AN ALGORITHM FOR GENERATING PERMUTATIONS IN SYMMETRIC GROUPS USING SOFT SPACES WITH GENERAL STUDY AND BASIC PROPERTIES OF PERMUTATION SPACES

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ABSTRACT

In this paper we introduced algorithm to find permutation in symmetric group using soft topological space to structure permutation topological space. Moreover, this class of permutation topology is called even (odd) permutation topology if its permutation is even (odd). Further, new notions in permutation topological spaces are investigated like splittable permutation spaces and ambivalent permutation spaces.

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1. INTRODUCTION

Soft sets were originally shown by Molodtsov [1]. Then they applied in many fields where they have rich possibility for applications. In 2011, the connotation of soft topological spaces (STSs) is shown by Muhammad and Munazza [2]. Some notions of (STS) and of its applications fundamental connotations of fuzzy soft topology and Intuitionistic fuzzy soft topology are studied by many mathematicians see ([3-9]). In 2014, Mahmood [10] introduced the notion of permutation topological space (PTS) using permutation \( \beta \) in symmetric group \( S_n \), where each permutation \( \beta \in S_n \) can be represented as a product of disjoint (separate) cycles. In other words, \( \beta = (b_1^1, b_2^1, ..., b_{\alpha_1}^1, b_1^2, b_2^2, ..., b_{\alpha_2}^2, ..., b_1^{c(\beta)}, b_2^{c(\beta)}, ..., b_{\alpha_{c(\beta)}}^{c(\beta)}) \) and \( \forall i \neq j \) satisfy \( \{b_1^i, b_2^i, ..., b_{\alpha_i}^i\} \cap \{b_1^j, b_2^j, ..., b_{\alpha_j}^j\} = \emptyset \) [11]. That means, \( \beta \) can be represented as \( \lambda_1 \lambda_2 ... \lambda_{c(\beta)} \), where \( \lambda_i \) separate cycles of length \( |\lambda_i| = \alpha_i \) and \( c(\beta) \) refers to the number of the product of separate cycles with the 1-cycles of \( \beta \). In 2015, some methods are introduced to generate fuzzy soft set and intuitionistic fuzzy soft set using different sets [12]. Now, the interesting question is there any an algorithm shows the relation between permutation space and soft space. In this work, this algorithm is given. We generate permutation topological space using soft topological space (STS). This class of permutation topology is called even (odd) permutation topology if its permutation is even (odd). Further, new notions in permutation topological spaces are investigated like splittable permutation spaces and ambivalent permutation spaces.

2. PRELIMINARIES AND BASIC RESULTS

We will show some past results and basic definitions in this section.

Definition 2.1 ([13])

A "cycle type" of \( \beta \) is the partition \( \alpha = \alpha(\beta) = (\alpha_1(\beta), \alpha_2(\beta), ..., \alpha_{c(\beta)}(\beta)) \).
Definition 2.2 ([13])

We refer to all the permutations in $S_n$ of cycle type $\alpha$ by $C^\alpha$.

Definition 2.3 ([13])

Let $\beta$ be a permutation in Alternating group $A_n$ and $\beta \in C^\alpha$. $A(\beta)$ conjugacy class of $\beta$ in $A_n$ is defined by $A(\beta) = \{ \gamma \in A_n \mid \gamma = t\beta t^{-1}; \text{ for some } t \in A_n \} = C^\alpha$, if $\beta \notin H_n$ \(\text{ or } C^{\alpha+}, \text{ if } \beta \in H_n\)

where $C^{\alpha+}$ and $C^{\alpha-}$ are two classes of equal order in Alternating group $A_n$ such that $C^\alpha = C^{\alpha+} \cup C^{\alpha-}$ and $H_n = \{ C^\alpha \mid n > 1, \text{ with all parts } \alpha_k \text{ of } \alpha \text{ different and odd} \}$.

Proposition 2.4 ([14])

The conjugacy classes $C^{\alpha \pm}$ of $A_n$ are ambivalent if $4 \mid (\alpha_i - 1)$ for each part $\alpha_i$ of $\alpha$.

Definition 2.5 ([10])

Let $\beta \in S_n$ with $\Omega = \{1, 2, \ldots, n\}$ and the "cycle type" of $\beta$ is $\alpha(\beta) = (\alpha_1, \alpha_2, \ldots, \alpha_c(\beta))$, and $\{\lambda_i\}_{i=1}^{c(\beta)}$ be a composite of "pairwise separate cycles" of $\beta$ where $\lambda_i = (b_1^i, b_2^i, \ldots, b_{\alpha_i}^i)$, $1 \leq i \leq c(\beta)$. Each $\lambda = (b_1, b_2, \ldots, b_k)$, $k$-cycle in $S_n$ we put $\beta$-set as $\lambda^\beta = \{b_1, b_2, \ldots, b_k\}$ and is said to be $\beta$-set of cycle $\lambda$. Also, $\beta$-sets of $\{\lambda_i\}_{i=1}^{c(\beta)}$ are investigated by $\{\lambda_i^\beta = \{b_1^i, b_2^i, \ldots, b_{\alpha_i}^i\} \mid 1 \leq i \leq c(\beta)\}$.

Remark 2.6

Suppose that $\lambda_i^\beta$ and $\lambda_j^\beta$ are $\beta$-sets in $\Omega$, where $|\lambda_i| = \sigma$ and $|\lambda_j| = \nu$. Then the known definitions will be written as following:

Definition 2.7 ([10])

Let $\lambda_i^\beta$ and $\lambda_j^\beta$ be $\beta$-sets in $\Omega$, they are called separate iff there exists $1 \leq d \leq \sigma$, for each $1 \leq r \leq \nu$ such that $b_d^i \neq b_r^j$\(\text{ and } \sum_{k=1}^{\sigma} b_k^i = \sum_{k=1}^{\nu} b_k^j\).

Definition 2.8 ([10])

Let $\lambda_i^\beta$ and $\lambda_j^\beta$ be $\beta$-sets in $\Omega$. We say they are equal iff there exists $1 \leq d \leq \sigma$, for each $1 \leq r \leq \nu$ such that $b_d^i = b_r^j$.

Definition 2.9: ([10])

We say $\lambda_i^\beta$ is contained in $\lambda_j^\beta$ and denoted $\lambda_i^\beta \subset \lambda_j^\beta$, iff $\sum_{k=1}^{\sigma} b_k^i < \sum_{k=1}^{\nu} b_k^j$.

Definition 2.10: ([15])

For any $\lambda^\beta = \{b_1, b_2, \ldots, b_r\}$ and $\eta^\beta = \{a_1, a_2, \ldots, a_0\}$ two subsets of $\Omega$, we call $\lambda^\beta$ and $\eta^\beta$ are similar $\beta$-sets in $\Omega$, iff $\sum_{k=1}^{r} b_k = \sum_{k=1}^{\nu} a_k$. 

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and there are two points say \( b_i, b_j \in \lambda^\beta \) such that \( b_i \in \eta^\beta \) and \( b_j \notin \eta^\beta \). Assume 
\[ \eta^\beta = \{ b_1, b_2, \ldots, b_r \}, \quad \lambda^\beta = \{ a_1, a_2, \ldots, a_b \} \]
are similar \( \beta \) - sets in \( \Omega \) and 
\[ \Delta = \text{Max}\{\text{Max}\{\eta^\beta - \omega\}, \text{Max}\{\lambda^\beta - \omega\}\} \]
where \( \omega = \{ b_1, b_2, \ldots, b_r \} \cap \{ a_1, a_2, \ldots, a_b \} \).

Then \( \lambda^\beta \preceq \eta^\beta \), if \( \Delta \in \eta^\beta \). Also, 
\[ \eta^\beta \preceq \lambda^\beta \] if \( \Delta \in \lambda^\beta \).

\[ \eta^\beta \wedge \lambda^\beta = \begin{cases} \lambda^\beta, & \text{if } \Delta \in \eta^\beta \\ \eta^\beta, & \text{if } \Delta \in \lambda^\beta \end{cases} \]

and 
\[ \eta^\beta \vee \lambda^\beta = \begin{cases} \eta^\beta, & \text{if } \Delta \in \eta^\beta \\ \lambda^\beta, & \text{if } \Delta \in \lambda^\beta \end{cases} \]

For any \( \lambda^\beta = \{ b_1, b_2, \ldots, b_r \} \) and \( \eta^\beta = \{ a_1, a_2, \ldots, a_b \} \) two subsets of \( \Omega \).

Then, 
\[ \lambda^\beta \wedge \eta^\beta = \begin{cases} \lambda^\beta, & \text{if } \left( \sum_{k=1}^{r} a_k < \sum_{k=1}^{r} b_k \right) \text{ Or } (\lambda^\beta \wedge \eta^\beta \text{ are similarand } \Delta \in \eta^\beta) \\ \eta^\beta, & \text{if } \left( \sum_{k=1}^{r} b_k < \sum_{k=1}^{r} a_k \right) \text{ Or } (\lambda^\beta \wedge \eta^\beta \text{ are similarand } \Delta \in \lambda^\beta) \end{cases} \]

and

\[ \lambda^\beta \vee \eta^\beta = \begin{cases} \lambda^\beta, & \text{if } \left( \sum_{k=1}^{r} b_k > \sum_{k=1}^{r} a_k \right) \text{ Or } (\lambda^\beta \vee \eta^\beta \text{ are similarand } \Delta \in \lambda^\beta) \\ \eta^\beta, & \text{if } \left( \sum_{k=1}^{r} a_k > \sum_{k=1}^{r} b_k \right) \text{ Or } (\lambda^\beta \vee \eta^\beta \text{ are similarand } \Delta \in \eta^\beta) \\ \mu^\beta, & \text{if } \lambda^\beta = \eta^\beta = \mu^\beta \\ \phi, & \text{if } \lambda^\beta \wedge \eta^\beta \text{ are disjoint} \end{cases} \]

\textbf{Definition 2.11: ([10])}

Let \( \{ \lambda_i^\beta \}_{i=1}^{I} = \{ b_1^i, b_2^i, \ldots, b_r^i \sigma_i \}_{i \in I} \) be a family of not separate \( \beta \) - sets. We define the intersection (union) of \( \{ \lambda_i^\beta \}_{i=1}^{I} \) respectively, by 
\[ \bigwedge_{i \in I} \lambda_i^\beta = \lambda_j^\beta, \quad \bigvee_{i \in I} \lambda_i^\beta = \lambda_j^\beta \], where
\[ \sum_{k=1}^{r} b_k^i = \text{inf} \{ \sum_{k=1}^{r} b_k^i ; i \in I \} \] and 
\[ \sum_{k=1}^{r} b_k^i = \text{sup} \{ \sum_{k=1}^{r} b_k^i ; i \in I \} . \]

\textbf{Definition 2.12: ([10])}

Assume \( \beta \) is permutation in \( S_n \), and let \( \{ \lambda_i^\beta \}_{i=1}^{\beta} \) be a family of product of separate cycles with the 1-cycles of \( \beta \), where \( |\lambda_i^\beta| = \alpha_i \), 
\( 1 \leq i \leq c(\beta) \), then permutation topology \( t_n^\beta \) is a family of \( \beta \) - sets of the collection \( \{ \lambda_i^\beta \}_{i=1}^{\beta} \) together with \( \Omega = \{1,2,\ldots,n\} \) and empty set. We say \( (\Omega, t_n^\beta) \) is permutation space.

\textbf{Definition 2.13: ([15])}

Assume \( (\Omega, t_n^\beta) \) is a permutation topological space. We say \( (\Omega, t_n^\beta) \) is a Permutation Single Space (PSS) iff all their proper open \( \beta \) - sets are singleton.

\textbf{Definition 2.14: ([15])}

Assume \( (\Omega, t_n^\beta) \) is a permutation topological space. We say \( (\Omega, t_n^\beta) \) is a Permutation Indiscrete Space (PIS) iff each open \( \beta \) - set is trivial \( \beta \) - set.

\textbf{Definition 2.15: ([11])}

Assume that \( E \) is a set of parameters, \( X \) is an initial universe set, \( P(X) \) is the power set of \( X \).
and $K$ is a subset of $E$. We say $(F, K)$ is a soft set over $X$ if $F$ is a multi-valued function of $K$ into $P(X)$.

**Definition 2.16 ([16])**

Assume $(F, A)$ and $(M, L)$ are soft sets over $X$, their union $(F, A) \cup (M, L)$ is a soft set such that for all $e \in C = A \cup L$, $H(e) = F(e)$ if $e \in A - L$, $H(e) = M(e)$ if $e \in L - A$, $F(e) \cup M(e)$ if $e \in A \cap L$. We write $(F, A) \cup (M, L) = (H, C)$. Also, their intersection is a soft set, denoted by $(F, A) \cap (M, L) = (K, E)$. We say $r_{F,K}$ is a soft set over $X$ if $F$ is a multi-valued function of $K$ into $P(X)$.

**Definition 2.17 ([2])**

A soft set $(g, b)^C$ is the complement of $(g, b)$ in $(X, E)$ and it is defined by $(g^C, K)$, where $K = E \setminus \{e \in E | G(e) = X\}$ and $\forall e \in K$

$$G^C(e) = \begin{cases} X \setminus G(e), & \text{if } e \in B \\ X, & \text{if } e \notin B. \end{cases}$$

**Definition 2.18: ([2])**

Assume $\tau$ is a family of soft sets over $X$. We say $\tau$ is a soft topology on $X$ if $\tau$ satisfies the following axioms:

(i) The intersection of any two soft sets in $\tau$ belongs to the family $\tau$.

(ii) The union of any number of soft sets in $\tau$ belongs to the family $\tau$.

(iii) $(X, E)$ and $\Phi$ belong to the family $\tau$.

We say $(X, E, \tau)$ is a soft topological space (STS) over $X$. Also, each member in the family $\tau$ is said to be a soft open set and its complement is said to be a soft closed set. Further, $(X, E, \tau)$ is said to be a soft topological space (STS) over $X$, if $\tau = \Phi((X, E))$. Also, $(X, E, \tau)$ is called a soft discrete topological space (SDTS) over $X$, if $\tau$ contains all soft sets over $X$.

**Some Results on Permutations 2.19: ([17, 18])**

1. $\beta = (b_1, b_2, \ldots, b_r)$ is even $\iff$ $r$ is odd.

2. $\beta \in S_n$ is even $\iff n - c(\beta)$ is even.

3. $\beta = (1)$ (Identity) $\iff \beta = (b)$ for some $b \in \Omega$.

**Remark 2.20:**

In this work, for any set $D = \{d_1, d_2, \ldots, d_k\}$ of $k$ distinct objects and for any cycle $B = (b_1, b_2, \ldots, b_m)$ we will use $|B| = m$ and $|D| = k$ to refer to the length of the cycle $B$ and the cardinality of set $D$.

3. AN ALGORITHM TO GENERATE PERMUTATION SPACES FROM SOFT SPACES

We will introduce in this section an algorithm to generate permutation topological spaces by analysis (STS) and this class of permutation topological spaces is called even (odd) permutation topology if its permutation is even (odd). Moreover, some basic properties of permutation spaces are studied.

**Steps of the work 3.1:**

Assume $(X, E, \Gamma)$ is a (STS), where $X = \{s_1, s_2, \ldots, s_k\}$, $E = \{e_1, e_2, \ldots, e_n\}$ and $\Gamma = \Phi((X, E), (E, \pi))$. Now, let $T_1 = \{F(e_1)\}_{i=1}^m$, $T_2 = \{F(e_2)\}_{i=1}^m$, $\ldots$, $T_n = \{F(e_n)\}_{i=1}^m$. For any $1 \leq i \leq n$, let $T_i = \{B \in T_i / B \neq \phi\}$.

$\forall (1 \leq i \leq n)$, let $\delta_i : X \rightarrow N$ be a map from $X$ into natural numbers $N$ defined by $\delta_i(s_j) = j + (i - 1)k$, for all $s_j \in X$ and $(1 \leq i \leq n)$ where $k = |X|$. Then $\sigma = \prod_{i=1}^n \sigma_i$ is called permutation in symmetric group $S_{nk}$ where for all $1 \leq i \leq n$, $\sigma_i = \prod_{L=1}^{L=\pi(s_g)} \delta_i(s_{s_{g_1}}) \delta_i(s_{s_{g_2}}) \ldots \delta_i(s_{s_{g_{L}}})$ is permutation which is product of $\prod_{L=\pi(s_g)}^{L=\pi(s_{g_{L}})}$ cyclic factors of the length $\omega_L$, where
\[ \omega_L = \{ s_1, s_2, \ldots, s_l \} \in T_i \] and 
\[ 1 \leq L \leq T_i. \]

Further, \( \sigma_i = (\delta_l(s_j)) \) if \( T_i = \phi \). Then
\( (\Omega, t_h) \) is called permutation topological space induced by soft topology \( \Gamma \), where \( \Omega = \{1, 2, \ldots, h\} \), \( h = nk \) and \( t_h \) is a family of \( \sigma \)-set of the family \( \{\sigma_i\}_{i=1}^{n} \) together with \( \Omega \) and empty set. Also, if \((X,E,\Gamma)\) is a soft indiscrete space. Then \((\Omega, t_h) \) is called permutation indiscrete space (PIS) induced by soft topology \( \Gamma \), where \( (\Omega, t_h) \) is a family of \( \sigma \)-set of the family \( \{\sigma_i\}_{i=1}^{n} \) together with \( \emptyset \) and empty set. Also, if \((X,E,\Gamma)\) is a soft indiscrete space. Then \( t_h \) is a family of \( \sigma \)-set of the family \( \{\sigma_i\}_{i=1}^{n} \). Finally, for any \((X,E,\Gamma)\) non-indiscrete (STS), where
\[ E = \{ e_1, e_2, \ldots, e_n \} \]
\[ X = \{ s_1, s_2, \ldots, s_k \} \]
we can generate permutation topological space \((\Omega, t_h) \) as follows:

(1) - Find \( T_i = \{ F_j(e_j) \}_{j=1}^{m} \), for all \( 1 \leq i \leq n \).

(2) - Find \( T_i = \{ \omega \in T_i / \omega \neq \phi \} \), for all \( 1 \leq i \leq n \).

(3) - Find \( \delta_l(s_j) = j + (i - 1)k \), \( \forall \) \( 1 \leq j \leq k \) and \( \forall \) \( 1 \leq i \leq n \).

(4) - Find \( \sigma_i \) for all \( 1 \leq i \leq n \), where
\[ \sigma_i = (\delta_l(s_j)) \] if \( T_i = \phi \) and
\[ \sigma_i = \prod_{L=1}^{m} (\delta_l(s_{g_1}) \delta_l(s_{g_2}) \ldots \delta_l(s_{g_L})) \]
if \( T_i \neq \phi \) where
\[ \omega_L = \{ s_1, s_2, \ldots, s_L \} \in T_i \]

(5) - Find \( \sigma = \prod_{i=1}^{n} \sigma_i \).

(6) - Find the "separate cycle factors including the 1-cycle" of \( \sigma \) say \( \lambda_1, \lambda_2, \ldots, \lambda_{\epsilon(\sigma)} \).

(7) - Find the \( \sigma \)-sets of \( \{ \lambda_i \}_{i=1}^{\epsilon(\sigma)} \).

(8) - Find \( \Omega \), where \( \Omega = \{1, 2, \ldots, h\} \) and \( h = nk \),

(9) - Find \( t_h \), where \( t_h = \{ \phi, \Omega, \lambda_1, \lambda_2, \ldots, \lambda_{\epsilon(\sigma)} \} \).

(10) - Find \( (\Omega, t_h) \) is (PTS).

**Remarks 3.2:**

(1) - For all \( 1 \leq i \leq n \), let \( T_i = \{ \phi, X, T_i \} \).

(2) - Find \( \delta_l(s_j) = j + (i - 1)k \), \( \forall \) \( 1 \leq j \leq k \) and \( \forall \) \( 1 \leq i \leq n \).

(3) - For any \( 1 \leq i \neq q \leq n \), we have \( \delta_l^{-1}(t) = s_c \), where \( c = t - k(i + 1) \).

(4) - If \( \delta_l(s_j) \neq \phi \), for some \( \forall \) \( \phi \in T_i \) for all \( \forall \) \( \phi \). Then \( \delta_l(s_j) \neq \phi \) for all \( \forall \) \( \phi \in T_i \) for all \( \forall \) \( \phi \).

**Permutation Subspaces Induced by Soft Topology \( \Gamma \) 3.3**

Let \( (\Omega, t_h) \) be a permutation space induced by soft topology \( \Gamma \), \( \lambda^\beta \subset \Omega \) and
\[ T_i^\beta = \lambda^\beta \cap \lambda_i^\beta, \]
for each proper \( \lambda_i^\beta \in t_h^\beta \).

\[ T_i^\beta = \{ b_1^i, b_2^i, \ldots, b_k^i \}, \]
if \( \lambda^\beta \) and \( \lambda_i^\beta \) are not separate
\[ \phi, \]
if \( \lambda^\beta \) and \( \lambda_i^\beta \) are separate
Let $\mathcal{R} = \{T_1^\beta \mid T_1^\beta$ nonempty open $\beta$-set$\}$. \forall T_1^\beta \\
\in \mathcal{R}$, set $i_k = \max\{b_1^i, b_2^i, \ldots, b_k^i\}$ and $m = \max\{b_1^i; T_1^\beta \in \mathcal{R}\}$. Let $B = \bigcap_{T_1^\beta \in \mathcal{R}} (\Omega' - T_1^\beta)$ where $\Omega' = \{1, 2, \ldots, m\}$ and $(\bigcap)$ is a normal intersection. \forall T_1^\beta \in \mathcal{R}$ we consider $T_1 = (b_1^i, b_2^i, \ldots, b_k^i)$ is $i_k$-cycle in $S_m$. In other words, the product of separate cycles of other permutation in symmetric group $S_m$ induced by $\lambda^\beta$ say $\gamma^\beta$ where $\gamma^\beta = \prod_{T_1^\beta \in \mathcal{R}} T_1^f$ and $\gamma^\beta = \prod_{T_1^\beta \in \mathcal{R}} T_1^f$ whenever $B = \phi$. Moreover, we say $(\Omega, t_m^\beta)_{\Gamma}$ is a permutation subspace induced by soft topology $\Gamma$ where $t_m^\beta\gamma$ is a family of all $\gamma^\beta -$ sets of product of separate cycles of $\gamma^\beta$ together with $\Omega'$ and empty set.

**Lemma 3.4:**

If each pair of different members in $\mathcal{R}$ are separate, then $B = \{b_1, b_2, \ldots, b_t\}$ has exactly $t$ points where $t = m - s$ and $\sum_{T_1^\beta \in \mathcal{R}} |T_1^\beta| = s$.

**Proof:**

Suppose that $T_1^\beta \cap T_1^\beta = \phi$, for any $T_1^\beta, T_1^\beta \in \mathcal{R}$ and $B = \{b_1, b_2, \ldots, b_k\}$ with $k \neq t$ where $t = m - s$ and $\sum_{T_1^\beta \in \mathcal{R}} |T_1^\beta| = s$. Since $B = \bigcap_{T_1^\beta \in \mathcal{R}} (\Omega' - T_1^\beta)$ where $\Omega' = \{1, 2, \ldots, m\}. \quad \text{Thus}$$|B| = |\bigcap_{T_1^\beta \in \mathcal{R}} (\Omega' - T_1^\beta)|. \text{Since } T_1^\beta \cap T_1^\beta = \phi$, for any $T_1^\beta, T_1^\beta \in \mathcal{R}$. This implies that $|B| = \bigcap_{T_1^\beta \in \mathcal{R}} (\Omega' - T_1^\beta) |\Omega' - t_m^\beta|$.

\[\sum_{T_1^\beta \in \mathcal{R}} |T_1^\beta| = m - s = t. \text{But } |B| = k \neq t \text{ and this contradiction. Therefore } B = \{b_1, b_2, \ldots, b_t\} \text{ has exactly } t \text{ points.}\]

**Example 3.5**

Let the set of cars under consideration be $X = \{a_1, a_2, a_3\}$. Let $E = \{\text{cheap } (e_1); \text{ dark color } (e_2); \text{ modern } (e_3); \text{ beautiful } (e_4)\}$ be set parameter set. Now, to buy a good car. Let $(F, A)$ be soft set describing the Mr. Z opinion and it is defined by $A = \{e_1, e_3, e_4\}$. $F(e_1) = \{a_1, a_3\}, F(e_3) = \{a_2\}, F(e_4) = X$

Further, we suppose that the good car in the opinion of his friend, say Mr. W, is described by $(G, B)$ and it is defined by $B = \{e_1, e_4\}$ $G(e_1) = \{a_3\}, G(e_4) = \{a_2, a_3\}$. We have: $\Gamma = \{\Phi, (U, E), (F, A), (G, B)\}$ is a soft topology. Find permutation space $(\Omega, \pi^\sigma_{t_m})_{\Gamma}$ induced by soft topology $\Gamma$. Also, find $(\Omega_1, \pi^\sigma_{t_m})$ and $(\Omega_2, \pi^\sigma_{t_m})_{\Gamma}$ where $\lambda^\sigma = \{1, 4\}$ and $\pi^\sigma = \{12\}$.

**Solution:** We consider that $T_1 = \{F(e_1), G(e_1)\}, T_2 = \{F(e_2), G(e_2)\}, T_3 = \{F(e_3), G(e_3)\}, T_4 = \{F(e_4), G(e_4)\}$

$T_1 = \{\{a_1, a_3\}, \{a_3\}\}, T_2 = \{\phi, \phi\}, T_3 = \{\{a_1, a_3\}, \{a_1, a_3\}\}$, $T_4 = \{\Phi, \{a_1, a_3\}\}$

$T_1 = \{\{a_1, a_3\}, \{a_3\}\}, T_2 = \{\phi, \phi\}, T_3 = \{\{a_2\}\}, T_4 = \{X, \{a_2, a_3\}\}$

\[T_1 = \{\phi, X, \{a_1, a_3\}\}, T_2 = \{\phi, X\}, T_3 = \{\phi, X, \{a_2\}\}, T_4 = \{\phi, \{a_2, a_3\}\}. \text{ Hence we have } (X, T_i) \text{ is}\]
a topological space for each \((1 \leq i \leq 4)\). Now, we consider that \((X_i, T_i)\) is a topological space for each \((1 \leq i \leq n)\).

\[
\sigma_i = \prod_{L=1}^{4} \delta_j(a_{\delta_1}) \delta_j(a_{\delta_2}) \delta_j(a_{\delta_3}) \delta_j(a_{\delta_4})
\]

which is product of \(T'_i\) cyclic factors of the length \(\omega_L\), where \(\omega_L \in T'_i\) and \(1 \leq L \leq T'_i\).

Hence we have:
\[
\begin{align*}
\sigma_1 &= (\delta_1(a_2)) \delta_1(a_3)) = (13)(3), \\
\sigma_2 &= (\delta_2(a_2)) = (5), \\
\sigma_3 &= (\delta_3(a_2)) = (8), \\
\sigma_4 &= (\delta_4(a_2) \delta_4(a_3)) = (11)(12)(10)
\end{align*}
\]

Then \(\sigma = \Pi_{i=1}^{4} \sigma_i = (13)(3)(5)(8)(11)(12)(10)\) is a permutation in symmetric group \(S_{12}\). Now, we can write \(\sigma \in S_n\) as \(\lambda_1 \lambda_2 \ldots \lambda_{c(\sigma)}\). Hence

\[
\begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
3 & 2 & 1 & 4 & 5 & 6 & 7 & 8 & 9 & 12 & 11 & 10
\end{pmatrix}
\]

is a permutation in symmetric group \(S_{12}\). Now, we can write \(\sigma \in S_n\) as \(\lambda_1 \lambda_2 \ldots \lambda_{c(\sigma)}\). Hence

\[
\begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
3 & 2 & 1 & 4 & 5 & 6 & 7 & 8 & 9 & 12 & 11 & 10
\end{pmatrix}
\]

is a permutation in symmetric group \(S_{12}\). Now, we can write \(\sigma \in S_n\) as \(\lambda_1 \lambda_2 \ldots \lambda_{c(\sigma)}\). Hence

\[
\begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
3 & 2 & 1 & 4 & 5 & 6 & 7 & 8 & 9 & 12 & 11 & 10
\end{pmatrix}
\]

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\end{pmatrix}
\]

is a permutation in symmetric group \(S_{12}\). Now, we can write \(\sigma \in S_n\) as \(\lambda_1 \lambda_2 \ldots \lambda_{c(\sigma)}\). Hence

\[
\begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
3 & 2 & 1 & 4 & 5 & 6 & 7 & 8 & 9 & 12 & 11 & 10
\end{pmatrix}
\]

is a permutation in symmetric group \(S_{12}\). Now, we can write \(\sigma \in S_n\) as \(\lambda_1 \lambda_2 \ldots \lambda_{c(\sigma)}\). Hence

\[
\begin{pmatrix}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\
3 & 2 & 1 & 4 & 5 & 6 & 7 & 8 & 9 & 12 & 11 & 10
\end{pmatrix}
\]
\( T_1^\sigma = \{1, 3\}, \quad T_2^\sigma = \{2\}, \quad T_3^\sigma = \{4\}, \quad T_4^\sigma = \{5\}, \)
\( T_5^\sigma = \{6\}, \quad T_6^\sigma = \{7\}, \quad T_7^\sigma = \{8\}, \quad T_8^\sigma = \{9\}, \)
\( T_9^\sigma = \{10\}, \quad T_{10}^\sigma = \{12\} \rightarrow \mathcal{R} = \{\{1, 3\}, \{2\}, \{4, 5\}, \{6\}, \{7\}, \{8, 9\}, \{10\}, \{12\}\} \)
\( \Rightarrow m = 12 \Rightarrow \Omega_2 = \Omega, \quad \sum_{T_i^\sigma \in \mathcal{R}} |T_i^\sigma| = s \)
\( = 11 \quad \text{and} \quad B = \bigcap_{T_i^\sigma \in \mathcal{R}} (\Omega_2 - T_i^\sigma) = \{11\}. \) This implies that \( t = |\beta| = 1, \) also \( m - s = 12 - 11 = 1. \) That means \( t = m - s. \)

Further, \( \gamma_{\pi^\sigma} = (1\ 3)(2\ 4)(5\ 6)(7\ 8)(9\ 10)(11\ 12) \)

is a permutation in symmetric group \( S_{12} \) induced by \( \pi^\sigma = \{12\} \) and \( (\Omega_2, \gamma_{\pi^\sigma})_\Gamma \) is a permutation subspace induced by soft topology \( \Gamma, \)

where
\( i_m^\sigma = i_{12} ^{\sigma} = \{\Omega_2, \phi, \{1, 3\}, \{2\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}, \{10\}, \{11\}, \{12\}\}. \) Moreover, (lemma 3.4) is hold for \( \pi^\sigma = \{12\} \) (since each pair of different members in \( \mathcal{R} \) are separate).

4. Some Notions of Permutation Spaces

**Definition 4.1:**

Assume \( (\Omega, \iota^\beta_n) \) is a (PTS), we say \( (\Omega, \iota^\beta_n) \) is even (odd) permutation space if its permutation \( \beta \) is even (odd) in \( S_n. \)

**Example 4.2:**

Assume \( (\Omega, \iota^\beta_9) \) is a (PTS), where \( \beta = (4 \ 1 \ 6 \ 2 \ 7 \ 3 \ 8 \ 9) \) \( (5). \) Then \( (\Omega, \iota^\beta_9) \) is an even permutation space since \( \beta \in S_9 \) is an even.

**Definition 4.3:**

Assume \( (\Omega, \iota^\beta_n) \) is a (PTS), we say \( (\Omega, \iota^\beta_n) \) is splittable permutation space if its permutation \( \beta \) satisfies \( \beta \in H_n. \)

**Example 4.4:**

Assume \( (\Omega, \iota^\beta_{10}) \) is a (PTS), where \( \beta = (5 \ 9 \ 6 \ 4 \ 3 \ 7 \ 1 \ 8 \ 10) \) \( (2). \) Therefore, we have \( (\Omega, \iota^\beta_{10}) \) is splittable permutation space since \( \beta \in H_n. \)

**Definition 4.5:**

Assume \( (\Omega, \iota^\beta_n) \) is a (PTS), we say \( (\Omega, \iota^\beta_n) \) is ambivalent permutation space if \( (\Omega, \iota^\beta_n) \) is splittable permutation space and for each part \( \alpha_i \) of \( \alpha(\beta) \) satisfies \( 4 \mid (\alpha_i - 1). \)

**Example 4.6:**

Assume \( (\Omega, \iota^\beta_6) \) be a (PTS), \( \beta = (6 \ 1 \ 5 \ 3 \ 2) \) \( (4). \) Then \( (\Omega, \iota^\beta_6) \) is ambivalent permutation space since \( (\Omega, \iota^\beta_6) \) is splittable permutation space and for each part \( \alpha_i \) of \( \alpha(\beta) \) satisfies \( 4 \mid (\alpha_i - 1). \)

**The Maps induced by soft topologies 4.7**

Suppose that \( (X, E, \tau_1), \ (X, E, \tau_2) \) and \( (X, E, \tau_3) \) are three (STSs) over the common universe \( X \) and the parameter set \( E \) with their permutations \( \beta, \mu \) and \( \delta \) in symmetric group \( S_n \) where \( n = |X| \times |E|. \) Hence, we consider that \( \delta: (\Omega, \iota^\beta_n)_{\tau_1} \to (\Omega, \iota^\mu_n)_{\tau_2} \) is a map, and \( \lambda^\beta = \{b_1, b_2, \ldots, b_k\} \) \( \beta \)-set, \( \delta(\lambda^\beta) \) is said to be \( \mu \)-set and it is defined as \( \delta(\lambda^\beta) = \{\delta(b_1), \delta(b_2), \ldots, \delta(b_k)\}. \) We say \( \delta \) is a permutation map induced by soft topology \( \tau_3. \)

**Definition 4.8**

Suppose that \( (X, E, \tau_1), \ (X, E, \tau_2) \) and \( (X, E, \tau_3) \) are three (STSs) over the common universe \( X \) and the parameter set \( E \) with their
Proof:

Let \( \omega_i = \{ x_{g_1}, x_{g_2}, \ldots, x_{g_{\Omega_i}} \} \),
\( \omega_j = \{ x_{h_1}, x_{h_2}, \ldots, x_{h_{\Omega_j}} \} \in T_i \), and \( \omega_j = \omega_i \), \( j = 1, 2 \). Since any pair \( \omega_i \neq \omega_j \), \( (1 \leq i \leq n) \) such that \( \omega_i \cap \omega_j = \phi \). Then \( \delta_i(x_{g_1}) \delta_i(x_{g_2}) \ldots \delta_i(x_{g_{\Omega_i}}) \)
\( (\delta_i(x_{g_1}) \delta_i(x_{g_2}) \ldots \delta_i(x_{g_{\Omega_i}})) \) and \( (\delta_j(x_{h_1}) \delta_j(x_{h_2}) \ldots \delta_j(x_{h_{\Omega_j}})) \) are separate cycles in symmetric group \( S_n \) [since \( \delta_i(x_{g_l}) \neq \delta_j(x_{h_p}) \), for any \( 1 \leq l \neq p \leq n \)].

Also, for any \( \omega_i = \{ x_{g_1}, x_{g_2}, \ldots, x_{g_{\Omega_i}} \} \in T_i \), and
\( \omega_i = \{ x_{h_1}, x_{h_2}, \ldots, x_{h_{\Omega_j}} \} \in T_j \), \( (1 \leq i \neq j \leq n) \).

Hence, we consider that \( (\delta_i(x_{g_1}) \delta_i(x_{g_2}) \ldots \delta_i(x_{g_{\Omega_i}})) \) and
\( (\delta_j(x_{h_1}) \delta_j(x_{h_2}) \ldots \delta_j(x_{h_{\Omega_j}})) \) are separate cycles in symmetric group \( S_{nk} \) [since
\( \delta_i(x_{g_l}) \neq \delta_j(x_{h_p}) \), for any \( 1 \leq i \neq l \leq n \) and \( x_{j}, x_{f} \in X \)]. In other side,
\( \beta = \prod_{i=1}^{n} \left( \prod_{j=1}^{n} (\delta_i(x_{g_1}) \delta_i(x_{g_2}) \ldots \delta_i(x_{g_{\Omega_i}})) \right) \in S_{nk} \) is a permutation for the permutation space \((\Omega, t^\beta)_{nk} \Gamma \). Thus, we consider that \( \sum_{i=1}^{n} |T_i^\beta| \) is the number of the product of separate cycles of \( \sum_{i=1}^{n} \left( \prod_{j=1}^{n} (\delta_i(x_{g_1}) \delta_i(x_{g_2}) \ldots \delta_i(x_{g_{\Omega_i}})) \right) \). Further, \( c(\beta) \) is the number of the product of separate cycles with the 1-cycles of \( \beta \). Thus \( c(\beta) \geq \frac{n}{\prod_{i=1}^{n} |T_i^\beta|} \) in general. Therefore either \( c(\beta) = \frac{n}{\prod_{i=1}^{n} |T_i^\beta|} \) or \( c(\beta) > \frac{n}{\prod_{i=1}^{n} |T_i^\beta|} \). If \( c(\beta) > \frac{n}{\prod_{i=1}^{n} |T_i^\beta|} \), then there is at least 1-cycle say \( (\delta_i(x_{j})) \) for some \( (x_{j} \in X \), \( 1 \leq i \leq n \)) with \( \delta_i(x_{j}) \neq \bigcup_{i} \psi \) and this contradiction with our hypothesis. Hence \( c(\beta) = \frac{n}{\prod_{i=1}^{n} |T_i^\beta|} \).

Theorem: 4.10

Let \( \omega_i = \{ x_{g_1}, x_{g_2}, \ldots, x_{g_{\Omega_i}} \} \) and for any pair \( \omega_1 \neq \omega_2 \in T_i \) \( (1 \leq i \leq n) \) such that \( \omega_1 \cap \omega_2 = \phi \) and \( \delta_i(x_{g_1}) \notin \bigcup_{i} \psi \), for any \( \omega_1 \cap \omega_2 = \phi \)
\( (1 \leq i \leq n) \). In other side,
\( (\Omega, t^\beta)_{nk} \Gamma \) is an even permutation space, if
\[ 2 \left( n - \sum_{i=1}^{n} |T_i^\beta| \right) \].

Proof:
By [Theorem (4.9)], we consider that 

$$\Omega_{t_{nk}} \Gamma$$ is a permutation space induced by soft topology \( \Gamma \) and its permutation \( \beta \) in symmetric group \( S_{nk} \) satisfies 

$$c(\beta) = 2 \left( nk - \sum_{i=1}^{n} |T_i| \right) \text{ and hence } nk - c(\beta) \text{ is even.}$$

Then \((\Omega_{t_{nk}} \Gamma)\) is an even permutation space.

**Lemma: 4.11**

Assume \((X, E, \Gamma)\) is a (STS). Then \((\Omega_{t_{n}} \Gamma)\) is odd permutation space, if \((X, E, \Gamma)\) is a (SITS) and \(2 \mid n\).

**Proof:**

Assume \((X, E, \Gamma)\) is a (SITS) and let \((\Omega_{t_{n}} \Gamma)\) be a permutation space induced by soft topology \( \Gamma \). Hence \( \beta = 1 2 3 \ldots n \) [since \((X, E, \Gamma)\) is (SITS)], thus \( c(\beta) = 1 \). Also, since \(2 \mid n\), then there is a positive integer number \( q \) such that \( n = 2q \) and hence \( n - c(\beta) = \) (even) - (odd). Hence \( \beta \) is odd permutation in \( S_n \).

Then \((\Omega_{t_{n}} \Gamma)\) is odd permutation space.

**Lemma: 4.12**

Assume \((X, E, \Gamma)\) is a soft discrete topological space. Then 

$$\sum_{i=1}^{n} |T_i| = |T_j|$$

for any \((1 \leq j \leq n)\).

**Proof:**

Assume \((X, E, \Gamma)\) is a soft discrete topological space. Then, we consider that 

$$T_i = T_j, \quad \forall(1 \leq i, j \leq n)$$

So 

$$|T_i| = |T_j|$$

\( \forall(1 \leq i, j \leq n) \) and hence  

$$\sum_{i=1}^{n} |T_i| = |T_1| + |T_2| + \cdots + |T_n| = |T_j| + |T_j| + \cdots + |T_j| = n|T_j|, \quad \forall(1 \leq j \leq n).$$

**Lemma: 4.13**

Let \((X = \{x_j\}_{j=1}^{k}, E = \{e_{r_{r=1}}\}, \Gamma)\) be a (STS).

Then \((\Omega_{t_{n}} \Gamma)\) is (PSS), if \( \omega_i = 1, \forall \omega_i \in T_i^\prime \), where \((1 \leq i \leq n)\).

**Proof:**

Let \((X, E, \Gamma)\) be a (STS). Then \((\Omega_{t_{n}} \Gamma)\) is (PSS), if \( \omega_i = 1, \forall \omega_i \in T_i^\prime \), and \((\Omega_{t_{n}} \Gamma)\) be a permutation space induced by soft topology \( \Gamma \). Since \( \omega_i = 1, \forall \omega_i \in T_i^\prime \), this implies that \( \omega_i = \{x_{g_i}\} \) for some \( x_{g_i} \in X \). Therefore \( \beta = \prod_{i=1}^{n} (\prod_{j=1}^{n} (\delta_i(x_{g_j}))) \) for some \( x_{g_i} \in X \). Then \((\Omega_{t_{n}} \Gamma)\) is (PSS) [since each proper open \( \beta \) - set is a singleton].

**Lemma: 4.14**

Let \((X = \{x_j\}_{j=1}^{k}, E = \{e_{r_{r=1}}\}, \Gamma)\) be a (STS).

Then \((\Omega_{t_{n}} \Gamma)\) is an even permutation space, if \( \omega_i = 1, \forall \omega_i \in T_i^\prime \), where \((1 \leq i \leq n)\).

**Proof:**

By [Lemma, (4.13)] we get \((\Omega_{t_{n}} \Gamma)\) is (PSS). Let \( \lambda_{i} (c(\beta)) \) be the family of the product of separate cycles with the 1-cycles of \( \beta \)

where \( \lambda_{i} = (\delta_i(x_{g_j})), \) for some \( x_{g_j} \in X \) and \( 1 \leq i \leq c(\beta) \) [since each proper open \( \beta \) - set is a singleton]. However,
\[ \beta = \prod_{i=1}^{n} \left( \prod_{j=1}^{k} (\delta_{ij}(x_{g_{ij}})) \right) = e \in S_{nk}. \]

is an identity element in \( S_{nk} \). Thus \( \beta = e = (1 \ 2 \ 3 \ ... \ (nk) ) \) and hence \( c(\beta) = nk \).

This implies \( nk - c(\beta) = 0 \) (even). Hence \( (\Omega, t_{nk}^{\beta}) \) is an even permutation space.

**Theorem: 4.15**

Let \( (X = \{ x_{j} \}_{j=1}^{k}, E = \{ e_{x_{j}} \}_{x_{j}=1}^{n}, \Gamma) \) be a (STS) and for any pair \( \omega_{i} \neq \omega_{j} \in T_{i} \) (1 \( \leq i \leq n \)) such that \( \omega_{i} \cap \omega_{j} = \emptyset \) and \( \delta_{i}(x_{j}) \in \bigcup_{\psi \in T_{i}-\{x_{j}\}} \psi \), for any \( (1 \leq i \leq n) \) and \( (1 \leq j \leq k) \). Then \( (\Omega, t_{nk}^{\beta}) \) is a splittable permutation space, if the following are hold.

1. \( 2 \mid \lfloor \omega_{i} \rfloor - 1 \), \( \forall \omega_{i} \in T_{i} \) (1 \( \leq i \leq n \)),
2. If \( \omega_{i} \neq \omega_{j} \), then \( |\omega_{i}| \neq |\omega_{j}| \), where \( \omega_{i} \in T_{i} \) and \( \omega_{j} \in T_{j} \) (1 \( \leq j, i \leq n \)).

**Proof:**

By [Theorem (4.9)], we consider that \( (\Omega, t_{nk}^{\beta}) \) is a permutation space induced by soft topology \( \Gamma \) and its permutation \( \beta \) in symmetric group \( S_{nk} \) satisfies \( c(\beta) = \sum_{i=1}^{n} t_{i}^{\beta} \). Moreover, \( \beta = \prod_{i=1}^{n} \sigma_{i} = \prod_{i=1}^{n} \left| t_{i}^{\beta} \right| = \left| \prod_{i=1}^{n} \sigma_{i} \right| = \left| \prod_{i=1}^{n} \sigma_{i} \right| \), where for all \( 1 \leq i \leq n \), \( \sigma_{i} = \prod_{L=L_{i}}^{n} \sigma_{L} \), \( \sigma_{L} = \prod_{L=L_{i}}^{n} \left( \delta_{ij}(x_{g_{ij}}) \right) \) is permutation \( \prod_{L=L_{i}}^{n} \left( \delta_{ij}(x_{g_{ij}}) \right) \) which is product of \( \left| t_{i}^{\beta} \right| \) cyclic factors of the length \( \left| \omega_{i} \right| \), where \( \omega_{i} \in T_{i} \) and \( 1 \leq l \leq \left| t_{i}^{\beta} \right| \).

For each \( \beta \in S_{n} \) can be written as uniquely product of separate cycles. Thus \( \beta = \lambda_{1} \lambda_{2} \ldots \lambda_{c(\beta)} \), where \( \lambda_{i} = a_{i} \), \( \{ \lambda_{i} \} = \{ c(\beta) \} \) are separate cycles and \( c(\beta) \) is the number of the product of separate cycles with the 1-cycles of \( \beta \).

Hence \( \alpha = \alpha(\beta) = (\alpha_{1}, \alpha_{2}, \ldots, \alpha_{c(\beta)}) \) is the "cycle type" of \( \beta \). Since \( \beta = \prod_{i=1}^{n} \sigma_{i} \) and \( c(\beta) = \sum_{i=1}^{n} t_{i}^{\beta} \). Then for any \( \lambda_{i} \) there exists \( \sigma_{i} \) satisfies \( \lambda_{i} = \sigma_{i} \), where \( (1 \leq i \leq n) \). The length of any cycle \( \sigma_{i} = (\delta_{i}(x_{g_{i1}}) \delta_{i}(x_{g_{i2}}) \ldots \delta_{i}(x_{g_{in}})) \) is \( \lfloor \omega_{i} \rfloor \) and this implies that \( \alpha_{1}, \alpha_{2}, \ldots, \alpha_{c(\beta)} \) is odd number for some \( (1 \leq i \leq n) \) and \( (1 \leq L \leq \left| t_{i}^{\beta} \right| ) \), thus \( \lfloor \omega_{i} \rfloor - 1 \mid 1 \leq i \leq n \). Also, for any \( (i \neq j) \) or \( (g \neq h) \) we have \( \omega_{i} \neq \omega_{j} \), where \( (1 \leq i, j \leq n) \), \( (1 \leq g \leq \left| t_{i}^{\beta} \right| ) \) and \( (1 \leq h \leq \left| t_{j}^{\beta} \right| ) \). Then \( \{ \omega_{i} \} \mid 1 \leq i \leq n \mid 1 \leq L \leq \left| t_{i}^{\beta} \right| \) are different sets and by (2) of our hypothesis we consider that \( \{ \omega_{i} \} \mid 1 \leq i \leq n \mid 1 \leq L \leq \left| t_{i}^{\beta} \right| \) are different too. Therefore \( \beta \in H_{n} \) and hence \( (\Omega, t_{nk}^{\beta}) \) is a splittable permutation space.
Theorem: 4.16

Let $X = \{x_j | j = 1, 2, \ldots, n\}$ be a (STS) and for any pair $\omega_1 \neq \omega_2 \in T_i$ \((1 \leq i \leq n)\) such that $\omega_i \cap \omega_2 = \emptyset$ and $\delta_i(x) \in \cup_{x \in T_i \setminus \{x_i\}} \psi$, for any \((1 \leq i \leq n)\) and \((1 \leq j \leq k)\). Then $(\Omega, t_e)$ is an ambivalent permutation space, if the following are hold.

1) $2 \mid (|\omega| - 1), \forall \omega \in T_i$ \((1 \leq i \leq n)\),
2) If $\omega_i \neq \omega_j$, then $|\omega_i| \neq |\omega_j|$, where $\omega_i \in T_i$ and $\omega_j \in T_j$ \((1 \leq j, i \leq n)\),
3) $4 \mid (|\omega| - 1), \forall \omega \in T_i$ \((1 \leq i \leq n)\).

Proof:

From (1) and (2) we consider that permutation space $(\Omega, t_e)$ is a splittable [By Lemma, (4.13)]. Also, from (1) and (2) that easy to show that \((\alpha_1, \alpha_2, \ldots, \alpha_c(\beta)) = (|\omega_i|, |\omega_1|, |\omega_2|, \ldots, |\omega_{c-1}|, |\omega_c|, \ldots, |\omega_{2c-1}|)\) . That means for any part $\alpha_i$ of $\alpha(\beta)$, there exists $\omega_{iL} \in T_i$ for some \((1 \leq i \leq n)\) and \((1 \leq L \leq |T_i|)\) satisfies $\alpha_i = |\omega_{iL}|$ and hence from (3) we have $4 \mid (|\alpha_L| - 1), \forall (1 \leq i \leq c(\beta))$.

Then $(\Omega, t_e)$ is an ambivalent permutation space.

5. PROPOSED IMPROVEMENTS

Due to the difficulties of finding link between two different topological spaces in different strictures, the present algorithm is the first link between soft space $(X, E, \Gamma)$ and permutation space $(\Omega, t_e)$, where they are two different topological spaces in different strictures. This algorithm will help us to study all of the notions in (STSs) that are given in past work in permutation topological spaces when they are induced by some soft topologies. This algorithm is highly recommended.

6. CONCLUSIONS AND OPEN PROBLEMS

In this work, an algorithm has been introduced to find the permutation in symmetric group using soft topological space to structure permutation topological space. Further, suppose that $(X, E, \tau_1)$, $(X, E, \tau_2)$ and $(X, E, \tau_3)$ are three (STSs) over the common universe $X$ the parameter set $E$ with their permutations $\beta$, $\mu$ and $\delta$ in symmetric group $S_n$ where $n = (|X| \times |E|)$. Hence the questions can be summarized as follows:

1) Is necessarily true, if $(\Omega, t_e)$ is a even (res. odd, splittable, ambivalent) permutation space and $T \rightarrow (\Omega, t_e)$ is continuous permutation map. Then the image under $\delta$ is even (res. odd, splittable, ambivalent) permutation space, too.
2) Is necessarily true, if $(X, E, \tau_1)$ and $(X, E, \tau_2)$ are two (STSs). Then $(\Omega, t_e)$ is a permutation space induced by soft topology $\tau$, where $T = \tau_1 \times \tau_2$.
3) Is necessarily true, if $(X, E, T)$ is a soft connected. Then $(\Omega, t_e)$ is a permutation connected induced by soft topology $T$.
4) Is necessarily true, if $(X, E, T)$ is a soft compact space. Then $(\Omega, t_e)$ is a permutation compact space induced by soft topology $T$.

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