Journal of Theoretical and Applied Information Technology
15th November 2019. Vol.97. No 22

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

INFORMATION TECHNOLOGY FOR NUMERICAL SIMULATION OF CONVECTIVE FLOWS OF A VISCOUS INCOMPRESSIBLE FLUID IN CURVILINEAR MULTIPLY CONNECTED DOMAINS

[1]NURLAN TEMIRBEKOV, [2]YERZHAN MALGAZHDAROV, [3]SAYA TOKANOVA, [3]FARIDA AMENOVA, [2]DOSSAN BAIGEREYEV, [2]AMANKELDY TURAROV

[1]Kazakhstan Engineering Technological University, Almaty, Kazakhstan

[2]D. Serikbayev East Kazakhstan State Technical University, Ust-Kamenogorsk, Kazakhstan

[3]S. Amanzholov East Kazakhstan State University, Ust-Kamenogorsk, Kazakhstan

E-mail: telec_saya@mail.ru

ABSTRACT

In this paper we describe a method for the numerical construction of curvilinear structured grids in doubly connected regions and numerical modeling of the convective flow of a non-uniformly heated liquid in a curvilinear coordinate system. The study is absolutely unique and conducted in accordance with modern scientific demands. Based on previous surveys and the latest findings in the study area, it brings the acute question of information technology for the numerical simulation of convective flows of a viscous incompressible fluid in curvilinear multiply connected domains to a significantly new level. The study is complex and attempts to analyze the theme thoroughly, taking into account all factors that may influence the final results. The paper presents a complete required set of multiple graphs, detailed equations and schemes in order to increase visualization of obtained results on a viscous incompressible fluid in curvilinear multiply connected domains and simplify the perception of the results for accurate scientific conclusions and further applied usage. In the numerical construction of curvilinear grids in doubly-connected domains, the implicit scheme and the method of fractional steps are used by the equidistribution method and Godunov-Thompson, and in the numerical realization of the equations of an incompressible fluid, an explicit scheme and a method of fractional steps are used. In the direction of the outer and inner boundaries, a cyclic run is used, and in the direction of the normal, a scalar run is used. Calculations were carried out for different cavity configurations, temperature regimes at the boundary. The graphs of numerical calculations of the temperature and current function are obtained. All this makes the current study an important contribution to the development of theoretical concepts and methodological approaches to the use of new information technologies in hydrodynamic studies that takes into account the specific features of the subject area, as well as the development, adaptation and approbation of tools in the process of modeling of natural and technogenic objects.

Keywords: *Computer Technology, Mathematical Modeling, Curvilinear Structured Grids, Doubly-Connected, Curvilinear Boundary*

1. INTRODUCTION

With the rapid development of computer technology, mathematical modeling of physical, chemical processes and mechanical systems in various branches of science is intensively developing. In recent years, it has become increasingly necessary to solve problems in complex regions with complex geometry and in zones of rapid changes in the characteristics of the physical medium (density, pressure). For modeling in complex areas, first of all, the physical area is to be discretized, that is, the stage of modeling physical geometries using a set of cells of difference grids. It should be noted that the use of non-uniform grids can cause non-physical sources of mass and momentum to appear in the calculation schemes, as well as the loss of important properties inherent in the approximated differential equations. The model equations recorded in curvilinear coordinates have a more complex form than the original equations, in particular, they contain

Journal of Theoretical and Applied Information Technology
15th November 2019. Vol.97. No 22

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

variable coefficients, additional terms, nonzero right-hand sides, and so on. Therefore, the question of approximation of equations on curvilinear grids is urgent and requires close attention. In addition, the diverse requirements imposed on difference grids make the construction of curvilinear grids a complex mathematical problem. In this regard, the development of theoretical concepts and methodological approaches to the use of advanced information technologies in hydrodynamic studies that take into account the specific features of the subject area, the development, adaptation of tools and approbation of them in the process of modeling of natural and technogenic objects of significant economic importance are very relevant.

Modern research in the field of computational and applied mathematics is aimed at the creation of automated computer programs for the construction of curvilinear adaptive structural and nonstructural difference grids, as well as the modernization of numerical algorithms for solving applied problems. A fundamental study on the justification, numerical realization of the construction of curvilinear adaptive grids, the problem of hydrodynamics, and also some experimental results were published in the works of N.T. Danaeva, Yu.I. Shokin, G.S. Khakimzyanova, N.M. Temirbekova, V.D. Liseikina, J.F. Thompson, Z.U.A. Warsi, C.W. Mastin, etc.

In this paper we consider the problem of constructing curvilinear grids on an arbitrary curvilinear boundary and inside a domain by a differential method. In these methods, differential equations of partial derivatives of various types are used, but differential methods for constructing grids based on solving equations of elliptic type are most widely used. A mathematical problem is also considered with respect to the vorticity variables ω, the stream function ψ, and the temperature θ describing the convective flow of a nonuniformly heated viscous fluid in an arbitrary doubly-connected D domain with a curvilinear boundary $\partial D = \Gamma_0 \cup \Gamma_1$ (Figure 1).

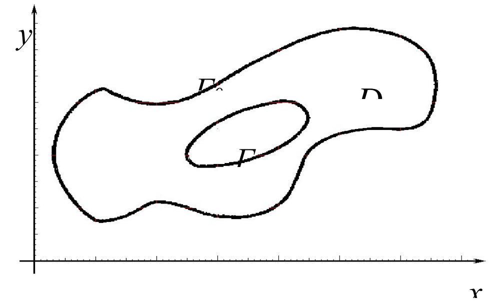


Figure 1: A Doubly Connected Physical Domain

2. FORMULATION OF THE PROBLEM

Methods for constructing curvilinear grids are considered. Differential methods for constructing curvilinear grids are used in this paper, since the physical domain under consideration is complex and has curvilinear boundaries. In these methods, partial differential equations of various types are used, but differential methods for constructing grids based on solving equations of elliptic type are most widely used.

The mapping of the physical region in the coordinate system (x, y) to the computational domain in the coordinate system (ξ,η) is performed by the method of cutting the region [1] (Figure 2). The curve of the outer boundary 1 is mapped onto the line $\eta = 0,\ 0 \le \xi \le 1$, and the inner boundary 3 is mapped onto the line $\eta = 1,\ 0 \le \xi \le 1$. The cut along the cutting line is made twice, the boundary 2 is mapped onto the line $\xi = 1,\ 0 \le \eta \le 1$, and the cut line 4 is mapped onto the line $\xi = 0,\ 0 \le \eta \le 1$.

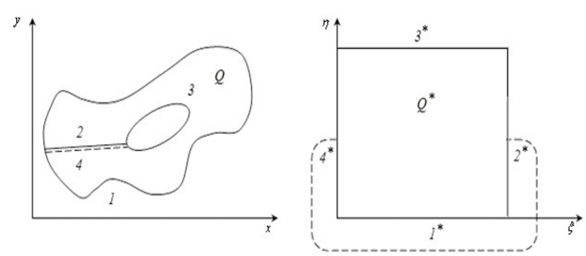


*Figure 2: The Mapping of A Doubly-Connected Curvilinear Domain Q To A Calculated Rectangle Q**

3. COMPUTATIONAL ALGORITHM

The construction of a grid in a two-dimensional domain begins with the construction of a grid on its boundary. Since the boundary of the domain is not monotonous, we describe the boundary with the help of equations in a given parametric form

Journal of Theoretical and Applied Information Technology
15th November 2019. Vol.97. No 22

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$x = f^1(p), \quad y = f^2(p), \quad 0 \le p \le l, \quad (1)$$

where l - is the length of the boundary.

To construct a grid on the boundaries, we use the one-dimensional equidistribution method, i.e. differential equation of the following form [1]:

$$\frac{\partial}{\partial \xi}\left(\vartheta(p)\frac{\partial p}{\partial \xi}\right) = 0, \xi \in (0,1) \quad (2)$$

$p(0) = 0, \quad p(1) = l$, where

$$\vartheta(p) = \sqrt{\left(\frac{\partial f^1(p)}{\partial p}\right)^2 + \left(\frac{\partial f^2(p)}{\partial p}\right)^2} > 0, \quad p \in [0,l].$$

The coordinates of the grid nodes at the boundaries are calculated by formula (1) using the values of found p. The equations of the equidistribution method are used to construct two-dimensional grids with the assumption of orthogonality of the required coordinate system [1]:

$$\frac{\partial}{\partial \xi}\left(g_{22}\frac{\partial \vec{x}}{\partial \xi}\right) + \frac{\partial}{\partial \eta}\left(g_{11}\frac{\partial \vec{x}}{\partial \eta}\right) = 0 \quad (3)$$

where $\vec{x} = (x, y)$ - is physical coordinates, $g_{11} = x_\xi^2 + y_\xi^2, \quad g_{22} = x_\eta^2 + y_\eta^2$ - are the components of the metric tensor.

The problem (1)-(3) for the construction of a grid on the boundary of the domain will be solved by a finite difference method. The finite difference scheme (2) has the following form:

$$\frac{1}{h_1}\left(\vartheta_{i+1/2}\frac{p_{i+1} - p_i}{h_1} + \vartheta_{i-1/2}\frac{p_i - p_{i-1}}{h_1}\right) = 0;$$

$$p_1 = 0, \quad p_{n_1} = l; i = 2,..,n_1 - 1 \quad (4)$$

where

$$\vartheta_{i+1/2} = \sqrt{\left(\frac{f^1(p_{i+1}) - f^1(p_i)}{p_{i+1} - p_i}\right)^2 + \left(\frac{f^2(p_{i+1}) - f^2(p_i)}{p_{i+1} - p_i}\right)^2}$$

If the boundary of the region is given as set of points $A_k(x_k, y_k)(k = 1,..,M)$, $(x_k, y_k) \in \Gamma_l \; (l = 1,2)$, then the length is defined as follows:

$$l_1 = 0; l_k = \sum_{i=2}^{k}\sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2};$$

$k = 2,..,M$

The parametric equation for determining the coordinates of nodes on the boundary for linear interpolation and $p_i \in [l_k, l_{k+1}]$ has the following form:

$$f^1(p_i) = x_k + \frac{x_{k+1} - x_k}{l_{k+1} - l_k}(p_i - l_k)$$

$$f^2(p_i) = y_k + \frac{y_{k+1} - y_k}{l_{k+1} - l_k}(p_i - l_k) \quad (5)$$

The resulting finite difference problem (4) is solved by an iterative method of successive approximations. A uniform grid on the interval $[0,l]$ is chosen as an initial approximation p_i^0. Let the grid p_i^n be built on the n-th iteration. Let us define

$$\vartheta_{i+1/2}^n = \sqrt{\left(\frac{f^1(p_{i+1}^n) - f^1(p_i^n)}{p_{i+1}^n - p_i^n}\right)^2 + \left(\frac{f^2(p_{i+1}^n) - f^2(p_i^n)}{p_{i+1}^n - p_i^n}\right)^2}$$

on the grid. The following successive approximation is found using them. For this, the following linear problem is solved

$$\frac{1}{h_1}\left(\vartheta_{i+1/2}^n\frac{p_{i+1}^{n+1} - p_i^{n+1}}{h_1} + \vartheta_{i-1/2}^n\frac{p_i^{n+1} - p_{i-1}^{n+1}}{h_1}\right) = 0$$

(6)

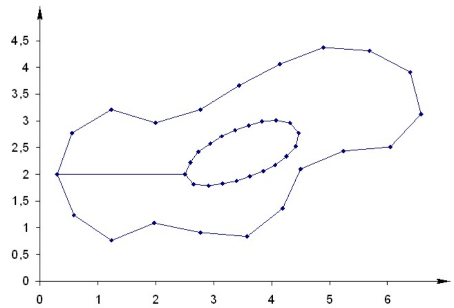
where $p_1^{n+1} = 0, \quad p_{n_1}^{n+1} = l; \; i = 2,..,n_1 - 1$.

The iterative process continues to the specified accuracy, that is, until the following condition is fulfilled:

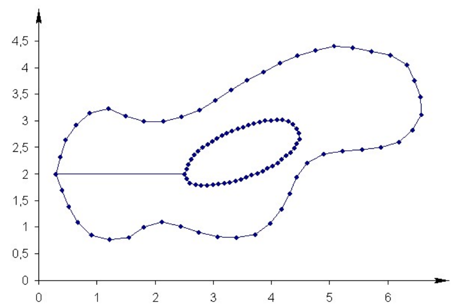
$$\max_{1 \le i \le n_1}\left|p_i^{n+1} - p_i^n\right| \le \varepsilon$$

The coordinates of the nodes on the boundary of the physical region are calculated using (5) based on the results of the last iteration approximation. Figures 3 show the results of the solution to the difference problem (6) and (5), where 20, 50 and 100 grid nodes are uniformly distributed at the boundaries.

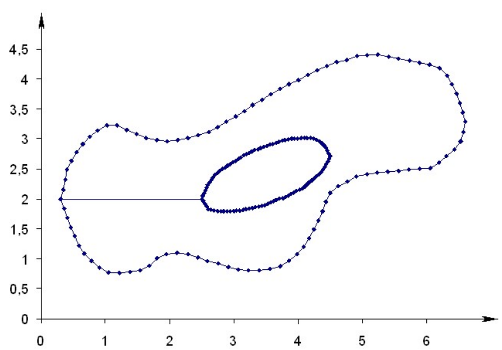
ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195



a) Evenly Distributed Mesh Nodes (20 Nodes)



b) Evenly Distributed Mesh Nodes (50 Nodes)



C) Evenly Distributed Mesh Nodes (100 Knots)

Figure 3. Uniformly Distributed Mesh Nodes

Let us write out the difference problem for determining the coordinates inside the domain. The finite difference scheme (3) has the following form:

$$\Lambda_{11}\vec{x}_{i,j} + \Lambda_{22}\vec{x}_{i,j} = 0, \qquad (7)$$

where

$$\Lambda_{11}\vec{x}_{i,j} = \frac{1}{h_1}\left(g_{22,i+1/2,j}\frac{\vec{x}_{i+1,j}-\vec{x}_{i,j}}{h_1} - g_{22,i-1/2,j}\frac{\vec{x}_{i,j}-\vec{x}_{i-1,j}}{h_1}\right),$$

$$\Lambda_{22}\vec{x}_{i,j} = \frac{1}{h_2}\left(g_{11,i,j+1/2}\frac{\vec{x}_{i,j+1}-\vec{x}_{i,j}}{h_2} - g_{11,i,j-1/2}\frac{\vec{x}_{i,j}-\vec{x}_{i,j-1}}{h_2}\right)$$

To determine the components of the metric tensor, the central differences in the integer nodes

$$x_{\xi,i,j} = \frac{x_{i+1,j}-x_{i-1,j}}{2h_1}, \quad x_{\eta,i,j} = \frac{x_{i,j+1}-x_{i,j-1}}{2h_2},$$

$$y_{\xi,i,j} = \frac{y_{i+1,j}-y_{i-1,j}}{2h_1}, \quad y_{\eta,i,j} = \frac{y_{i,j+1}-y_{i,j-1}}{2h_2},$$

$$g_{11,i,j} = x^2_{\xi,i,j} + y^2_{\xi,i,j}, \quad g_{22,i,j} = x^2_{\eta,i,j} + y^2_{\eta,i,j},$$

and in the center of the faces the cells are determined by averaging in the following way:

$$g_{11,i+1/2,j} = \frac{g_{11,i+1,j}+g_{11,i,j}}{2}, \quad g_{11,i-1/2,j} = \frac{g_{11,i,j}+g_{11,i-1,j}}{2}.$$

The remaining coefficients were determined similarly. The alternating directions method is used to find the numerical solution of (6). Let us consider the algorithm of the alternating directions method:

$$\frac{\vec{x}^{n+1/2}_{i,j}-\vec{x}^{n}_{i,j}}{0.5\tau} = \Lambda^n_{11}\vec{x}^{n+1/2}_{i,j} + \Lambda^n_{22}\vec{x}^{n}_{i,j} \qquad (8)$$

$$\frac{\vec{x}^{n+1}_{i,j}-\vec{x}^{n+1/2}_{i,j}}{0.5\tau} = \Lambda^n_{11}\vec{x}^{n+1/2}_{i,j} + \Lambda^n_{22}\vec{x}^{n+1}_{i,j} \qquad (9)$$

Here, n is the number of the iteration, τ is the iteration parameter. Since the components of the metric tensor depend on the solution, the coefficients g_{11}, g_{22} are calculated with the help of the n-th iteration solution. Since the domain under consideration is doubly connected and the grid nodes must coincide on the cut line (see Figure 2), then in the ξ direction we need to apply the cyclic sweep method [5], with periodic conditions:

$$A_{i+n_1} = A_i, B_{i+n_1} = B_i, C_{i+n_1} = C_i, F_{i+n_1} = F_i,$$
$$i = 1,..,n_1 - 1 \qquad (10)$$

If conditions (10) are satisfied, the solution of equations (8) is also periodic with period $n_1 - 1$, i.e.

$$\vec{x}_i = \vec{x}_{i+n_1-1}.$$

Journal of Theoretical and Applied Information Technology
15th November 2019. Vol.97. No 22

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

Therefore, it is sufficient to find a solution $\vec{x}_i$, $i=\overline{1,n_1-1}$. In this case, equation (8) with periodic conditions can be written as follows:

$$\begin{cases} -A_1\vec{x}_{n_1-1,j}^{n+1/2}+C_1\vec{x}_{1,j}^{n+1/2}-B_1\vec{x}_{2,j}^{n+1/2}=F_1, i=1 \\ -A_i\vec{x}_{i-1,j}^{n+1/2}+C_i\vec{x}_{i,j}^{n+1/2}-B_i\vec{x}_{i+1,j}^{n+1/2}=F_i, 2\le i\le n_1-1 \\ \vec{x}_{n_1,j}^{n+1/2}=\vec{x}_{1,j}^{n+1/2} \end{cases} \tag{11}$$

where $A_i=\dfrac{\tau}{2}\dfrac{g_{22,i-1/2,j}^{n}}{h_1^2}$,

$$B_i=\frac{\tau}{2}\frac{g_{22,i+1/2,j}^{n}}{h_1^2},$$

$$C_i=1+A_i+B_i,$$

$$F_i=\vec{x}_{i,j}^{n}+\frac{\tau}{2}\Lambda_{22}\vec{x}_{i,j}^{n}.$$

To determine the running coefficients, the following formulas are used [5]:

$$\alpha_{i+1}=\frac{B_i}{C_i-\alpha_iA_i}; \qquad \beta_{i+1}=\frac{F_i+A_i\beta_i}{C_i-\alpha_iA_i};$$

$$\gamma_{i+1}=\frac{A_i\gamma_i}{C_i-\alpha_iA_i};\ i=2,..,n_1-1$$

$$\alpha_2=\frac{B_1}{C_1};\ \beta_2=\frac{F_1}{C_1};\ \gamma_2=\frac{A_1}{C_1};$$

$$p_i=\alpha_{i+1}p_{i+1}+\beta_{i+1}; \qquad q_i=\alpha_{i+1}q_{i+1}+\gamma_{i+1};$$

$$i=n_1-2,\dots,1; \qquad p_{n_1-1}=\beta_{n_1};$$

$$q_{n_1-1}=\alpha_{n_1}+\gamma_{n_1};$$

$$\vec{x}_{n_1,j}^{n+1/2}=\frac{\beta_{n_1+1}+\alpha_{n_1+1}p_1}{1-\alpha_{n_1+1}q_1-\gamma_{n_1+1}};$$

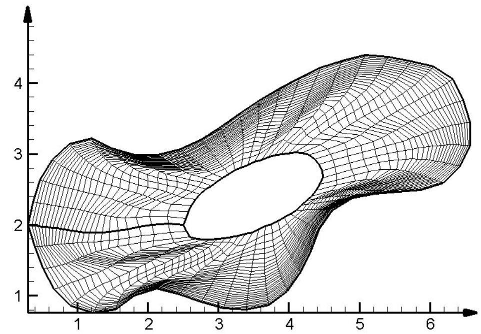
$$\vec{x}_{i,j}^{n+1/2}=p_i+q_i\vec{x}_{n_1,j}^{n+1/2};\ i=2,..,n_1-1.$$

On the cut line, the following periodic boundary conditions are taken into account:

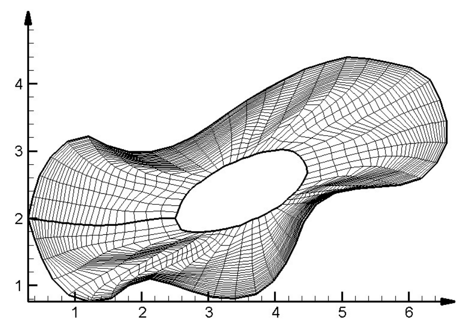
$$A_{n_1}=A_1,\ B_{n_1}=B_1,\ C_{n_1}=C_1,\ F_{n_1}=F_1.$$

A scalar sweep with fixed boundary values found with the help of (5) is used for equation (9) in the direction η. Methodical calculations of the construction of curvilinear grids are considered based on the method described above.

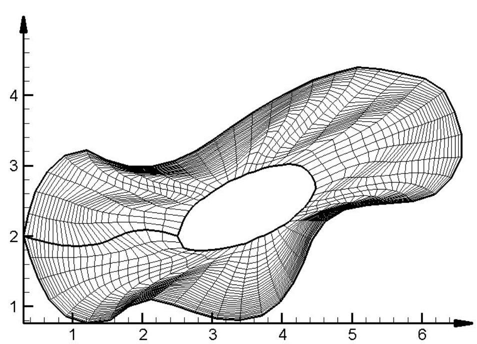
Since the conditions of periodicity were used in one direction, the results are also periodic. In order to determine the most optimal grid, we used the estimation of the grids quality according to the methods proposed in [1]. The work [1] considers four types of estimates that are orthogonality, local uniformity, non-extension and convexity of the constructed grid.



a)



b)



c)

Figure 4: Results of Calculations for the Construction of Curvilinear Grids

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

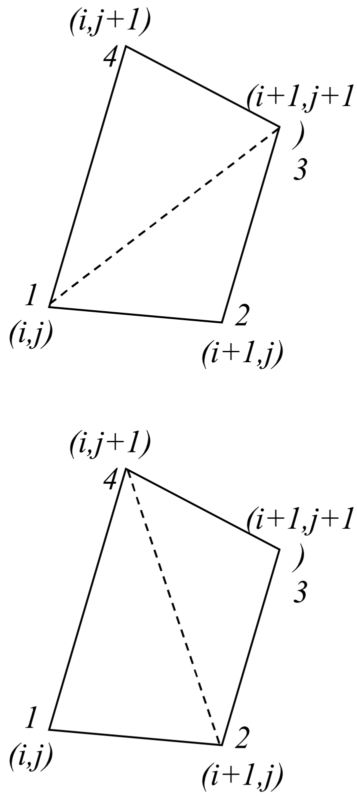


Figure 5: Cell Division into Triangles

Each grid cell is considered and is divided diagonally into triangles. The following value corresponds to the convexity criterion (see equation 12): where ,

1)

$$S_{(i,j),(i+1,j),(i+1,j+1)}=\frac{1}{2}\left[\left(x_{i+1,j}-x_{i,j}\right)\left(y_{i+1,j+1}-y_{i,j}\right)-\left(x_{i+1,j+1}-x_{i,j}\right)\left(y_{i+1,j}-y_{i,j}\right)\right]$$

2)

$$S_{(i,j),(i,j+1),(i+1,j+1)}=\frac{1}{2}\left[\left(x_{i+1,j+1}-x_{i,j}\right)\left(y_{i,j+1}-y_{i,j}\right)-\left(x_{i,j+1}-x_{i,j}\right)\left(y_{i+1,j+1}-y_{i,j}\right)\right]$$

3)

$$S_{(i,j),(i+1,j),(i,j+1)}=\frac{1}{2}\left[\left(x_{i+1,j}-x_{i,j}\right)\left(y_{i,j+1}-y_{i,j}\right)-\left(x_{i,j+1}-x_{i,j}\right)\left(y_{i+1,j}-y_{i,j}\right)\right]$$

4)

$$S_{(i+1,j),(i,j+1),(i+1,j+1)}=\frac{1}{2}\left[\left(x_{i+1,j+1}-x_{i+1,j}\right)\left(y_{i,j+1}-y_{i+1,j}\right)-\left(x_{i,j+1}-x_{i+1,j}\right)\left(y_{i+1,j+1}-y_{i+1,j}\right)\right]$$

- the areas of the corresponding triangles formed by the diagonals. The value $Q_{i,j}^{1}$ can lie in the interval $(-\infty,1]$, for a convex cell $0<Q_{i,j}^{1}\le 1$, for degenerate in a triangle and self-intersecting cells $-\infty<Q_{i,j}^{1}\le 0$.

To determine the estimate of the orthogonality criterion, use the minimum value of the sine of the angle as follows:

$$Q_{i,j}^{2}=\min_{k=(i,j),(i+1,j),(i,j+1),(i+1,j+1)}\{\sin\varphi_k\} \tag{13}$$

where

$$\sin\varphi_{i,j}=\frac{2S_{(i,j),(i+1,j),(i,j+1)}}{l_{(i,j),(i+1,j)}l_{(i,j),(i,j+1)}},$$

$$\sin\varphi_{i+1,j}=\frac{2S_{(i,j),(i+1,j),(i+1,j+1)}}{l_{(i,j),(i+1,j)}l_{(i+1,j),(i+1,j+1)}},$$

$$\sin\varphi_{i,j+1}=\frac{2S_{(i,j),(i,j+1),(i+1,j+1)}}{l_{(i,j),(i,j+1)}l_{(i,j+1),(i+1,j+1)}},$$

$$\sin\varphi_{i+1,j+1}=\frac{2S_{(i+1,j),(i,j+1),(i+1,j+1)}}{l_{(i+1,j),(i+1,j+1)}l_{(i,j+1),(i+1,j+1)}},$$

and the lengths of the sides are $l_{(i,j),(i+1,j)}=\sqrt{\left(x_{i+1,j}-x_{i,j}\right)^2+\left(y_{i+1,j}-y_{i,j}\right)^2}$, etc.

Values of functions $Q_{i,j}^{2}$ can take values from a segment $[-1,1]$, for convex cells it takes positive values, and for degenerate ones it is equal to zero, and negative values for a nonconvex and self-intersecting cell. The next criterion of grid quality is the elongation of the cell, which is defined as follows:

$$Q_{i,j}^{3}=\frac{\min\limits_{k=[(i,j),(i+1,j)],[(i+1,j),(i+1,j+1)],[(i+1,j+1),(i,j+1)],[(i,j+1),(i,j)]}\{l_k\}}{\max\limits_{k=[(i,j),(i+1,j)],[(i+1,j),(i+1,j+1)],[(i+1,j+1),(i,j+1)],[(i,j+1),(i,j)]}\{l_k\}} \tag{14}$$

Values $Q_{i,j}^{3}$ vary in the interval $[0,1]$. One of the main requirements is local uniformity, i.e., all cells in the area should be evenly distributed. Adaptive grids are considered in [1], therefore they use a control function. Without the control function, the criterion of local uniformity is defined as follows:

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$Q^4_{i,j} = \min\left\{\frac{S_{i+1/2,j+1/2}}{\tilde{S}}, \frac{\tilde{S}}{S_{i+1/2,j+1/2}}\right\} \qquad (15)$$

where $S_{i+1/2,j+1/2}$ is the area of the cell surrounded by nodes

$$\{(i,j),(i+1,j),(i+1,j+1),(i,j+1)\},$$

and a is the average area of one cell. The values of vary in the interval [0.1]

$$\tilde{S} = \frac{\sum_{i=1}^{n1-1}\sum_{j=1}^{n2-1} S_{i+1/2,j+1/2}}{(n_1-1)(n_2-1)}.$$

In order to determine the best variant of the grid, the criteria for the quality of the grid were determined at each iteration according to the methods described above. Based on the defined criteria for the quality of the grid at each iteration, the worst (minimum score) was determined, and the best ones were chosen from the worst ones. Thus, the most optimal grid was defined by convexity, since convexity and orthogonality are related criteria.

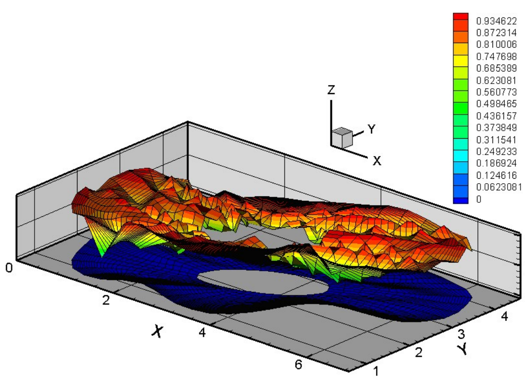


Figure 6: Graphical Representation of the Convexity Criterion in Space

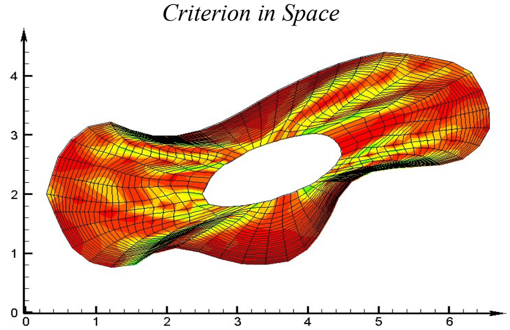


Figure 7: Graphical Representation of the Convexity Criterion in the Plane

It can be seen from Figures 6 and 7 that all grid cells are convex, since the values of the estimates are in intervals $0 < Q^1_{i,j} \le 1$. Thus, we have the most suitable and mutually orthogonal curvilinear grid in a doubly-connected domain. To simulate a convective flow, the equation of an incompressible fluid is used in the vorticity ω, stream function ψ, and temperature θ variables with corresponding initial and boundary conditions [4] in curvilinear coordinate systems. A mathematical problem describing the convective flow of a non-uniformly heated viscous fluid in an arbitrary doubly-connected domain D with a curvilinear boundary is considered $\partial D = \Gamma_0 \cup \Gamma_1$ (Figure 1).

This problem in Cartesian coordinate systems, in a fixed bounded two-dimensional domain, can be formulated as follows:

$$\frac{\partial\omega}{\partial t} + u\frac{\partial\omega}{\partial x} + v\frac{\partial\omega}{\partial y} = \mu_u\frac{\partial}{\partial x}\left(\frac{\partial\omega}{\partial x}\right) + \mu_u\frac{\partial}{\partial y}\left(\frac{\partial\omega}{\partial y}\right) + \beta\frac{\partial\theta}{\partial x} \qquad (16)$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = -\omega \qquad (17)$$

$$\frac{\partial\theta}{\partial t} + u\frac{\partial\theta}{\partial x} + v\frac{\partial\theta}{\partial y} = \mu_\theta\frac{\partial}{\partial x}\left(\frac{\partial\theta}{\partial x}\right) + \mu_\theta\frac{\partial}{\partial y}\left(\frac{\partial\theta}{\partial y}\right) \qquad (18)$$

with the following initial and boundary conditions:

$$\left.\begin{array}{l} \omega = 0, \quad \theta = \varphi(x,y), \quad (x,y)\in\overline{D}, \quad t = 0 \\ \psi = 0, \quad \dfrac{\partial\psi}{\partial n} = 0, \quad \theta = \phi_1(x,y,t), \quad (x,y)\in\Gamma_0, \quad t\in(0,T] \\ \psi = \lambda(t), \quad \dfrac{\partial\psi}{\partial n} = 0, \quad \theta = \phi_2(x,y,t) \quad (x,y)\in\Gamma_1, \quad t\in(0,T] \end{array}\right\} \qquad (19)$$

where x, y are the Cartesian coordinates, t is time, $u = \frac{\partial\psi}{\partial y}$, $v = -\frac{\partial\psi}{\partial x}$ are the components of the velocity vector, μ_u, μ_θ are the coefficients of kinematic viscosity and thermal diffusivity, β is the coefficient of thermal density variation, Γ_0, Γ_1 are the disjoint contours, $\overline{n}$ is the direction of the

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

outer normal to the boundary ∂D, ϕ_i is the specified functions. An analogous problem was considered in [3] on a closed rectangular cavity D with a boundary Γ_0 containing a rectangular body with a boundary Γ_1 on a uniform rectangular grid taking into account the uniqueness of the pressure condition. The use of a Cartesian coordinate system in finite difference methods is complicated in a doubly-connected domain with a curvilinear boundary, as in Figure 1, because of the description of the computational grid at the boundaries and the necessary interpolation procedures for obtaining boundary conditions. The problem (16)-(19) in the curvilinear coordinate system when mapping the physical plane (x, y) to the computational plane (ξ, η) is as follows:

$$\frac{\partial\omega}{\partial t}+a_1\frac{\partial\omega}{\partial\xi}+a_2\frac{\partial\omega}{\partial\eta}=J\mu_u\frac{\partial}{\partial\xi}\left(a_{11}\frac{\partial\omega}{\partial\xi}\right)+J\mu_u\frac{\partial}{\partial\eta}\left(a_{22}\frac{\partial\omega}{\partial\eta}\right)-$$
$$-J\mu_u\frac{\partial}{\partial\xi}\left(a_{12}\frac{\partial\omega}{\partial\eta}\right)-J\mu_u\frac{\partial}{\partial\eta}\left(a_{12}\frac{\partial\omega}{\partial\xi}\right)+J\beta\left[y_\eta\frac{\partial\theta}{\partial\xi}-y_\xi\frac{\partial\theta}{\partial\eta}\right] \quad (20)$$

$$\frac{\partial}{\partial\xi}\left(a_{11}\frac{\partial\psi}{\partial\xi}\right)-\frac{\partial}{\partial\xi}\left(a_{12}\frac{\partial\psi}{\partial\eta}\right)-\frac{\partial}{\partial\eta}\left(a_{12}\frac{\partial\psi}{\partial\xi}\right)+\frac{\partial}{\partial\eta}\left(a_{22}\frac{\partial\psi}{\partial\eta}\right)=-\frac{\omega}{J} \quad (21)$$

$$\frac{\partial\theta}{\partial t}+a_1\frac{\partial\theta}{\partial\xi}+a_2\frac{\partial\theta}{\partial\eta}=J\mu_\theta\frac{\partial}{\partial\xi}\left(a_{11}\frac{\partial\theta}{\partial\xi}\right)+J\mu_\theta\frac{\partial}{\partial\eta}\left(a_{22}\frac{\partial\theta}{\partial\eta}\right)-$$
$$-J\mu_\theta\frac{\partial}{\partial\xi}\left(a_{12}\frac{\partial\theta}{\partial\eta}\right)-J\mu_\theta\frac{\partial}{\partial\eta}\left(a_{12}\frac{\partial\theta}{\partial\xi}\right) \quad (22)$$

where $a_1 = uJy_\eta - vJx_\eta$, $a_2 = -uJy_\xi + vJx_\xi$,

$a_{11} = J\left(y_\eta^2 + x_\eta^2\right)$, $a_{12} = J\left(y_\xi y_\eta + x_\xi x_\eta\right)$,

$a_{22} = J\left(y_\xi^2 + x_\xi^2\right)$,

$J = \dfrac{1}{x_\xi y_\eta - x_\eta y_\xi}$ is the Jacobian of transformation.

The use of transformation into a curvilinear coordinate system allows us to consider problem (20) - (22) on a uniform rectangular grid and obtain qualitative pictures of simulated processes at moderate amounts of grid nodes. A metric conversion factors can be calculated analytically or numerically, depending on the definition or specification of the curvilinear boundary. An explicit scheme and an iterative method of successive upper relaxation is used to solve the problem numerically. The differential problem is replaced by a difference analogue of the following form:

$$\frac{\omega_{i,j}^{n+1}-\omega_{i,j}^{n}}{\tau}+\Lambda_{1h}\omega_{i,j}^{n}+\Lambda_{2h}\omega_{i,j}^{n}=$$
$$=\mu_{u,i,j}\Lambda_{11h}\omega_{i,j}^{n}+\mu_{u,i,j}\Lambda_{22h}\omega_{i,j}^{n}$$
$$-\mu_{u,i,j}\Lambda_{12h}\omega_{i,j}^{n}-\mu_{u,i,j}\Lambda_{21h}\omega_{i,j}^{n}$$
$$+\beta_{i,j}\Phi_h\theta_{i,j}^{n} \quad (23)$$

$$\Lambda_{11,h}\psi_{i,j}^{n+1}+\Lambda_{11,h}\psi_{i,j}^{n+1}-\Lambda_{12,h}\psi_{i,j}^{n+1}-\Lambda_{21,h}\psi_{i,j}^{n+1}=-\frac{\omega_{i,j}^{n+1}}{J_{i,j}} \quad (24)$$

$$u_{i,j}^{n+1}=J_{i,j}\left[-x_{\eta,i,j}\frac{\psi_{i+1,j}^{n+1}-\psi_{i-1,j}^{n+1}}{2h_1}+x_{\xi,i,j}\frac{\psi_{i,j+1}^{n+1}-\psi_{i,j-1}^{n+1}}{2h_2}\right] \quad (25$$

$$v_{i,j}^{n+1}=J_{i,j}\left[-y_{\eta,i,j}\frac{\psi_{i+1,j}^{n+1}-\psi_{i-1,j}^{n+1}}{2h_1}+y_{\xi,i,j}\frac{\psi_{i,j+1}^{n+1}-\psi_{i,j-1}^{n+1}}{2h_2}\right] \quad (26)$$

$$\frac{\theta_{i,j}^{n+1}-\theta_{i,j}^{n}}{\tau}+\Lambda_{1h}\theta_{i,j}^{n}+\Lambda_{2h}\theta_{i,j}^{n}=$$
$$=\mu_{\theta,i,j}\Lambda_{11h}\theta_{i,j}^{n}+\mu_{\theta,i,j}\Lambda_{22h}\theta_{i,j}^{n}$$
$$-\mu_{\theta,i,j}\Lambda_{12h}\theta_{i,j}^{n}-\mu_{\theta,i,j}\Lambda_{21h}\theta_{i,j}^{n} \quad (27)$$

where

$$a_{1,i,j}=J_{i,j}\left(y_{\eta,i,j}u_{i,j}^{n}-x_{\eta,i,j}v_{i,j}^{n}\right),$$
$$a_{2,i,j}=J_{i,j}\left(x_{\xi,i,j}v_{i,j}^{n}-y_{\xi,i,j}u_{i,j}^{n}\right),$$
$$a_{11,i,l}=J_{i,j}\left(y_{\eta,i,j}^{2}+x_{\eta,i,j}^{2}\right),$$
$$a_{22}=J_{i,j}\left(y_{\xi,i,j}^{2}+x_{\xi,i,j}^{2}\right),$$
$$a_{12,i,j}=a_{21,i,j}=J_{i,j}\left(y_{\xi,i,j}y_{\eta,i,j}+x_{\xi,i,j}x_{\eta,i,j}\right).$$

The difference analogues of the corresponding differential operators have the following form (see equation 28):

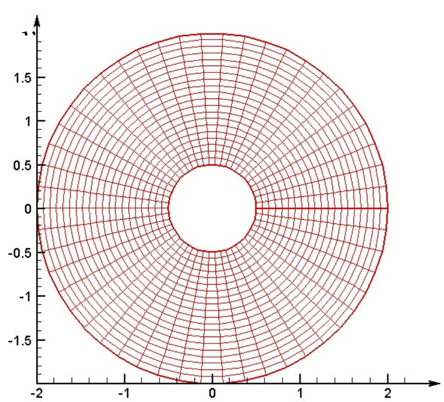
The algorithm of numerical implementation is carried out in the following way:

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

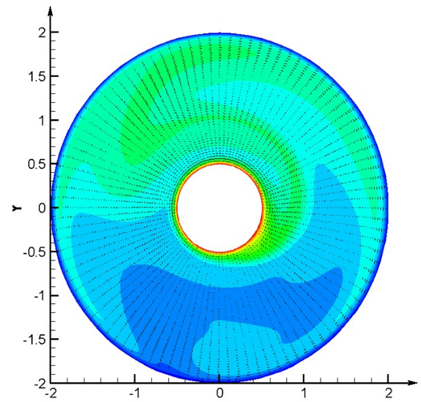
first of all, $\omega_{i,j}^{n+1}$ is found by (23); then $\psi_{i,j}^{n+1}$ is found by (24); $u_{i,j}^{n+1}$ and $v_{i,j}^{n+1}$ are determined from (25), (26) using the found values of $\psi_{i,j}^{n+1}$; the values of $\theta_{i,j}^{n+1}$ are calculated using the new values of $u_{i,j}^{n+1}$ and $v_{i,j}^{n+1}$ from (27). The iteration process continued until the following condition is met:

$$\max_{\substack{1\le i\le n_1\\ 1\le j\le n_2}}\left|\omega_{i,j}^{n+1}-\omega_{i,j}^{n}\right|\le\varepsilon .$$

The use of an explicit scheme and slowly convergent iterative methods is explained by the fact that rapidly convergent cost-effective methods, using implicit schemes, require self-adjointness and positive definiteness of the matrix of differential operators. This complicates the problem in the presence of the metric tensors coefficients. According to the algorithm described above, numerous methodological calculations are carried out in various doubly connected domains. Dimensionless quantities of velocity, length, temperature, and time were used in the calculations. In the exampl, the temperature was assumed to be $\theta = 0$ in the outer boundary, and $\theta = 1$ in the inner boundary A ring was specially chosen in the first example, where the boundaries are described by the equations of a circle, the grid is constructed by an algebraic method, and the components of the metric tensor are determined analytically (Figure 8a). Figure 8b shows the results of calculating the temperature change. It can be seen from the figure that the cold liquid is based in the lower part of the region, since it is known from the physics course that the density of the liquid is inversely proportional to the temperature.

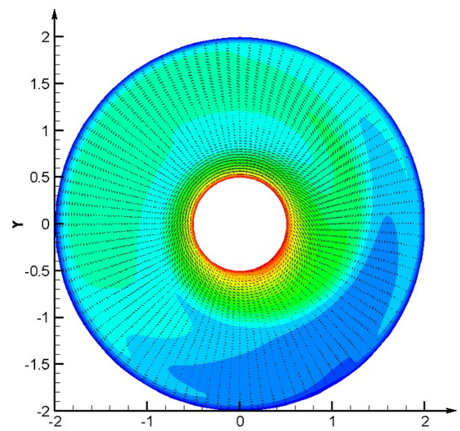


a)

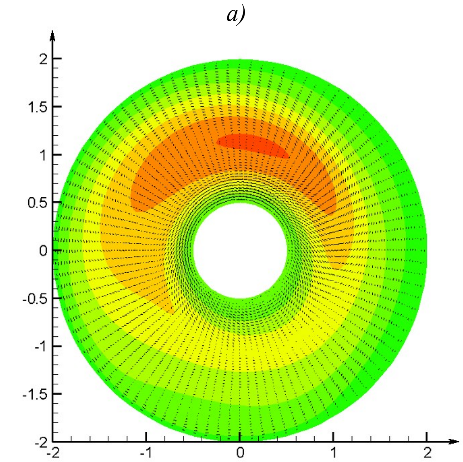


b)

Figure 8: Estimated Grid and the Results of Numerical Solution of the Temperature Change



a)



b)

Figure 9: The Results of a Numerical Solution of a) Temperature Change, b) the Current Function at The Same Time

Since the process is not stationary, the norms of velocity and temperature also do not tend to a stationary regime in calculations. According to this, the vortex regimes constantly change and the cooler liquid swings in the lower part of the calculated domain. Figure 10 shows the results of calculating the temperature and the current function at the same time.

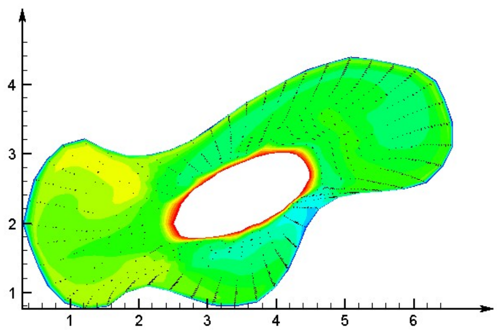


Figure 10: Results of the Numerical Solution of the Temperature Variation in the Curvilinear Domain

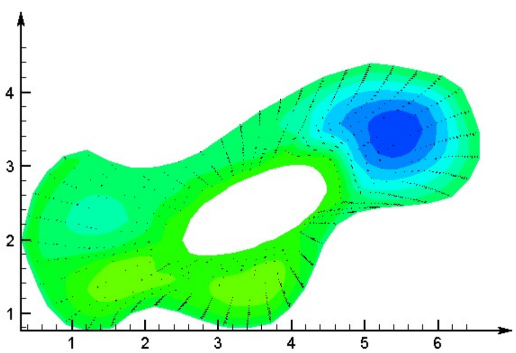


Figure 11: Results of the Numerical Solution of the Current Function in the Curvilinear Domain

Figures 10 and 11 show the results of numerical calculations of the temperature and current functions at the same time. It is seen from the figures that vortex motions are formed in the curves of the boundary, and a warm liquid accumulates in the upper parts of the bend. Since the process is non-stationary, the modes of vortices are constantly changing, and the temperature of the fluid is constantly transferred. In conclusion, it can be seen that the construction of a curvilinear grid for the description of convective flow helps to obtain a qualitative description of physical processes.

The results obtained after investigations are principally different from previously conducted surveys in the field of computational and applied mathematics aimed at the creation of automated computer programs for the construction of curvilinear adaptive structural and nonstructural difference grids, as well as the modernization of numerical algorithms for solving applied problems. Previous studies in this area usually lack a holistic approach, whereas the current study is presented in the form of relevant feedback on urgent scientific demands for theoretical concepts and methodological approaches to the use of advanced information technologies in hydrodynamic studies. It takes into account the specific features of the subject area, as well as the development, adaptation and approbation of tools in the process of modeling of natural and technogenic objects of significant importance.

4. CONCLUSIONS

The findings of the study have been reflected and confirmed by the graphs, allowing us to conclude that applied methods for the numerical construction of curvilinear structured grids in doubly connected domains are effective. They present a scientific opportunity to develop theoretical concepts and methodological approaches to the use of new information technologies in hydrodynamic studies, taking into account the specific features of the subject area, as well as the development, adaptation and approbation of tools in the process of modeling of natural and technogenic objects. Numerical modeling of the convective flow of a non-uniformly heated liquid in a curvilinear coordinate system is performed. The implicit scheme and the method of fractional steps are used in the numerical construction of curvilinear grids in doubly connected domains by equidistribution methods and the method of Godunov-Thompson. An explicit scheme and the method of fractional steps are used in the numerical implementation of the equations of an incompressible fluid. A cyclic run is used in the direction of the outer and inner boundaries, and a scalar run is used in the normal direction. Calculations were carried out for different cavity configurations and temperature regimes at the boundary. The graphs of numerical calculations of the temperature and current functions have been obtained, namely: the estimated grid and the numerical solution of the temperature change; the numerical solution of the temperature change and the current function at the same time; the numerical solution of the temperature variation in the curvilinear domain; the numerical solution of the current function in the curvilinear domain. However, further investigations need to be performed in this area in order to develop and implement the detailed scheme of the results obtained in the current study.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

REFERENCES:

[1] J.F. Thompson, Z.U.A. Warsi, and C.W.Mastin, *"Numerical grid generation, foundations and applications"*, New York: Elsevier, 1985.
[2] Y.I. Shokin, N.T. Danaev, G.S. Khakimzyanov, and N.Yu. Shokina, *"Lectures on difference schemes on moving grids"*, Part 2, Almaty, 2008.
[3] S.K. Godunov, A.V. Zabrodin, and M.Ya. Ivanov, "*Numerical solution of multidimensional problems of gas dynamics*", Moscow: Nauka, 1976.
[4] V.P. Sirochenko, "Numerical modeling of convective viscous fluid flows in multiply connected regions", *Proceedings of the International Conference RDAMM,* Spec. Issue Vol.6, Part 2, 2001, pp. 554-562.
[5] G.P. Prokopov, *"On the organization of comparison of algorithms and programs for constructing regular two-dimensional difference grids",* Moscow, 1989.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

APPENDIX

1) Equation 12

$$Q_{i,j}^{1}=\frac{\min\left\{S_{(i,j),(i+1,j),(i+1,j+1)},S_{(i,j),(i,j+1),(i+1,j+1)},S_{(i,j),(i+1,j),(i,j+1)},S_{(i+1,j),(i,j+1),(i+1,j+1)}\right\}}{0.5\left(S_{(i,j),(i+1,j),(i+1,j+1)}+S_{(i,j),(i,j+1),(i+1,j+1)}\right)} \tag{12}$$

2) Equation 28

$$\Lambda_{1,h}\omega_{i,j}^{n}=\frac{1}{2}\left[\left(a_{1,i+1/2,j}-\left|a_{1,i+1/2,j}\right|\right)\frac{\omega_{i+1,j}^{n}-\omega_{i,j}^{n}}{h_1}+\left(a_{1,i-1/2,j}+\left|a_{1,i-1/2,j}\right|\right)\frac{\omega_{i,j}^{n}-\omega_{i-1,j}^{n}}{h_1}\right],$$

$$\Lambda_{2,h}\omega_{i,j}^{n}=\frac{1}{2}\left[\left(a_{2,i,j+1/2}-\left|a_{2,i,j+1/2}\right|\right)\frac{\omega_{i,j+1}^{n}-\omega_{i,j}^{n}}{h_2}+\left(a_{2,i,j-1/2}+\left|a_{2,i,j-1/2}\right|\right)\frac{\omega_{i,j}^{n}-\omega_{i,j-1}^{n}}{h_2}\right],$$

$$\Lambda_{11,h}\omega_{i,j}^{n}=\frac{J_{i,j}}{h_1}\left(a_{11,i+1/2,j}\frac{\omega_{i+1,j}^{n}-\omega_{i,j}^{n}}{h_1}-a_{11,i-1/2,j}\frac{\omega_{i,j}^{n}-\omega_{i-1,j}^{n}}{h_1}\right),$$

$$\Lambda_{22,h}\omega_{i,j}^{n}=\frac{J_{i,j}}{h_2}\left(a_{22,i,j+1/2}\frac{\omega_{i,j+1}^{n}-\omega_{i,j}^{n}}{h_2}-a_{22,i,j-1/2}\frac{\omega_{i,j}^{n}-\omega_{i,j-1}^{n}}{h_2}\right),$$

$$\Lambda_{12,h}\omega_{i,j}^{n}=\frac{J_{i,j}}{2h_1}\left(a_{12,i+1,j}\frac{\omega_{i+1,j+1}^{n}-\omega_{i+1,j-1}^{n}}{2h_2}-a_{12,i-1,j}\frac{\omega_{i-1,j+1}^{n}-\omega_{i-1,j-1}^{n}}{2h_2}\right),$$

$$\Lambda_{21,h}\omega_{i,j}^{n}=\frac{J_{i,j}}{2h_2}\left(a_{12,i,j+1}\frac{\omega_{i+1,j+1}^{n}-\omega_{i-1,j+1}^{n}}{2h_1}-a_{12,i,j-1}\frac{\omega_{i+1,j-1}^{n}-\omega_{i-1,j-1}^{n}}{2h_1}\right),$$

$$\Phi_{h}\theta_{i,j}^{n}=J_{i,j}\left[y_{\eta,i,j}\frac{\theta_{i+1,j}^{n}-\theta_{i-1,j}^{n}}{2h_1}-y_{\xi,i,j}\frac{\theta_{i,j+1}^{n}-\theta_{i,j-1}^{n}}{2h_2}\right] \tag{28}$$