1680	1680
p5.3.k	495	495	495	495	495	495	495	495	495	495	495	495
p5.3.l	595	595	595	595	595	595	595	595	595	595	595	595
p5.3.n	755	755	755	755	755	755	755	755	755	755	755	755
p5.3.o	870	870	870	870	870	870	870	870	870	870	870	870
p5.3.q	1070	1070	1070	1070	1065	1065	1065	1070	1070	1070	1070	1070
p5.3.r	1125	1125	1125	1125	1120	1125	1125	1125	1125	1125	1125	1125
p5.3.s	1190	1190	1190	1190	1190	1190	1185	1190	1190	1190	1190	1190
p5.3.t	1260	1260	1260	1260	1250	1255	1260	1260	1260	1260	1260	1260
p5.3.u	1345	1345	1345	1345	1330	1335	1335	1345	1345	1345	1345	1345
p5.3.v	1425	1425	1425	1425	1425	1425	1420	1425	1425	1425	1425	1425
p5.3.w	1485	1485	1475	1485	1465	1465	1465	1485	1485	1485	1485	1485
p5.3.x	1555	1555	1540	1540	1535	1540	1540	1555	1555	1555	1555	1555
p5.3.y	1595	1595	1580	1590	1590	1590	1590	1590	1595	1595	1595	1595
p5.3.z	1635	1635	1630	1635	1635	1635	1635	1635	1635	1635	1635	1635

p5.4.m	555	555	555	555	555	555	555	555	555	555	555	555
p5.4.o	690	690	690	690	690	690	690	690	690	690	690	690
p5.4.p	765	765	765	765	760	760	760	760	765	765	765	765
p5.4.q	860	860	860	860	860	860	860	860	860	860	860	860
p5.4.r	960	960	960	960	960	960	960	960	960	960	960	960
p5.4.s	1030	1030	1030	1030	1030	1030	1030	1030	1030	1030	1030	1030
p5.4.t	1160	1160	1160	1160	1160	1160	1160	1160	1160	1160	1160	1160
p5.4.u	1300	1300	1300	1300	1300	1300	1300	1300	1300	1300	1300	1300
p5.4.v	1320	1320	1320	1320	1320	1320	1320	1320	1320	1320	1320	1320
p5.4.w	1390	1390	1390	1390	1380	1390	1380	1380	1385	1390	1390	1390
p5.4.x	1450	1445	1445	1450	1450	1450	1450	1450	1450	1450	1450	1450
p5.4.y	1520	1520	1520	1520	1510	1510	1500	1520	1520	1520	1520	1520
p5.4.z	1620	1620	1595	1620	1620	1575	1580	1620	1620	1620	1620	1620

Table 7. Best results of SS, ISS, and HISS-SD compared with other algorithms for test set 6.

Ins.	HISS-SD	ISS	SS	ASe	AD C	ARC	ASi	MAI 0	PSOM A	PSOi A	PM A	Best Known
p6.2.d	192	192	192	192	192	192	192	192	192	192	192	192
p6.2.j	948	948	948	948	948	948	948	948	948	948	948	948
p6.2.l	1116	111	111	111	111	111	111	1116	1116	1116	111	1116
p6.2.m	1188	118	118	118	118	118	118	1188	1188	1188	118	1188
p6.2.n	1260	126	126	126	126	125	126	1260	1260	1260	126	1260
p6.3.g	282	282	282	282	282	282	282	282	282	282	282	282
p6.3.h	444	444	444	444	444	438	438	444	444	444	444	444
p6.3.i	642	642	642	642	642	642	642	642	642	642	642	642
p6.3.k	894	894	894	894	888	888	894	894	894	894	894	894
p6.3.l	1002	100	100	100	100	100	100	1002	1002	1002	100	1002
p6.3.m	1080	108	108	108	107	108	108	1080	1080	1080	108	1080
p6.3.n	1170	117	117	117	116	116	116	1170	1170	1170	117	1170
p6.4.j	366	366	366	366	366	366	366	366	366	366	366	366
p6.4.k	528	528	528	528	528	528	528	528	528	528	528	528
p6.4.l	696	696	696	696	696	696	696	696	696	696	696	696

Table 8. Best results of SS, ISS, and HISS-SD compared with other algorithms for test set 6.

Ins.	HISS-SD	ISS	SS	ASe	AD C	ARC	ASi	MAI 0	PSOM A	PSOi A	PM A	Best Known
p7.2.d	190	190	190	190	190	190	190	190	190	190	190	190
p7.2.e	290	290	290	290	290	290	290	290	290	290	290	290
p7.2.f	387	387	387	387	387	387	387	387	387	387	387	387
p7.2.g	459	459	459	459	459	459	459	459	459	459	459	459
p7.2.h	521	521	521	521	521	521	521	521	521	521	521	521
p7.2.i	580	579	580	580	579	579	579	580	580	580	580	580
p7.2.j	646	646	646	646	646	646	646	646	646	646	646	646
p7.2.k	705	704	705	705	704	704	704	705	705	705	705	705
p7.2.l	767	767	761	767	767	767	767	767	767	767	767	767
p7.2.m	827	827	827	827	827	827	827	827	827	827	827	827
p7.2.n	888	888	888	888	878	878	878	888	888	888	888	888
p7.2.o	945	945	945	945	945	940	941	945	945	945	945	945
p7.2.p	1002	100	100	100	991	993	993	1002	1002	1002	100	1002
p7.2.q	1043	104	104	104	104	104	104	1044	1044	1044	104	1044

p7.2.r	1094	108 9	109 2	109 4	109 3	108 8	109 4	1094	1094	1094	109 4	1094
p7.2.s	1136	113 4	112 8	113 6	113 6	113 4	113 1	1136	1136	1136	113 6	1136
p7.2.t	1179	117 4	116 4	117 9	117 9	117 9	117 9	1179	1179	1179	117 9	1179
p7.3.h	425	425	425	425	425	425	425	425	425	425	425	425
p7.3.i	487	487	487	487	487	486	487	487	487	487	487	487
p7.3.j	564	564	564	564	564	564	564	564	564	564	564	564
p7.3.k	633	633	633	633	632	633	633	633	633	633	633	633
p7.3.l	684	684	682	684	683	684	684	684	683	684	684	684
p7.3.m	762	762	762	762	762	762	762	762	762	762	762	762
p7.3.n	820	820	820	820	819	819	820	820	820	820	820	820
p7.3.o	874	874	874	874	874	874	874	874	874	874	874	874
p7.3.p	927	929	927	929	925	926	925	929	927	929	929	929
p7.3.q	987	987	987	987	987	987	987	987	987	987	987	987
p7.3.r	1026	102 4	102 4	102 6	102 4	102 1	102 2	1026	1026	1026	102 6	1026
p7.3.s	1081	108 1	108 1	108 1	108 1	108 1	107 7	1081	1081	1081	108 1	1081
p7.3.t	1120	111 8	111 6	111 8	111 7	110 3	111 7	1120	1120	1120	112 0	1120
p7.4.g	217	217	217	217	217	217	217	217	217	217	217	217
p7.4.h	285	285	285	285	285	285	285	285	285	285	285	285
p7.4.i	366	366	366	366	366	366	366	366	366	366	366	366
p7.4.k	520	520	520	520	520	520	520	520	520	520	520	520
p7.4.l	590	590	590	590	590	590	590	590	590	590	590	590
p7.4.m	646	646	646	646	644	646	646	646	646	646	646	646
p7.4.n	730	726	730	730	725	725	726	726	726	730	730	730
p7.4.o	781	781	781	781	778	781	778	781	781	781	781	781
p7.4.p	846	846	846	846	846	838	842	846	846	846	846	846
p7.4.q	909	908	909	909	909	909	909	909	909	909	909	909
p7.4.r	970	970	970	970	970	970	970	970	970	970	970	970
p7.4.s	1022	102 2	102 2	102 2	101 9	102 1	101 9	1022	1022	1022	102 2	1022
p7.4.t	1077	107 7	107 7	107 7	107 2	107 7	107 7	1077	1077	1077	107 7	1077

Table 9: Performance comparison based on RPE average for each data set of the associated instance.

Data set	RPE average for each dataset										
	HISS-SD	ISS	SS	ASe	ADC	ARC	ASi	MA10	PSOMA	PSOiA	PMA
Test Set 4	0.003	0.090	0.196	0.312	0.903	0.861	0.775	0.030	0.026	0.002	0.002
Test Set 5	0.000	0.015	0.246	0.036	0.231	0.257	0.284	0.061	0.015	0.000	0.000
Test Set 6	0.000	0.000	0.000	0.000	0.152	0.201	0.124	0.000	0.000	0.000	0.000
Test Set 7	0.007	0.058	0.097	0.006	0.146	0.188	0.150	0.013	0.021	0.000	0.000
RPE AVG	0.003	0.041	0.132	0.089	0.358	0.377	0.333	0.026	0.016	0.000	0.000
Eq. Best	153	123	115	128	80	80	84	146	146	156	156

The efficiency of the algorithm was measured by the relative percentage error (RPE) and the average RPE (ARPE). The RPE is clarified as the relative error between the best known results obtained in scientific literature and the best results obtained by the proposed algorithm. It shows the performance of the proposed algorithm over a number of runs, as in Equation (11). On the other hand, the ARPE is

clarified as the relative error between the best known results obtained in scientific literature and the average results obtained by the proposed algorithm over a number of runs. It shows the robustness of the proposed algorithm, as in Equation (12).

$$RPE = \frac{x_{best} - x_{max}}{x_{best}} \cdot 100 \quad (11)$$

$$ARPE = \frac{x_{best} - x_{avg}}{x_{best}} \cdot 100 \quad (12)$$

where x_{best} is the best known result found in literature, x_{max} is the best results obtained by the proposed algorithm over all the independent runs, and x_{avg} is the average of the obtained results by the proposed algorithm over a number of runs.

Tables 9 presents the RPE average value for each set of standard benchmark instances. SS, ISS, and HISS-SD are compared with state-of-the-art of algorithms based on the RPE average value. Table 8 evidently shows that the RPE average values of SS,

ISS, and HISS-SD are 0.132, 0.041, and 0.003, respectively. SS outperforms 3 algorithms (ADC, ARC, and ASi) out of 8 algorithms. Meanwhile, ISS outperforms 4 algorithms (ASe, ADC, ARC, and ASi) out of 8 algorithms. HISS-SD outperforms 6 algorithms (ASe, ADC, ARC, ASi, MA10, PSOMA) out of 8 algorithms. The last row in Table 8 presents the number of solutions that is equal to the best known results.

Table 10 presents the ARPE average for each set of standard benchmark instances. SS, ISS, and HISS-SD are compared with the state-of-the-art algorithms, wherein the average results are reported in scientific literature based on ARPE average value. The table shows that SS and ISS outperform 4 compared algorithms (ASe, ADC, ARC, and ASi) out of 8 algorithms. HISS-SD outperforms 7 compared algorithms (ASe, ADC, ARC, ASi, MA10, PSOMA, and PMA) out of 8 algorithms.

Table 10: Performance comparison based on ARPE average for each data set of the associated instance.

Data set	ARPE average for each dataset										
	HISS-SD	ISS	SS	ASe	ADC	ARC	ASi	MA10	PSOMA	PSOiA	PMA
Test Set 4	0.166	0.893	1.066	1.866	1.867	2.063	1.703	0.207	0.285	0.110	0.318
Test Set 5	0.050	0.232	0.712	0.823	1.161	1.107	1.181	0.095	0.090	0.034	0.090
Test Set 6	0.000	0.017	0.056	1.071	1.170	1.210	1.065	0.017	0.000	0.000	0.293
Test Set 7	0.056	0.389	0.439	0.512	0.617	2.532	2.457	0.106	0.179	0.030	0.128
ARPE AVG	0.068	0.383	0.568	1.068	1.204	1.728	1.601	0.106	0.139	0.044	0.208

The average values obtained by SS, ISS, HISS-SD, and the state-of-the-art algorithms are used to compare the performances of these algorithms through a Friedman statistical test. The p -values obtained through the Friedman and Iman–Davenport statistical tests are less than the critical level (0.05), which indicates the statistically different performances of the compared algorithms. Hence, the difference among the compared algorithms is detected through a Holm and Hochberg post-hoc statistical test (see Derrac, et al. [35] for more details).

Table 10 summarizes the average ranking (lower is better) for each algorithm (SS, ISS, and HISS-SD are included in the ranking). The ranking shows that SS is ranked 6.66, ISS is 6.05, and HISS-SD is 3.3. Therefore, PSOiA becomes the controlled method in the Holm and Hochberg statistical test because it is ranked as the 1st algorithm. The p -values computed by the Friedman test is 0.000, which is way below the significance interval of 95% ($\alpha = 0.05$). This result shows that there is a significant difference among the observed results listed in Tables 5-9.

Next, the Holm and Hochberg statistical test [35] is performed to obtain the adjusted p-values for each comparison between PSOiA (the control method) and other algorithms. The adjusted p-value of the Holm and Hochberg statistical test is summarized in Table 12. PSOiA algorithm is statistically better than ARC, ADC, ASi, ASe, SS, ISS, PMA, PSOMA, and MA10 with a critical level of 0.05 (adjusted p-value < 0.05). Whereas, the results also reveal that HISS-SD is comparable to PSOiA with a critical level of 0.05 (adjusted p-value > 0.05). Notably, SS and ISS are included in the comparison with the state-of-the-art algorithms.

Table 11: Average rank obtained through a Friedman test for the compared algorithms.

Algorithm	Ranking
ASe	8.15
ADC	8.78
ARC	9.05
ASi	8.58
MA10	3.86
PSOMA	4.30
PSOiA	2.80
PMA	4.42

SS	6.66
ISS	6.05
HISS	3.30

In summary, multiple comparison statistical tests indicate that HISS-SD an effective and efficient solution method for the TOP compared with the state-of-the-art algorithms.

3	ASi	0	0	0
4	ASe	0	0	0
5	SS	0	0	0
6	ISS	0	0	0
7	PMA	0.000	0.000	0.000
8	PSOMA	0.000	0.000	0.000
9	MA10	0.004	0.009	0.009
10	HISS	0.188	0.188	0.188

Table 12: Adjusted p-value obtained through a Holm and Hochberg statistical test for the compared algorithms

i	Algorithm m	Unadjusted P	pHol m	pHochber g
1	ARC	0	0	0
2	ADC	0	0	0

The average execution CPU times for SS, ISS, and HISS are then compared with the algorithms for datasets 1–7 in Table 5.13. The execution times for some of the algorithms (ARC and ASi) are reported for datasets 4–7.

Table 13: Average execution time for each standard benchmark dataset.

	HISS-SD	ISS	SS	ASe	ADC	ARC	ASi	MA10	PSOMA	PSOiA	PMA
Test Set 1	0.17	0.08	0.149	4.877	5.238	-	-	1.95	0.18	2.15	6.81
Test Set 2	0.00	0.01	0.009	2.776	2.994	-	-	0.24	0.01	0.41	1.43
Test Set 3	0.15	0.08	0.163	5.607	5.923	-	-	2.06	0.49	3.18	9.56
Test Set 4	80.39	69.19	145.782	31.469	32.735	8.6	367.4	182.36	83.89	218.58	109.34
Test Set 5	50.24	46.34	54.974	14.331	15.132	2.9	119.9	35.33	14.72	49.5	22.86
Test Set 6	62.02	58.34	68.402	13.813	14.465	2.1	89.6	39.07	7.59	47.08	26.38
Test Set 7	120.39	111.34	138.521	23.312	24.652	6.3	272.8	112.75	49.09	97.47	54.56
AVG Time	44.77	40.77	58.286	13.741	14.448	4.975	212.425	53.394	22.281	59.767	35.58

6. CONCLUSION

We have proposed a Scatter Search algorithm for Team Orienteering problem. SS follows the general framework of the population-based metaheuristic [31] while it has five prominent components: (1) *A Diversification Generation Method*: SS employ this component to generate good starting solutions to solve the TOP. (2) *An Improvement Method*: The aim of this operation is to explore the neighborhood of the generated solutions by modifying it [21]. In our algorithm, we used a Steepest Descent (SD) heuristic as the improvement method to search for a better quality solution. (3) *A reference set update method*: this component is the heart of a SS procedure, which may result in important modifications during the search process because of its initial composition [21]. In this work, a Queen Bee (QB) strategy is used to build and update the reference set. (4) *Subset generation method (SGM)*: plays an important role in selecting pairs from a reference set for the combination method to generate a new solution [23]. In the present work, greedy select parents (GSP) is employed as selection mechanism to generate a subset of pairs for the combination method. (5) *A solution combination method*: The main goal of this component is to

generate one or more trial solutions from various regions of the solution search space.

We introduced new improved SS called ISS. In the ISS, two main modifications were employed: a selection strategy within the subset generation method (greedy select parents (GSP)) and an updating strategy within the reference set update method (Queen Bee (QB) strategy). The performance of the ISS can be further enhanced because the SS is a population-based algorithm and concentrates more on exploration (diversification) than on exploitation (intensification) [31]. Furthermore, a hybridization of the ISS algorithm with Steepest Descent (SD), namely, HISS-SD was presented for the TOP. Generally, the hybridization in this paper managed to improve the quality of the obtained results by ISS further. The comparison between the proposed algorithms showed that HISS-SD is the best algorithm among the proposed algorithms in terms of yielding the best results. To further investigate the performance of HISS-SD, we compare the HISS-SD with state-of-the-art algorithms. The results were compared in terms of maximum obtained value, relative percentage error, average relative percentage error, and average CPU time. A statistical test was conducted to determine

the algorithm that performed better compared with the others. The results revealed that SS and ISS only outperformed some of the state-of-the-art algorithms. By contrast, HISS-SD outperformed all state-of-the-art algorithms and was comparable to one algorithm.

The proposed algorithms have been tested on TOP benchmark datasets. The proposed algorithms could be tested and validated with respect to other variants of the orienteering problem, such as team orienteering problem with time windows, multi-constraint team orienteering problem with (multiple) time windows, orienteering problem with hotel selection, and orienteering problem with hotel selection and time windows.

In future work, we intend to test the proposed HISS-SD on other combinatorial optimization problems. And the performance of the proposed algorithms could be improved by dynamically changing the parameter of SS during the search process based on objective function value, which generates interesting results.

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