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# MODULAR REPRESENTATIONS OF THE $F_pW_n$-SPECHT MODULES $S_K(\lambda,\mu)$ AS LINEAR CODES

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**ABSTRACT**

We will find in this paper a generating matrix of the subspace representing the Specht module $S_K((1,1,1),(2))$ for each field $K$ of characteristic 0, and for each field $K = F_p = GF(p)$, where $p = 3$, 5. We will also find the representation matrices of two kinds of transpositions and give the way to find the representation of any permutation $w$ belongs to the Weyl group $W_n$ of type $B_n$. The main aim of this paper is finding the linear codes of the subspaces which represent the Specht modules.

We mention that some of the ideas of this work in this paper has been influenced by that of Adalbert Kerber and Axel Kohnert [11], even though that their paper is about the symmetric group and this paper is about the Weyl groups of type $B_n$.

**Keywords:** Field of characteristic 0 (infinite field), Finite field $F_p = GF(p)$, Weyl group $W_n$ of type $B_n$, group ring $F_pW_n$, $F_pW_n$-module, $F_pW_n$-submodule, pair of partitions $(\lambda,\mu)$ of a positive integer $n$, Specht polynomial, Specht module, $(\lambda,\mu)$-tableau, row standard $(\lambda,\mu)$-tableau, standard $(\lambda,\mu)$-tableau, vector space , subspace, generating matrix, linear code.

**Remarks :** Throughout this paper, let:

*i-* $F_p$ be the Galois field (finite field) of order $p$ ([6], p.429), that is $F_p = GF(p)$ ([7],p.2).

*ii-* $K$ be a field which is infinite (of characteristic 0) or finite of order a prime number $p \geq 3$, and $x_1, x_2, \ldots, x_n$ be independent indeterminates over $K$.

*iii-* $W_n$ be the Weyl group of type $B_n$, which is the group of all permutations $w$ of $\{x_1, x_2, \ldots, x_n, -x_1, -x_2, \ldots, -x_n\}$, such that $w(-x_i) = -w(x_i)$, for each $i = 1, 2, \ldots, n$.

*iv-* $KW_n$ be the group ring of $W_n$ with coefficients in $K$.

## 1. PRELIMINARIES

**Definition 1.1.** Let $(\lambda,\mu) = ((\lambda_1, \lambda_2, \ldots, \lambda_s), (\mu_1, \mu_2, \ldots, \mu_t))$ be a pair of partitions of a positive integer n, and let $Z^{(\lambda,\mu)}$ be any $(\lambda,\mu)$-tableau, then:

$$g(Z^{(\lambda,\mu)}) = \prod_{i=1}^{s}\prod_{j=1}^{\lambda_i} Z^{(\lambda,\mu)}(i,j,1)^{2i-1} \cdot \prod_{i=1}^{t}\prod_{j=1}^{\mu_i} Z^{(\lambda,\mu)}(i,j,2)^{2i-2}.$$

$g(Z^{(\lambda,\mu)})$ is called the row position monomial of $Z^{(\lambda,\mu)}$ ([1], p.14, [2], p.7 and [4], p.13).

**Example 1.2.** Let $Z^{((3,2),(2,2))}$ be the following $((3,2),(2,2))$-tableau:

$$\begin{matrix} -x_3 & x_2 & -x_5 \\ -x_9 & x_1 & \end{matrix} \quad ; \quad \begin{matrix} x_8 & x_4 \\ x_7 & -x_6 \end{matrix} \quad .$$

Then the row position monomial $g(Z^{((3,2),(2,2))}) = x_1^3 x_2^1 (-x_3)^1 x_4^0 (-x_5)^1 (-x_6)^2 x_7^2 x_8^0 (-x_9)^3 = -x_1^3 x_2 x_3 x_5 x_6^2 x_7^2 x_9^3$.

**Theorem 1.3.** Let $(\lambda,\mu) = ((\lambda_1,\lambda_2,\ldots,\lambda_s), (\mu_1,\mu_2,\ldots,\mu_t))$ be a pair of partitions of a positive integer n, and let $Z^{(\lambda,\mu)}$ be any $(\lambda,\mu)$-tableau. Then the $KW_n$-module $M_K(\lambda,\mu) = KW_n\, g(Z^{(\lambda,\mu)})$ has a $K$-basis $B(\lambda,\mu) = \{g(Z^{(\lambda,\mu)}) \mid Z^{(\lambda,\mu)}$ is a row standard $(\lambda,\mu)$-tableau$\}$, and $\dim_K M_K(\lambda,\mu) = \dfrac{n!}{\lambda_1!\cdots\lambda_s!\cdot\mu_1!\cdots\mu_t!}$ ([2], p.7 and [4], p.14).

**Definition 1.4.** Let $\{y_1,\ldots,y_r\} \subseteq \{\pm x_1,\ldots,\pm x_n\}$, such that $y_i \neq \pm y_j$ for each $i,j = 1,\ldots,r$ and $i \neq j$, then we define

$$\Delta_1(y_1,\ldots,y_r) = \begin{cases} \prod_{1\le i<j\le r} (y_j^2 - y_i^2) \prod_{\ell=1}^{r} y_\ell, & if\ r>1 \\ y_1, & if\ r=1 \end{cases}$$

$$\Delta_2(y_1,\ldots,y_r) = \begin{cases} \prod_{1\le i<j\le r} (y_j^2 - y_i^2) & if\ r>1 \\ 1 & if\ r=1 \end{cases}$$

([2], p.8 and [4], p.15).

**Example 1.5.** $\Delta_2(x_3,x_9,x_2) = (x_2^2 - x_3^2)(x_2^2 - x_9^2)(x_9^2 - x_3^2) = x_2^4 x_9^2 - x_2^4 x_3^2 + x_2^2 x_3^4 - x_2^2 x_9^4 + x_3^2 x_9^4 - x_3^4 x_9^2$, and $\Delta_1(x_3,x_9,x_2) = x_2 x_3 x_9 \cdot (\Delta_2(x_3,x_9,x_2)) = x_2^5 x_3 x_9^3 - x_2^5 x_3^3 x_9 + x_2^3 x_3^5 x_9 - x_2^3 x_3 x_9^5 + x_2 x_3^3 x_9^5 - x_2 x_3^5 x_9^3$.

**Definition 1.6.** Let $(\lambda,\mu) = ((\lambda_1,\lambda_2,\ldots,\lambda_s), (\mu_1,\mu_2,\ldots,\mu_t))$ be a pair of partitions of a positive integer $n$, and let $Z^{(\lambda,\mu)}$ be any $(\lambda,\mu)$-tableau, then:

$$f(Z^{(\lambda,\mu)}) = \begin{cases} f_1(Z^{\lambda}) & if\ |\mu|=0 \\ f_2(Z^{\mu}) & if\ |\lambda|=0 \\ f_1(Z^{\lambda}).\, f_2(Z^{\mu}) & \text{otherwise} \end{cases}$$

such that:

$$f_1(Z^{\lambda}) = \prod_{j=1}^{\lambda_1} \Delta_1(Z^{(\lambda,\mu)}(1,j,1),\ldots,Z^{(\lambda,\mu)}(\lambda'_j,j,1))$$

where $\lambda'_j$ is the number of the indeterminates in the $j^{th}$ column of the first tableau $Z^{\lambda}$, and

$$f_2(Z^{\mu}) = \prod_{j=1}^{\mu_1} \Delta_2(Z^{(\lambda,\mu)}(1,j,2),\ldots,Z^{(\lambda,\mu)}(\mu'_j,j,2))$$

where $\mu'_j$ is the number of the indeterminates in the $j^{th}$ column of the second tableau $Z^{\mu}$, $f(Z^{(\lambda,\mu)})$ is called the Specht polynomial of $(\lambda,\mu)$-tableau $Z^{(\lambda,\mu)}$ ([2], p.9 and [4], p.15).

**Theorem 1.7.** Let $(\lambda,\mu) = ((\lambda_1,\lambda_2,\ldots,\lambda_s), (\mu_1,\mu_2,\ldots,\mu_t))$ be a pair of partitions of a positive integer n, and let $Z^{(\lambda,\mu)}$ be any $(\lambda,\mu)$-tableau, then the Specht polynomial of the $(\lambda,\mu)$-tableau $Z^{(\lambda,\mu)}$ can be expressed as $f(Z^{(\lambda,\mu)}) = \sum_{\tau\in \underline{C}(Z^{(\lambda,\mu)})} \operatorname{sgn}(\tau)\cdot \tau\, g(Z^{(\lambda,\mu)})$, where $\underline{C}(Z^{(\lambda,\mu)})$ is the set of all positive column permutations of $Z^{(\lambda,\mu)}$ ([2], p.10 and [4], p.16).

**Corollary 1.8.** $|\underline{C}(Z^{(\lambda,\mu)})| = \lambda'_1!\ldots\lambda'_{\lambda_1}!\,\mu'_1!\ldots\mu'_{\mu_1}!$, where $(\lambda,\mu)' = (\lambda',\mu') = ((\lambda'_1,\lambda'_2,\ldots,\lambda'_{\lambda_1}), (\mu'_1,\mu'_2,\ldots,\mu'_{\mu_1}))$ is the conjugate pair of partitions of the pair of partitions $(\lambda,\mu) = ((\lambda_1,\lambda_2,\ldots,\lambda_s), (\mu_1,\mu_2,\ldots,\mu_t))$ ([4], p.12).

**Corollary 1.9.** $f(Z^{(\lambda,\mu)})$ consists of exactly

$|\underline{C}(Z^{(\lambda,\mu)})| = \lambda'_1! \ldots \lambda'_{\lambda_1}!\, \mu'_1! \ldots \mu'_{\mu_1}!$ monomials where $(\lambda,\mu)' = (\lambda',\mu') = ((\lambda'_1,\lambda'_2,\ldots,\lambda'_{\lambda_1}),(\mu'_1,\mu'_2,\ldots,\mu'_{\mu_1}))$ is the conjugate pair of partitions of the pair of partitions $(\lambda,\mu) = ((\lambda_1,\lambda_2,\ldots,\lambda_s),(\mu_1,\mu_2,\ldots,\mu_t))$.

**Example 1.10.** Let $Z^{((2,1),(1,1))}$ be the following $((2,1),(1,1))$-tableau

$$\begin{matrix} x_3 & x_1 \\ x_5 & \end{matrix} \; ; \; \begin{matrix} x_4 \\ -x_2 \end{matrix}.$$

Then $g(Z^{((2,1),(1,1))}) = x_1 x_2^2 x_3 x_5^3$, and $\underline{C}(Z^{((2,1),(1,1))}) = \{i, (x_3\ x_5), (x_4\ -x_2), (x_3\ x_5)(x_4\ -x_2)\}$, and $|\underline{C}(Z^{((2,1),(1,1))})| = \lambda'_1! \cdot \lambda'_2! \cdot \mu'_1! = 2! \cdot 1! \cdot 2! = 4$.

$f(Z^{((2,1),(1,1))}) = \prod_{j=1}^{\lambda_1} \Delta_1(Z^{((2,1),(1,1))}(1,j,1), \ldots, Z^{((2,1),(1,1))}(\lambda'_j, j, 1)) \prod_{j=1}^{\mu_1} \Delta_2(Z^{((2,1),(1,1))}(1,j,2), \ldots, Z^{((2,1),(1,1))}(\mu'_j, j, 2))$

$= \Delta_1(Z^{((2,1),(1,1))}(1,1,1), Z^{((2,1),(1,1))}(2,1,1)) \cdot \Delta_1(Z^{((2,1),(1,1))}(1,2,1)) \cdot \Delta_2(Z^{((2,1),(1,1))}(1,1,2), Z^{((2,1),(1,1))}(2,1,2)) = \Delta_1(x_3, x_5) \cdot \Delta_1(x_1) \cdot \Delta_2(x_4, -x_2) = (x_5^2 - x_3^2)\, x_3 x_5 \cdot x_1 \cdot (x_2^2 - x_4^2)$

$= x_1 x_2^2 x_3 x_5^3 - x_1 x_2^2 x_3^3 x_5 - x_1 x_3 x_4^2 x_5^3 + x_1 x_3^3 x_4^2 x_5$

$= i\, g(Z^{((2,1),(1,1))}) - (x_3\ x_5) g(Z^{((2,1),(1,1))}) - (x_4 - x_2)\, g(Z^{((2,1),(1,1))}) + (x_3\ x_5)(x_4\ -x_2)\, g(Z^{((2,1),(1,1))})$

$= \sum_{\tau \in \underline{C}(Z^{((2,1),(1,1))})} \operatorname{sgn}(\tau) \cdot \tau\, g(Z^{((2,1),(1,1))})$.

**Definition 1.11.** Let $(\lambda,\mu) = ((\lambda_1,\lambda_2,\ldots,\lambda_s), (\mu_1,\mu_2,\ldots,\mu_t))$ be a pair of partitions of a positive integer n, and let $Z^{(\lambda,\mu)}$ be any $(\lambda,\mu)$-tableau. Then the cyclic $KW_n$-module $S_K(\lambda,\mu)$ generated over $KW_n$ by $f(Z^{(\lambda,\mu)})$ (i.e., $S_K(\lambda,\mu) = KW_n\, f(Z^{(\lambda,\mu)})$) is called the Specht module over $K$ corresponding to the pair of partitions $(\lambda,\mu)$ of $n$ ([2], p.10 and [4], p.16).

**Theorem 1.12.** Let $(\lambda,\mu) = ((\lambda_1,\lambda_2,\ldots,\lambda_s), (\mu_1,\mu_2,\ldots,\mu_t))$ be a pair of partitions of a positive integer n. Then there are exactly $\frac{n!}{H_{\lambda,\mu}}$ distinct $(\lambda,\mu)$-standard tableaux where $H_{\lambda,\mu} = H_\lambda . H_\mu$, such that $H_\lambda = \prod_{i=1}^{s} \prod_{j=1}^{\lambda_i} h_{ij}$, where $h_{ij} = \lambda_i + \lambda'_j - i - j + 1$, and $H_\mu = \prod_{i=1}^{t} \prod_{j=1}^{\mu_i} e_{ij}$ where $e_{ij} = \mu_i + \mu'_j - i - j + 1$ ([2], p.20 & p.21 and [4], p.13).

**Theorem 1.13.** Let $(\lambda,\mu) = ((\lambda_1,\lambda_2,\ldots,\lambda_s), (\mu_1,\mu_2,\ldots,\mu_t))$ be a pair of partitions of a positive integer $n$. Then the Specht module $S_K(\lambda,\mu)$ has a $K$-basis $B(\lambda,\mu) = \{f(Z^{(\lambda,\mu)}) \mid Z^{(\lambda,\mu)}$ is a standard $(\lambda,\mu)$-tableau$\}$, and $\dim_K S_K(\lambda,\mu) = \frac{n!}{H_{\lambda,\mu}}$ ([2], p.21, [3], p.305 and [4], p.17).

## 2. A GENERATOR MATRIX OF THE SUBSPACE REPRESENTING THE SPECHT MODULE $S_K((1,1,1),(2))$ WHEN K IS A FIELD OF CHARACTERISTIC 0

$$\dim_K S_K((1,1,1),(2)) = \frac{5!}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} =$$

10, and thus we have ten standard ((1,1,1),(2))-tableaux, which are:

$$Z_1^{((1,1,1),(2))} = \begin{matrix} x_1 \\ x_2 \\ x_3 \end{matrix} \; ; \; \begin{matrix} x_4 & x_5 \end{matrix},$$

$$Z_2^{((1,1,1),(2))} = \begin{matrix} x_1 & x_3 & x_5 \\ x_2 & & \\ x_4 & & \end{matrix} ,$$

$$Z_3^{((1,1,1),(2))} = \begin{matrix} x_1 & x_3 & x_4 \\ x_2 & & \\ x_5 & & \end{matrix} ,$$

$$Z_4^{((1,1,1),(2))} = \begin{matrix} x_1 & x_2 & x_5 \\ x_3 & & \\ x_4 & & \end{matrix} ,$$

$$Z_5^{((1,1,1),(2))} = \begin{matrix} x_1 & x_2 & x_4 \\ x_3 & & \\ x_5 & & \end{matrix} ,$$

$$Z_6^{((1,1,1),(2))} = \begin{matrix} x_1 & x_2 & x_3 \\ x_4 & & \\ x_5 & & \end{matrix} ,$$

$$Z_7^{((1,1,1),(2))} = \begin{matrix} x_2 & x_1 & x_5 \\ x_3 & & \\ x_4 & & \end{matrix} ,$$

$$Z_8^{((1,1,1),(2))} = \begin{matrix} x_2 & x_1 & x_4 \\ x_3 & & \\ x_5 & & \end{matrix} ,$$

$$Z_9^{((1,1,1),(2))} = \begin{matrix} x_2 & x_1 & x_3 \\ x_4 & & \\ x_5 & & \end{matrix} ,$$

$$Z_{10}^{((1,1,1),(2))} = \begin{matrix} x_3 & x_1 & x_2 \\ x_4 & & \\ x_5 & & \end{matrix} .$$

The corresponding Specht polynomials are:

$$f\left(Z_1^{((1,1,1),(2))}\right) = (x_2^2 - x_1^2)(x_3^2 - x_1^2)(x_3^2 - x_2^2)x_1x_2x_3$$
$$= -x_1^5x_2^3x_3 + x_1^3x_2^5x_3 + x_1^5x_2x_3^3 - x_1x_2^5x_3^3 - x_1^3x_2x_3^5 + x_1x_2^3x_3^5 ,$$

$$f\left(Z_2^{((1,1,1),(2))}\right) = (x_2^2 - x_1^2)(x_4^2 - x_1^2)(x_4^2 - x_2^2)x_1x_2x_4$$
$$= -x_1^5x_2^3x_4 + x_1^3x_2^5x_4 + x_1^5x_2x_4^3 - x_1x_2^5x_4^3 - x_1^3x_2x_4^5 + x_1x_2^3x_4^5 ,$$

$$f\left(Z_3^{((1,1,1),(2))}\right) = (x_2^2 - x_1^2)(x_5^2 - x_1^2)(x_5^2 - x_2^2)x_1x_2x_5$$
$$= -x_1^5x_2^3x_5 + x_1^3x_2^5x_5 + x_1^5x_2x_5^3 - x_1x_2^5x_5^3 - x_1^3x_2x_5^5 + x_1x_2^3x_5^5 ,$$

$$f\left(Z_4^{((1,1,1),(2))}\right) = (x_3^2 - x_1^2)(x_4^2 - x_1^2)(x_4^2 - x_3^2)x_1x_3x_4$$
$$= -x_1^5x_3^3x_4 + x_1^3x_3^5x_4 + x_1^5x_3x_4^3 - x_1x_3^5x_4^3 - x_1^3x_3x_4^5 + x_1x_3^3x_4^5 ,$$

$$f\left(Z_5^{((1,1,1),(2))}\right) = (x_3^2 - x_1^2)(x_5^2 - x_1^2)(x_5^2 - x_3^2)x_1x_3x_5$$
$$= -x_1^5x_3^3x_5 + x_1^3x_3^5x_5 + x_1^5x_3x_5^3 - x_1x_3^5x_5^3 - x_1^3x_3x_5^5 + x_1x_3^3x_5^5 ,$$

$$f\left(Z_6^{((1,1,1),(2))}\right) = (x_4^2 - x_1^2)(x_5^2 - x_1^2)(x_5^2 - x_4^2)x_1x_4x_5$$
$$= -x_1^5x_4^3x_5 + x_1^3x_4^5x_5 + x_1^5x_4x_5^3 - x_1x_4^5x_5^3 - x_1^3x_4x_5^5 + x_1x_4^3x_5^5 ,$$

$$f\left(Z_7^{((1,1,1),(2))}\right) = (x_3^2 - x_2^2)(x_4^2 - x_2^2)(x_4^2 - x_3^2)x_2x_3x_4$$
$$= -x_2^5x_3^3x_4 + x_2^3x_3^5x_4 + x_2^5x_3x_4^3 - x_2x_3^5x_4^3 - x_2^3x_3x_4^5 + x_2x_3^3x_4^5 ,$$

$$f\left(Z_8^{((1,1,1),(2))}\right) = (x_3^2 - x_2^2)(x_5^2 - x_2^2)(x_5^2 - x_3^2)x_2x_3x_5$$
$$= -x_2^5x_3^3x_5 + x_2^3x_3^5x_5 + x_2^5x_3x_5^3 - x_2x_3^5x_5^3 - x_2^3x_3x_5^5 + x_2x_3^3x_5^5 ,$$

$$f\left(Z_9^{((1,1,1),(2))}\right) = (x_4^2 - x_2^2)(x_5^2 - x_2^2)(x_5^2 - x_4^2)x_2x_4x_5$$
$$= -x_2^5x_4^3x_5 + x_2^3x_4^5x_5 + x_2^5x_4x_5^3 - x_2x_4^5x_5^3 - x_2^3x_4x_5^5 + x_2x_4^3x_5^5 ,$$

$$f\left(Z_{10}^{((1,1,1),(2))}\right) = (x_4^2 - x_3^2)(x_5^2 - x_3^2)(x_5^2 - x_4^2)x_3x_4x_5$$
$$= -x_3^5x_4^3x_5 + x_3^3x_4^5x_5 + x_3^5x_4x_5^3 - x_3x_4^5x_5^3 - x_3^3x_4x_5^5 + x_3x_4^3x_5^5 .$$

The above polynomials give the following generator matrix ([8], p.2 and [9], p.49) for the subspace $V^{((1,1,1),(2))}$ (which represents the Specht module $S_K((1,1,1),(2))$) of the vector space $K^{60}$ (which represents the module $M_K((1,1,1),(2))$ whose dimension is $\dim_K M_K((1,1,1),(2)) = \frac{5!}{1!\cdot 1!\cdot 1!\cdot 2!} = 60$), where $K$ is a field of characteristic 0:

$\mu^{((1,1,1),(2))} =$

$$\begin{bmatrix}
-1&1&1&-1&-1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&-1&1&0&0&0&0&1&-1&0&0&0&0&-1&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&-1&0&1&0&0&0&1&0&-1&0&0&0&-1&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&-1&0&1&0&0&0&1&0&-1&0&0&0&-1&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&-1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{bmatrix}$$

$$\begin{bmatrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&-1&0&0&0&0&0&0&0&0&0&0&-1&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&-1&0&0&0&0&0&0&0&0&0&-1&0&1&0&0&0&0&0&0&0\\
-1&0&0&1&0&0&0&0&0&0&0&0&1&0&0&-1&0&0&0&0&0&0&0&0&-1&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&-1&0&0&0&0&0&0&0&0&0&-1&0&1&0&0&0&0&0&0\\
0&-1&0&0&1&0&0&0&0&0&0&0&0&1&0&0&-1&0&0&0&0&0&0&0&0&-1&0&0&1&0\\
0&0&-1&0&0&1&0&0&0&0&0&0&0&0&1&0&0&-1&0&0&0&0&0&0&0&0&-1&0&0&1
\end{bmatrix}$$

## 3. THE SUBSPACE $V_{(3)}^{((1,1,1),(2))}$ OF $F_3^{60}$ AS A LINEAR CODE

$f\left(Z_1^{((1,1,1),(2))}\right), \ldots, f\left(Z_{10}^{((1,1,1),(2))}\right) \bmod 3$ will be:

$$f_{(3)}\left(Z_1^{((1,1,1),(2))}\right) = 2x_1^5x_2^3x_3 + x_1^3x_2^5x_3 + x_1^5x_2x_3^3 + 2x_1x_2^5x_3^3 + 2x_1^3x_2x_3^5 + x_1x_2^3x_3^5,$$

$$f_{(3)}\left(Z_2^{((1,1,1),(2))}\right) = 2x_1^5x_2^3x_4 + x_1^3x_2^5x_4 + x_1^5x_2x_4^3 + 2x_1x_2^5x_4^3 + 2x_1^3x_2x_4^5 + x_1x_2^3x_4^5,$$

$$f_{(3)}\left(Z_3^{((1,1,1),(2))}\right) = 2x_1^5x_2^3x_5 + x_1^3x_2^5x_5 + x_1^5x_2x_5^3 + 2x_1x_2^5x_5^3 + 2x_1^3x_2x_5^5 + x_1x_2^3x_5^5,$$

$$f_{(3)}\left(Z_4^{((1,1,1),(2))}\right) = 2x_1^5x_3^3x_4 + x_1^3x_3^5x_4 + x_1^5x_3x_4^3 + 2x_1x_3^5x_4^3 + 2x_1^3x_3x_4^5 + x_1x_3^3x_4^5,$$

$$f_{(3)}\left(Z_5^{((1,1,1),(2))}\right) = 2x_1^5x_3^3x_5 + x_1^3x_3^5x_5 + x_1^5x_3x_5^3 + 2x_1x_3^5x_5^3 + 2x_1^3x_3x_5^5 + x_1x_3^3x_5^5,$$

$$f_{(3)}\left(Z_6^{((1,1,1),(2))}\right) = 2x_1^5x_4^3x_5 + x_1^3x_4^5x_5 + x_1^5x_4x_5^3 + 2x_1x_4^5x_5^3 + 2x_1^3x_4x_5^5 + x_1x_4^3x_5^5,$$

$$f_{(3)}\left(Z_7^{((1,1,1),(2))}\right) = 2x_2^5x_3^3x_4 + x_2^3x_3^5x_4 + x_2^5x_3x_4^3 + 2x_2x_3^5x_4^3 + 2x_2^3x_3x_4^5 + x_2x_3^3x_4^5,$$

$$f_{(3)}\left(Z_8^{((1,1,1),(2))}\right) = 2x_2^5x_3^3x_5 + x_2^3x_3^5x_5 + x_2^5x_3x_5^3 + 2x_2x_3^5x_5^3 + 2x_2^3x_3x_5^5 + x_2x_3^3x_5^5,$$

$$f_{(3)}\left(Z_9^{((1,1,1),(2))}\right) = 2x_2^5x_4^3x_5 + x_2^3x_4^5x_5 + x_2^5x_4x_5^3 + 2x_2x_4^5x_5^3 + 2x_2^3x_4x_5^5 + x_2x_4^3x_5^5,$$

$$f_{(3)}\left(Z_{10}^{((1,1,1),(2))}\right) = 2x_3^5x_4^3x_5 + x_3^3x_4^5x_5 + x_3^5x_4x_5^3 + 2x_3x_4^5x_5^3 + 2x_3^3x_4x_5^5 + x_3x_4^3x_5^5.$$

The above polynomials give the following generator matrix for the subspace $V_{(3)}^{((1,1,1),(2))}$ (which represents the Specht module $S_{F_3}((1,1,1),(2))$) of the vector space $F_3^{60} = (GF(3))^{60}$ (which represents the module $M_{F_3}((1,1,1),(2))$):

$\mu_{(3)}^{((1,1,1),(2))} =$

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$$\left[\begin{matrix}
2&1&1&2&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&2&1&0&0&0&0&1&2&0&0&0&0&2&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&0&0&0&0\\
0&0&0&0&0&0&0&0&2&0&1&0&0&0&1&0&2&0&0&0&2&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&2&0&1&0&0&0&1&0&2&0&0&0&2&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{matrix}\right.$$

$$\left.\begin{matrix}
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&2&0&0&0&0&0&0&0&0&0&0&2&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&2&0&0&0&0&0&0&0&0&0&2&0&1&0&0&0&0&0&0&0\\
2&0&0&1&0&0&0&0&0&0&0&0&1&0&0&2&0&0&0&0&0&0&0&0&2&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&2&0&0&0&0&0&0&0&0&0&2&0&1&0&0&0&0&0&0\\
0&2&0&0&1&0&0&0&0&0&0&0&0&1&0&0&2&0&0&0&0&0&0&0&0&2&0&0&1&0\\
0&0&2&0&0&1&0&0&0&0&0&0&0&0&1&0&0&2&0&0&0&0&0&0&0&0&2&0&0&1
\end{matrix}\right]$$

The rows of the above matrix $\mu_{(3)}^{((1,1,1),(2))}$ form a basis for the subspace $V_{(3)}^{((1,1,1),(2))}$ of the vector space $F_3^{60}$. We can consider this ten-dimensional subspace $V_{(3)}^{((1,1,1),(2))}$ as a linear (60,10,6,3)-code ([5], p.16), where 60 means that each vector of this subspace has 60 coordinates, 10 means that the dimension of this subspace is 10, 6 means that any nonzero vector of this subspace $V_{(3)}^{((1,1,1),(2))}$ has at least 6 nonzero coordinates (which means that the minimum distance of this code is 6 ([10], p.195)), and 3 means that this subspace $V_{(3)}^{((1,1,1),(2))}$ is over a field of order 3.

## 4. THE SUBSPACE $V_{(5)}^{((1,1,1),(2))}$ OF $F_5^{60}$ AS A LINEAR CODE

$f\left(Z_1^{((1,1,1),(2))}\right)$, … , $f\left(Z_{10}^{((1,1,1),(2))}\right)$ mod 5 will be:

$$f_{(5)}\left(Z_1^{((1,1,1),(2))}\right) = 4x_1^5x_2^3x_3 + x_1^3x_2^5x_3 + x_1^5x_2x_3^3 + 4x_1x_2^5x_3^3 + 4x_1^3x_2x_3^5 + x_1x_2^3x_3^5,$$

$$f_{(5)}\left(Z_2^{((1,1,1),(2))}\right) = 4x_1^5x_2^3x_4 + x_1^3x_2^5x_4 + x_1^5x_2x_4^3 + 4x_1x_2^5x_4^3 + 4x_1^3x_2x_4^5 + x_1x_2^3x_4^5,$$

$$f_{(5)}\left(Z_3^{((1,1,1),(2))}\right) = 4x_1^5x_2^3x_5 + x_1^3x_2^5x_5 + x_1^5x_2x_5^3 + 4x_1x_2^5x_5^3 + 4x_1^3x_2x_5^5 + x_1x_2^3x_5^5,$$

$$f_{(5)}\left(Z_4^{((1,1,1),(2))}\right) = 4x_1^5x_3^3x_4 + x_1^3x_3^5x_4 + x_1^5x_3x_4^3 + 4x_1x_3^5x_4^3 + 4x_1^3x_3x_4^5 + x_1x_3^3x_4^5,$$

$$f_{(5)}\left(Z_5^{((1,1,1),(2))}\right) = 4x_1^5x_3^3x_5 + x_1^3x_3^5x_5 + x_1^5x_3x_5^3 + 4x_1x_3^5x_5^3 + 4x_1^3x_3x_5^5 + x_1x_3^3x_5^5,$$

$$f_{(5)}\left(Z_6^{((1,1,1),(2))}\right) = 4x_1^5x_4^3x_5 + x_1^3x_4^5x_5 + x_1^5x_4x_5^3 + 4x_1x_4^5x_5^3 + 4x_1^3x_4x_5^5 + x_1x_4^3x_5^5,$$

$$f_{(5)}\left(Z_7^{((1,1,1),(2))}\right) = 4x_2^5x_3^3x_4 + x_2^3x_3^5x_4 + x_2^5x_3x_4^3 + 4x_2x_3^5x_4^3 + 4x_2^3x_3x_4^5 + x_2x_3^3x_4^5,$$

$$f_{(5)}\left(Z_8^{((1,1,1),(2))}\right) = 4x_2^5x_3^3x_5 + x_2^3x_3^5x_5 + x_2^5x_3x_5^3 + 4x_2x_3^5x_5^3 + 4x_2^3x_3x_5^5 + x_2x_3^3x_5^5,$$

$$f_{(5)}\left(Z_9^{((1,1,1),(2))}\right) = 4x_2^5x_4^3x_5 + x_2^3x_4^5x_5 + x_2^5x_4x_5^3 + 4x_2x_4^5x_5^3 + 4x_2^3x_4x_5^5 + x_2x_4^3x_5^5,$$

$$f_{(5)}\left(Z_{10}^{((1,1,1),(2))}\right) = 4x_3^5x_4^3x_5 + x_3^3x_4^5x_5 + x_3^5x_4x_5^3 + 4x_3x_4^5x_5^3 + 4x_3^3x_4x_5^5 + x_3x_4^3x_5^5.$$

The above polynomials give the following generator matrix for the subspace $V_{(5)}^{((1,1,1),(2))}$ (which represents the Specht module $S_{F_5}((1,1,1),(2))$) of the vector space $F_5^{60} = (GF(5))^{60}$ (which represents the module $M_{F_5}((1,1,1),(2))$):

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$\mu_{(5)}^{((1,1,1),(2))} =$

$$\begin{bmatrix} 4&1&1&4&4&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&4&1&0&0&0&0&1&4&0&0&0&0&4&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&4&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&4&0&1&0&0&0&1&0&4&0&0&0&4&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&4&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&4&0&1&0&0&0&1&0&4&0&0&0&4&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&4&0&1 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \end{bmatrix}$$

$$\begin{bmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&4&0&0&0&0&0&0&0&0&0&0&4&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&4&0&0&0&0&0&0&0&0&0&4&0&1&0&0&0&0&0&0&0 \\ 4&0&0&1&0&0&0&0&0&0&0&0&1&0&0&4&0&0&0&0&0&0&0&0&4&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&4&0&0&0&0&0&0&0&0&0&4&0&1&0&0&0&0&0&0 \\ 0&4&0&0&1&0&0&0&0&0&0&0&0&1&0&0&4&0&0&0&0&0&0&0&0&4&0&0&1&0 \\ 0&0&4&0&0&1&0&0&0&0&0&0&0&0&1&0&0&4&0&0&0&0&0&0&0&0&4&0&0&1 \end{bmatrix}$$

The rows of the above matrix $\mu_{(5)}^{((1,1,1),(2))}$ form a basis for the subspace $V_{(5)}^{((1,1,1),(2))}$ of the vector space $F_5^{60}$. We can consider this ten-dimensional subspace $V_{(5)}^{((1,1,1),(2))}$ as a linear (60,10,6,5)-code.

## 5. THE REPRESENTATION MATRICES OF THE POSITIVE TRANSPOSITION $(x_1\ x_3)$

We find the representation matrix for the positive transposition $(x_1\ x_3) \in W_5$ by letting $(x_1\ x_3)$ act on the Specht polynomials of the ten standard ((1,1,1),(2))-tableaux, $f\left(Z_1^{((1,1,1),(2))}\right)$, $f\left(Z_2^{((1,1,1),(2))}\right)$, …, $f\left(Z_{10}^{((1,1,1),(2))}\right)$ as follows:

$$\begin{aligned}(x_1\ x_3) f\left(Z_1^{((1,1,1),(2))}\right) &= -x_1 x_2^3 x_3^5 + x_1 x_2^5 x_3^3 + \\ &\quad x_1^3 x_2 x_3^5 - x_1^3 x_2^5 x_3 - \\ &\quad x_1^5 x_2 x_3^3 + x_1^5 x_2^3 x_3 \\ &= -f\left(Z_1^{((1,1,1),(2))}\right),\end{aligned}$$

$$\begin{aligned}(x_1\ x_3) f\left(Z_2^{((1,1,1),(2))}\right) &= -x_2^3 x_3^5 x_4 + x_2^5 x_3^3 x_4 + \\ &\quad x_2 x_3^5 x_4^3 - x_2^5 x_3 x_4^3 - \\ &\quad x_2 x_3^3 x_4^5 + x_2^3 x_3 x_4^5 \\ &= -f\left(Z_7^{((1,1,1),(2))}\right),\end{aligned}$$

$$\begin{aligned}(x_1\ x_3) f\left(Z_3^{((1,1,1),(2))}\right) &= -x_2^3 x_3^5 x_5 + x_2^5 x_3^3 x_5 + \\ &\quad x_2 x_3^5 x_5^3 - x_2^5 x_3 x_5^3 - \\ &\quad x_2 x_3^3 x_5^5 + x_2^3 x_3 x_5^5 \\ &= -f\left(Z_8^{((1,1,1),(2))}\right),\end{aligned}$$

$$\begin{aligned}(x_1\ x_3) f\left(Z_4^{((1,1,1),(2))}\right) &= -x_1^3 x_3^5 x_4 + x_1^5 x_3^3 x_4 + \\ &\quad x_1 x_3^5 x_4^3 - x_1^5 x_3 x_4^3 - \\ &\quad x_1 x_3^3 x_4^5 + x_1^3 x_3 x_4^5 \\ &= -f\left(Z_4^{((1,1,1),(2))}\right),\end{aligned}$$

$$\begin{aligned}(x_1\ x_3) f\left(Z_5^{((1,1,1),(2))}\right) &= -x_1^3 x_3^5 x_5 + x_1^5 x_3^3 x_5 + \\ &\quad x_1 x_3^5 x_5^3 - x_1^5 x_3 x_5^3 - \\ &\quad x_1 x_3^3 x_5^5 + x_1^3 x_3 x_5^5 \\ &= -f\left(Z_5^{((1,1,1),(2))}\right),\end{aligned}$$

$$\begin{aligned}(x_1\ x_3) f\left(Z_6^{((1,1,1),(2))}\right) &= -x_3^5 x_4^3 x_5 + x_3^3 x_4^5 x_5 + \\ &\quad x_3^5 x_4 x_5^3 - x_3 x_4^5 x_5^3 - \\ &\quad x_3^3 x_4 x_5^5 + x_3 x_4^3 x_5^5 \\ &= f\left(Z_{10}^{((1,1,1),(2))}\right),\end{aligned}$$

$$\begin{aligned}(x_1\ x_3) f\left(Z_7^{((1,1,1),(2))}\right) &= -x_1^3 x_2^5 x_4 + x_1^5 x_2^3 x_4 + \\ &\quad x_1 x_2^5 x_4^3 - x_1^5 x_2 x_4^3 - \\ &\quad x_1 x_2^3 x_4^5 + x_1^3 x_2 x_4^5 \\ &= -f\left(Z_2^{((1,1,1),(2))}\right),\end{aligned}$$

$$\begin{aligned}(x_1\ x_3) f\left(Z_8^{((1,1,1),(2))}\right) &= -x_1^3 x_2^5 x_5 + x_1^5 x_2^3 x_5 + \\ &\quad x_1 x_2^5 x_5^3 - x_1^5 x_2 x_5^3 - \\ &\quad x_1 x_2^3 x_5^5 + x_1^3 x_2 x_5^5 \\ &= -f\left(Z_3^{((1,1,1),(2))}\right),\end{aligned}$$

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$$(x_1\ x_3) f\left(Z_9^{((1,1,1),(2))}\right) = -x_2^5 x_4^3 x_5 + x_2^3 x_4^5 x_5 + x_2^5 x_4 x_5^3 - x_2 x_4^5 x_5^3 - x_2^3 x_4 x_5^5 + x_2 x_4^3 x_5^5 = f\left(Z_9^{((1,1,1),(2))}\right),$$

$$(x_1\ x_3) f\left(Z_{10}^{((1,1,1),(2))}\right) = -x_1^5 x_4^3 x_5 + x_1^3 x_4^5 x_5 + x_1^5 x_4 x_5^3 - x_1 x_4^5 x_5^3 - x_1^3 x_4 x_5^5 + x_1 x_4^3 x_5^5 = f\left(Z_6^{((1,1,1),(2))}\right),$$

which give the following representation matrix for $(x_1\ x_3)$:

$$\eta^{((1,1,1),(2))}(x_1\ x_3) = \begin{bmatrix} -1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&0&-1&0&0\\ 0&0&0&-1&0&0&0&0&0&0\\ 0&0&0&0&-1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1\\ 0&-1&0&0&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0&0&0 \end{bmatrix}$$

acting on the generator matrix $\mu^{((1,1,1),(2))}$ of the subspace $V^{((1,1,1),(2))}$ of the vector space $K^{60}$, and the representation matrix mod 3 for $(x_1\ x_3)$ will be:

$$\eta_{(3)}^{((1,1,1),(2))}(x_1\ x_3) = \begin{bmatrix} 2&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&2&0&0&0\\ 0&0&0&0&0&0&0&2&0&0\\ 0&0&0&2&0&0&0&0&0&0\\ 0&0&0&0&2&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1\\ 0&2&0&0&0&0&0&0&0&0\\ 0&0&2&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0&0&0 \end{bmatrix}$$

acting on the generator matrix $\mu_{(3)}^{((1,1,1),(2))}$ of the subspace $V_{(3)}^{((1,1,1),(2))}$ of the vector space $F_3^{60}$, and the representation matrix mod 5 for $(x_1\ x_3)$ will be:

$$\eta_{(5)}^{((1,1,1),(2))}(x_1\ x_3) = \begin{bmatrix} 4&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&4&0&0&0\\ 0&0&0&0&0&0&0&4&0&0\\ 0&0&0&4&0&0&0&0&0&0\\ 0&0&0&0&4&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1\\ 0&4&0&0&0&0&0&0&0&0\\ 0&0&4&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0&0&0 \end{bmatrix}$$

acting on the generator matrix $\mu_{(5)}^{((1,1,1),(2))}$ of the subspace $V_{(5)}^{((1,1,1),(2))}$ of the vector space $F_5^{60}$.

## 6. THE REPRESENTATION MATRICES OF THE NEGATIVE TRANSPOSITION $\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix}$

We compute the representation matrix for the negative transposition $\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} \in W_5$, by letting $\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix}$ act on the Specht polynomials of the ten standard ((1,1,1),(2))-tableaux, $f\left(Z_1^{((1,1,1),(2))}\right)$, $f\left(Z_2^{((1,1,1),(2))}\right)$, … , $f\left(Z_{10}^{((1,1,1),(2))}\right)$, as follows:

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_1^{((1,1,1),(2))}\right) = x_1^5 x_2^3 x_3 - x_1^3 x_2^5 x_3 - x_1^5 x_2 x_3^3 + x_1 x_2^5 x_3^3 + x_1^3 x_2 x_3^5 - x_1 x_2^3 x_3^5 = -f\left(Z_1^{((1,1,1),(2))}\right),$$

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_2^{((1,1,1),(2))}\right) = -x_1^5 x_2^3 x_4 + x_1^3 x_2^5 x_4 + x_1^5 x_2 x_4^3 - x_1 x_2^5 x_4^3 - x_1^3 x_2 x_4^5 + x_1 x_2^3 x_4^5 = f\left(Z_2^{((1,1,1),(2))}\right),$$

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_3^{((1,1,1),(2))}\right) = -x_1^5 x_2^3 x_5 + x_1^3 x_2^5 x_5 + x_1^5 x_2 x_5^3 - x_1 x_2^5 x_5^3 - x_1^3 x_2 x_5^5 + x_1 x_2^3 x_5^5 = f\left(Z_3^{((1,1,1),(2))}\right),$$

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_4^{((1,1,1),(2))}\right) = x_1^5 x_3^3 x_4 - x_1^3 x_3^5 x_4 - x_1^5 x_3 x_4^3 + x_1 x_3^5 x_4^3 + x_1^3 x_3 x_4^5 - x_1 x_3^3 x_4^5 = -f\left(Z_4^{((1,1,1),(2))}\right),$$

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_5^{((1,1,1),(2))}\right) = x_1^5 x_3^3 x_5 - x_1^3 x_3^5 x_5 - x_1^5 x_3 x_5^3 + x_1 x_3^5 x_5^3 + x_1^3 x_3 x_5^5 - x_1 x_3^3 x_5^5 = -f\left(Z_5^{((1,1,1),(2))}\right),$$

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_6^{((1,1,1),(2))}\right) = -x_1^5 x_4^3 x_5 + x_1^3 x_4^5 x_5 + x_1^5 x_4 x_5^3 - x_1 x_4^5 x_5^3 - x_1^3 x_4 x_5^5 + x_1 x_4^3 x_5^5 = f\left(Z_6^{((1,1,1),(2))}\right),$$

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_7^{((1,1,1),(2))}\right) = x_2^5 x_3^3 x_4 - x_2^3 x_3^5 x_4 - x_2^5 x_3 x_4^3 + x_2 x_3^5 x_4^3 + x_2^3 x_3 x_4^5 - x_2 x_3^3 x_4^5 = -f\left(Z_7^{((1,1,1),(2))}\right),$$

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_8^{((1,1,1),(2))}\right) = x_2^5 x_3^3 x_5 - x_2^3 x_3^5 x_5 - x_2^5 x_3 x_5^3 + x_2 x_3^5 x_5^3 + x_2^3 x_3 x_5^5 - x_2 x_3^3 x_5^5 = -f\left(Z_8^{((1,1,1),(2))}\right),$$

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_9^{((1,1,1),(2))}\right) = -x_2^5 x_4^3 x_5 + x_2^3 x_4^5 x_5 + x_2^5 x_4 x_5^3 - x_2 x_4^5 x_5^3 - x_2^3 x_4 x_5^5 + x_2 x_4^3 x_5^5 = f\left(Z_9^{((1,1,1),(2))}\right),$$

$$\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} f\left(Z_{10}^{((1,1,1),(2))}\right) = x_3^5 x_4^3 x_5 - x_3^3 x_4^5 x_5 - x_3^5 x_4 x_5^3 + x_3 x_4^5 x_5^3 + x_3^3 x_4 x_5^5 - x_3 x_4^3 x_5^5 = -f\left(Z_{10}^{((1,1,1),(2))}\right),$$

which give the following representation matrix for $\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix}$:

$$\eta^{((1,1,1),(2))}\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

acting on the generator matrix $\mu^{((1,1,1),(2))}$ of the subspace $V^{((1,1,1),(2))}$ of the vector space $K^{60}$, and the representation matrix mod 3 for $\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix}$ will be:

$$\eta_{(3)}^{((1,1,1),(2))}\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \end{bmatrix}$$

acting on the generator matrix $\mu_{(3)}^{((1,1,1),(2))}$ of the subspace $V_{(3)}^{((1,1,1),(2))}$ of the vector space $F_3^{60}$, and the representation matrix mod 5 for $\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix}$ will be

$$\eta_{(5)}^{((1,1,1),(2))}\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

acting on the generator matrix $\mu_{(5)}^{((1,1,1),(2))}$ of the subspace $V_{(5)}^{((1,1,1),(2))}$ of the vector space $F_5^{60}$.

In the same way as above, we can find the representation matrix for each positive transposition and each negative transposition, and hence we can find the representation matrix for each permutation

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$w \in W_5$ since any permutation $w \in W_5$ can be written as a product of positive transpositions followed by a product of negative transpositions ([4], p.3), and the representation of the permutation $w = w_1 \cdot w_2 \cdots w_m$ will be the product of the representation matrix of $w_1$ by the representation matrix of $w_2$ multiplied by $\cdots$ multiplied by the representation matrix of $w_m$ (i.e., $\eta^{((1,1,1),(2))}(w) = \eta^{((1,1,1),(2))}(w_1) \cdot \eta^{((1,1,1),(2))}(w_2) \cdots \eta^{((1,1,1),(2))}(w_m)$), and the same way will be applied for the representation matrix of $w$ mod $p$ for each prime $p = 3,5$.

**Example 6.1.** Let $w = (x_1 \;\; -x_3 \;\; -x_1 \;\; x_3)$. Then $w = \begin{pmatrix} x_3 \\ -x_3 \end{pmatrix}(x_1 \;\; x_3)$, and the representation matrix of $w$ is:

$$\eta^{((1,1,1),(2))}(x_1 \;\; -x_3 \;\; -x_1 \;\; x_3) = \eta^{((1,1,1),(2))}\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} \cdot \eta^{((1,1,1),(2))}(x_1 \;\; x_3)$$

$$= \begin{bmatrix} -1&0&0&0&0&0&0&0&0&0\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&-1&0&0&0&0&0&0\\ 0&0&0&0&-1&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&0&-1&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&-1 \end{bmatrix} \cdot \begin{bmatrix} -1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&0&-1&0&0\\ 0&0&0&-1&0&0&0&0&0&0\\ 0&0&0&0&-1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1\\ 0&-1&0&0&0&0&0&0&0&0\\ 0&0&-1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&1&0&0&0&0 \end{bmatrix}$$

$$= \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&-1&0&0&0\\ 0&0&0&0&0&0&0&-1&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&-1&0&0&0&0 \end{bmatrix},$$

acting on the generator matrix $\mu^{((1,1,1),(2))}$ of the subspace $V^{((1,1,1),(2))}$ of the vector space $K^{60}$ (where $K$ is a field of characteristic 0), while the representation matrix of $w$ mod 3 will be:

$$\eta_{(3)}^{((1,1,1),(2))}(x_1 \;\; -x_3 \;\; -x_1 \;\; x_3) = \eta_{(3)}^{((1,1,1),(2))}\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} \cdot \eta_{(3)}^{((1,1,1),(2))}(x_1 \;\; x_3)$$

$$= \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&2&0&0&0\\ 0&0&0&0&0&0&0&2&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&2&0&0&0&0 \end{bmatrix},$$

acting on the generator matrix $\mu_{(3)}^{((1,1,1),(2))}$ of the subspace $V_{(3)}^{((1,1,1),(2))}$ of the vector space $F_3^{60}$, and the representation matrix of $w$ mod 5 will be:

$$\eta_{(5)}^{((1,1,1),(2))}(x_1 \;\; -x_3 \;\; -x_1 \;\; x_3) = \eta_{(5)}^{((1,1,1),(2))}\begin{pmatrix} x_3 \\ -x_3 \end{pmatrix} \cdot \eta_{(5)}^{((1,1,1),(2))}(x_1 \;\; x_3)$$

$$= \begin{bmatrix} 1&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&4&0&0&0\\ 0&0&0&0&0&0&0&4&0&0\\ 0&0&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&1\\ 0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&4&0&0&0&0 \end{bmatrix},$$

acting on the generator matrix $\mu_{(5)}^{((1,1,1),(2))}$ of the subspace $V_{(5)}^{((1,1,1),(2))}$ of the vector space $F_5^{60}$.

## 7. THE *p*-MODULAR REPRESENTATIONS FOR THE SPECHT MODULES OVER $KW_5$ CORRESPONDING TO ALL PAIRS OF PARTITIONS $(\lambda,\mu)$ OF 5 AS LINEAR CODES WHEN THE FIELD $K = F_p$, *p* IS PRIME NUMBER AND $p \geq 3$

In this section, we will find the linear codes of the representations of the Specht modules corresponding to all pairs of partitions $(\lambda,\mu)$ of 5, when $p \geq 3$ as follows:

1) For the pair of partitions $((5),(\ ))$, we have that $m(M) = \dim_{F_p} M_{F_p}((5),(\ )) = \frac{5!}{5!} = 1$ by theorem 1.3, and we have that $k_p$ = $\dim_{F_p} S_{F_p}((5),(\ ))$ = $\frac{5!}{5\cdot4\cdot3\cdot2\cdot1}$ = 1 by theorem 1.13, and the minimum distance (which is equal the number of monomials that each Specht polynomial consists) is $d_p = 1!\cdot1!\cdot1!\cdot1!\cdot1! = 1$ by corollary 1.9, since the conjugate of the pair of partitions $((5),(\ ))$ is the pair of partitions $((1,1,1,1,1),(\ ))$. Therefore, the subspace $V_{(p)}^{((5),(\ ))}$ (which represents the Specht module $S_{F_p}((5),(\ ))$) can be considered as a linear (1,1, 1, *p*)-code, $\forall\ p \geq 3$.
2) For the pair of partitions $((4,1),(\ ))$, we have that $m(M) = \dim_{F_p} M_{F_p}((4,1),(\ )) = \frac{5!}{4!\cdot1!}$ =5, $k_p = \dim_{F_p} S_{F_p}((4,1),(\ )) = \frac{5!}{5\cdot3\cdot2\cdot1\cdot1}$ $= 4$, and the minimum distance is $d_p = 2!\cdot1!\cdot1!\cdot1! = 2$, since $((4,1),(\ ))' = ((2,1,1,1),(\ ))$. Therefore, the subspace $V_{(p)}^{((4,1),(\ ))}$ (which represents the Specht module $S_{F_p}((4,1),(\ ))$) can be considered as a linear (5, 4, 2, *p*)-code, $\forall\ p \geq 3$.
3) For the pair of partitions $((3,2),(\ ))$, we have that $m(M) = \dim_{F_p} M_{F_p}((3,2),(\ )) = \frac{5!}{3!\cdot2!}$ = 10, $k_p = \dim_{F_p} S_{F_p}((3,2),(\ )) = \frac{5!}{4\cdot3\cdot1\cdot2\cdot1}$ = 5, and the minimum distance is $d_p = 2!\cdot2!\cdot1! = 4$, since $((3,2),(\ ))' = ((2,2,1),(\ ))$. Therefore, the subspace $V_{(p)}^{((3,2),(\ ))}$ (which represents the Specht module $S_{F_p}((3,2),(\ ))$) can be considered as a linear (10, 5, 4, *p*)-code, $\forall\ p \geq 3$.
4) For the pair of partitions $((3,1,1),(\ ))$, we have that $m(M) = \dim_{F_p} M_{F_p}((3,1,1),(\ )) = \frac{5!}{3!\cdot1!\cdot1!} = 20$, $k_p = \dim_{F_p} S_{F_p}((3,1,1),(\ )) = \frac{5!}{5\cdot2\cdot1\cdot2\cdot1} = 6$, and the minimum distance is $d_p = 3!\cdot1!\cdot1! = 6$, since $((3,1,1),(\ ))' = ((3,1,1),(\ ))$. Therefore, the subspace $V_{(p)}^{((3,1,1),(\ ))}$ (which represents the Specht module $S_{F_p}((3,1,1),(\ ))$) can be considered as a linear (20, 6, 6, *p*)-code, $\forall\ p \geq 3$.
5) For the pair of partitions $((2,2,1),(\ ))$, we have that $m(M) = \dim_{F_p} M_{F_p}((2,2,1),(\ )) = \frac{5!}{2!\cdot2!\cdot1!} = 30$, $k_p = \dim_{F_p} S_{F_p}((2,2,1),(\ ))$ $= \frac{5!}{4\cdot2\cdot3\cdot1\cdot1} = 5$, and the minimum distance is $d_p = 3!\cdot2! = 12$, since $((2,2,1),(\ ))' = ((3,2),(\ ))$. Therefore, the subspace $V_{(p)}^{((2,2,1),(\ ))}$ (which represents the Specht module $S_{F_p}((2,2,1),(\ ))$) can be considered as a linear (30, 5, 12, *p*)-code, $\forall\ p \geq 3$.

6) For the pair of partitions $((2,1,1,1),(\ ))$, we have that $m(M) = \dim_{F_p} M_{F_p}((2,1,1,1),(\ ))$ $= \frac{5!}{2!\cdot 1!\cdot 1!\cdot 1!} = 60$, $k_p = \dim_{F_p} S_{F_p}((2,1,1,1),(\ ))$ $= \frac{5!}{5\cdot 1\cdot 3\cdot 2\cdot 1} = 4$, and the minimum distance is $d_p = 4!\cdot 1! = 24$, since $((2,1,1,1),(\ ))' = ((4,1),(\ ))$. Therefore, the subspace $V_{(p)}^{((2,1,1,1),(\ ))}$ (which represents the Specht module $S_{F_p}((2,1,1,1),(\ ))$) can be considered as a linear (60, 4, 24, $p$)-code, $\forall\ p \geq 3$.

7) For the pair of partitions $((1,1,1,1,1),(\ ))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1,1,1,1,1),(\ )) =$ $\frac{5!}{1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!} = 120$, $k_p = \dim_{F_p} S_{F_p}((1,1,1,1,1),(\ )) = \frac{5!}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 1$, and the minimum distance is $d_p = 5! = 120$, since $((1,1,1,1,1),(\ ))' = ((5),(\ ))$. Therefore, the subspace $V_{(p)}^{((1,1,1,1,1),(\ ))}$ (which represents the Specht module $S_{F_p}((1,1,1,1,1),(\ ))$) can be considered as a linear (120, 1, 120, $p$)-code, $\forall\ p \geq 3$.

8) For the pair of partitions $((4),(1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((4),(1)) = \frac{5!}{4!\cdot 1!} =$ 5, $k_p = \dim_{F_p} S_{F_p}((4),(1)) = \frac{5!}{4\cdot 3\cdot 2\cdot 1\cdot 1} =$ 5, and the minimum distance is $d_p = 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1! = 1$, since $((4),(1))' = ((1,1,1,1),(1))$. Therefore, the subspace $V_{(p)}^{((4),(1))}$ (which represents the Specht module $S_{F_p}((4),(1))$) can be considered as a linear (5, 5, 1, $p$)-code, $\forall\ p \geq 3$.

9) For the pair of partitions $((3,1),(1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((3,1),(1)) =$ $\frac{5!}{3!\cdot 1!\cdot 1!} = 20$, $k_p = \dim_{F_p} S_{F_p}((3,1),(1)) =$ $\frac{5!}{4\cdot 2\cdot 1\cdot 1\cdot 1} = 15$, and the minimum distance is $d_p = 2!\cdot 1!\cdot 1!\cdot 1! = 2$, since $((3,1),(1))' = ((2,1,1),(1))$. Therefore, the subspace $V_{(p)}^{((3,1),(1))}$ (which represents the Specht module $S_{F_p}((3,1),(1))$) can be considered as a linear (20, 15, 2, $p$)-code, $\forall\ p \geq 3$.

10) For the pair of partitions $((2,2),(1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((2,2),(1)) =$ $\frac{5!}{2!\cdot 2!\cdot 1!} = 30$, $k_p = \dim_{F_p} S_{F_p}((2,2),(1)) =$ $\frac{5!}{3\cdot 2\cdot 2\cdot 1\cdot 1} = 10$, and the minimum distance is $d_p = 2!\cdot 2!\cdot 1! = 4$, since $((2,2),(1))' = ((2,2),(1))$. Therefore, the subspace $V_{(p)}^{((2,2),(1))}$ (which represents the Specht module $S_{F_p}((2,2),(1))$) can be considered as a linear (30, 10, 4, $p$)-code, $\forall\ p \geq 3$.

11) For the pair of partitions $((2,1,1),(1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((2,1,1),(1)) =$ $\frac{5!}{2!\cdot 1!\cdot 1!\cdot 1!} = 60$, $k_p = \dim_{F_p} S_{F_p}((2,1,1),(1)) =$ $\frac{5!}{4\cdot 1\cdot 2\cdot 1\cdot 1} = 15$, and the minimum distance is $d_p = 3!\cdot 1!\cdot 1! = 6$, since $((2,1,1),(1))' = ((3,1),(1))$. Therefore, the subspace $V_{(p)}^{((2,1,1),(1))}$ (which represents the Specht module $S_{F_p}((2,1,1),(1))$) can be considered as a linear (60, 15, 6, $p$)-code, $\forall\ p \geq 3$.

12) For the pair of partitions $((1,1,1,1),(1))$, we

have that $m(M) = \dim_{F_p} M_{F_p}((1,1,1,1),(1)) = \frac{5!}{1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!} = 120$, $k_p = \dim_{F_p} S_{F_p}((1,1,1,1),(1)) = \frac{5!}{4\cdot 3\cdot 2\cdot 1\cdot 1} = 5$, and the minimum distance is $d_p = 4!\cdot 1! = 24$, since $((1,1,1,1),(1))' = ((4),(1))$. Therefore, the subspace $V_{(p)}^{((1,1,1,1),(1))}$ (which represents the Specht module $S_{F_p}((1,1,1,1),(1))$) can be considered as a linear (120, 5, 24, $p$)-code, $\forall\ p \geq 3$.

13) For the pair of partitions $((3),(2))$, we have that $m(M) = \dim_{F_p} M_{F_p}((3),(2)) = \frac{5!}{3!\cdot 2!} = 10$, $k_p = \dim_{F_p} S_{F_p}((3),(2)) = \frac{5!}{3\cdot 2\cdot 1\cdot 2\cdot 1} = 10$, and the minimum distance is $d_p = 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1! = 1$, since $((3),(2))' = ((1,1,1),(1,1))$. Therefore, the subspace $V_{(p)}^{((3),(2))}$ (which represents the Specht module $S_{F_p}((3),(2))$) can be considered as a linear (10, 10, 1, $p$)-code, $\forall\ p \geq 3$.

14) For the pair of partitions $((2,1),(2))$, we have that $m(M) = \dim_{F_p} M_{F_p}((2,1),(2)) = \frac{5!}{2!\cdot 1!\cdot 2!} = 30$, $k_p = \dim_{F_p} S_{F_p}((2,1),(2)) = \frac{5!}{3\cdot 1\cdot 1\cdot 2\cdot 1} = 20$, and the minimum distance is $d_p = 2!\cdot 1!\cdot 1!\cdot 1! = 2$, since $((2,1),(2))' = ((2,1),(1,1))$. Therefore, the subspace $V_{(p)}^{((2,1),(2))}$ (which represents the Specht module $S_{F_p}((2,1),(2))$) can be considered as a linear (30, 20, 2, $p$)-code, $\forall\ p \geq 3$.

15) For the pair of partitions $((1,1,1),(2))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1,1,1),(2)) = \frac{5!}{1!\cdot 1!\cdot 1!\cdot 2!} = 60$, $k_p = \dim_{F_p} S_{F_p}((1,1,1),(2)) = \frac{5!}{3\cdot 2\cdot 1\cdot 2\cdot 1} = 10$, and the minimum distance is $d_p = 3!\cdot 1!\cdot 1! = 6$, since $((1,1,1),(2))' = ((3),(1,1))$. Therefore, the subspace $V_{(p)}^{((1,1,1),(2))}$ (which represents the Specht module $S_{F_p}((1,1,1),(2))$) can be considered as a linear (60, 10, 6, $p$)-code, $\forall\ p \geq 3$.

16) For the pair of partitions $((3),(1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((3),(1,1)) = \frac{5!}{3!\cdot 1!\cdot 1!} = 20$, $k_p = \dim_{F_p} S_{F_p}((3),(1,1)) = \frac{5!}{3\cdot 2\cdot 1\cdot 2\cdot 1} = 10$, and the minimum distance is $d_p = 1!\cdot 1!\cdot 1!\cdot 2! = 2$, since $((3),(1,1))' = ((1,1,1),(2))$. Therefore, the subspace $V_{(p)}^{((3),(1,1))}$ (which represents the Specht module $S_{F_p}((3),(1,1))$) can be considered as a linear (20, 10, 2, $p$)-code, $\forall\ p \geq 3$.

17) For the pair of partitions $((2,1),(1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((2,1),(1,1)) = \frac{5!}{2!\cdot 1!\cdot 1!\cdot 1!} = 60$, $k_p = \dim_{F_p} S_{F_p}((2,1),(1,1)) = \frac{5!}{3\cdot 1\cdot 1\cdot 2\cdot 1} = 20$, and the minimum distance is $d_p = 2!\cdot 1!\cdot 2! = 4$, since $((2,1),(1,1))' = ((2,1),(2))$. Therefore, the subspace $V_{(p)}^{((2,1),(1,1))}$ (which represents the Specht module $S_{F_p}((2,1),(1,1))$) can be considered as a linear (60, 20, 4, $p$)-code, $\forall\ p \geq 3$.

18) For the pair of partitions $((1,1,1),(1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1,1,1),(1,1))$

$= \dfrac{5!}{1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!}$ =120, $k_p = \dim_{F_p} S_{F_p}((1,1,1)$ $,(1,1)) = \dfrac{5!}{3\cdot 2\cdot 1\cdot 2\cdot 1} = 10$, and the minimum distance is $d_p = 3!\cdot 2! = 12$, since $((1,1,1),(1,1))' = ((3),(2))$. Therefore, the subspace $V_{(p)}^{((1,1,1),(1,1))}$ (which represents the Specht module $S_{F_p}((1,1,1),(1,1))$ ) can be considered as a linear (120, 10, 12, $p$)-code, $\forall\, p \ge 3$.

19) For the pair of partitions $((2),(3))$, we have that $m(M) = \dim_{F_p} M_{F_p}((2),(3)) = \dfrac{5!}{2!\cdot 3!} = 10$, $k_p = \dim_{F_p} S_{F_p}((2),(3)) = \dfrac{5!}{2\cdot 1\cdot 3\cdot 2\cdot 1} = 10$, and $d_p = 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1! = 1$, since $((2),(3))' = ((1,1),(1,1,1))$. Therefore, the subspace $V_{(p)}^{((2),(3))}$ (which represents the Specht module $S_{F_p}((2),(3))$) can be considered as a linear (10, 10, 1, $p$)-code, $\forall\, p \ge 3$.

20) For the pair of partitions $((1,1),(3))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1,1),(3)) = \dfrac{5!}{1!\cdot 1!\cdot 3!} = 20$, $k_p = \dim_{F_p} S_{F_p}((1,1),(3)) = \dfrac{5!}{2\cdot 1\cdot 3\cdot 2\cdot 1} = 10$, and $d_p = 2!\cdot 1!\cdot 1!\cdot 1! = 2$, since $((1,1),(3))' = ((2),(1,1,1))$. Therefore, the subspace $V_{(p)}^{((1,1),(3))}$ (which represents the Specht module $S_{F_p}((1,1),(3))$) can be considered as a linear (20, 10, 2, $p$)-code, $\forall\, p \ge 3$.

21) For the pair of partitions $((2),(2,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((2),(2,1)) = \dfrac{5!}{2!\cdot 2!\cdot 1!} = 30$, $k_p = \dim_{F_p} S_{F_p}((2),(2,1)) = \dfrac{5!}{2\cdot 1\cdot 3\cdot 1\cdot 1} = 20$, and $d_p = 1!\cdot 1!\cdot 2!\cdot 1! = 2$, since $((2),(2,1))' = ((1,1),(2,1))$. Therefore, the subspace $V_{(p)}^{((2),(2,1))}$ (which represents the Specht module $S_{F_p}((2),(2,1))$) can be considered as a linear (30, 20, 2, $p$)-code, $\forall\, p \ge 3$.

22) For the pair of partitions $((1,1),(2,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1,1),(2,1)) = \dfrac{5!}{1!\cdot 1!\cdot 2!\cdot 1!} = 60$, $k_p = \dim_{F_p} S_{F_p}((1,1),(2,1)) = \dfrac{5!}{2\cdot 1\cdot 3\cdot 1\cdot 1} = 20$, and $d_p = 2!\cdot 2!\cdot 1! = 4$, since $((1,1),(2,1))' = ((2),(2,1))$. Therefore, the subspace $V_{(p)}^{((1,1),(2,1))}$ (which represents the Specht module $S_{F_p}((1,1),(2,1))$ ) can be considered as a linear (60, 20, 4, $p$)-code, $\forall\, p \ge 3$.

23) For the pair of partitions $((2),(1,1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((2),(1,1,1)) = \dfrac{5!}{2!\cdot 1!\cdot 1!\cdot 1!} = 60$, $k_p = \dim_{F_p} S_{F_p}((2),(1,1,1)) = \dfrac{5!}{2\cdot 1\cdot 3\cdot 2\cdot 1} = 10$, and $d_p = 1!\cdot 1!\cdot 3! = 6$, since $((2),(1,1,1))' = ((1,1),(3))$. Therefore, the subspace $V_{(p)}^{((2),(1,1,1))}$ (which represents the Specht module $S_{F_p}((2),(1,1,1))$ ) can be considered as a linear (60, 10, 6, $p$)-code, $\forall\, p \ge 3$.

24) For the pair of partitions $((1,1),(1,1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1,1),(1,1,1)) = \dfrac{5!}{1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!} = 120$, $k_p = \dim_{F_p} S_{F_p}((1,1),(1,1,1)) = \dfrac{5!}{2\cdot 1\cdot 3\cdot 2\cdot 1} =$

10, and $d_p = 2!\cdot 3! = 12$, since $((1,1),(1,1,1))' = ((2),(3))$. Therefore, the subspace $V_{(p)}^{((1,1),(1,1,1))}$ (which represents the Specht module $S_{F_p}((1,1),(1,1,1))$) can be considered as a linear (120, 10, 12, $p$)-code, $\forall\ p \geq 3$.

25) For the pair of partitions $((1),(4))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1),(4)) = \frac{5!}{1!\cdot 4!} = 5$, $k_p = \dim_{F_p} S_{F_p}((1),(4)) = \frac{5!}{1\cdot 4\cdot 3\cdot 2\cdot 1} = 5$, and $d_p = 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1! = 1$, since $((1),(4))' = ((1),(1,1,1,1))$. Therefore, the subspace $V_{(p)}^{((1),(4))}$ (which represents the Specht module $S_{F_p}((1),(4))$) can be considered as a linear (5, 5, 1, $p$)-code, $\forall\ p \geq 3$.

26) For the pair of partitions $((1),(3,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1),(3,1)) = \frac{5!}{1!\cdot 3!\cdot 1!} = 20$, $k_p = \dim_{F_p} S_{F_p}((1),(3,1)) = \frac{5!}{1\cdot 4\cdot 2\cdot 1\cdot 1} = 15$, and $d_p = 1!\cdot 2!\cdot 1!\cdot 1! = 2$, since $((1),(3,1))' = ((1),(2,1,1))$. Therefore, the subspace $V_{(p)}^{((1),(3,1))}$ (which represents the Specht module $S_{F_p}((1),(3,1))$) can be considered as a linear (20, 15, 2, $p$)-code, $\forall\ p \geq 3$.

27) For the pair of partitions $((1),(2,2))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1),(2,2)) = \frac{5!}{1!\cdot 2!\cdot 2!} = 30$, $k_p = \dim_{F_p} S_{F_p}((1),(2,2)) = \frac{5!}{1\cdot 3\cdot 2\cdot 2\cdot 1} = 10$, and $d_p = 1!\cdot 2!\cdot 2! = 4$, since $((1),(2,2))' = ((1),(2,2))$. Therefore, the subspace $V_{(p)}^{((1),(2,2))}$ (which represents the Specht module $S_{F_p}((1),(2,2))$) can be considered as a linear (30, 10, 4, $p$)-code, $\forall\ p \geq 3$.

28) For the pair of partitions $((1),(2,1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1),(2,1,1)) = \frac{5!}{1!\cdot 2!\cdot 1!\cdot 1!} = 60$, $k_p = \dim_{F_p} S_{F_p}((1),(2,1,1)) = \frac{5!}{1\cdot 4\cdot 1\cdot 2\cdot 1} = 15$, and $d_p = 1!\cdot 3!\cdot 1! = 6$, since $((1),(2,1,1))' = ((1),(3,1))$. Therefore, the subspace $V_{(p)}^{((1),(2,1,1))}$ (which represents the Specht module $S_{F_p}((1),(2,1,1))$) can be considered as a linear (60, 15, 6, $p$)-code, $\forall\ p \geq 3$.

29) For the pair of partitions $((1),(1,1,1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((1),(1,1,1,1)) = \frac{5!}{1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!} = 120$, $k_p = \dim_{F_p} S_{F_p}((1),(1,1,1,1)) = \frac{5!}{1\cdot 4\cdot 3\cdot 2\cdot 1} = 5$, and $d_p = 1!\cdot 4! = 24$, since $((1),(1,1,1,1))' = ((1),(4))$. Therefore, the subspace $V_{(p)}^{((1),(1,1,1,1))}$ (which represents the Specht module $S_{F_p}((1),(1,1,1,1))$) can be considered as a linear (120, 5, 24, $p$)-code, $\forall\ p \geq 3$.

30) For the pair of partitions $((\ ),(5))$, we have that $m(M) = \dim_{F_p} M_{F_p}((\ ),(5)) = \frac{5!}{5!} = 1$, $k_p = \dim_{F_p} S_{F_p}((\ ),(5)) = \frac{5!}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 1$, and $d_p = 1!\cdot 1!\cdot 1!\cdot 1!\cdot 1! = 1$, since $((\ ),(5))' = ((\ ),(1,1,1,1,1))$. Therefore, the subspace $V_{(p)}^{((\ ),(5))}$ (which represents the

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Specht module $S_{F_p}((\ ),(5)))$ can be considered as a linear (1, 1, 1, $p$)-code, $\forall\ p \geq 3$.

31) For the pair of partitions $((\ ),(4,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((\ ),(4,1)) = \dfrac{5!}{4!\cdot 1!} = 5$, $k_p = \dim_{F_p} S_{F_p}((\ ),(4,1)) = \dfrac{5!}{5\cdot 3\cdot 2\cdot 1} = 4$, and $d_p = 2!\cdot 1!\cdot 1!\cdot 1! = 2$, since $((\ ),(4,1))' = ((\ ),(2,1,1,1))$. Therefore, the subspace $V_{(p)}^{((\ ),(4,1))}$ (which represents the Specht module $S_{F_p}((\ ),(4,1)))$ can be considered as a linear (5, 4, 2, $p$)-code, $\forall\ p \geq 3$.

32) For the pair of partitions $((\ ),(3,2))$, we have that $m(M) = \dim_{F_p} M_{F_p}((\ ),(3,2)) = \dfrac{5!}{3!\cdot 2!} = 10$, $k_p = \dim_{F_p} S_{F_p}((\ ),(3,2)) = \dfrac{5!}{4\cdot 3\cdot 1\cdot 2\cdot 1} = 5$, and $d_p = 2!\cdot 2!\cdot 1! = 4$, since $((\ ),(3,2))' = ((\ ),(2,2,1))$. Therefore, the subspace $V_{(p)}^{((\ ),(3,2))}$ (which represents the Specht module $S_{F_p}((\ ),(3,2)))$ can be considered as a linear (10, 5, 4, $p$)-code, $\forall\ p \geq 3$.

33) For the pair of partitions $((\ ),(3,1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((\ ),(3,1,1)) = \dfrac{5!}{3!\cdot 1!\cdot 1!} = 20$, $k_p = \dim_{F_p} S_{F_p}((\ ),(3,1,1)) = \dfrac{5!}{5\cdot 2\cdot 1\cdot 2\cdot 1} = 6$, and $d_p = 3!\cdot 1!\cdot 1! = 6$, since $((\ ),(3,1,1))' = ((\ ),(3,1,1))$. Therefore, the subspace $V_{(p)}^{((\ ),(3,1,1))}$ (which represents the Specht module $S_{F_p}((\ ),(3,1,1)))$ can be considered as a linear (20, 6, 6, $p$)-code, $\forall\ p \geq 3$.

34) For the pair of partitions $((\ ),(2,2,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((\ ),(2,2,1)) = \dfrac{5!}{2!\cdot 2!\cdot 1!} = 30$, $k_p = \dim_{F_p} S_{F_p}((\ ),(2,2,1)) = \dfrac{5!}{4\cdot 2\cdot 3\cdot 1\cdot 1} = 5$, and $d_p = 3!\cdot 2! = 12$, since $((\ ),(2,2,1))' = ((\ ),(3,2))$. Therefore, the subspace $V_{(p)}^{((\ ),(2,2,1))}$ (which represents the Specht module $S_{F_p}((\ ),(2,2,1)))$ can be considered as a linear (30, 5, 12, $p$)-code, $\forall\ p \geq 3$.

35) For the pair of partitions $((\ ),(2,1,1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((\ ),(2,1,1,1)) = \dfrac{5!}{2!\cdot 1!\cdot 1!\cdot 1!} = 60$, $k_p = \dim_{F_p} S_{F_p}((\ ),(2,1,1,1)) = \dfrac{5!}{5\cdot 1\cdot 3\cdot 2\cdot 1} = 4$, and $d_p = 4!\cdot 1! = 24$, since $((\ ),(2,1,1,1))' = ((\ ),(4,1))$. Therefore, the subspace $V_{(p)}^{((\ ),(2,1,1,1))}$ (which represents the Specht module $S_{F_p}((\ ),(2,1,1,1)))$ can be considered as a linear (60, 4, 24, $p$)-code, $\forall\ p \geq 3$.

36) For the pair of partitions $((\ ),(1,1,1,1,1))$, we have that $m(M) = \dim_{F_p} M_{F_p}((\ ),(1,1,1,1,1)) = \dfrac{5!}{1!\cdot 1!\cdot 1!\cdot 1!\cdot 1!} = 120$, $k_p = \dim_{F_p} S_{F_p}((\ ),(1,1,1,1,1)) = \dfrac{5!}{5\cdot 4\cdot 3\cdot 2\cdot 1} = 1$, and $d_p = 5! = 120$, since $((\ ),(1,1,1,1,1))' = ((\ ),(5))$. Therefore, the subspace $V_{(p)}^{((\ ),(1,1,1,1,1))}$ (which represents the Specht module $S_{F_p}((\ ),(1,1,1,1,1)))$ can be considered as a linear (120, 1, 120, $p$)-code, $\forall\ p \geq 3$.

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Finally, we summarize the above linear codes in the following Table 1:

**Table 1**

| No. | $(\lambda,\mu)$ of $n$=5 | $(\lambda,\mu)'=(\lambda',\mu')$ of $n$=5 | $m(M)$ | $k_p, p\geq 3$ | $d_p, p\geq 3$ |
|---|---|---|---|---|---|
| 1 | ((5),( )) | ((1,1,1,1,1),( )) | 1 | 1 | 1 |
| 2 | ((4,1),( )) | ((2,1,1,1),( )) | 5 | 4 | 2 |
| 3 | ((3,2),( )) | ((2,2,1),( )) | 10 | 5 | 4 |
| 4 | ((3,1,1),( )) | ((3,1,1),( )) | 20 | 6 | 6 |
| 5 | ((2,2,1),( )) | ((3,2),( )) | 30 | 5 | 12 |
| 6 | ((2,1,1,1),( )) | ((4,1),( )) | 60 | 4 | 24 |
| 7 | ((1,1,1,1,1),( )) | ((5),( )) | 120 | 1 | 120 |
| 8 | ((4),(1)) | ((1,1,1,1),(1)) | 5 | 5 | 1 |
| 9 | ((3,1),(1)) | ((2,1,1),(1)) | 20 | 15 | 2 |
| 10 | ((2,2),(1)) | ((2,2),(1)) | 30 | 10 | 4 |
| 11 | ((2,1,1),(1)) | ((3,1),(1)) | 60 | 15 | 6 |
| 12 | ((1,1,1,1),(1)) | ((4),(1)) | 120 | 5 | 24 |
| 13 | ((3),(2)) | ((1,1,1),(1,1)) | 10 | 10 | 1 |
| 14 | ((2,1),(2)) | ((2,1),(1,1)) | 30 | 20 | 2 |
| 15 | ((1,1,1),(2)) | ((3),(1,1)) | 60 | 10 | 6 |
| 16 | ((3),(1,1)) | ((1,1,1),(2)) | 20 | 10 | 2 |
| 17 | ((2,1),(1,1)) | ((2,1),(2)) | 60 | 20 | 4 |
| 18 | ((1,1,1),(1,1)) | ((3),(2)) | 120 | 10 | 12 |
| 19 | ((2),(3)) | ((1,1),(1,1,1)) | 10 | 10 | 1 |
| 20 | ((1,1),(3)) | ((2),(1,1,1)) | 20 | 10 | 2 |
| 21 | ((2),(2,1)) | ((1,1),(2,1)) | 30 | 20 | 2 |
| 22 | ((1,1),(2,1)) | ((2),(2,1)) | 60 | 20 | 4 |
| 23 | ((2),(1,1,1)) | ((1,1),(3)) | 60 | 10 | 6 |
| 24 | ((1,1),(1,1,1)) | ((2),(3)) | 120 | 10 | 12 |
| 25 | ((1),(4)) | ((1),(1,1,1,1)) | 5 | 5 | 1 |
| 26 | ((1),(3,1)) | ((1),(2,1,1)) | 20 | 15 | 2 |
| 27 | ((1),(2,2)) | ((1),(2,2)) | 30 | 10 | 4 |
| 28 | ((1),(2,1,1)) | ((1),(3,1)) | 60 | 15 | 6 |
| 29 | ((1),(1,1,1,1)) | ((1),(4)) | 120 | 5 | 24 |
| 30 | (( ),(5)) | (( ),(1,1,1,1,1)) | 1 | 1 | 1 |
| 31 | (( ),(4,1)) | (( ),(2,1,1,1)) | 5 | 4 | 2 |
| 32 | (( ),(3,2)) | (( ),(2,2,1)) | 10 | 5 | 4 |
| 33 | (( ),(3,1,1)) | (( ),(3,1,1)) | 20 | 6 | 6 |
| 34 | (( ),(2,2,1)) | (( ),(3,2)) | 30 | 5 | 12 |
| 35 | (( ),(2,1,1,1)) | (( ),(4,1)) | 60 | 4 | 24 |
| 36 | (( ),(1,1,1,1,1)) | (( ),(5)) | 120 | 1 | 120 |

where $(\lambda,\mu)'=(\lambda',\mu')$ is the conjugate pair of partitions of the pair of partitions $(\lambda,\mu)$ of $n$ = 5, $m(M)$ is the dimension of the vector space $F_p^{m(M)}$, which represents the $F_pW_5$-module $M_{F_p}(\lambda,\mu)=F_pW_5\,g(Z^{(\lambda,\mu)})$, for each prime number $p\geq 3$, $k_p$ is the dimension of the subspace $V_{(p)}^{(\lambda,\mu)}$ of $F_p^{m(M)}$, where $V_{(p)}^{(\lambda,\mu)}$

represents the Specht module $S_{F_p}(\lambda,\mu) = F_p W_5\, f_{(p)}(Z^{(\lambda,\mu)})$, and $d_p$ is the minimum distance, which is the least number of the nonzero coordinates in a nonzero vector of the subspace $V_{(p)}^{(\lambda,\mu)}$.

## 8. CONCLUSIONS

In general, we conclude that for any pair of partitions $(\lambda,\mu) = ((\lambda_1,\lambda_2,\ldots,\lambda_s),(\mu_1,\mu_2,\ldots,\mu_t))$, whose conjugate pair of partitions $(\lambda,\mu)' = (\lambda',\mu') = ((\lambda'_1,\lambda'_2,\ldots,\lambda'_{\lambda_1}),(\mu'_1,\mu'_2,\ldots,\mu'_{\mu_1}))$ of a positive integer n, the subspace $V_{(p)}^{(\lambda,\mu)}$ (which represents the Specht module $S_{F_p}(\lambda,\mu)$) can be considered as a linear

$$\left(\frac{n!}{\lambda_1!\cdot\lambda_2!\cdots\lambda_s!\cdot\mu_1!\cdot\mu_2!\cdots\mu_t!},\ \frac{n!}{\prod_{i=1}^{s}\prod_{j=1}^{\lambda_i}(\lambda_i+\lambda'_j-i-j+1)\cdot\prod_{i=1}^{t}\prod_{j=1}^{\mu_i}(\mu_i+\mu'_j-i-j+1)},\ \lambda'_1!\cdot\lambda'_2!\cdots\lambda'_{\lambda_1}!\cdot\mu'_1!\cdot\mu'_2!\cdots\mu'_{\mu_1}!,\ p\right)\text{-code},$$

$\forall\, p \geq 3$.

As example, for the pair of partitions $(\lambda,\mu) = ((3,2,2),(4,1))$ whose conjugate pair of partitions is $(\lambda,\mu)' = (\lambda',\mu') = ((3,3,1),(2,1,1,1))$ of the positive integer $n = 12$, the subspace $V_{(p)}^{((3,2,2),(4,1))}$ (which represents the Specht module $S_{F_p}((3,2,2),(4,1))$) can be considered as a linear

$$\left(\frac{12!}{3!\cdot 2!\cdot 2!\cdot 4!\cdot 1!},\frac{12!}{5\cdot 4\cdot 1\cdot 3\cdot 2\cdot 2\cdot 1\cdot 5\cdot 3\cdot 2\cdot 1\cdot 1},\ 3!\cdot 3!\cdot 1!\cdot 2!\cdot 1!\cdot 1!\cdot 1!,\ p\right)\text{-code},\ \forall\, p \geq 3,$$

which is the linear (831600, 66528, 72, $p$)-code, $\forall\, p \geq 3$.

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