Journal of Theoretical and Applied Information Technology
31st December 2019. Vol.97. No 24

ISSN: **1992-8645** www.jatit.org E-ISSN: **1817-3195**

3D OBJECT RECONSTRUCTION FROM 3D POINT CLOUD BY SUPERSHAPES USING PSO

REMMACH HASSNAE[1] , SADGAL MOHAMMED[2]

[1]Department of Computer Science, Computer Systems Engineering Laboratory,
Cadi Ayyad University, Marrakesh, Morocco
E-mail: [1]remmach.hassnae@gmail.com, [2]sadgal@uca.ma

ABSTRACT

In this paper, we apply the PSO method 'Particle Swarm Optimization' to reconstruct a 3d object from a 3d point cloud using supershapes. Reconstructing a 3d object from a 3d point cloud obtained from different devices is very important in many fields. For instance, the use of 3d scanners is very common in the field of medicine. Thus, a good reconstruction of the 3d point cloud given by the device can be very helpful. This problematic can be summed up in finding the surface that approximate the best the point cloud provided at the beginning. The rarity of works applying optimization methods and especially metaheuristics to this kind of issues in the literature makes the originality of this work. We have opted in our work to use a population-based metaheuristic method. The parametric surfaces employed in our work are the recent forms introduced recently by Gielis; called supershapes. We have also used the radial Euclidean distance in the definition of the fitness function. This function will serve as an indicator of dissimilarities between the original form and the reconstructed one. Our approach has been quite successful in providing very satisfactory results compared to the existing results in the literature.

Keywords: *PSO, 3D Reconstruction, Supershapes, Fitness Function, Point Cloud.*

1. INTRODUCTION

Computer vision is a very rich element of artificial intelligence. Its role consists on equipping the machine with the necessary abilities to analyze and exploit the data present in images or video sequences captured by a system of acquisition. There are different techniques employed for acquiring 3d images in the literature, using various acquisition devices such as scanners, stereo cameras ... Acquisition techniques produce crude data modeled as a 3D point cloud. The exploitation of the data present in 3d point cloud remains complex.

Concretely, it is question of finding the 3d structure of the environment perceived by the 3d sensor employed to model it in the form of 3d point cloud. The problematic is then summarized in the three-dimensional reconstruction of a scene or a 3d object from a 3d point cloud. The information obtained by three-dimensional reconstruction is crucial for the three-dimensional perception of the outside world by the machine. Several researchers have studied this issue and have proposed several methods to solve it. In the recent years, one approach stands out from the different methods proposed by researchers to reconstruct a 3d object from a 3d point cloud. The approach in question consists of considering this problematic as an optimization problem. It is then a matter of minimizing carefully a chosen error function until obtaining a final reconstructed object. This object must have a very close resemblance to the reference object.

In our approach, we try to reconstruct a 3d object using parametric surfaces and using a good optimization method. The literature is rich in terms of parametric surfaces. The parametric surfaces that we will use have been introduced very recently by Gielis [1] and are called supershapes. It is an extension of superquadric using rational and irrational symmetries [2]. A supershape has six parameters. By manipulating these parameters, we can obtain a variety of forms. The issue can be then interpreted as follow: adjust the six parameters until finding the shape that perfectly matches the 3d point cloud of the 3d object provided at the beginning. This task will be achieved with an optimization function. There are several different reconstruction algorithms in the literature.

In this paper, 3d reconstruction is perceived as an optimization problem. Optimization methods are classified into two types: deterministic methods and

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

approximate methods [3]. We will look into the approximate methods considering their good performance. Metaheuristics [4] belong to the family of approximate optimization method and are characterized by their iterative generic aspect applicable to problems of different domains. They are also popular for their simplicity of implementation. They stood out from the family of approximate optimization methods by their speed of convergence and their efficiency. Metaheuristic are divided into two groups : single-solution searches and population-based solutions searches. In recent decades, researchers have found their source of inspiration; to define some metaheuristics; in animals that usually live in community. Researchers were inspired by the social behavior of different species in nature to define the principle of several optimization algorithms. Among the various algorithms defined, we quote few methods : the 'Particle Swarm Optimization' PSO method inspired by the birds flocking[5], the 'Ant Colony Optimization' ACO method inspired by the social behavior of the ant colonies[6], the 'Artificial Bee Colony' ABC method inspired by the organization a colony of bees[7], etc.

The work presented in this paper consists of reconstructing a 3D object from a 3D point cloud using a well-adapted metaheuristic to the problem and using also parametric surfaces. For parametric surfaces, our preferred choice would be supershapes for their ease of representation. And concerning the chosen metaheuristic, we opted for the PSO method 'Particle Swarm Optimization' to reconstruct our shape for the following reasons : its good compatibility with continuous values optimization (which is the case of parametric surfaces) and its good performance.

In the next section, we present a literature review of common methods in three-dimensional reconstruction and also existing parametric surfaces used to represent a 3d object. The dilemma of the importance degree given to the choice of the optimization method and the error function will also be discussed. Some works using optimization methods for three-dimensional reconstruction will be cited. Then, we will expose in the following section our proposed approach. We will begin by introducing briefly the concepts used: supershapes and PSO. Next, we will detail the adaptation of the optimization method to our problem and explain the choice of the fitness function. After, we will present the results obtained by our approach and we will compare them with other methods existing in the literature. Finally, we will finish our article with a conclusion on the performance and effectiveness of our method and discuss potential improvements that we can apply to our approach in our future research.

2. LITERATURE REVIEW

To perform the three-dimensional reconstruction, we can use very advanced technologies able to provide 3D reconstruction such as laser scanners which are very efficient but very expensive. For this reason, the researchers were engaged in the implementation of algorithms and approaches applicable to less expensive technologies that are within everyone's reach. A multitude of three-dimensional reconstruction methods are proposed in the literature and vary according to the nature of the input data. The most famous classical method of 3d reconstruction in the literature is the Delaunay triangulation. Several methods have been introduced based on the Delaunay triangulation and the Varonoi diagram, for instance the Crust algorithm proposed by Amenta [8]. The main drawbacks of this sort of algorithms are their vulnerability to the interfering elements, their inefficiency when it comes to reconstruct an object from a large point cloud and also their important execution time. All these disadvantages make impossible the implementation of this kind of algorithm in real-time applications. There is another type of reconstruction algorithms defined in the literature, it is the methods Shape from X. The user can choose the most adequate method to his needs and more specifically to the type of the used image. The Shape From X methods are numerous, here are the most popular and most employed methods:

- "Shape From Shading" (SFS) : this method is based on the information of shadow to reconstruct the object [9].
- "Shape From Motion" (SFM) : this method is based on the information of movement between the object and the camera [10][11], it is used especially in the mobile robotics domain where there is a capital need to reconstruct the scene observed by a robot in its navigation[12].
- "Shape From Silhouette": this method uses different images taken from different views to extract the silhouette of the object in order to reconstruct it.
- "Shape From Focus / Defocus": this method is based respectively on the information of sharpness or the information of optical blur to reconstruct the object[13].
- "Shape From Texture": this method acts on textured objects by exploiting the gradient and

the deformations of the object's textures; caused by three-dimensional projection [14].

The major inconvenience of this family of algorithms is the obligation to integrate external constraints to the object that we want to reconstruct in order to obtain a good reconstruction.

There are several models in the literature for representing 3d objects. The polyhedral surfaces represent the first basic model created in the 90s [15], it has been used and developed by several authors [16] [17]. Subdivision models are also proposed in the literature to represent 3d objects. They are based in their definition on the polyhedral models and proceed to the 3d object representation through subdivision steps and scheme. The subdivision surfaces appeared in 1978 [18], then several works of different researchers were proposed to generalize the use of these surfaces and also to improve their performance in the 3d object representation. Therefore, a multitude of subdivision schemes has been proposed and enriched the literature.

There is also another type of parametric surfaces effective in the 3d object representation, it is superquadrics[19]. The supershapes; being the most recent extension of the superquadrics; will be the surfaces chosen to represent at best our 3d object. It is question of finding the supershape that matches perfectly the 3d point cloud provided as input in order to reconstruct the object in question. In other words, we will have to find the six parameters of the suitable supershape [20], that is, the closest to the shape of the point cloud. The problem will then be directed to the optimization of the supershape's parameters. Therefore, it will be necessary to establish a judicious choice of an objective function to well determine the optimal values of the supershape's parameters. This objective function, called also the error function, will act as an error indicator by checking the dissimilarities and similarities between the object to be reconstructed and the point cloud [21] [22] .

The determination of the importance degree of the objective function definition and the optimization method was and still is a debate for researchers. Some researchers admit that the definition of the objective function is more important for the purpose of having a good rendering, and others believe that the choice of the optimization method is capital and more important to obtain good results. As far as we are concerned, we consider that both have the same degree of importance. If the objective function is poorly determined, then the problem is ill-defined. And if the choice of the optimization method is made incorrectly, then the chances of finding the right solution diminish. Being the first step in solving an optimization problem, does not mean that it is the most important step. All steps must be treated with the same importance degree. Some works in the literature focused either on the good determination of objective function or on the use of the appropriate optimization method. They obtained very good results, but if the same interest was focused on both, it is strongly believed that the results would be better.

On one hand, the majority of the researches dedicated to the determination of the best performant objective function in the representation of superquadrics spotlights the objective functions based on their definition on Euclidean distance. The researchers opted to use the radial Euclidean distance instead of working with the real Euclidean distance in order to simplify the complexity of the computations required and, thus, to minimize the computation time. Various comparative studies of different objective functions have been carried out. We mention for example the works of Gross and Boult [23], the works of Solina and Bajcsy [21] and the works of Van Dop and Regtien [24]. The two functions star providing the best results in the experiments achieved in the literature are : the function of Gross and Boult, and the function of Solina and Bajcsy. According to the study established by Zhang [25] comparing the performance of these two functions under different conditions, it was concluded that the function of Gross and Boult shows better results than the function of Solina and Bajcsy; given the different conditions applied during the experiments.

On the other hand, the most popular optimization method used in the superquadric representations is the deterministic algorithm Levenberg-Marquardt [26]. Given that the supershapes are recent forms, rare are the works employing them. There is an application of a stochastic algorithm on surface reconstruction using supershapes, it is about the genetic algorithm [27]. The results were quite satisfactory and promising. In general, population-based methods [28] have encouraging results in different domains.

The resolution of our problem will then be summarized in the good definition of the objective function as well as the good choice of the optimization process. To our knowledge, the use of population-based metaheuristic methods is not

Journal of Theoretical and Applied Information Technology
31st December 2019. Vol.97. No 24

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

widely used in 3d reconstruction using supershapes. Our contribution will then be to propose a population-based optimization model to reconstruct a 3d object from a given 3d point cloud using supershapes.

3. THE PROPOSED APPROACH

To solve the problematic of 3d object reconstruction by supershapes, we propose to use the PSO (Particle Swarm Optimization) method, which is a population-based metaheuristic. This optimization method will have the role of minimizing the objective function defined smartly in order to reconstruct the 3d object from its 3d point cloud. The reconstructed form is supposed to approximate the shape of the parametric surface. Before detailing our approach, we first define the notion of supershapes and the operation mode of the PSO algorithm. Then we will continue with the explanation of the different steps followed to solve our problem.

3.1. Supershapes

Supershapes, also called Gielis surfaces, are parametric surfaces based on superquadric formulation, introduced in the literature recently by Gielis [2]. The distinctive feature of these new surfaces is the control that we can have on the number of symmetries and, thus, the ability of generating an infinity of forms. Contrary to superquadrics, supershapes are represented by six parameters instead of two. This contributes to the diversification of the shapes formed, and increases the chances of reconstructing successfully various 3d objects. Until today, works using supershapes in image processing remains rare. But these two characteristics of supershapes were the object of our motivation in the use of these new surfaces in the three-dimensional reconstruction:

- Their simplicity in representation and use,
- Their parametric representation which facilitates the approximation of surfaces. The parametric representation of supershapes in 3D is expressed as follows [29] :

$$\begin{pmatrix} x(\theta,\emptyset) \\ y(\theta,\emptyset) \\ z(\theta,\emptyset) \end{pmatrix} = \begin{pmatrix} r_1(\theta) r_2(\emptyset) \cos\theta \cos\emptyset \\ r_1(\theta) r_2(\emptyset) \sin\theta \cos\emptyset \\ r_2(\emptyset) \sin\emptyset \end{pmatrix} \quad (1)$$

With:

x, y and z are the points on the surface of the supershape;

θ: represents the longitude with $-\pi \leq \theta \leq \pi$;

∅: represents the latitude with $-\pi / 2 \leq \emptyset \leq \pi / 2$;

To generate a supershape, we use two generating polygons. Each polygon produces 3 different coefficients of the supershape. To model this mathematically, a supershape is the spherical product of two superpolygons; in other words, it is the spherical product of the rays of two superpolygons represented respectively in polar coordinates as follows:

$$r1(\theta) = \frac{1}{\sqrt[n_1]{\left|\frac{1}{a}\cos\left(\frac{m\theta}{4}\right)\right|^{n_2} + \left|\frac{1}{b}\sin\left(\frac{m\theta}{4}\right)\right|^{n_3}}} \quad (2)$$

$$r2(\emptyset) = \frac{1}{\sqrt[N_1]{\left|\frac{1}{a}\cos\left(\frac{M\emptyset}{4}\right)\right|^{N_2} + \left|\frac{1}{b}\sin\left(\frac{M\emptyset}{4}\right)\right|^{N_3}}} \quad (3)$$

With:

$a, b \in R^+$: These parameters control the dimensions of the polygon;

m, M $\in R_*^+$: These parameters control the number of symmetries;

$n_1, n_2, n_3 \in R$: These are the coefficients of the supershape (same for N1,N2 and N3).

The longitude and the latitude are expressed as follow :

$$\begin{cases} \theta = \theta(x,y) = \arctan\left(\frac{y}{x}\right) \\ \phi = \phi(x,y,z,n_1,n_2,n_3) \\ \quad = \arctan\left(\frac{z r_1(\theta)\sin(\theta)}{y}\right) \\ \quad = \arctan\left(\frac{z r_1(\theta)\cos(\theta)}{x}\right) \end{cases} \quad (4)$$

By adjusting these parameters (refer to Table 1), we produce the corresponding supershapes, as shown in Figure 1:

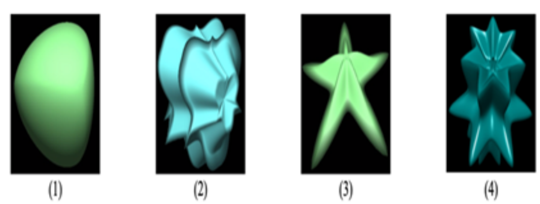


Figure 1:Examples of 3d supershapes

Table 1:Parameters of supershapes represented in Figure 1 (a=b=1)

SUPERSHAPE	m	n₁	n₂	n₃	M	N₁	N₂	N₃
(1)	3	0.5	1.7	1.7	2	10	10	10
(2)	5.7	0..5	1	2.5	10	3	0.2	1
(3)	5	0.1	1.7	1.7	1	0.3	0.5	0.5
(4)	7	0.2	1.7	1.7	7	0.2	1.7	1.7

Journal of Theoretical and Applied Information Technology
31st December 2019. Vol.97. No 24

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

3.2. Particle Swarm Optimization

The Particle Swarm Optimization [30] is an optimization algorithm belonging to the family of metaheuristics population-based solutions, introduced in the literature by Kennedy and Eberhart in 1995. They were inspired in its definition by the social behavior of migratory birds. These birds communicate with each other for two purposes : the first one is to optimize their energy while they move, and the second one is to find the best path to their destination. We qualify this type of metaheuristic as a swarm intelligence method. Over the years, the PSO has been successfully applied to different domains, and has proved its effectiveness in terms of cost and quality of the provided solutions.

The PSO algorithm is inspired by migratory birds by considering each bird as a particle that moves in the search space and transmits information about its position to neighboring particles.

The steps of the basic algorithm are described in the following flowchart:

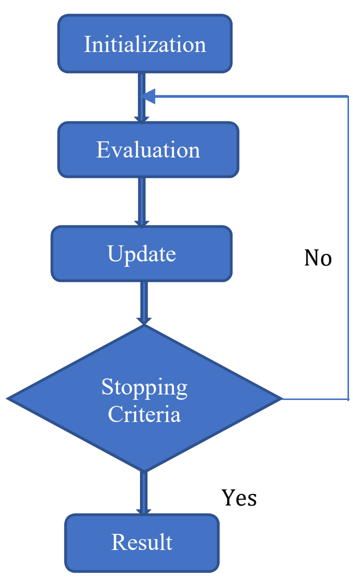


Figure 2:Flowchart of PSO method

Step1 : Initialization

The algorithm starts with the initialization of the swarm that represents the research space, its dimension is n. It initializes also the particles. The particles represent the individuals of the swarm. These are the candidate solutions.

Each particle i is characterized by:

- Its position $\vec{X_i} = (x_{i1}, x_{i2}, \dots, x_{in})$: this vector determines the location of the particle i in the search space;
- Its velocity $\vec{V_i} = (v_{i1}, v_{i2}, \dots, v_{in})$: this vector determines the distance achieved by the particle i from its current position.

The following two values are also initialized when launching the algorithm:

- $\boldsymbol{P_{best}}$: represents the best position of the particle i
  $$P_{best\ i} = (pbest_{i1}, pbest_{i2}, \dots, pbest_{in})$$
- $\boldsymbol{G_{best}}$: represents the best position of the swarm, that is to say the particle having the minimum fitness value of the whole swarm. It is the leader of the research agents of the swarm.
  $$G_{best} = (gbest_1, gbest_2, \dots, gbest_n)$$

Step2 : Evaluation

The swarm evaluation stage consists of evaluating objective function values for all the particles in the swarm in order to determine the best values and the value of the global best (G_{best}).

Step 3 : Update (refer to Figure 3)

At each iteration the following values are updated:

- The position of the particle: The new position of each particle is defined according to the following equation:

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \qquad (5)$$

$$with\ 1 \le j \le n$$

With : $x_{ij}(t)$ representing the position of the particle i at time t and $v_{ij}(t+1)$ representing the velocity of the the particle i at the time t+1; it is calculated at each iteration thanks to the following formula :

$$v_{ij}(t+1) = \omega\, v_{ij}(t) + c_1 r_1 \left[P_{best\ i,j}(t) - x_{ij}(t)\right] + c_2 r_2 \left[G_{best\ i}(t) - x_{ij}(t)\right] \qquad (6)$$

With:
ω: coefficient of inertia defined by the user;
$v_{ij}(t)$: the velocity of the particle i at time t;
c_1 and c_2 : coefficients of acceleration defined by the user;
r_1 and r_2: values chosen randomly from the interval [0,1] at each iteration;
$P_{best\ i,j}(t)$: the best position by which the particle has passed;
$G_{best\ i}(t)$: the best position defined by the swarm;
The first term of the equation represents the current motion; it is also a weighting term that controls the direction of particle's motion; the second term represents the personal influence of the particle, and the third term represents

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

the social influence of the group on the particle.

- The values of the vectors P_{best} and G_{best} :

At each iteration, the comparison of the fitness values of each particle with the values of P_{best} is performed to establish the choice of the best value between the two.

In our case, we try to minimize our objective function, so the best value chosen will be the minimum fitness value. This is modeled as follows:

$$P_{best\ i,j}(t+1) = \begin{cases} x_{ij}(t+1)\ ,if\ f\left(x_{ij}(t+1)\right) < P_{best\ i,j}(t) \\ P_{best\ i,j}(t), otherwise \end{cases} \quad (7)$$

$$G_{best\ i}(t) = \arg min_{P_{best\ i}} f\left(P_{best\ i}(t+1)\right) \quad with \quad 1 \le i \le n \quad (8)$$

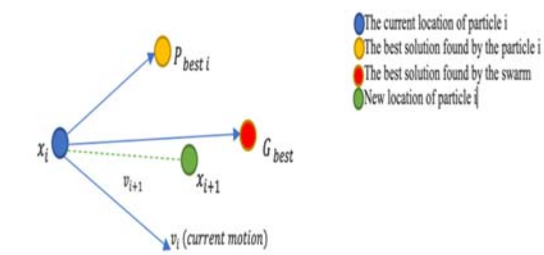


Figure 3:Illustration of Position and Velocity updates

Step 4 : Stopping criteria

These steps are repeated until satisfaction of the defined stopping criteria which can be either the attainment of the convergence or the attainment of a maximum number of iterations. Once the stopping criteria are satisfied, the final result can be displayed.

3.3. The Optimization Problem

In this article, our problematic of reconstructing a 3D object using supershapes from a 3d point cloud is considered as an optimization problem.

To solve an optimization problem, we must first define an objective function to optimize, then we must formulate the constraints or stopping criteria if necessary, and finally we must choose the appropriate optimization algorithm to optimize the defined objective function. All of these factors have a primordial role in the program's good performance.

The definition of the objective function in our case is based on the projection of the 3d point cloud provided initially on the supershape. We seek to find the supershape that perfectly matches the shape of this point cloud; in other words, we seek to find the set of parameters of the supershape which corresponds the most to the parameters of the supershape that we want to reconstruct. Then the objective function represents in our case the distance separating each point of the point cloud from the supershape along the line passing through the point and the center of the supershape (refer to Figure 9). This is called radial Euclidean distance.

The choice of the optimization method strongly depends on the nature of the defined objective function.

We propose to solve our reconstruction problem by using the optimization process illustrated in the following figure:

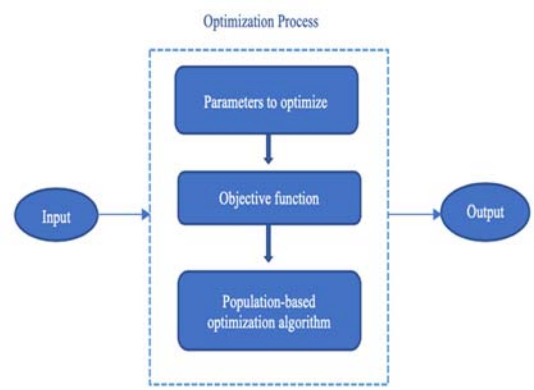


Figure 4:The Optimization Procedure

First of all, we must define all the parameters that we are trying to optimize. In our case this set of parameters is simply the set of parameters defining the supershape. Then as indicated, comes the turn of the definition of the objective function, it will take care of evaluating all the candidate solutions present in the research space. It is necessary to correctly analyze the problem of optimization, and to take into account all the parameters that we will optimize in order to perfect the definition of the objective function. Finally, a good choice of the optimization method is required. We were directed to population-based metaheuristic methods. The optimization process takes as input a 3D point cloud and at the end provides the reconstructed image that best fits with that point cloud. A 3D point cloud can be defined as being the light version of a 3d object, it represents a set of multidimensional points, i.e., represented in a 3D coordinate system: (x, y, z).

Journal of Theoretical and Applied Information Technology
31st December 2019. Vol.97. No 24

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

There are different devices available to obtain 3D points cloud such as 3d scanners, stereo cameras, LiDARs, etc. They can also be generated by a computer software such as meshlab, matlab or afanche 3D.

3D points cloud have been used in several fields in order to simplify the applied treatments, for example: medical imaging, civil engineering, aerial imagery, robotics, etc. Sometimes this goal is not always reached because the 3D points cloud remains voluminous which is consequent for calculation operations. In this case, a reduction of the number of points contained in the points cloud is necessary until a minimum number of points representing the initial shape is reached. This operation is called The re-sampling (refer to Figure 5).

Figure 5:The Re-Sampling Of 3d Point Cloud

3.3.1 Parameters to optimize

The purpose of our approach is to find the supershape which corresponds to the 3d point cloud provided at the beginning. So it's all about optimizing the set of parameters of the supershape in question. Following the definitions given in section (3-a), a supershape depends on 10 parameters : [a, b, m, n1, n2, n3, M, N1, N2, N3].

We can add to the supershape's parameters a set of transformation parameters. Common transformations in different applications are affine transformations: translation, rotation and scaling. There are also other transformations existing in the literature that can be associated with affine transformations and also added to the parameters of the supershape, so as to be able to diversify and increase the number of forms.

We relied on Wyvill's work [31] and studied the following set of warping transformations:

It should be noted that the transformations are applied around the z axis, i.e. only the x and y components change, and for each point P at the coordinates (x, y, z) the warping function is applied (we note it $w(P)$) to obtain the coordinates of point P'.

- Bending :

$$P\left|\begin{matrix} x \\ y \\ z \end{matrix}\right. \Rightarrow P'\left|\begin{matrix} -\sin(\theta) * \left(y - \frac{1}{k}\right) + x_0 \\ cos(\theta) * \left(y - \frac{1}{k}\right) + \frac{1}{k} \\ z \end{matrix}\right. \quad (9)$$

$$\text{with } \begin{cases} \theta = (x - x_0) * k \\ k \text{ is the bending rate} \\ \frac{1}{k} \text{ is the bending axis} \end{cases}$$

Figure 6:Illustration Of Bending Transformation

- Tapering :

$$P\left|\begin{matrix} x \\ y \\ z \end{matrix}\right. \Rightarrow P'\left|\begin{matrix} x * (1 + z * w) \\ y \\ z \end{matrix}\right. \quad (10)$$

with w the warping factor

Figure 7:Illustration Of Tapering Transformation

- Twisting :

$$P\left|\begin{matrix} x \\ y \\ z \end{matrix}\right. \Rightarrow P'\left|\begin{matrix} x * cos(\theta) - y * \sin(\theta) \\ x * \sin(\theta) + y * cos(\theta) \\ z \end{matrix}\right. \quad (11)$$

with $\theta = z * w$

Figure 8:Illustration Of Twisting Transformation

Applying all these transformations to the supershape, its number of parameters will increase and becomes 28 instead of 10 parameters. The parameter vector of the supershape will be expressed as follows:
[a,b,m,n_1, n_2, n_3,M, $N_1, N_2, N_3, T_x, T_y, T_z, R_x, R_y, R_z, S_x, S_y, S_z, g_x, g_y, g_z, t_x, t_y, t_z, p_x, p_y, p_z$] (12) With:

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

T_x, T_y, T_z : the translation factors according the 3 directions (x, y, z);

R_x, R_y, R_z : the rotation factors according the 3 directions (x, y, z);

S_x, S_y, S_z : the scaling factors according the 3 directions (x, y, z);

g_x, g_y, g_z : the tapering factors according the 3 directions (x, y, z);

t_x, t_y, t_z : the twisting factors according the 3 directions (x, y, z);

p_x, p_y, p_z : the bending factors according the 3 directions (x, y, z).

3.3.2 Objective function

The good definition of the objective function will help us find the best possible solution for our optimization problem. To succeed in this task, we must understand very well the problem that we will optimize, and also the role of the objective function.

In our problematic, we seek to approach the 3d point cloud by a supershape in order to successfully establish the three-dimensional reconstruction of this point cloud. It is then question of identifying the supershape that coincides the best with the form of the point cloud. From the parametric representation of supershapes presented in equation (1), the potential function can be determined as follows:

$$F(x,y,z) = 1 - \frac{x^2+y^2+r_1^2(\theta)z^2}{\left(r_2^2(\emptyset)*r_1^2(\theta)\right)} \quad (13)$$

To demonstrate that a point P belonging to the point cloud corresponds to a point on the surface of the supershape, we use the radial Euclidean distance. The distance separating a point P from the surface of the supershape can be defined as follows:

$$d(P) = 1 - \frac{||\overrightarrow{OP}||}{||\overrightarrow{OI}||} \quad (14)$$

O: the center of the supershape;

I: the intersection between the segment $\overrightarrow{OP}$ (passing through the center O and the point P) and the supershape.

This distance is nothing but the potential function defined in the equation (15), it is illustrated in the following figure:

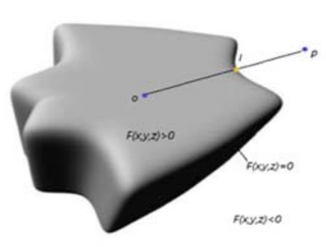


Figure 9:Projection And Position Of A Point With Respect To A Supershape

We can determine the position of any point belonging to the point cloud with respect to the supershape using to the sign of the potential function, we distinguish three cases (refer to Figure 9):

- The point is positioned on the surface, in this case the potential function is null;
- The point is outside the surface, in this case the potential function is negative;
- The point is inside the surface, in this case the potential function is positive.

We define the objective function as follow [22]:

$$Err(V) = \sum_{i=1}^{n} F^2(P_i) \quad (15)$$

So the solution of our problem is then to find the vector of the supershape's parameters which minimizes this error function. To model this mathematically, it is assumed generally that a supershape is expressed as a vector of m parameters: $S = [p_1, p_2, \ldots, p_m]$. The error function that we want to optimize will therefore be the following function:

$$Err(S) = Err(p_1, p_2, \ldots, p_m) \quad (16)$$

4. EXPERIMENTATION

The general structure of de proposed approach is illustrated in the flowchart below :

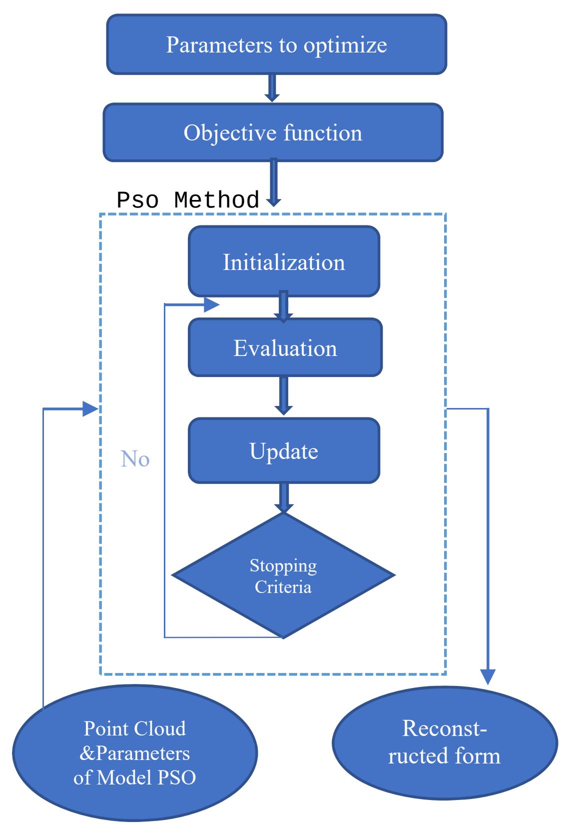


Figure 10:Flowchart of the proposed approach

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

To illustrate our approach, we apply it on known parametric surfaces as supershapes.

In the PSO algorithm, the parameters (ω, c_1, c_2) are defined by the user. The values assigned to these parameters can strongly impact the convergence of the algorithm. For this, many studies have been established to limit the definition interval of these parameters. It was concluded that $\omega \in]-1,1[$ and that $c_1 + c_2 < 4(1+\omega)$. Some authors have selected a different set of values for these parameters displayed in the following table [32]:

Table 2:Examples of the PSO parameters (ω, c1, c2)

	ω	c1	c2
Clerc & Kennedy	0.729	1.494	1.494
Trelea	0.6	1.7	1.7
Carlisle & Dozier	0.729	2.041	0.948
Jiang & Luo & Yang	0.715	1.7	1.7

In our experiment, we chose to test 8 sets of the parameters (ω, c_1, c_2). In addition to the 4 sets presented in the table above, we also work with the following sets:

$$(\omega, c_1, c_2) = (0.4, 0.5, 0.3)$$
$$(\omega, c_1, c_2) = (0.99, 2, 2)$$
$$(\omega, c_1, c_2) = (-0.25, 2{,}5, 0.5)$$
$$(\omega, c_1, c_2) = (-0.3488, -0.2746, 4.8976)$$

We initialize our swarm with a population of 83 particles and fix the maximum number of iterations at 4000.

The lower and upper bounds and the evolution step for each parameter are also fixed. We should be careful while defining the lower and upper bounds because these bounds can easily influence the results positively or negatively; it represents the limits of the variables required by the PSO method. The choice of the bounds must be wide enough containing a satisfactory minimum but not too wide in order to avoid a rapid convergence. The evolution step plays an important role in the search for absolute extrema.

For each supershape, we tried to determine specific values of the lower and the upper bounds. The evolution step value was fixed at : Epsillon=0.05.

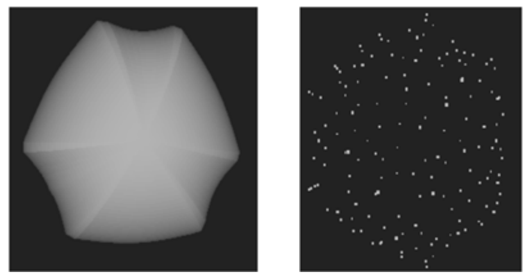
We tested our approach on several supershapes to illustrate its performance. We chose to present the results obtained of the two following supershapes:

4.1. Supershape 1

The parameters of this supershape are :

$$S_1 = (6{,}4{,}1000{,}390{,}390{,}2{,}2{,}2)$$

Once we get the supershape, we generate its point cloud. Then we simplify the complexity of the generated point cloud by re-sampling it. Note that our method is able to reconstruct the 3d object even if the number of points present in the point cloud provided is reduced after the re-sampling.



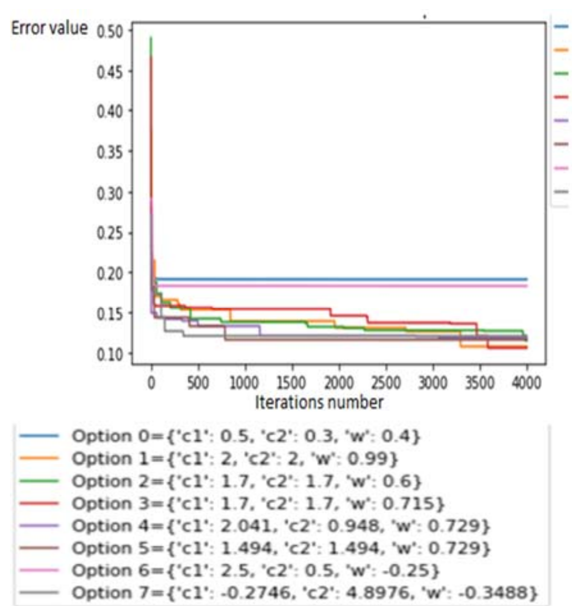
Supershape1 After Re-sampling the point cloud

Figure 11:The re-sampling of S1's point cloud

The lower and upper bounds (LB and UB respectively) are defined as follows :

$$LB = [3{,}9.e^2, 3.e^2, 3{,}1{,}1{,}1{,}5.e^{-1}, 5.e^1]$$
$$UB = [8{,}1100{,}450{,}450{,}5{,}4{,}4{,}4{,}2{,}2]$$

The following graphs illustrate the evolution of the error value along the 4000 iterations and according to the 8 defined PSO parameters.



ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Figure 12 :Graphical representation of the error value depending on the number of iterations

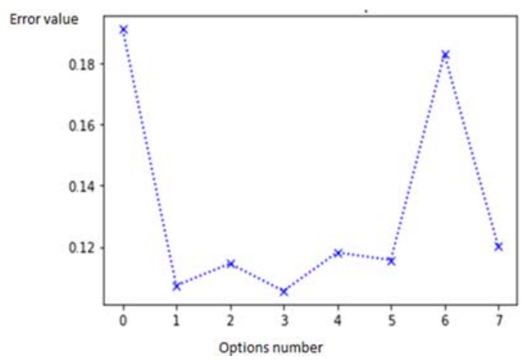
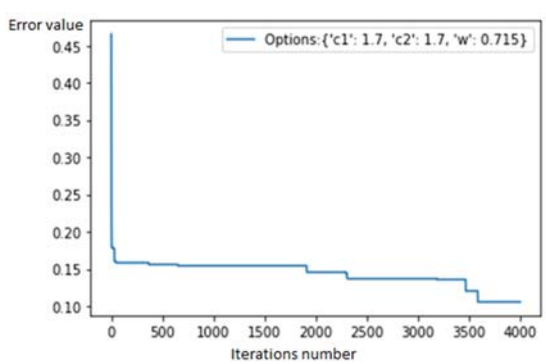


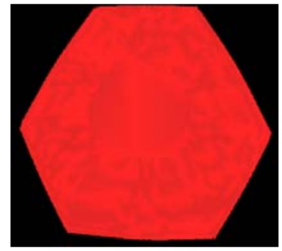
Figure 13:Graphical representation illustrating the best option

As we can see in the figures 12 and 13, the option number 3 (parameters defined by Jiang & Luo & Yang) shows the best performance. This set of parameters helps the algorithm to provide the best error value.

According to the best result obtained in the figure 13, we present the evolution curve of the error according to the number of iterations and the final reconstructed form in the following figure :



a- Curve of the error value according to the number of iterations in the case of option 3



b- The reconstructed form

Figure 14:Results in the case of option 3

The value of the error stabilizes around the iteration 3600.

The parameters of the reconstructed form are :

$$R\_S_1 = (5.027, 3.01, 1098.95, 357.38, 371.14, 2.12, 3.94, 3.71)$$

And the best error value obtained is :

$$Err = 0.10$$

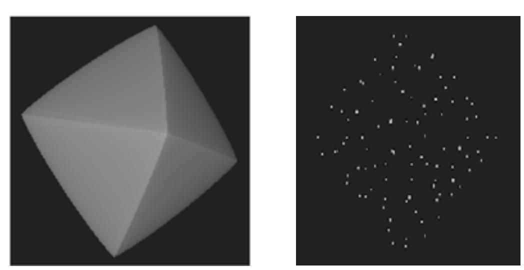
It is noted that the parameters values obtained of the reconstructed form R_S_1 are very close to the supershape's parameters S_1 provided at the beginning. This observation is confirmed in the figure 14-b representing the reconstructed form of the point cloud corresponding to the supershape S_1. The shape obtained matches very well the point cloud and reflects the initial provided shape.

4.2. Supershape 2

The parameters of this supershape are :

$$S_2 = (3, 6, 100, 198, 100, 39, 39, 39)$$

Before applying our approach on the supershape, we proceed first to the re-sampling of the point cloud to simplify all the following operations, and also to demonstrate the effectiveness of our approach with limited number of points.



Supershape2 *After Re-sampling*

Figure 15:The re-sampling of S2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

The lower and upper bounds (LB and UB respectively) are defined as follows :

$$LB = [2,9.e^1, 1.8e^2, 3,9.e^1, 3.e^1, 3.e^1, 1.e^{-2}, 1.e^{-2}]$$

$$UB = [4.5,110,200,200,7,110,60,60,1.3,1.4]$$

The following graphs illustrate the evolution of the error value along the 4000 iterations and according to the 8 defined PSO parameters.

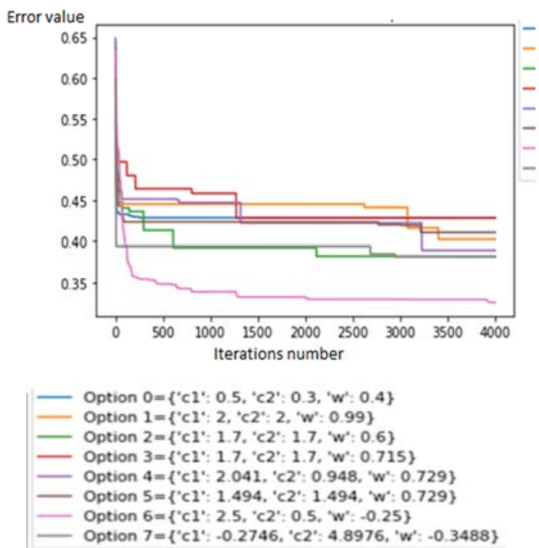


Figure 16:Graphical Representation Of The Error Value Depending On The Number Of Iterations

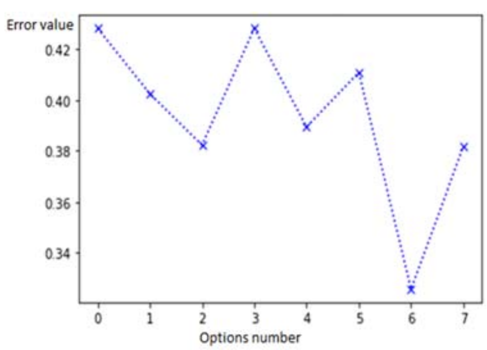
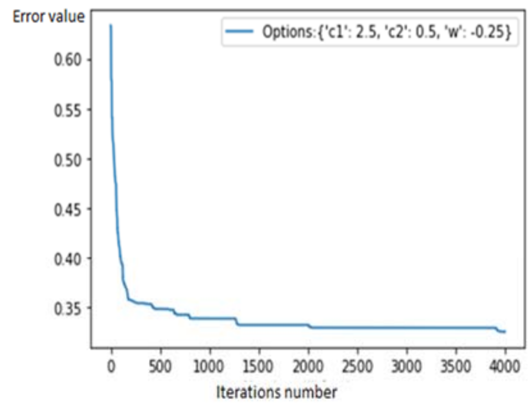


Figure 17:Graphical Representation Illustrating The Best Option

As we can see in the figures 16 and 17, this time the option number 6 shows the best performance.

As we can see here, the fact of changing PSO parameters can influence strongly the quality of the results. We cannot determine the same set of parameters for all the shapes. So, for each shape, a specific set of parameters may prove its effectiveness and present best results.

According to the best result obtained, here is the evolution curve of the error according to the number of iterations and the final reconstructed form. The value of the error stabilizes around the iteration 3900.



a. Curve of the error value according to the number of iterations in the case of option 6

b. The reconstructed form

Figure 18:Results In The Case Of Option 6

The parameters of the reconstructed form are :

$$R\_S_2 = (2.80,6.98,100,189.67,186.89,91.19,58.72,53.52)$$

And the best error value obtained is :

$$\boldsymbol{Err = 0.32}$$

By comparing the values of the reconstructed form R_S_2 with the parameters of the supershape S_2, we note that the algorithm succeeded very well in approaching the first part of the parameters and succeeded less in approaching the second part. Hence the margin of error increases in comparison with the first error value obtained corresponding to the supershape S_1. On the other hand, we obtained a rather good reconstruction of the point cloud as we can see in figure 18-b.

4.3. Discussion

Our approach consists in performing the 3D reconstruction of a 3d object from its point cloud

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

using a population-based method. The goal would be to find the shape that perfectly matches the point cloud provided at the beginning. To illustrate this, we worked with supershapes. We generated the 3d point cloud of the supershapes. After that, we proceeded to the re-sampling of the point cloud. We also tried to make the right choice of the initialization parameters of our algorithm by performing several tests. Since the compromise between the response time and the quality of the solution provided is respected, we can admit that the set of choices made has been judicious.

There are little works involved in three-dimensional reconstruction of a 3d object by supershapes using optimization methods. Fougerolle [29] carried out works on three-dimensional reconstruction of Gielis surfaces using Levenberg-marquardt (LM) algorithm [26]. The error function used by Fougerolle is also based on its definition on the Euclidean radial distance. And there is also another work of the same author employing the genetic algorithm, which is a metaheuristic belonging to the family of evolutionary algorithms [27].

In order to determine the effectiveness of our approach, we will compare our results with existing results in the literature. The following table compares the results obtained by our approach and those obtained by Fourgerolle's approaches. Note that the presented results concern the two supershapes presented previously in this section:

Table 3:Comparative Table

	Error Value		
	LM Algorithm	Genetic Algorithm	**PSO**
Supershape1	3.54	0.24	**0.10**
Supershape2	1.35	0.76	**0.32**

In the case of the two supershapes, there is a considerable difference in the error values. The values obtained by our approach are very small compared to those obtained by Fougerolle's first approach using the LM algorithm which is a deterministic method. Fougerolle's second approach improved the performance of its 3d reconstruction method by using a metaheuristic. The difference between the results of his two methods is glaring. Our approach offers a better reconstruction than the two approaches proposed by Fougerolle given the obtained error values. We can conclude that the fact of using a metaheuristic method is more judicious than using a deterministic method for the resolution of this kind of problematics. And the PSO algorithm has a better performance in the 3d reconstruction by supershapes in comparison with the genetic algorithm.

5. CONCLUSION AND FUTURE WORK

In this paper, we considered the problem of three-dimensional reconstruction of a 3d object as an optimization problem. We choose to employ the recent forms of Gielis in our work to illustrate the 3d object reconstruction. We proposed then a method of reconstruction focused on the use of supershapes and also on the PSO method. We give a lot of interest to the choice of the optimization method and the error function given their importance in the success of the approach.

Our approach has been applied on several models of supershapes. The results obtained showed that our approach was effective enough to solve this problem. The three dimensional reconstruction was successful by using an optimization method and more specifically by employing metaheuristics methods.

The results are very encouraging and leads us to further develop our approach in our future research work. It can be generalized and adapted to more complex shapes. 3d object reconstruction by supershapes can be accomplished by dividing the object in question into supershapes that we can reconstruct easily and regroup them to obtain the 3d object. Transformation parameters such as warping parameters or others can be added to supershapes. There is a wide choice of existing swarm optimization methods in the literature. Other methods can be applied than the PSO, and a comparative study on the performance of each method can be achieved to designate the most efficient method for three-dimensional reconstruction of an object by supershapes from a 3D point cloud. We can also work on the best method selected and adapt its structure to perfect its performance.

REFRENCES:

[1] J. Gielis, 2003, "A generic geometric transformation that unifies a wide range of natural and abstract shapes," Am. J. Bot.

[2] J. Gielis, B. Beirinckx, and E. Bastiaens, 2003, "Superquadrics with rational and irrational symmetry," Proc. eighth ACM Symp. Solid Model. Appl. - SM ’03, no. January, p.262.

[3]X. Yang, 2010, "Engineering Optimization, Hoboken," NJ, USA:John Wiley & Sons, Inc.

[4] P. S. et E. T. J. Dréo, A. Pétrowski, 2003, "Métaheuristiques pour l’optimisation difficile," Eyrolles.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

[5] Y. Shi and R. C. Eberhart, 1998, "Parameter selection in particle swarm optimization," pp. 591–600.

[6] A. Colorni, M. Dorigo, and V. Maniezzo, 1992, "An investigation of some properties of an ‚Ant algorithm," Ppsn 92.

[7] B. Basturk, D.Karaboga, May 12-14, 2006, "An Artificial Bee Colony (ABC) Algorithm for Numeric function Optimization," IEEE Swarm Intelligence Symposium 2006, Indianapolis, Indiana, USA.

[8] N. Amenta, 1999, "The CRust Algorithm for 3D Surface Reconstruction," in ACM Symposium on Computational Geometry.

[9] E. Prados and O. Faugeras, 2006, "Shape from shading," in Handbook of Mathematical Models in Computer Vision.

[10] M. C. Bakkay, March 2016, 2018, "Reconstruction Volumétrique dans des Environnements Multi-caméras".

[11] N. G. J. M. T. Rodriguez P. Sturm, 2003, "VISIRE : Photorealistic 3D reconstruction from video sequences," in IEEE International Conference on Image Processing.

[12] T. Bailey, 2002, "Mobile Robot Localisation and Mapping in Extensive Outdoor Environments," Philosophy.

[13] P. Grossmann, 1987, "Depth from focus," Pattern Recognit. Lett.

[14] J. Aloimonos, 1988, "Shape from texture," Biol. Cybern.

[15] Boissonnat, J.-D. and Yvinec, M., 1995, "Géométrie algorithmique. Ediscience international," Paris.

[16] Greca, R. L. , 2005, " Approche déclarative de la modélisation de surfaces," PhD thesis, Université de la Méditerranée.

[17] Botsch, M., Pauly, M., Kobbelt, L., Alliez, P., Levy, B., Bischoff, S., and Rössl, C., 2008, "Geometric modeling based on polygonal meshes," Eurographics Course Notes.

[18] Catmull, E. and Clark, J., 1978, "Recursively generated B-spline surfaces on arbitrary topological meshes," Computer-Aided Design, pages 350–355.

[19]A. H. Barr, 1981, "Superquadrics and Angle-Preserving Transformations," in IEEE Computer Graphics and Applications.

[20] L. Chevalier, F. Jaillet, and A. Baskurt, 2003, "Segmentation and superquadric modeling of 3D objects," UNION Agency--Science Press.

[21] F. Solina and R. Bajcsy, 1990, "Recovery of parametric models from range images: The case for superquadrics with global deformations," The Visual Computer 12(2), pp. 131–1476.

[22] A. Jaklic, A. Leonardis, and F. Solina, 2000, "Segmentation and recovery of superquadrics," Kluwer AcademicPublisher, Dordrecht.

[23] A. Gross and T. Boult, 1988, "Error of fit measures for recovering parametric solids," in Proc. Internat. Conf.Computer Vision, pp. 690–694.

[24] Van Dop, E., Regitien, P.L., 1998, "Fitting undeformed super- quadrics to range data: Improving model recovery and classification," In: Internat. Conf. Computer Vision and Pattern Recognition, Vol. 2, pp. 396–401.

[25] Y. Zhang, 2003, "Experimental comparison of superquadric fitting objective functions," Pattern Recognition Let-ter 24, pp. 2185–2193.

[26] 4. Y. D. Fougerolle, A. Gribok, S. Foufou, F. Truchetet, and M. A. Abidi, October 2005, "Implicit surface modeling using supershapes and R-functions," in Proc. Pacific Graphics'05, (Macao, China), pp. 169–172.

[27] Y. Bokhabrine, Y. D. Fougerolle, S. Foufou, and F.Truchetet, 2007, "Genetic algorithms for gielis surface recovery from 3D data sets," in Proceedings -International Conference on Image Processing, ICIP.

[28] N. El and H. Contribution, 2012, "Contribution aux méthodes hybrides d'optimisation heuristique : Distribution et application à l'interopérabilité des systèmes d'information Norelislam El Hami".

[29] Y. D. Fougerolle, A. Gribok, S. Foufou, E. Truchetet, and M. A. Abidi, 2006, "Supershape recovery from 3D data sets," in Proceedings - International Conference on Image Processing, ICIP.

[30] J. Kennedy and R. Eberhart, 1995, "Particle Sawrm Optimization," in Neural Networks Proceedings., IEEE International Conference.

[31] B. Wyvill, A. Guy, and E. Galin, 1999, "Extending the CSG tree. Warping, blending and Boolean operations in an implicit surface modeling system," Comput. Graph. Forum, vol. 18, no. 2, pp. 149–158.

[32] C. Ströÿner , 2014, "Particle Swarm Optimization & Parameter Selection," pp 7.