

HYBRID GPS + UWB POSITIONING THROUGH UKF WITH RSS AND TOA

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ABSTRACT

The Global Positioning System (GPS) is an accurate positioning system. However, this system encounters difficulties in so-called constrained environments (indoor environments, presence of obstructions, etc.) where GPS signals are very often masked or reflected. To mitigate this situation, another technique such as Infra Red (IR) technology or GPS repeaters are used to perform well for constrained positioning. In this document, we propose a hybrid GPS - UWB positioning system that provide better positioning accuracy as compared to UWB or GPS system only. For this purpose, we use pseudo-range metrics for the GPS system and Received Signal Strength (RSS) for Ultra Wideband (UWB) technology that we simultaneously couple through the Unscented Kalman Filter (UKF). The performance of this hybrid positioning technique is highlighted in different scenarios. Simulation results show that the proposed algorithm is more accurate than GPS alone. Also, it is better than the results of GPS and UWB coupling through the Extended Kalman Filter (EKF).

Keywords: GPS, UWB, Hybrid positioning, UKF, EKF.

1. INTRODUCTION

For many business cases or for security scenarios including real-time tracking, navigation and clock synchronization, several positioning systems have been proposed and implemented in recent years. Among these systems, Global Positioning System (GPS) based on a constellation of satellites is the most used. However, GPS signals are deteriorated due the obstacles encountered in the constrained environment because of the phenomena of reflection, multipath fading and diffraction [1]. To mitigate this situation, several indoor systems such as Infrared (IR), Ultrasound (US) and Radio Frequency (RF) systems have been developed. UWB (Ultra Wideband) technology falls into this last category. (see Figure1).

The current localizations approaches used the hybridization techniques by combining two or more positioning systems and then provide independently the estimation of the position. In general, the GPS is used in the outdoor environment and another technique like UWB is used for indoor positioning.

Ultra Wide Band (UWB) is a key technique that has proven effective in indoor positioning and new

algorithms have been developed to improve its performance [2].

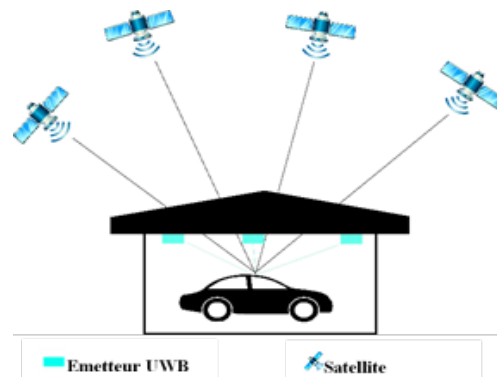


Figure 1: Example Of Model Representation

This work is motivated by the fact that UWB is the most promising technology for indoor positioning and tracking. The nature of the application in question plays a major role in determining a appropriate solution for achieving certain qualities attributes. Hybrid positioning approaches have

future potential because they combine features of different mechanism to improve performance.

The performance of the hybrid system is a promising topic of research. UWB technology was used to improve GPS positioning by relying on filters such as the particle filter [3]. In this kind of filtering system, the level of error is beyond 1 m for other sensors despite the use of UWB signals.

In the solution we proposed, it is assessed with the Unscented Kalman Filter in which we develop and propose a new design of a hybrid system UWB sensors, used in both outdoor/indoor localization.

The rest of the document is organized as follows: Section 2 presents the Unscented Kalman filter. Part 3 instructs us on the different positioning techniques based on UKF. Section 4 presents the performance of our algorithm in different scenarios. Section 5 is for conclusions and future research.

2. UNSCENTED KALMAN FILTER

The unscented Kalman Filter is described in [3][4][5]. This filter, based on deterministic sampling is appreciated for its estimation accuracy, robustness and ease of implementation [5]. The UKF uses the non-linear model of the system and assumed the distribution of the random state model through the Unscented Transformation. The non-linear state model is expressed as follows:

$$X_k = f(X_{k-1}) + w_k \quad (1)$$

where $w_k \square \mathcal{N}(0, Q_k)$ is zero-mean gaussian white noise and covariance Q_k affecting the state vector X_k at the instant t_k . The random function f is used to determine the predicted state from the previous state.

The observation model ζ_k is expressed as a function of the state vector X_k as follow:

$$\zeta_k = h(X_k) + V_k \quad (2)$$

Where $V_k \square \mathcal{N}(0, R_k)$ is a gaussian white noise of zero mean and covariance R_k . The observation function h links the measurements to the status vector. The unscented kalman Filter (UKF) consists of two parts: the unscented transformation and the algorithm.

2.1 Unscented Transformation (UT)

It is a method for calculating the statistics of a random variable that undergoes a non-linear evolution. It consists in depriving oneself of the step of linearization of the non-linear function as it is the case with EKF by passing in a deterministic way a series of finite points called sigma points in the measurement function to finally obtain the mean and the variance of the state which one seeks. We want to calculate the mean and covariance of the random variable $\zeta = f(X)$ with a non-linear function f .

We assume that the variable X has mean \bar{X} and covariance P_X .

To do this, a set of $2L+1$ points (with L the size of the state) is chosen in a deterministic way so that their mean and covariance are \bar{X} and P_X respectively.

The choice of these $2L+1$ points called sigma points is made as follows:

$$\chi_0 = \bar{X}, i = 0 \quad (3)$$

$$\chi_i = \bar{X} + (\sqrt{(L+\lambda)P_X})_i, i = 1, \dots, L \quad (4)$$

$$\chi_i = \bar{X} - (\sqrt{(L+\lambda)P_X})_i, i = L+1, \dots, 2L \quad (5)$$

The weights of χ_0 for mean ω_0^m and covariance ω_0^c are defined:

$$\omega_0^m = \frac{\lambda}{L+\lambda} \text{ and } \omega_0^c = \frac{\lambda}{L+\lambda} + (1-\alpha^2 + \beta) \quad (6)$$

β is used to incorporate prior knowledge of the distribution of X . It's equal to 2 for a Gaussian distribution.

Also, for χ_i we have:

$$\omega_i^m = \omega_i^c = \frac{1}{2(L+\lambda)}, i = 1, \dots, 2L \quad (7)$$

where $\lambda = \alpha^2(L+k) - L$ is a scaling parameter;

$\alpha = [10^{-4}; 1]$ determines the spread of the sigma points around \bar{X} ;

k is another scaling parameter which is usually set to 0 (details in [6]).

At each sigma point propagated through the nonlinear function f , we have:

$$\gamma_i = f(X_i), i = 0, \dots, 2L \quad (8)$$

The mean, covariance and inter-covariance of the non-linear observable ζ are obtained using a weighted sample mean and covariance of the posterior sigma points:

$$\bar{\zeta} \approx \sum_{i=0}^{2L} \omega_i^m \gamma_i \quad (9)$$

$$P_{\zeta\zeta} \approx \sum_{i=0}^{2L} \omega_i^c (\gamma_i - \bar{\zeta})(\gamma_i - \bar{\zeta})^T \quad (10)$$

$$P_{X\zeta} \approx \sum_{i=0}^{2L} \omega_i^c (\chi_i - \bar{X})(\gamma_i - \bar{\zeta})^T \quad (11)$$

UKF algorithm is composed of three (3) main phases [3] after initialization: 'the prediction phase', 'the observation phase' and 'the state update phase' described as follows:

2.2 Unscented Kalman Filter Algorithm

1) Initialisation

To initialize our filter we need the initial position X_0 of the mobile, its covariance P_{X_0} assigned to this position as well as the covariances of the noises of the state vector Q and the observation R as defined:

$$\bar{X}_0 = E[X_0], \quad P_{X_0} \approx E[(X_0 - \bar{X}_0)(X_0 - \bar{X}_0)^T] \quad (12)$$

$$Q_k = E[w_k w_k^T]; \quad R_k = E[V_k V_k^T] \quad (13)$$

For $k = 1, \dots, \infty$, we generate sigma points:

$$\chi_{k-1} = [\bar{X}_{k-1} \quad \bar{X}_{k-1} + \sqrt{(L+\lambda)P_{k-1}} \quad \bar{X}_{k-1} - \sqrt{(L+\lambda)P_{k-1}}] \quad (14)$$

2) Prediction transformation

Each sigma point is predicted by the non-linear function f . Since the state noise w is additive and of zero mean, it can be dissociated temporally from the prediction function and one obtains:

$$\chi_{k|k-1} = f(\chi_{k-1}) \quad (15)$$

Once the sigma points are transformed, the mean \bar{X} and covariance P of the state are calculated using the weighted sums of the sigma points.

$$\bar{X}_{k|k-1} = \sum_{i=0}^{2L} \omega_i^m \chi_{i,k|k-1} \quad (16)$$

$$P_{k|k-1} \approx Q_{k-1} + \sum_{i=0}^{2L} \omega_i^c (\chi_{i,k|k-1} - \bar{X}_{k|k-1})(\chi_{i,k|k-1} - \bar{X}_{k|k-1})^T \quad (17)$$

The covariance processing noise Q_k of the state vector is added to the covariance matrix after transformation of nonlinear function.

3) Observation transformation

The transformed sigma points give new sigma points which are:

$$\gamma_{k|k-1} = [\bar{X}_k \quad \bar{X}_k + \sqrt{(L+\lambda)P_k} \quad \bar{X}_k - \sqrt{(L+\lambda)P_k}] \quad (18)$$

These sigma points propagate through the observation function h . As far as the prediction is concerned, for its additive character and zero average, the measurement noise V_{k-1} is not considered by the observation function.

$$\gamma_{k|k-1} = h(\chi_{k|k-1}) \quad (19)$$

Here $\gamma_{k|k-1}$ is the matrix from the sigma points. This matrix is used to predict the mean $\bar{\zeta}_{k|k-1}$, covariance $P_k^{\zeta\zeta}$ of this prediction as follows:

$$\bar{\zeta}_{k|k-1} = \sum_{i=0}^{2L} \omega_i^m \gamma_{i,k|k-1} \quad (20)$$

$$P_k^{\zeta\zeta} \approx R_k + \sum_{i=0}^{2L} \omega_i^c (\gamma_{i,k|k-1} - \bar{\zeta}_{k|k-1})(\gamma_{i,k|k-1} - \bar{\zeta}_{k|k-1})^T \quad (21)$$

It is noted that the measurement noise covariance R_k is added to the covariance.

The inter-covariance $P_k^{X\zeta}$ which is the covariance between the measurement and the estimated state is:

$$P_k^{X\zeta} \approx \sum_{i=0}^{2L} \omega_i^c (\chi_{i,k|k-1} - \bar{X}_{k|k-1})(\gamma_{i,k|k-1} - \bar{\zeta}_{k|k-1})^T \quad (22)$$

4) Measurement Update

These covariance matrices allow us to calculate the gain of the kalman filter K_k with:

$$K_k = P_k^{X\zeta} (P_k^{\zeta\zeta})^{-1} \quad (23)$$

From this Kalman filter gain we update the state vector \bar{X}_k and the covariance P_k respectively as follows:

$$\bar{X}_k = \bar{X}_{k|k-1} + K_k (\zeta_k - \bar{\zeta}_{k|k-1}) \quad (24)$$

$$P_k = P_{k|k-1} - K_k P_k^{\zeta\zeta} K_k^T \quad (25)$$

3. POSITIONING TECHNIQUES

The estimation of the position of a mobile in a sensor network can be obtained by calculating the distance between the mobile and the different sensors. It is the same principle used by the GPS satellites. Depending on the sensor array technology, different types of measurements can be processed by the Unscented Kalman Filter (UKF). The estimation of the distance based on RSS (Received Signal Strength) as well as the pseudodistance depends strongly on the actual distance between the mobile and the transmitter of the signal as well as the nature of the transmitters. For this purpose, the measurement error increases with distance.

3.1 Positioning Through distance calculation

Positioning is mainly based on the GPS satellite system. The distance between the GPS satellite and the mobile called pseudorange is expressed as follows:

$$D = d + c.\Delta t + V_{gps} \quad (26)$$

where c is the signal speed and d is the geometric distance between the mobile and the satellite.

UKF estimates the position of the mobile based on this distance between the mobile and the satellite. By standing in space, a satellite S of known position has the coordinates:

$$X_{si} = [x_{si}, y_{si}, z_{si}]^T \quad (27)$$

with $i = 1, \dots, n$ where n is the number of satellites accessible by the mobile. This number depends on

the nature of the environment in which we find ourselves. Also

$$X_{M,k} = [x_{M,k}, y_{M,k}, z_{M,k}]^T \quad (28)$$

constitutes the coordinates of the mobile M now t_k . Thus, the measurement model defined by the UKF is:

$$\zeta_{s,k} = [d_{s1,k}, d_{s2,k}, \dots, d_{sn,k}]^T + c.\Delta t + V_{gps} \quad (29)$$

where $d_{si,k}$ represents the estimated distance between the mobile M and the satellite i at the time t_k and is expressed as follows:

$$d_{si,k} = \sqrt{(x_{M,k} - x_{si})^2 + (y_{M,k} - y_{si})^2 + (z_{M,k} - z_{si})^2} \quad (30)$$

3.2 Positioning Through Received Signal Strength

Received Signal Strength (RSS) is a measure of the strength of the received signal. We will apply this measurement to UWB sensors in our indoor case to locate a mobile.

The RSS measurement model is formulated as follows [7]:

$$RSS = \frac{p_j^t}{p_j^r} + V_{RSS} \quad (31)$$

where p_j^t is the emitted power, p_j^r the received power by the mobile.

Based on Friis formula, it is estimated as follows:

$$p_j^t = p_j^r d_j^\alpha \quad \text{avec } j = 1, \dots, m \quad (32)$$

where d_j is the geometric distance between the mobile and the $j^{i\text{eme}}$ sensor, m is the number of sensors, α is a constant coefficient which determines the nature of the environment varying from 2 to 5 (equal to 2 in free space). So finally, the RSS expression is:

$$RSS = d_j^\alpha + V_{RSS} \quad (33)$$

In space we note the coordinates of the UWB sensor by:

$$X_{Uj} = [x_{Uj}, y_{Uj}, z_{Uj}] \quad (34)$$

As a result, the geometric distance between the sensor and the mobile is expressed by:

$$d_{Uj,k} = \sqrt{(x_{M,k} - x_{Uj})^2 + (y_{M,k} - y_{Uj})^2 + (z_{M,k} - z_{Uj})^2} \quad (35)$$

The measurement model in this case will be defined as follows:

$$\zeta_{Uj,k} = [d_{U1,k}, d_{U2,k}, \dots, d_{Um,k}]^T + V_{RSS} \quad (36)$$

3.3 Hybridization

Positioning accuracy can be achieved by the intelligent combination of several measurements from different technologies [7].

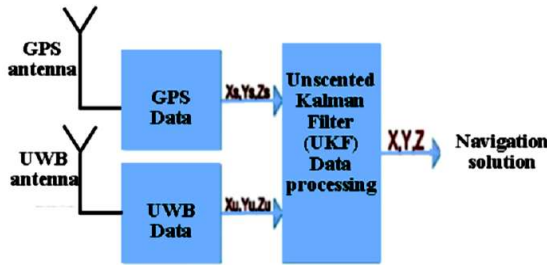


Figure 2: Architecture Of Hybrid GPS/UWB System

The hybrid technique proposed in this document combines pseudorange based on GPS satellite measurements and RSS from UWB sensors. For this purpose, we adopt the tight coupling which consists in collecting simultaneously the raw data of the GPS and the UWB in the same filter without any pre-processing step as it is the case for the loose coupling. Considering n (GPS satellites) and m (UWB sensors), the hybrid observation model for the Unscented Kalman Filter is:

$$\zeta_k = \begin{bmatrix} \zeta_{si,k} \\ \zeta_{Uj,k} \end{bmatrix} \quad (37)$$

Starting from equations (2), (29) and (36), the Hybrid covariance matrix for the observation model is:

$$R_k = \begin{bmatrix} R_{gps,k} & 0_{n \times m} \\ 0_{m \times n} & R_{RSS,k} \end{bmatrix} \quad (38)$$

where $0_{n \times m}$ and $0_{m \times n}$ are zero matrix of respective dimension $n \times m$ and $m \times n$. Then $R_{gps,k}$ and $R_{RSS,k}$ are the covariances of the respective GPS and UWB measurement noise.

4. SIMULATION

The mobile we want to locate and track is equipped with UWB sensor receiving signals from UWB transmitters and GPS receiver to receive signals from GPS satellites. These different data received according to the metrics are processed by the UKF simultaneously. In our simulation we use data from [8]. In the different scenarios we estimate the following values:

$$X_0 = (10m, 5m, 10m) \quad (39)$$

which is the initial position of the mobile covariance, matrix identity:

$$P_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (40)$$

As for the covariance of the state vector noise, we express it by diagonal matrix

$$Q_k = \text{diag}(0.05ms^{-1}, 0.05ms^{-1}, 0.05ms^{-1}) \quad (41)$$

$$= \begin{bmatrix} 0.05 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.05 \end{bmatrix}$$

It constitutes the covariance of the localization error on the state vector for a duration $N = 600s$; with a period $T = 0.1s$;

For a number n of GPS satellites and m of UWB transmitters we will evaluate the RMSE in different scenarios.

The Root Mean Square Error (RMSE) is used to evaluate the mobile tracking error with our algorithm and is defined as follows:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X}_i)^2} \quad (42)$$

The different test scenarios are listed in Table1 below.

In the table above, we noticed that the error is very high for GPS location compared to that of UWB. The more UWB signals we use to couple with GPS, there is an improvement in positioning. The error function is defined as follows:

$$E(x, y, z) = \sqrt{(x_{ii} - x_i)^2 + (y_{ii} - y_i)^2 + (z_{ii} - z_i)^2} \quad (43)$$

with (x_{ii}, y_{ii}, z_{ii}) the real coordinates of the mobile at each time and (x_i, y_i, z_i) the coordinates estimated by the algorithm in different scenarios.

Table 1: RMSE Location In Different Scenarios

Metric	Type and number of sensors	RMSE (m)
Pseudorange	Scenario 1: 4 GPS	21.8937
RSS	Scenario 2: 4 UWB	0.0498
Hybrid (Pseudorange + RSS)	Scenario 3: 4 GPS + 2 UWB	33.0624
	Scenario 4: 4 GPS + 3 UWB	0.1945
	Scenario 5: 3 GPS + 3 UWB	0.3155

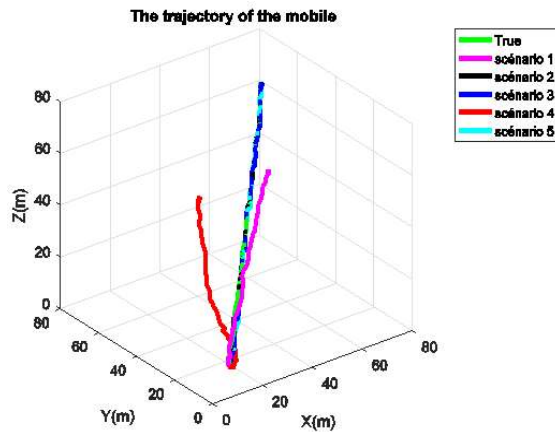


Figure 3: Comparative Trajectory Of the Mobile In Different Scenarios With The Real Trajectory.

We present on fig4, in different scenarios, the evolution of the location error.

We notice on figure 4 that in three scenarios the positioning error is less than 1 m. These are cases in which the UWB technology intervenes but with at least three received signals.

We compare the errors of the UKF and EKF filters in three scenarios that require GPS and UWB hybridization, i.e. scenarios 3, 4 and 5 results are shown on figures 5, 6 and 7 respectively.

On these different figures below we have in pink the evolution of the error with the EKF and in green the error with UKF in a hybrid context. We clearly observe that the UKF is better compared to the EKF because the errors with the UKF are lower.

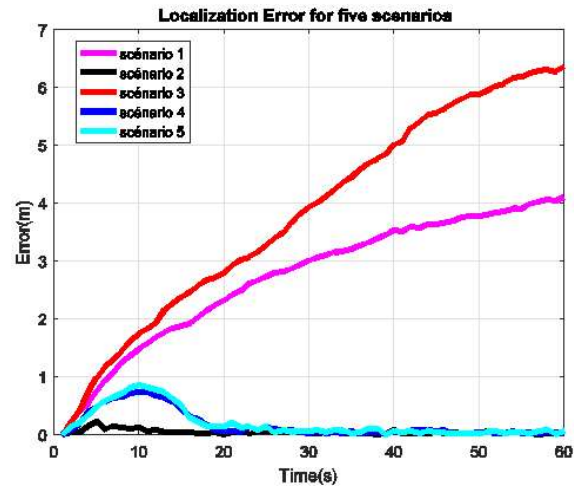


Figure 4: Mobile Location Error In Different scenarios

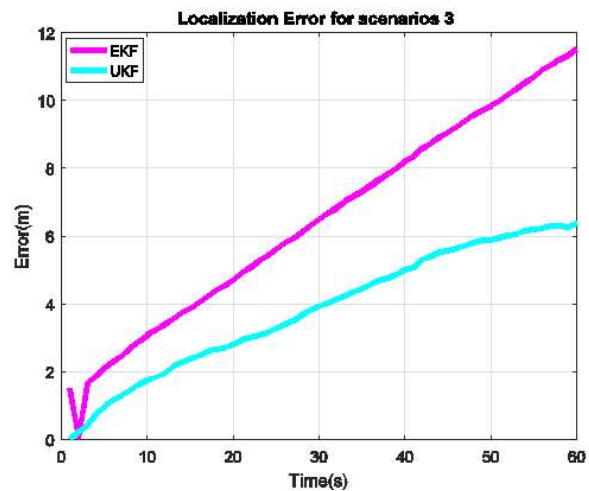


Figure 5: Mobile Location Error Compared with UKF (green) And EKF (pink) In Scenario 3

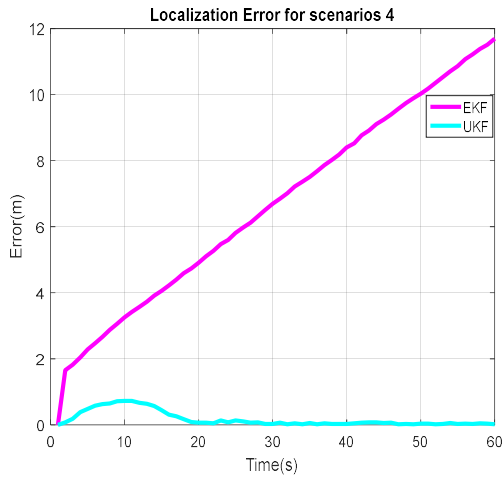


Figure 6: Mobile Location Error Compared with UKF (green) And EKF (pink) In Scenario 4

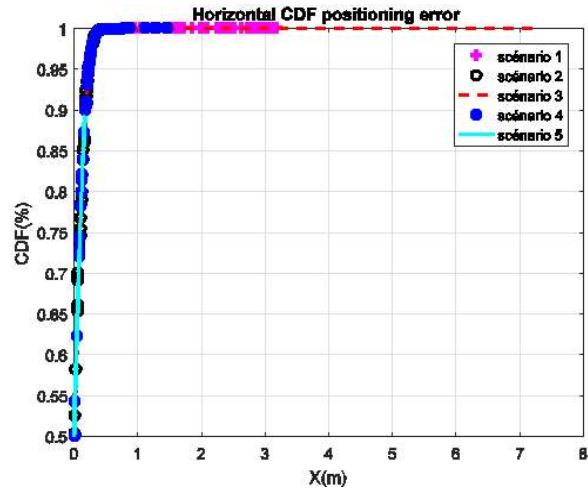


Figure 8: Horizontal CDF Positioning Error

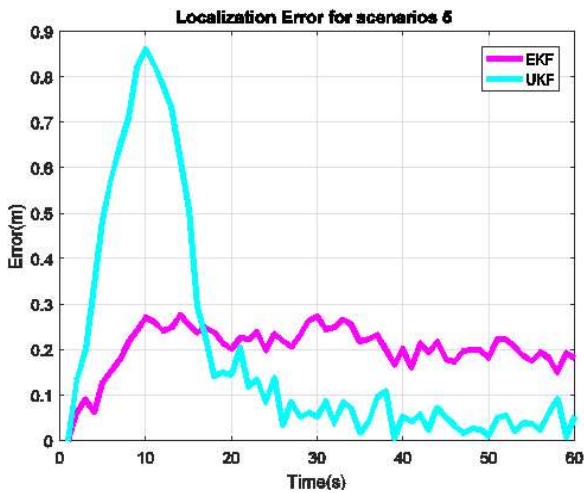


Figure 7: Mobile Location Error Compared with UKF (green) And EKF (pink) In Scenario 5

In figure 9 which concerns the vertical error, 94% of the positioning errors for scenario 5 are around 0.5 m.

In the other cases, 100 % of the positioning errors are at least equal to 0.7 m. But one can go up to 2.3 m (colour blue) in scenario 4; 7.1 m in pink for scenario 1 and 7.5 m (red colour) for scenario 3.

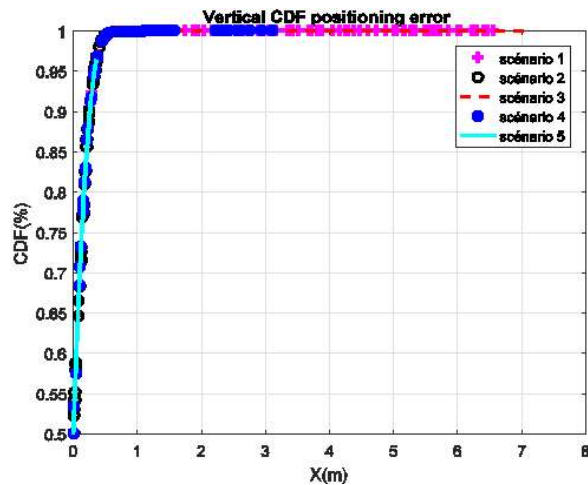


Figure 9: Vertical CDF Positioning Error

We observe on figure 8 that in the fifth scenario (green colour in figure 8) 86 % of localization errors are around 0.2 m.

But from there, the algorithm does not give us any more information on the movement of the mobile and this is explained by the fact that the GPS signals are weak.

For the other cases, 100 % of the errors are greater or equal to 0.5 m. But in scenario 3 (red colour) we can go up to 7.6 m. Then to 3.1 m for GPS only in pink and 1.3 m for scenario 4 (4GPS+3UWB).

This section proves that the localization accuracy can be further improved by fusing GPS data and UWB radios measurements. Our proposed UKF system show best performance compared to that of EKF approach in different scenarios tested.

Our system proposed could be evaluated in others scenarios like constant speed and variable speed of the mobile or scenarios of more number of sensors.

The performance of our system have been simulated with only one metrics, pseudorange for GPS and RSS for UWB. The use of other metrics like TOA can help us prove the accuracy of our UKF approach.

5. CONCLUSION

In this work, after a theoretical study of an Unscented Kalman Filter algorithm, we presented a new approach for positioning system. This approach is based in combining data from GPS and UWB in an Unscented Kalman Filter (UKF) in order to estimate, mobile's position. The metrics used here are pseudo range for GPS and Received Signal Strength (RSS) for UWB. This UKF-based approach provides more accurate positioning than a system based on one technology and hence the interest of the UWB in improving GPS applications

The localization error of our proposed UKF system has been compared to another integrated system based on the Extended Kalman Filter (EKF) in the same five simulation scenarios. The performance of our UKF approach are better than the EKF approach generally used in hybrid positioning system to integrate the nonlinear information from different sensors and confirm our choice of UKF.

REFERENCES :

- [1] T. Qiu, Y. Zhou, F. Xia, NG. Jin, L. Feng, "A localization strategy based on n-times trilateral centroid with weight" *International Journal of Communication Systems*, school of software, Dalian University of Technology (China), 2012
- [2] M. A. Al-Ammar, S. Alhadhrami, A. Al-Salman, A. Alarifi, H. S. Al-Khalifa, A. Alnafessa, M. Alsaleh "Comparative Survey of Indoor Positioning Technologies, Techniques, and Algorithms" *In Proceedings of the 2014 International Conference on Cyberworlds (CW)*, Santander, Spain, 6–8 October 2014; pp. 1–8.
- [3] A. Belakbir, M. Amghar, N. Sbiti, "An indoor-outdoor positioning system based on the combination of gps and uwb sensors" *Journal of Theoretical and Applied Information Technology*, Vol. 61, N°1, 10 March 2014, pp. 67-72.
- [4] S. Julier, J. K. Uhlmann "Unscented filtering and nonlinear estimation" *Proceedings of the IEEE*, Vol.92, N°3, Mars 2004
- [5] S. Julier, J. K. Uhlmann "A new Extension of kalman filter to nonlinear systems" *in Proceedings of AeroSense*, The 11th Int. Symposium. A.D.S.S.C., 1997.
- [6] R. v. der Merwe, E. A. Wan " Efficient Derivative-Free Kalman Filters for Online Learning" *European Symposium on Artificial Neural Networks*, Oregon Graduate Institute of science and Technology(USA), April 25-27, 2001, pp 205-210.
- [7] M. Rhudy, Y. Gu, "Understanding Nonlinear Kalman Filters, Part II: An Implementation Guide" *Interactive Robotics Letters*, West Virginia University (USA), June 2013, pp.1-18
- [8] S. Tiwari, D. Wang, M. Fattouche, F. Ghannouchi "A Hybrid RSS/TOA Method for 3D Positioning in an Indoor Environment" *International Scholarly Research Network*, The iRadio Lab, Department of Electrical and Computer Engineering, University of Calgary (Canada), 2012.
- [9] Xi Chen, W. Gao, J. Wang "Robust all- source positioning of UAVs based on belief propagation" *Eurasip journal on advances in signal processing*, september 2013.