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NEW MODEL OF TURING MACHINES WITH GENETIC ALGORITHMS

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ABSTRACT

In this paper we introduced new model of Turing machine, and represented the Turing Machine in the space of matrices size $\alpha * \lambda$, is called Turing Matrices Space (TMS), denoted by M(α , λ),and construct the formulate which translate the Turing machine to Turing Matrices Space Then the space is called Standard Turing Matrices Space (Standard TMS). In the second phase of the work we using the Turing Machines with crossover operators to accelerated the work of the genetic algorithms then we have provided many of theorems and lemma to use as a mathematical form in this new model of Turing machine

Key-Words: - *Genetic Algorithms, Crossover Method, Turing Machine, Multidimensional Turing Machines*

1. INTRODUCTION

A Turing machine is a mathematical model of computation that we can used a predefined set ofrules, which to determine the result on a strip of tape according to a table of rules .In 1936 the notion of Turing machines appeared introduced by Alan M. Turing [1]. Claude Shannon was studied the problem which called now the discretional complexity of Turing machines in the 1950s [2]. Philipp K. Hooper first studied The immortality problem, by consists from finding an initial infinite configuration on which the Turing machine never halts, whatever the initial state during 1966,[5]. Yurii Rogozhin's introduced in 1982 the smallest Turing machine, nothing changed during the next 10 years. In 1992, only improved universal machine by Rogozhin[6].Stephen Wolfram in 2002 introduced very small weakly universal Turing machines [3]. Pavlotskaya in 2003 proved that a Turing machine instructions has a decidable halting problem even coupled with a finite automaton [4].In 2007, by Wolfram the tape of the Turing machine is initially fixed. Its initial configuration is not exactly periodic, but it is "regular" in the sense that the infinite word written on the tape [2]

2. Multidimensional Turing Machines [7, 8]

Multidimensional Turing machine, the Turing machine tape is viewed as having the ability to extend infinitely in more than one dimension [9]. A two–dimensional machine has a transition defined as 7–tuple (S, Σ, Γ, δ, S_0, B, F) where,

- S is a set of finite states.
- Σ is the set of input symbols (alphabet).
- Γ is the tape alphabet.
- $B \in \Gamma$ The blank symbol.
- δ is The transition function for an t–tape Turing machine can be defined $\delta: S \times \Gamma^t \rightarrow S \times \Gamma^t \times \{L,R,U,D\}^t$ where L ,R ,U, and D are left, right , up, and down respectively which indicates the direction to move the read/write head.
- $S_0 \in S$ is the start state.
- F is the set of final (accepting) states

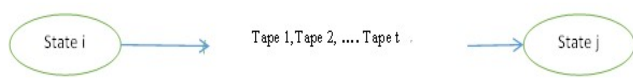

Figure (1)

Definition: HALT

In computability theory, During analysis a Turing Machine halts if there is no leaving arc from the current state for the character read from the tape 1 and the character read from tape 2 or if a 'HALT' state is reached. A sentence is accepted if a machine ends in the 'HALT' state. [10, 11]

3. 2-2: Example

This example about one-domination Turing machine, where t=2 tapes, the direction to move the read/write head is denoted by R (Right), L (Left), and N (Null), S={S0,S1,HALT}, Σ ={a,b}, and Γ={a,b,B}, for example the sentence input is "aabb" placed in the tape one , see figure(3), the head position initial "a" , with each the position of tape two are blank, the current position of the head to two tapes its bold.

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Figure (2)

Table 1 shows the analysis of a sentence ('aabb') for the Turing machine acceptor existing in Figure (3). Initially the sentence 'aabb' is placed on Tape (1) with the head pointing to the initial 'a' in the sentence. Blank symbols are placed to the right and left of the sentence. Tape(2). Since the input characters are 'a' and 'B' from Tape 1 and Tape 2 respectively, the Turing machine follows the change with the corresponding input characters. The change followed is shown in Figure 3. The head moves right on Tape 1. The character 'a' is written to Tape 2 and the head moves right. Table 1 shows each change made, and the contents of the tape after each change. The arrows indicate the characters read from the tapes. The machine ends in the 'HALT' state which involves that the sentence is accepted

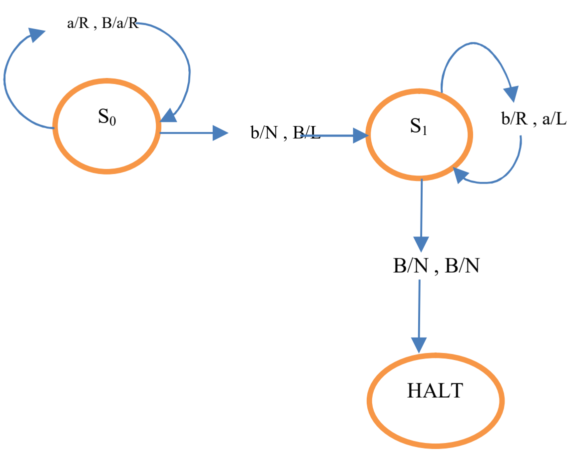

Figure (3)

Table (1)

State	Tape One / Tape Two	Transition State
S0	Tape One: ... B a a b b B Tape Two: .. B ...	S0
S0	Tape One: ... B a a b b B Tape Two: B a B	S0
S0	Tape One: ... B a a b b B Tape Two: B a a B	S1
S1	Tape One: ... B a a b b B Tape Two: B a a B	S1
S1	Tape One: ... B a a b b B Tape Two: B a a B	S1
S1	Tape One: ... B a a b b B Tape Two: B a a B	HALT

2-4 :(N-dimensional) Turing Machines

N-dimensional Turing machine. The Turing machine tape is viewed as having the ability to extend infinitely in more than one dimension [9]. A N–dimensional machine has a transition defined as 7–tuple ($S, \Sigma, \Gamma, \delta, S_0, B, F$) where,

- S is a set of finite states.
- Σ is the set of input symbols (alphabet).
- Γ is the tape alphabet.
- $B \in \Gamma$ The blank symbol.
- δ is The transition function for an t–tape Turing machine can be defined $\delta: S \times \Gamma^t \rightarrow S \times \Gamma^t \times \{d_0, d_1, d_2, \ldots d_{N-1}\}^t$ where $d_0, d_1, d_2, \ldots d_{N-1}$ which indicates the direction to move the read/write head.
- $S_0 \in S$ is the start state.
- F is the set of final (accepting) states.

2-5:Example

Suppose $d_h = h\frac{2\pi}{|D|}$ h=0,1,2 …, $|D|$ -1 for example if h=0,1 then d0= 0 ,direction right (R) and $d_1 = \pi$,direction left (L) if h=0,1,2,3 then d0= 0 ,direction right (R) , $d_1 = \frac{\pi}{2}$,direction up (U)

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, $d_2= \pi$ direction left (L) and $d_3= \frac{3\pi}{2}$,direction down (D)

2-6: Definition

The space of each matrices of size $\alpha * \lambda$, it's representation of Turing Machine, is called ***Turing Matrices Space (TMS)***, denoted by $M(\alpha , \lambda)$,
Where α=|Γ| (|S|-1), λ= t |S|, and t denoted to the number of tapes.

2-7: Example

From the example (2-2), |Γ| =3, |S|=3, and t=2, then from definition (2-6) α=6, and λ=6.
Also from the example (2-3), |Γ| =3, |S|=4, and t=2, then α=9, and λ=8.

2-8: Definition (Label Read/Write/Diction)

The label X/Y/d (Read/Write/Diction) in tape τ from state q_i to q_j , X is represent of the read X∈Γ from tape τ , Y is represent of the write Y∈Γ in tape τ , and d move the head in direction d, we have a transition defined by $\delta(q_i,X)=(q_j,Y,d)$ which replaces X with Y, transitions from q_i to q_j state , and moves the "read head" in direction d (left, right, up, …) to read the next input.

2-9: Definition

Tape set of integer numbers Γ', is define the function ω to convert the elements of tape alphabet symbols Γ to integer numbers from 3 to |Γ|+2.
$\omega: \Gamma \rightarrow \Gamma'$
If Γ={$\alpha_1,\alpha_2,\ldots, \alpha_{|\Gamma|-1}$,B}, and B is a blank element, then $\Gamma' = \{3,4,\ldots,|\Gamma|+2\}$, where $\alpha_i \epsilon \Gamma$ corresponding $\omega(\alpha_i) = (i+2) \epsilon \Gamma'$, and B corresponding to ω (B)=|Γ|+2.

2-10: Example

From example (2-2) and (2-3) , Γ={a, b, B}, then $\Gamma' = \{3,4,5\}$

2-11:Definition: ***Standard Turing Matrices Space(Standard TMS).***

If $A \in M(\alpha , \lambda)$, where A=[$a_{ij}$], and $\delta(k_1,x)=(k_2,u,d)$, for the label x/u/d (read/write/direction) between the states S_{k1}, and S_{k2} , such that

$$k_1 = \left\lfloor \frac{(i-1)}{|\Gamma|} \right\rfloor,$$

$$k_2 = (j-1) \bmod |S| ,$$

Where S is a set of finite states, and Γ is the tape alphabet Γ={B} ∪ Σ, Then when read x∈Γ in τ tape

x=ω^{-1}(y), where y=((i-1) mod | Γ |)+3, with the following

1. a_{ij}=0 if there no transition between the state S_{k1}, and state S_{k2} .
2. a_{ij}=+1 if doesn't write on tape (that mean null (N)), and doesn't move head on tape between state S_{k1}, and state S_{k2}.
3. a_{ij}=+2 if doesn't write on tape, and right move head on tape
4. a_{ij}=-2 if doesn't write on tape, and left move head on tape.
5. $a_{ij} \geq +3$ if to write x=ω^{-1}(a_{ij}) on tape, and right move head on tape where $x \in \Gamma$(Tape integer numbers).
6. $a_{ij} \leq -3$ if to write x=ω^{-1}(-a_{ij}) on tape, and left move head on tape where $x \in \Gamma$ (Tape integer numbers).

Then the space TMS is called Standard Turing Matrices Space (Standard TMS).

2-12:Example

Suppose the graph of Standard TMS with S= {S0, HALT}, Γ= {0,1,B},and t=2, see figure (4)

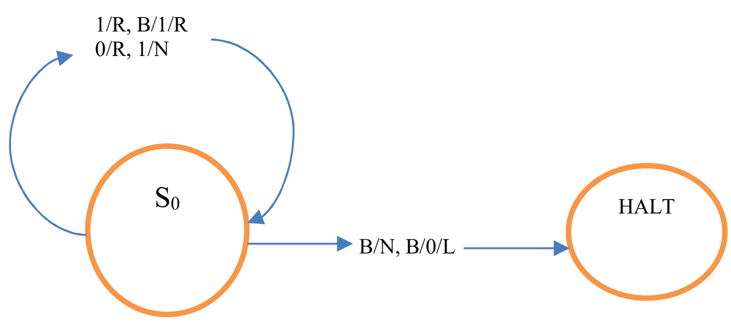

Figure (4)

Then $A \in M(3 , 4)$,

$$A = S0 \begin{array}{c} \\ \begin{cases} 0 \\ 1 \\ B \end{cases} \end{array} \begin{array}{c} \begin{array}{cccc} S0 & HALT & S0 & HALT \end{array} \\ \begin{bmatrix} +2 & 0 & | & 0 & 0 \\ +2 & 0 & | & +1 & 0 \\ 0 & +1 & | & +4 & -3 \end{bmatrix} \end{array}$$

2-13: Definition (6): The rows of a matrix $A \in M(\alpha,\lambda)$, and corresponding to the specific states S_i (state of index i) is called **state rows** denoted by (SR_i).

2-14: Example

In the figure (4) the state rows of index 0 denoted by SR_0, it's corresponding to the state S_0 such that:

$$SR_0=\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & -2 & 0 \end{pmatrix}$$

Where S= {S_0,S_1,HALT}, Σ ={a,b}, and Γ={a,b,B},

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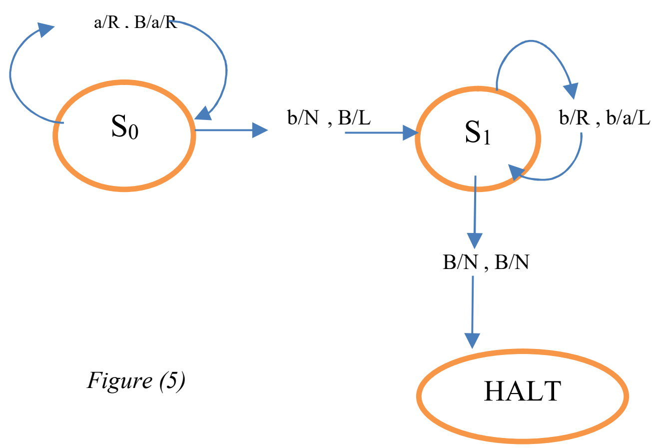

Figure (5)

2-15: Lemma : If $A \in M(\alpha, \lambda)$, represent the space of Multidimensional Turing machine of Standard TMS , then for each transition between S_{k1} and S_{K2} with x/y/z in tape τ , where $1 \le \tau \le t$, and $\tau \in N$, it's correspond to $a_{ij} \in A$, such that

$i = |\Gamma|\, K_1 + (\omega(x) - 2)$ ....(1)

$j = (\tau-1)|S| + (K_2+1)$ ....(2)

where 'x' is the symbol read from the tape τ, and 'y' is the symbol to be written to the tape τ, and 'z' denoted to the movement of the head in the direction z of tape τ.

proof:

Since $K_1 = \left\lfloor \frac{(i-1)}{|\Gamma|} \right\rfloor$ from definition (2-7) and the reminder of $\frac{(i-1)}{|\Gamma|}$ is (i-1) mod $|\Gamma|$, and r= (i-1) mod $|\Gamma|$, then

$K_1 = \frac{(i-1)-r}{|\Gamma|}$

$(i-1) - r = |\Gamma| k_1$

$i = |\Gamma| k_1 + r + 1$

From definition (4), $y = \omega(x) = ((i-1) \bmod |\Gamma|) + 3$,

Then r=y-3

$i = |\Gamma|\, K_1 + (\omega(x) - 2)$.

also from definition (5) , $k2 = (j-1) \bmod |S|$

$j - 1 = q|S| + k2$, $q \in Z$, $q \ge 0$

$j = q|S| + k2 + 1$

the value q is a number of multiple of tapes, because the number of columns in standard Turing matrix is a multiple of tapes ($\lambda = t\,|S|$), in the first tape q=0, in the second tape q=1, then in the τ'th tape $q = \tau - 1$, then

$j = (\tau - 1)|S| + k2 + 1$

$j = (\tau-1)|S| + (K2+1)$ ■

2-16:remark (1) : case $\tau = 1$ then $j = (K_2+1)$ from lemma (2-15)

2-17: Example (7):

Suppose S={ S_0, S_1, S_2,S_3,S_4,S_5,HALT}, Γ={a ,b ,c ,B}, and t=2, then $\alpha = |\Gamma|\,(|S|-1) = 24$, and $\lambda = t\,|S| = 14$.

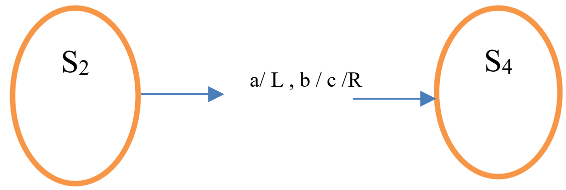

Figure (6)

From above for first tape $\tau=1$: (a/L), and by using lemma (1)

Then $i = |\Gamma|\, K1 + (\omega(x)-2) = 4 * 2 + (\omega(a)-2) = 8 + (3-2) = 9$

$j = (t-1)\,|S| + (K2+1) = (1-1) * 7 + (4+1) = 5$

then from definition(5), $a_{95} = -2$.

2-18: Example

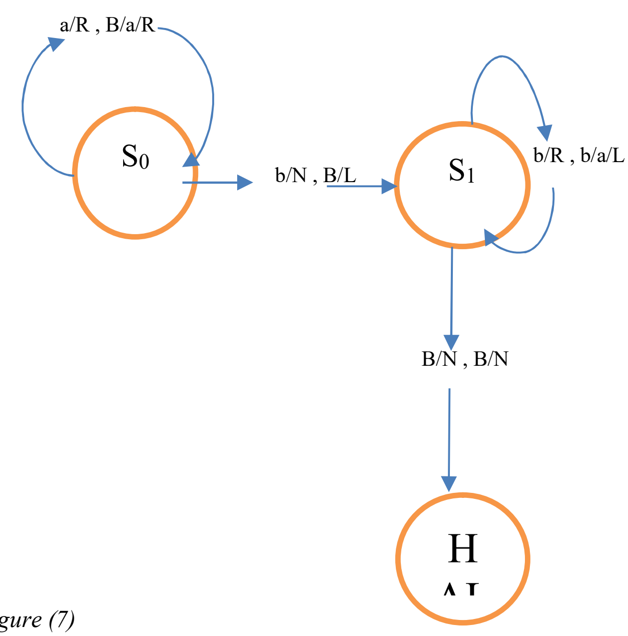

Figure (7)

From figure (7) S= {S_0, S1, HALT}

Γ= {a, b, B}, t=2, $\alpha = |\Gamma|\,(|S|-1) = 3\,(3-1) = 6$, and $\lambda = t\,|S| = 2 \times 3 = 6$,

$\Gamma' = \{3,4,5\}$, If $A \in M(6,6)$, then by definition (2-7)

$$A = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & -3 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

For example if $a_{55} = -3$, i=5, and j=5 then

$k1 = \left\lfloor \frac{(i-1)}{|\Gamma|} \right\rfloor = \left\lfloor \frac{(5-1)}{3} \right\rfloor = 1$,

$K2 = (j-1) \bmod |S| = (5-1) \bmod 3 = 1$

From state S_1 to same state S_1, then

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- The first tape read x , when $x=\omega^{-1}(y)$, and y=(i-1) mod | Γ |)+3,

  y= ((5-1) mod 3) +3=4, then $x=\omega^{-1}(4)$ =b,
- from definition(5) if $a_{55} \leq -3$, the head in the second tape write

$x=\omega^{-1}(-a_{55}) = \omega^{-1}(3)$ =a, and move left in the first tape. That mean is b/a/L.

2-19: Definition (7):(Elect vector)

The vector $X=(x_1,x_2,\ldots,x_{|S|-1})$ of zero-one, and the size | S |-1, where $x_1,x_2,\ldots,x_{|S|-1}$ corresponding to states $S_0,S_1, \ldots, S_{|S|-2}$, $x_1=0$, and the number of ones in X is less than, $\frac{|S|-1}{2}$ called *Elect vector*.

2-20: Definition (8):(The Norm of Elect vector)

The number of ones in the elect vector called norm of Elect vector, denoted by ‖ X‖ , where x is Elect vector, and

$$\|X\| = \sum_{i=1}^{|S|-1} x_i$$

2-21: Remark(2): From definition (2-19) and definition (2-20) the norm of Elect vector is $\|X\| \leq \frac{|S|-1}{2}$

2-22: Definition (9): (Shave Matrix)

The matrix of size α×α, denoted by I(X), where X is Elect vector, then

$$I_{ij} = \begin{cases} 1 & x_k = 1, i = j, (k)|\Gamma| + 1 \leq i \leq (k+1)|\Gamma|, and\ k = 1,2,\ldots,|S|-1 \\ 0 & Otherwise \end{cases}$$

The matrix I(X) called *shave matrix*.

2-23:Example(9):

Let S={S0,S1,S2,H}, Γ={a, b,B}, t=3, and X=(0,0,1), then α=9, λ=12, and k=2 by definition of the Shave Matrix I(X), the nonzero element is $I_{77}= I_{88}= I_{99}=1$, because $7 \leq i \leq 9$, then

= diag((1,1,…,1)-($x_1,x_2,\ldots,x_{|S|-1}$))

= diag(1,1,…,1)- diag($x_1,x_2,\ldots,x_{|S|-1}$)

= I-I(X) ■

2-28:Example(11):

From example (2-23) the complement of shave matrix I(X) is

I(X′)=diag(1,1,1,1,1,1,0,0,0,1,1,1).

3. Genetic Programming Operators.

Genetic operators are used to transform the population of the individuals from one generation to another. Genetic operator consists of three types of operator's mutation, crossover and selection which must work in conjunction with one another in order for the algorithm to be successful. Genetic operators are used to transform the population of the individuals from one generation to another [12].now we use Turing machine with Genetic operators to acceleration the work of Genetic algorithms

3-1: Crossover [13]

Crossover is the process of taking more than one parent solutions and producing a child solution from them. After the selection (reproduction) process, the population is supplemented with better individuals. By recombining operator that proceeds in three steps

1. The selection operator choosing at random a pair of two individual strings for the mating.
2. A cross location is selected at random along the string length.
3. The position values are swapped between the two strings following the Cross location.

3-2:Definition (11) (Transformation Row Echelon)

If $I_\alpha=diag(I_1,I_2,\ldots,I_{|S|-1})$ is Identity matrix of size α, and I_j identity matrix of size |Γ| (where j=1,2,…, |S|-1), and $I_\alpha=(SR_1,SR_2,\ldots SR_{|S|-1})$, where SR_j is |Γ| rows that include I_j, then the definition of transformation row echelon T(X,Y), where ‖X‖=1 with one nonzero element of X at position h, and ‖Y‖=1 with one nonzero element of Y at position k, where

1- if h=k, then $T(X,Y)=I_\alpha$
2- if h≠k, then

$T(X, Y) = (SR_1,\ldots, SR_{k-1}, SR_h, SR_{k+1},\ldots, SR_{h-1}, SR_k, SR_{h+1},\ldots,SR_{|S|-1})$, k<h

or

$T(X, Y) = (SR_1\ldots SR_{h-1}, SR_k, SR_{h+1},\ldots SR_{k-1}, SR_h, SR_{k+1},\ldots,SR_{|S|-1})$, h<k

3-3: Remark (3): From definition (3-2), suppose a=|S|-1, then

$$T_{ij} = \begin{cases} 1 & \begin{cases} i = (h-1)a + r, and\ j = (k-1)a + r, & r = 1,2,\ldots,a \\ or \\ i = (k-1)a + r, and\ j = (h-1)a + r, & r = 1,2,\ldots,a \end{cases} \\ 0 & \begin{cases} i = j = (h-1)a + r, r = 1,2,\ldots,a \\ or \\ i = j = (k-1)a + r, r = 1,2,\ldots,a \end{cases} \\ I_{ij} & Otherwise \end{cases}$$

3-4: Example (12):

Suppose S={ S0,S1,S2 ,H} ,|S|=4 , Γ={a,b.B} ,and |Γ|=3

Let X=(0,1,0) and Y=(0,0.1) ,h= 2 ,k=3 , a=|S|-1=3

$$T(X,Y)=\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

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3-5:Definition(12) : the parent matrices represented of Turing Machine of size $\alpha * \lambda$, it's belong to $M(\alpha , \lambda)$,(Turing Matrices Space (TMS)),

3-6: Definition(13) : the child matrices which introduce from exchanging segments in the parents matrices

3-7: Theorem (1):

Let $A \in M(\alpha 1, \lambda 1)$ and $B \in M(\alpha 2, \lambda 2)$, are Standard TMS, and the parent matrices , if $\alpha = \alpha 1 = \alpha 2$, $\lambda = \lambda 1 = \lambda 2$,X ≠ Y (where X, and Y are Elect vectors for A, and B respectively), and ||X||=||Y||=1,then child matrices C, and D are

C=I (X′) A+ T(X, Y) I(Y) B (3)

and

D= I (Y′) B+ T(Y, X) I(X) A (4)

Proof :

Suppose the Elect vector X from graph T1 corresponding to matrix A where

$X=(0_0,0_1,\ldots.0_{k-1},1_k,0_{k+1},..,0_\alpha)$ and $A=\begin{pmatrix} SR_0^A \\ \vdots \\ SR_\alpha^A \end{pmatrix}$ and

Elect vector Y from graph T2 corresponding to matrix B where $Y=(0_0,0_1,\ldots.\ 0_{h-1},1_h,0_{h+1},..,0_\alpha)$, $B=\begin{pmatrix} SR_0^B \\ \vdots \\ SR_\alpha^B \end{pmatrix}$,

From the left side of (3), then

$$C=\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_{k-1}^A \\ SR_k^B \\ SR_{k+1}^A \\ \vdots \\ SR_\alpha^A \end{pmatrix} \Rightarrow C=\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_{k-1}^A \\ 0 \\ SR_{k+1}^A \\ \vdots \\ SR_\alpha^A \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ SR_k^B \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

$$C=I(X')\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_\alpha^A \end{pmatrix} + T(X,Y)\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ SR_h^B \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$ (because T(X,Y) swap between SR_h^B and SR_k^B)

$$C=I(X')\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_\alpha^A \end{pmatrix} + T(X,Y)\, I(Y)\begin{pmatrix} SR_0^B \\ SR_1^B \\ \vdots \\ SR_\alpha^B \end{pmatrix}$$

C=I(X′) A+ T(X,Y) I(Y)B

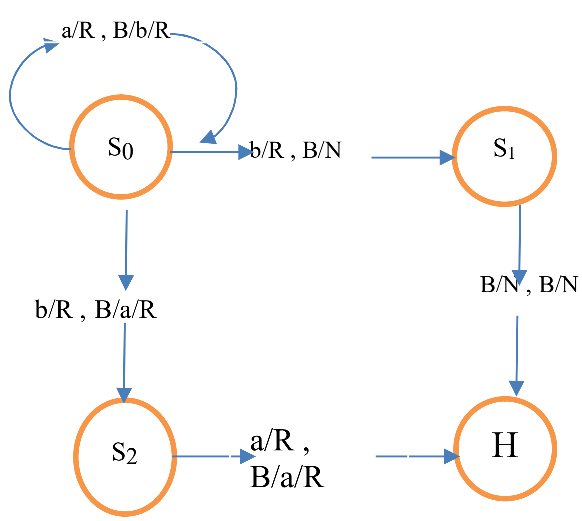
By the same way we can prove D= I(Y′)B+ T(Y,X)(I(X)A ■

3-8: Example (13)

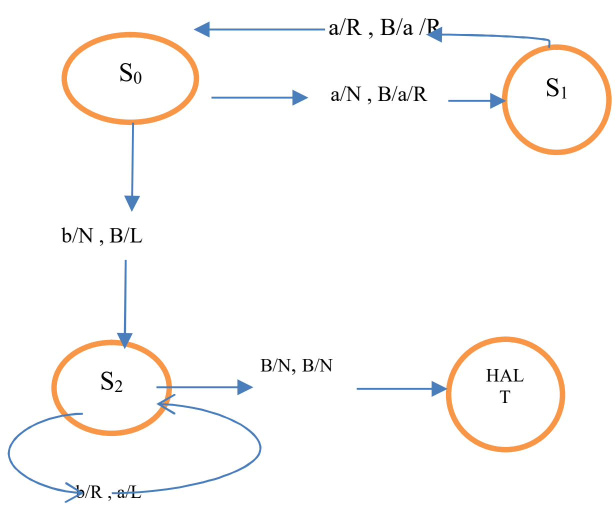
in this example choose state S_2 as Elect vector from graph (p1) and choose state S_1 from graph (p2) as Elect vector and swap him such that insert S_2 to graph (p2), insert S_1 to graph (p1)

$$P1=\begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

$$P2=\begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -2 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$



Graph (p1)



Graph (p2)

By proposition (2-24) , I(X)= diag (0,0,0,0,0,0,1,1,1) , I(Y)= diag (0,0,0,1,1,1,0,0,0) and I(X′)= diag (1,1,1,1,1,1,0,0,0) then C=I(X′) A+ T(X,Y) I(Y)B ,and D= I(Y′)B+ T(Y,X)(I(X)A

$$SR_0^A = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 1 & 3 & 0 \end{pmatrix}$$

$$, SR_1^A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$SR_2^A = \begin{pmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

then $A=\begin{pmatrix} SR_0^A \\ SR_1^A \\ SR_2^A \end{pmatrix}$

$$SR_0^B = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & -2 & 0 \end{pmatrix}$$

$$, SR_1^B = \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$SR_2^B = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

then $B=\begin{pmatrix} SR_0^B \\ SR_1^B \\ SR_2^B \end{pmatrix}$

C=I(X′) A+ T(X,Y) I(Y)B

$$=\mathrm{diag}(1,1,1,1,1,1,0,0,0)\begin{pmatrix} SR_0^A \\ SR_1^A \\ SR_2^A \end{pmatrix} + T(X,Y)$$

$$(\mathrm{diag}\ (0,0,0,1,1,1,0,0,0)) \begin{pmatrix} SR_0^B \\ SR_1^B \\ SR_2^B \end{pmatrix}$$

$$=\begin{pmatrix} SR_0^A \\ SR_1^A \\ 0 \end{pmatrix} + T(X,Y) \begin{pmatrix} 0 \\ SR_2^B \\ 0 \end{pmatrix}$$

$$=\begin{pmatrix} SR_0^A \\ SR_1^A \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ SR_2^B \end{pmatrix} = \begin{pmatrix} SR_0^A \\ SR_1^A \\ SR_2^B \end{pmatrix}$$

$$C== \begin{pmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

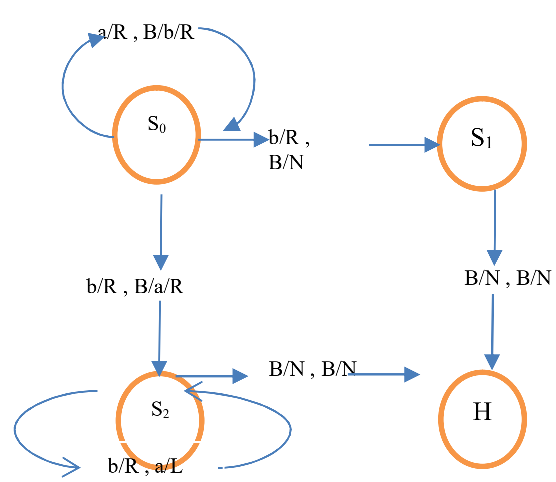


Figure (C)

3-9:lemma (2):

Let $A \in M(\alpha 1, \lambda 1)$ and $B \in M(\alpha 2, \lambda 2)$, are Standard TMS, and parent

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matrices, if $\alpha = \alpha1 = \alpha2$, $\lambda = \lambda1 = \lambda2$, X = Y, and ||X||=||Y||=1,then child matrices are

$C=I(X')A+I(Y)B \quad (5)$

and

$D=I(Y')B+I(X)A \quad (6)$

Proof :

By then definition (3-2) if h=k, then $T(X,Y)=I_\alpha$ and by theorem (3-7)

$C=I(X')A+T(X,Y)I(Y)B$

$D=I(Y')B+T(Y,X)I(X)A$

This impels $C=I(X')A+I(Y)B$

and

$D=I(Y')B+I(X)A$ ■

3-10:Example (14)

From example (3-4) and suppose choose $S_1$from graph T1 and from graph T2,

I(X)= diag (0,0,0,1,1,1,0,0,0) , I(Y)= diag (0,0,0,1,1,1,0,0,0) and I(X′)= diag (1,1,1,0,0,0,1,1,1) and from example (3-8)

$$A=\begin{pmatrix} SR_0^A \\ SR_1^A \\ SR_2^A \end{pmatrix} B=\begin{pmatrix} SR_0^B \\ SR_1^B \\ SR_2^B \end{pmatrix}$$

Then $C=I(X')A+I(Y)B$

$$C= \text{diag}\ (1,1,1,0,0,0,1,1,1)\begin{pmatrix} SR_0^A \\ SR_1^A \\ SR_2^A \end{pmatrix} + \text{diag}\ (0,0,0,1,1,1,0,0,0)\begin{pmatrix} SR_0^B \\ SR_1^B \\ SR_2^B \end{pmatrix}$$

$$=\begin{pmatrix} SR_0^A \\ 0 \\ SR_2^A \end{pmatrix} + \begin{pmatrix} 0 \\ SR_1^B \\ 0 \end{pmatrix} = \begin{pmatrix} SR_0^A \\ SR_1^B \\ SR_2^A \end{pmatrix}$$ by the same way to fined $D=I(Y')B+I(X)A$.

3-11:Definition (12) (Echelon block)

if $X=(0,0,0,\ldots,1_h,0,..0)$ and $Y=(0,0,0,\ldots,1_k,0,..0)$ are elect vectors ,||X||=||Y||=1 then

$T_{block}(X,Y)=(b_1,b_2,\ldots b_r)$ where (r=|S|-1) such that $b_h=k$,$b_k=h$ and h ǂ k

3-12:Remark(4): If h = k then $T_{block}(X,Y)=I$

3-13:Example (15):

Suppose S={ S_0,S_1,S_2,S_3 ,H} ,|S|=5 , Γ={a,b.B} ,and |Γ|=3

Let X=(0,1,0,0)

and Y=(0,0,1,0)

h= 2 ,k=3 , a=|S|-1=4

$T_{block}(X,Y)=(1,3,2,4)$

3-14:Remark: The AND operator is a Boolean operator denoted by (∧) used to implement a logical conjunction on two logical expressions, where the AND operator choose a value of one if both its operands are ones, otherwise the value is zero [9].

3-15:Definition (14) : If X,Y are elect vectors and $\|X\wedge Y\|=0$, $X=X_1+X_2$,$Y=Y_1+Y_2$, $\|X_1\wedge Y_1\|=0$, $\|X_1\wedge Y_1\|=0$, $T_{block}(X,Y)=(c_1,c_2,\ldots c_r)$, $T_{block}(X_1,Y_1)=(a_1,a_2,\ldots a_r)$, $T_{block}(X_2,Y_2)=(b_1,b_2,\ldots b_r)$ Where r= =|S|-1, and $a_i,b_i,c_i=1,2,\ldots,r$, and $1\le i\le r$ then

$$c_i=\begin{cases} i & a_i=b_i=i \\ b_i & i=a_i\,, a_i\neq b_i \\ a_i & i=b_i \quad , a_i\neq b_i \end{cases}$$

$T_{block}(X,Y)=T_{block}(X_1,Y_1)+T_{block}(X_2,Y_2)$

Note: the case a_iǂi and b_iǂ i there are not exist because $\|X\wedge Y\|=0$

3-17:Example (16):

Suppose S={ S_0,S_1,S_2,S_3 ,S_4,S_5 ,H} ,|S|=7 , Γ={a, b, B} ,and |Γ|=3

Let $X_1=(0,0,1,0,0,0)$, $X_2=(0,0,0,0,1,0)$

and $Y_1=(0,1,0,0,0,0)$, $Y_2=(0,0,0,1,0,0)$

$T_{block}(X_1,Y_1)=(1,3,2,4,5,6)$

$T_{block}(X_2,Y_2)=(1,2,3,5,4,6)$

If X=X1+X2, Y=Y1+Y2, then by definition (3-11) then $T_{block}(X,Y)=(1,3,2,5,4,6)$

3-18: Theorem (2):

If $I_\alpha=\text{diag}(I_1,I_2,\ldots,I_{|S|-1})$ is Identity matrix of size α, and I_j identity matrix of size |Γ| (where j=1,2,…, |S|-1), $I_\alpha=(SR_1,SR_2,\ldots SR_{|S|-1})$, where SR_j is |Γ| rows that include I_j, and $\|X\|=\|Y\|>1$,$\|X\wedge Y\|=0$ $X=X_1+X_2+\ldots+X_n$, $Y=Y_1+Y_2+\ldots+Y_n$,(where n=||X||), and $\|X_i\|=\|Y_i\|=1$,

$X_i\neq X_j$, $Y_i\neq Y_j$,where i≠ j, ∀ i, j =1,2,….n

then

$$T_{block}(X,Y)=\sum_{i=1}^{n} T_{block}(X_i,Y_i)$$

proof:

Since X, and Y are elect vectors , such that $\|X\|=\|Y\|>1$and $\|X\wedge Y\|=0$,

Suppose

$X=\sum_{i=1}^{n} X_i$, and $Y=\sum_{i=1}^{n} Y_i$,

where

, $\|X_i\wedge Y_i\|=0$, ∀ i=1,2,….n

Let $T_{block}(X,Y)=(c_1,c_2,\ldots c_r)$, and $T_{block}(X_i,Y_i)=(a_{i1},a_{i2},\ldots a_{ir})$, where r=|S|-1, and i= 1,2,…,|S|-1, see definition (3-15), by induction proof, let the base case n=2, is true see definition

(2-4), now suppose for n=z is true, it's easy proof for n=z+1. ■

3-19:Remark(5): there is no case $\|X\| \neq \|Y\|$

3-20:Lemma (3):

Let $A \in M(\alpha 1,\lambda 1)$ and $B \in M(\alpha 2,\lambda 2)$, are Standard TMS, and parent matrices, if $\alpha=\alpha 1=\alpha 2$, $\lambda=\lambda 1=\lambda 2$, X = Y, and $\|X\|=\|Y\|>1$, then child matrices are

$C=I(X')\,A+T_{block}\,(X,Y)\;I(Y)\;B$ (7)

and

$D=I(Y')B+T_{block}\,(Y,X)\;I(X)A$ (8)

Proof:

Suppose the Elect vector X from graph T1 corresponding to matrix A where $X=\sum_{i=1}^{n} X_i$, and $A=\begin{pmatrix} SR_0^A \\ \vdots \\ SR_\propto^A \end{pmatrix}$ and Elect vector Y from graph T2 corresponding to matrix B where $Y=\sum_{i=1}^{n} Y_i$, $B=\begin{pmatrix} SR_0^B \\ \vdots \\ SR_\propto^B \end{pmatrix}$ then $I(X_i)=diag(0,0,0,1_i,\ldots,0)$, i=1,2,….,n $I(Y_i)=diag(0,0,0,1_i,\ldots,0)$, i=1,2,….,n ,then $X'=\sum_{i=1}^{n}(X_i')$,and $Y'=\sum_{i=1}^{n}(Y_i')$

$$=\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_{k-1}^A \\ SR_h^B \\ SR_{k+1}^A \\ \vdots \\ SR_l^B \\ SR_{k+s}^A \\ \vdots \\ SR_z^B \\ SR_{k+m}^A \\ \vdots \\ SR_\propto^A \end{pmatrix} \Longrightarrow C=\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_{k-1}^A \\ 0 \\ SR_{k+1}^A \\ \vdots \\ 0 \\ SR_{k+s}^A \\ \vdots \\ 0 \\ SR_{k+m}^A \\ \vdots \\ SR_\propto^A \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ \vdots \\ SR_h^B \\ 0 \\ SR_l^B \\ 0 \\ \vdots \\ SR_z^B \\ 0 \end{pmatrix}$$

$$C=\left(\begin{pmatrix} SR_0^A \\ 0 \\ \vdots \\ 0 \\ 0 \\ SR_{k+1}^A \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ SR_\propto^A \end{pmatrix}+\begin{pmatrix} 0 \\ SR_1^A \\ \vdots \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ SR_{k+s}^A \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}+\ldots+\begin{pmatrix} 0 \\ 0 \\ \vdots \\ SR_{k-1}^A \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ \vdots \\ 0 \\ SR_{k+m}^A \\ \vdots \\ 0 \end{pmatrix}\right)+\left(\begin{pmatrix} 0 \\ 0 \\ \vdots \\ SR_h^B \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ SR_l^B \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}+\ldots+\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ SR_z^B \\ 0 \end{pmatrix}\right)$$

$$C=\left(I(X_1')\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_\propto^A \end{pmatrix}+I(X_2')\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_\propto^A \end{pmatrix}+\ldots+I(X_n')\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_\propto^A \end{pmatrix}\right)+\left(T_{block}\,(X_1,Y_1)\begin{pmatrix} 0 \\ 0 \\ \vdots \\ SR_h^B \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}+T_{block}\,(X_2,Y_2)\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ SR_{k+s}^B \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix}\right.$$

$$\left.+\ldots+T_{block}\,(X_n,Y_n)\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ SR_{k+m}^B \\ 0 \end{pmatrix}\right)$$

$$C=(I(X_1')+I(X_2')+\ldots+I(X_n'))\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_\propto^A \end{pmatrix}+T_{block}$$

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$$(X_1, Y_1)I(Y_1)\begin{pmatrix} SR_0^B \\ SR_1^B \\ \vdots \\ SR_\infty^B \end{pmatrix} + T_{block}\,(X_2, Y_2)I(Y_2)\begin{pmatrix} SR_0^B \\ SR_1^B \\ \vdots \\ SR_\infty^B \end{pmatrix}$$

$$+\ldots+ T_{block}\,(X_n, Y_n)I(Y_n)\begin{pmatrix} SR_0^B \\ SR_1^B \\ \vdots \\ SR_\infty^B \end{pmatrix}$$

$$C=(I(X_1')+I(X_2')+\ldots+I(X_n'))\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_\infty^A \end{pmatrix}+$$

$$(T_{block}\,(X_1, Y_1)I(Y_1) + T_{block}\,(X_2, Y_2)I(Y_2)$$

$$+\ldots+ T_{block}\,(X_n, Y_n)I(Y_n))\begin{pmatrix} SR_0^B \\ SR_1^B \\ \vdots \\ SR_\infty^B \end{pmatrix}$$

$$C= \sum_{i=1}^{n} I(X_i')\begin{pmatrix} SR_0^A \\ SR_1^A \\ \vdots \\ SR_\infty^A \end{pmatrix} + \left(\sum_{i=1}^{n} Tblock\,(X_i, Y_i)I(Y_i)\right)\begin{pmatrix} SR_0^B \\ SR_1^B \\ \vdots \\ SR_\infty^B \end{pmatrix}$$

Suppose $[\sum_{i=1}^{n} T_{block}\,(X_i, Y_i)\, I(Y_i)]= T_{block}\,(X,Y)I(Y)$ then

Then $C= I(X')A+ T_{block}\,(X,Y)I(Y)\, B$

By the same way we can prove $D= I(Y')B+ T(Y,X)(I(X)A$ ■

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