ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

AN ALGORITHM FOR GENERATING PERMUTATIONS IN SYMMETRIC GROUPS USING SOFT SPACES WITH GENERAL STUDY AND BASIC PROPERTIES OF PERMUTATION SPACES

[1]SHUKER MAHMOOD KHALIL, [2]FATIMA HAMEED KHADEER

Department of Mathematics, College of Science, Basrah University, Basrah 61004, Iraq.
E-mail: shuker.alsalem@gmail.com

ABSTRACT

In this paper we introduced algorithm to find permutation in symmetric group using soft topological space to structure permutation topological space. Moreover, this class of permutation topology is called even (odd) permutation topology if its permutation is even (odd). Further, new notions in permutation topological spaces are investigated like splittable permutation spaces and ambivalent permutation spaces.

Keywords: *Soft Set Theory, Symmetric Groups, Splitting, Ambivalent, Permutation Topological Spaces.*
MSC 2010: 54A10, 54C10, 54A05, 20G05, 20D06, 20B30

1. INTRODUCTION

Soft sets were originally shown by Molodtsov [1]. Then they applied in many fields where they have rich possibility for applications. In 2011, the connotation of soft topological spaces (STSs) is shown by Muhammad and Munazza [2]. Some notions of (STS) and of its applications fundamental connotations of fuzzy soft topology and Intuitionistic fuzzy soft topology are studied by many mathematicians see ([3-9]). In 2014, Mahmood [10] introduced the notion of permutation topological space (PTS) using permutation β in symmetric group S_n, where each permutation $\beta \in S_n$ can be represented as a product of disjoint (separate) cycles. In other words, $\beta = (b_1^1, b_2^1, \ldots, b_{\alpha_1}^1)(b_1^2, b_2^2, \ldots, b_{\alpha_2}^2) \ldots (b_1^{c(\beta)}, b_2^{c(\beta)}, \ldots, b_{\alpha_{c(\beta)}}^{c(\beta)})$ and $\forall\, i \neq j$ satisfy $\{b_1^i, b_2^i, \ldots, b_{\alpha_i}^i\} \cap \{b_1^j, b_2^j, \ldots, b_{\alpha_j}^j\} = \phi$ [11]. That means, β can be represented as $\lambda_1 \lambda_2 \cdots \lambda_{c(\beta)}$, where λ_i separate cycles of length $|\lambda_i| = \alpha_i$ and $c(\beta)$ refers to the number of the product of separate cycles with the 1-cycles of β. In 2015, some methods are introduced to generate fuzzy soft set and intuitionistic fuzzy soft set using different sets [12]. Now, the interesting question is there any an algorithm shows the relation between permutation space and soft space. In this work, this algorithm is given. We generate permutation topological space using soft topological space (STS). This class of permutation topology is called even (odd) permutation topology if its permutation is even (odd). Further, new notions in permutation topological spaces are investigated like splittable permutation spaces and ambivalent permutation spaces.

2. PRELIMINARIES AND BASIC RESULTS

We will show some past results and basic definitions in this section.

Definition 2.1 ([13])

A "cycle type" of β is the partition $\alpha = \alpha(\beta) = (\alpha_1(\beta), \alpha_2(\beta), \ldots, \alpha_{c(\beta)}(\beta))$.

Definition 2.2 ([13])

We refer to all the permutations in S_n of cycle type α by C^{α}.

Definition 2.3 ([13])

Let β be a permutation in Alternating group A_n and $\beta \in C^{\alpha}$. $A(\beta)$ conjugacy class of β in A_n is defined by $A(\beta) = \{\gamma \in A_n \mid \gamma = t\beta t^{-1}$; for some $t \in A_n\}$ =

$$\begin{cases} C^{\alpha}, & (\text{if } \beta \notin H_n) \\ C^{\alpha+} \text{ or } C^{\alpha-}, & (\text{if } \beta \in H_n) \end{cases}$$

where $C^{\alpha\mp}$ are two classes of equal order in Alternating group A_n such that $C^{\alpha} = C^{\alpha+} \cup C^{\alpha-}$ and $H_n = \{C^{\alpha}$ of $S_n \mid n > 1$, with all parts α_k of α different and odd$\}$.

Proposition 2.4 ([14])

The conjugacy classes $C^{\alpha\pm}$ of A_n are ambivalent if $4 \mid (\alpha_i - 1)$ for each part α_i of α.

Definition 2.5 ([10])

Let $\beta \in S_n$ with $\Omega = \{1,2,\ldots,n\}$ and the "cycle type" of β is $\alpha(\beta) = (\alpha_1, \alpha_2, \ldots, \alpha_{c(\beta)})$, and $\{\lambda_i\}_{i=1}^{c(\beta)}$ be a composite of "pairwise separate cycles" of β where $\lambda_i = (b_1^i, b_2^i, \ldots, b_{\alpha_i}^i)$, $1 \le i \le c(\beta)$. Each $\lambda = (b_1, b_2, \ldots, b_k)$, k – cycle in S_n we put β – set as $\lambda^{\beta} = \{b_1, b_2, \ldots, b_k\}$ and is said to be β – set of cycle λ. Also, β – sets of $\{\lambda_i\}_{i=1}^{c(\beta)}$ are investigated by $\{\lambda_i^{\beta} = \{b_1^i, b_2^i, \ldots, b_{\alpha_i}^i\} \mid 1 \le i \le c(\beta)\}$.

Remark 2.6

Suppose that λ_i^{β} and λ_j^{β} are β – sets in Ω, where $|\lambda_i| = \sigma$ and $|\lambda_j| = \upsilon$. Then the known definitions will be written as following:

Definition 2.7 ([10])

Let λ_i^{β} and λ_j^{β} be β – sets in Ω, they are called separate iff there exists $1 \le d \le \sigma$, for each $1 \le r \le \upsilon$ such that $b_d^i \ne b_r^j$ and $\sum_{k=1}^{\sigma} b_k^i = \sum_{k=1}^{\upsilon} b_k^j$.

Definition 2.8 ([10])

Let λ_i^{β} and λ_j^{β} be β – sets in Ω. We say they are equal iff there exists $1 \le d \le \sigma$, for each $1 \le r \le \upsilon$ such that $b_d^i = b_r^j$.

Definition 2.9: ([10])

We say λ_i^{β} is contained in λ_j^{β} and denoted

$\lambda_i^{\beta} \hat{\subset} \lambda_j^{\beta}$, iff $\sum_{k=1}^{\alpha_i} b_k^i < \sum_{k=1}^{\alpha_j} b_k^j$.

Definition 2.10: ([15])

For any $\lambda^{\beta} = \{b_1, b_2, \ldots, b_r\}$ and $\eta^{\beta} = \{a_1, a_2, \ldots, a_{\upsilon}\}$ two subsets of Ω, we call λ^{β} and η^{β} are similar β – sets in Ω, iff

$$\sum_{k=1}^{r} b_k = \sum_{k=1}^{\upsilon} a_k$$

Journal of Theoretical and Applied Information Technology
15th May 2018. Vol.96. No 09

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

and there are two points say $b_i, b_j \in \lambda^\beta$ such that $b_i \in \eta^\beta$ and $b_j \notin \eta^\beta$. Assume $\lambda^\beta = \{b_1, b_2, \ldots, b_r\}$, $\eta^\beta = \{a_1, a_2, \ldots, a_\upsilon\}$ are similar $\beta-$sets in Ω and $\Delta = Max\{Max\{\eta^\beta - \omega\}, Max\{\lambda^\beta - \omega\}\}$, where $\omega = \{b_1, b_2, \ldots, b_r\} \cap \{a_1, a_2, \ldots, a_\upsilon\}$.

Then $\lambda^\beta \hat{\subset} \eta^\beta$, if $\Delta \in \eta^\beta$. Also, $\eta^\beta \hat{\subset} \lambda^\beta$ if $\Delta \in \lambda^\beta$,

$$\eta^\beta \wedge \lambda^\beta = \begin{cases} \lambda^\beta, \ if \ \ \Delta \in \eta^\beta \\ \eta^\beta, \ if \ \ \Delta \in \lambda^\beta \end{cases}$$

and

$$\eta^\beta \vee \lambda^\beta = \begin{cases} \eta^\beta, \ if \ \ \Delta \in \eta^\beta \\ \lambda^\beta, \ if \ \ \Delta \in \lambda^\beta \end{cases}$$

For any $\lambda^\beta = \{b_1, b_2, \ldots, b_r\}$ and $\eta^\beta = \{a_1, a_2, \ldots, a_\upsilon\}$ two subsets of Ω.

Then,

$$\lambda^\beta \wedge \eta^\beta = \begin{cases} \lambda^\beta, \text{ if } (\sum_{k=1}^{r} b_k < \sum_{k=1}^{\upsilon} a_k) \text{ Or } (\lambda^\beta \,\&\, \eta^\beta \text{ are similar and } \Delta \in \eta^\beta) \\ \eta^\beta, \text{ if} (\sum_{k=1}^{r} b_k > \sum_{k=1}^{\upsilon} a_k) \text{Or } (\lambda^\beta \,\&\, \eta^\beta \text{ are similar and } \Delta \in \lambda^\beta) \\ \mu^\beta, \text{ if } \lambda^\beta = \eta^\beta = \mu^\beta \\ \phi, \text{if } \lambda^\beta \,\&\, \eta^\beta \text{ are disjoint} \end{cases}$$

and

$$\lambda^\beta \vee \eta^\beta = \begin{cases} \lambda^\beta, \text{ if } (\sum_{k=1}^{r} b_k > \sum_{k=1}^{\upsilon} a_k) \text{ Or } (\lambda^\beta \& \eta^\beta \text{ are similar and } \\ \quad \Delta \in \lambda^\beta) \\ \eta^\beta, \text{ if} (\sum_{k=1}^{r} b_k < \sum_{k=1}^{\upsilon} a_k) \text{Or } (\lambda^\beta \& \eta^\beta \text{ are similar and } \\ \quad \Delta \in \eta^\beta) \\ \mu^\beta, \text{ if } \lambda^\beta = \eta^\beta = \mu^\beta \\ \Omega, \text{if } \lambda^\beta \& \eta^\beta \text{ are disjoint} \end{cases}$$

Definition 2.11: ([10])

Let $\{\lambda_i^\beta = \{b_1^i, b_2^i, \ldots, b_{\sigma_i}^i\}\}_{i\in I}$ be a family of not separate $\beta-$sets. We define the intersection (union) of $\{\lambda_i^\beta\}_{i\in I}$ respectively, by $\underset{i\in I}{\wedge} \lambda_i^\beta = \lambda_j^\beta$, where $\sum_{k=1}^{\sigma_j} b_k^j = \inf\{\sum_{k=1}^{\sigma_i} b_k^i \ ; \ i \in I\}$ and $\underset{i\in I}{\vee} \lambda_i^\beta = \lambda_j^\beta$, where $\sum_{k=1}^{\sigma_j} b_k^j = \sup\{\sum_{k=1}^{\sigma_i} b_k^i \ ; \ i \in I\}$.

Definition 2.12: ([10])

Assume β is permutation in S_n, and let $\{\lambda_i\}_{i=1}^{c(\beta)}$ be a family of product of separate cycles with the 1-cycles of β, where $|\lambda_i| = \alpha_i$, $1 \le i \le c(\beta)$, then permutation topology t_n^β is a family of $\beta-$sets of the collection $\{\lambda_i\}_{i=1}^{c(\beta)}$ together with $\Omega = \{1,2,\ldots,n\}$ and empty set. We say (Ω, t_n^β) is permutation space.

Definition: 2.13: ([15])

Assume (Ω, τ_n^β) is a permutation topological space. We say (Ω, τ_n^β) is a Permutation Single Space (PSS) iff all their proper open $\beta-$sets are singleton.

Definition: 2.14: ([15])

Assume (Ω, τ_n^β) is a permutation topological space. We say (Ω, τ_n^β) is a Permutation Indiscrete Space (PIS) iff each open $\beta-$set is trivial $\beta-$set.

Definition 2.15: ([1])

Assume that E is a set of parameters, X is an initial universe set, $P(X)$ is the power set of X

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

and K is a subset of E. We say (F,K) is a soft set over X if F is a multi-valued function of K into $P(X)$.

Definition 2.16 ([16])

Assume (F,A) and (M,L) are soft sets over X, their union (H,C) is a soft set such that for all $e \in C = A \bigcup L$, $H(e) = F(e)$ if $e \in A - L$, $H(e) = M(e)$ if $e \in L - A$, $F(e) \bigcup M(e)$ if $e \in A \bigcap B$. we write $(F,A) \coprod (M,L) = (H,C)$. Also, their intersection is a soft set, denoted by $(H,C) = (F,A) \prod (M,L)$ where $C = A \bigcap L$, and it is defined as $H(e) = F(e) \bigcap M(e)$ for all $e \in C$.

Definition 2.17 ([2])

A soft set $(G,B)^c$ is the complement of (G,B) in (X,E) and it is defined by (G^c, K), where $K = E \setminus \{e \in B \mid G(e) = X\}$ and $\forall e \in K$

$$G^c(e) = \begin{cases} X \setminus G(e), & if \ e \in B \\ X, & if \ e \notin B. \end{cases}$$

Definition 2.18: ([2])

Assume τ is a family of soft sets over X. We say τ is a soft topology on X if τ satisfies the following axioms:
(i)The intersection of any two soft sets in τ belongs to the family τ.
(ii) The union of any number of soft sets in τ belongs to the family τ.
(iii) (X,E) and Φ belong to the family τ.

We say (X,E,τ) is a soft topological space (STS) over X. Also, each member in the family τ is said to be a soft open set and its complement is said to be a soft closed set. Further, (X,E,τ) is said to be a soft topological indiscrete space (SITS) over X, if $\tau = \{\Phi,(X,E)\}$. Also, (X,E,τ) is called a soft discrete topological space (SDTS) over X, if τ contains of all soft sets over X.

Some Results on Permutations 2.19: ([17, 18])

(1) $\beta = (b_1, b_2, ..., b_r)$ is even $\Leftrightarrow$ r is odd.

(2) $\beta \in S_n$ is even $\Leftrightarrow$ $n - c(\beta)$ is even,

(3) $\beta = (1)$ (Identity) $\Leftrightarrow \beta = (b)$ for some $b \in \Omega$.

Remark 2.20:

In this work, for any set $D = \{d_1, d_2, \ldots, d_k\}$ of k distinct objects and for any cycle $B = (b_1 \, b_2 \ldots b_m)$ we will use $|B| = m$ and $|D| = k$ to refer to the length of the cycle B and the cardinality of set D.

3. AN ALGORITHM TO GENERATE PERMUTATION SPACES FROM SOFT SPACES

We will introduce in this section an algorithm to generate permutation topological space by analysis (STS) and this class of permutation topological spaces is called even (odd) permutation topology if its permutation is even (odd). Moreover, some basic properties of permutation spaces are studied.

Steps of the work 3.1:

Assume (X,E,Γ) is a (STS), where $X = \{s_1, s_2, \ldots, s_k\}$, $E = \{e_1, e_2, ..., e_n\}$ and $\Gamma = \{\Phi, (X,E), \{(F_i, E)\}_{i=1}^m\}$. Now, let $T_1 = \{F_i(e_1)\}_{i=1}^m$, $T_2 = \{F_i(e_2)\}_{i=1}^m, \ldots, T_n = \{F_i(e_n)\}_{i=1}^m$. For any $1 \le i \le n$, let $T_i' = \{B \in T_i \,/\, B \ne \phi\}$. $\forall$ $(1 \le i \le n)$, let $\delta_i : X \to N$ be a map from X into natural numbers N defined by $\delta_i(s_j) = j + (i-1)k$, for all $s_j \in X$ and $(1 \le i \le n)$ where $k = |X|$. Then $\sigma = \prod_{i=1}^{n} \sigma_i$ is called permutation in symmetric group S_{nk} where for all $1 \le i \le n$, $\sigma_i = \prod_{L=1}^{|T_i'|} (\delta_i(s_{g_1}) \ \delta_i(s_{g_2}) \ldots \delta_i(s_{g_{|\omega_L|}}))$ is permutation which is product of $|T_i'|$ cyclic factors of the length $|\omega_L|$, where

Journal of Theoretical and Applied Information Technology
15th May 2018. Vol.96. No 09

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$\omega_L = \{s_{g_1}, s_{g_2}, \dots, s_{g_{|\omega_L|}}\} \in T_i'$ and $1 \le L \le |T_i'|$. Further, $\sigma_i = (\delta_i(s_i))$ if $T_i' = \phi$. Then $(\Omega, t_h^{\sigma})_\Gamma$ is called permutation topological space induced by soft topology Γ, where $\Omega = \{1,2,\dots,h\}$, $h = nk$ and t_h^{σ} is a family of $\sigma-$set of the family $\{\sigma_i\}_{i=1}^{n}$ together with Ω and empty set. Also, if (X,E,Γ) is a soft indiscrete space. Then $(\Omega, t_h^{\sigma})_\Gamma$ is called permutation indiscrete space (PIS) induced by soft topology Γ, where $\sigma = (1\ 2\ 3 \dots h)$. Finally, for any (X,E,Γ) non-indiscrete (STS), where $\Gamma = \{\Phi, (X,E), \{(F_i,E)\}_{i=1}^{m}\}$, $E = \{e_1, e_2, \dots, e_n\}$ and $X = \{s_1, s_2, \dots, s_k\}$, we can generate permutation topological space $(\Omega, t_h^{\sigma})_\Gamma$ as follows:

(1)- Find $T_i = \{F_j(e_i)\}_{j=1}^{m}$, for all $1 \le i \le n$,

(2)- Find $T_i' = \{\omega \in T_i \,/\, \omega \ne \phi\}$, for all $1 \le i \le n$,

(3)- Find $\delta_i(s_j) = j + (i-1)k$, $\forall\ (1 \le j \le k)$ and $\forall\ (1 \le i \le n)$,

(4)- Find σ_i for all $1 \le i \le n$, where

$$\sigma_i = (\delta_i(s_i)), \quad \text{if } T_i' = \phi \text{ and}$$

$$\sigma_i = \prod_{L=1}^{|T_i'|} (\delta_i(s_{g_1})\ \delta_i(s_{g_2})\ \dots \delta_i(s_{g_{|\omega_L|}})),$$

if $T_i' \ne \phi$ where

$$\omega_L = \{s_{g_1}, s_{g_2}, \dots, s_{g_{|\omega_L|}}\} \in T_i'$$

(5)-Find $\sigma = \prod_{i=1}^{n} \sigma_i$,

(6)-Find the "separate cycle factors including the 1-cycle" of σ say $\lambda_1 \lambda_2 \dots \lambda_{c(\sigma)}$,

(7)-Find the $\sigma-$sets of $\{\lambda_i\}_{i=1}^{c(\sigma)}$

(8)- Find Ω, where $\Omega = \{1,2,\dots h\}$ and $h = nk$,

(9)- Find t_h^{σ}, where $t_h^{\sigma} = \{\phi, \Omega, \lambda_1^{\sigma}, \lambda_2^{\sigma}, \dots, \lambda_{c(\sigma)}^{\sigma}\}$.

(10) Then $(\Omega, t_h^{\sigma})_\Gamma$ is (PTS).

Remarks 3.2:

(1) For all $1 \le i \le n$, let $T_i^{*} = \{\phi, X\} \cup T_i'$. Here we used normal union ($\cup$), normal intersection ($\cap$) and empty set (ϕ) then we have (X, T_i^{*}) is a topological space for each ($1 \le i \le n$).

(2) We consider that $\delta_i : (X, T_i^{*}) \to (X_i, T_i^{*/})$ is an isomorphic, where $X_i = \delta_i(X)$ and $T_i^{*/} = \{Y \mid Y = \delta_i(Z); Z \in T_i^{*}\}$. In other words, $\forall\ 1 \le t \le mk$, we have $\delta_i^{-1}(t) = s_c$, where $c = t - k(i+1)$.

(3) For any $(1 \le i \ne q \le n)$ and $(1 \le j \le k)$, we have $\delta_i(s_j) \notin X_q$ [Since $\delta_i(s_j) = j + (i-1)k \ne j + (q-1)k$, $\forall j = 1,2,..,k$]

(4) If $\delta_i(s_j) \notin \bigcup_{\psi \in T_i^{*/} - \{X_i\}} \psi$, for some $(1 \le i \le n)$ and $(1 \le j \le k)$. Then $\delta_i(s_j) \notin \bigcup_{\psi \in T_q^{*/} - \{X_q\}} \psi$ for all $(1 \le q \le n)$.

Permutation Subspaces Induced by Soft Topology Γ 3.3

Let $(\Omega, t_n^{\beta})_\Gamma$ be a permutation space induced by soft topology Γ, $\lambda^{\beta} \subset \Omega$ and $T_i^{\beta} = \lambda^{\beta} \wedge \lambda_i^{\beta}$, for each proper $\lambda_i^{\beta} \in t_n^{\beta}$,

$$T_i^{\beta} = \begin{cases} \{b_1^i, b_2^i, \dots, b_{i_k}^i\}, & \text{if } \lambda^{\beta} \ \& \ \lambda_i^{\beta} \text{ are not separate} \\ \phi, & \text{if } \lambda^{\beta} \ \& \ \lambda_i^{\beta} \text{ are separate} \end{cases}$$

Let $\Re = \{T_i^{\beta} \mid T_i^{\beta}$ nonempty open $\beta -$set$\}$. $\forall\ T_i^{\beta} \in \Re$, let $b_k^i = Max\{b_1^i, b_2^i, ..., b_{i_k}^i\}$ and $m = Max\{b_k^i ; T_i^{\beta} \in \Re\}$. Let $B = \bigcap_{T_i^{\beta} \in \Re} (\Omega' - T_i^{\beta})$ where $\Omega' = \{1,2,..., m\}$ and ($\bigcap$) is a normal intersection. $\forall\ T_i^{\beta} \in \Re$ we consider $T_i = (b_1^i, b_2^i, ..., b_{i_k}^i)$ is $i_k -$cycle in S_m. In other words, the product of separate cycles of other permutation in symmetric group S_m induced by λ^{β} say $\gamma^{\lambda^{\beta}}$ where $\gamma^{\lambda^{\beta}} = \prod_{T_i^{\beta} \in \Re} T_i \prod_{r=1}^{t} (b_r)$ and $\gamma^{\lambda^{\beta}} = \prod_{T_i^{\beta} \in \Re} T_i$ whenever $B = \phi$. Moreover, we say $(\Omega', t_m^{\gamma^{\lambda^{\beta}}})_\Gamma$ is a permutation subspace induced by soft topology Γ where $t_m^{\gamma^{\lambda^{\beta}}}$ is a family of all $\gamma^{\lambda^{\beta}}$ – sets of product of separate cycles of $\gamma^{\lambda^{\beta}}$ together with Ω' and empty set.

Lemma 3.4:

If each pair of different members in $\Re$ are separate, then $B = \{b_1, b_2, ..., b_t\}$ has exactly t points where $t = m - s$ and $\sum_{T_i^{\beta} \in \Re} |T_i| = s$.

Proof:

Suppose that $T_i^{\beta} \cap T_j^{\beta} = \phi$, for any $T_i^{\beta}, T_j^{\beta} \in \Re$ and $B = \{b_1, b_2, ..., b_k\}$ with $k \neq t$ where $t = m - s$ and $\sum_{T_i^{\beta} \in \Re} |T_i| = s$. Since $B = \bigcap_{T_i^{\beta} \in \Re} (\Omega' - T_i^{\beta})$ where $\Omega' = \{1,2,..., m\}$. Thus $| B | = | \bigcap_{T_i^{\beta} \in \Re} (\Omega' - T_i^{\beta}) |$. since $T_i^{\beta} \cap T_j^{\beta} = \phi$, for any $T_i^{\beta}, T_j^{\beta} \in \Re$. This implies that $| B | = | \bigcap_{T_i^{\beta} \in \Re} (\Omega' - T_i^{\beta}) | = | \Omega' | - \sum_{T_i^{\beta} \in \Re} |T_i| = m - s = t$. But $| B | = k \neq t$ and this contradiction. Therefore $B = \{b_1, b_2, ..., b_t\}$ has exactly t points.

Example 3.5

Let the set of cars under consideration be $X = \{a_1, a_2, a_3\}$. Let $E = \{$ cheap (e_1); dark color (e_2); modern (e_3); beautiful (e_4)$\}$ be set parameter set. Now, to buy a good car. Let (F, A) be soft set describing the Mr. Z opinion and it is defined by

$A = \{e_1, e_3, e_4\}$

$F(e_1) = \{a_1, a_3\}$, $F(e_3) = \{a_2\}$, $F(e_4) = X$

Further, we suppose that the good car in the opinion of his friend, say Mr.W, is described by (G, B) and it is defined by

$B = \{e_1, e_4\}$

$G(e_1) = \{a_3\}$, $G(e_4) = \{a_2, a_3\}$

We have:

$\Gamma = \{\Phi, (U, E), (F, A), (G, B)\}$ is a soft topology.

Find permutation space $(\Omega, t_n^{\sigma})_\Gamma$ induced by soft topology Γ. Also, find $(\Omega'_1, t_m^{\gamma^{\lambda^{\sigma}}})_\Gamma$ and $(\Omega'_2, t_m^{\gamma^{\pi^{\sigma}}})_\Gamma$ where $\lambda^{\sigma} = \{1,4\}$ and $\pi^{\sigma} = \{12\}$.

Solution: We consider that

$T_1 = \{F(e_1), G(e_1)\}, T_2 = \{F(e_2), G(e_2)\}, T_3 = \{F(e_3), G(e_3)\}, T_4 = \{F(e_4), G(e_4)\} \Rightarrow$

$T_1 = \{\{a_1, a_3\}, \{a_3\}\}, T_2 = \{\phi, \phi\}, T_3 = \{\{a_1, a_3\}, \{a_1, a_2, a_3\}\}, T_4 = \{\Phi, \{a_1, a_3\}\} \Rightarrow$

$T_1' = \{\{a_1, a_3\}, \{a_3\}\}, T_2' = \phi, T_3' = \{a_2\}, T_4' = \{X, \{a_2, a_3\}\} \Rightarrow$

$\overset{*}{T_1} = \{\phi, X, \{a_3\}, \{a_1, a_3\}\}, \overset{*}{T_2} = \{\phi, X\}, \overset{*}{T_3} = \{\phi, X, \{a_2\}\}, \overset{*}{T_4} = \{\phi, X, \{a_2, a_3\}\}$. Hence we have $(X, \overset{*}{T_i})$ is a topological space for each ($1 \le i \le 4$). Now, we consider that $(X_i, T_i^{/*})$ is a topological space for each ($1 \le i \le n$).

where $(X_1, T_1^{/*}) = (\{1,2,3\}, \{\phi, \{1,2,3\}, \{3\}, \{1,3\}\})$,

$(X_2, T_2^{/*}) = (\{4,5,6\}, \{\phi, \{4,5,6\}\})$,

$(X_3, T_3^{/*}) = (\{7,8,9\}, \{\phi, \{7,8,9\}, \{8\}\})$,

$(X_4, T_4^{/*}) = (\{10,11,12\}, \{\phi, \{10,11,12\} \{11,12\}\})$.

Moreover, for all $1 \le i \le n$, $\sigma_i = \prod_{L=1}^{|T_i'|} (\delta_i(a_{g_1})\, \delta_i(a_{g_2}) \ldots \delta_i(a_{g_{|\omega_L|}}))$ is permutation which is product of $|T_i'|$ cyclic factors of the length $|\omega_L|$, where $\omega_L \in T_i'$ and $1 \le L \le |T_i'|$. Hence we have

$$\sigma_1 = (\delta_1(a_2), \delta_1(a_3))(\delta_1(a_3)) = (1\ 3)(3),$$

$$\sigma_2 = (\delta_2(a_2)) = (5),$$

$$\sigma_3 = (\delta_3(a_2)) = (8),$$

$$\sigma_4 = (\delta_4(a_2)\ \delta_4(a_3)) = (11\ 12)(10\ 11\ 12).$$

Then $\sigma = \prod_{i=1}^{4} \sigma_i =$ $(1\ 3)(3)(5)(8)(11\ 12)(10\ 11\ 12) =$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & 2 & 1 & 4 & 5 & 6 & 7 & 8 & 9 & 12 & 11 & 10 \end{pmatrix}$$

is a permutation in symmetric group S_{12}. Now, we can write $\sigma \in S_n$ as $\lambda_1 \lambda_2 \ldots \lambda_{c(\sigma)}$. Hence

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 3 & 2 & 1 & 4 & 5 & 6 & 7 & 8 & 9 & 12 & 11 & 10 \end{pmatrix} =$$

$(1\ 3)\ (2)\ (4)(5)\ (6)\ (7)\ (8)(9)(11)(10\ 12)$.

Therefore (Ω, t_n^{σ}) is a permutation space induced by soft topology Γ, where $t_n^{\sigma} = t_{12}^{\sigma} = \{\Omega, \phi, \{1,3\}, \{2\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}, \{11\}, \{10,12\}\}$ and $\Omega = \{1,2,3,4,5,6,7,8,9,10,11,12\}$. Now to find the permutation subspace for,

we consider that

$T_1^{\sigma} = \{1,3\}$, $T_2^{\sigma} = \{2\}$, $T_3^{\sigma} = \{4\}$, $T_4^{\sigma} = \phi$,
$T_5^{\sigma} = \{1,4\}$, $T_6^{\sigma} = \{1,4\}$, $T_7^{\sigma} = \{1,4\}$,
$T_8^{\sigma} = \{1,4\}$, $T_9^{\sigma} = \{1,4\}$, $T_{10}^{\sigma} = \{1,4\} \Rightarrow$
$\Re = \{\{1,3\}, \{2\}, \{4\}, \{1,4\}\} \Rightarrow$
$Max\,\{Max\,\{1,3\}, Max\,\{2\}, Max\,\{4\}, Max\,\{1,4\}\} = Max\,\{3,2,4\} = 4 = m \Rightarrow$
$\Omega_1' = \{1,2,3,4\}$, $B = \bigcap_{T_i^{\sigma} \in \Re} (\Omega_1' - T_i^{\sigma}) = \phi$

Then

$$\gamma^{\lambda^{\sigma}} = (1\ 3)(2)(4)(1\ \ 4) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 2 & 1 & 3 \end{pmatrix} = (1\ \ 4\ \ 3)(2)$$

is a permutation in symmetric group S_4 induced by $\lambda^{\sigma} = \{1,4\}$ and $(\Omega_1', t_4^{\gamma^{\lambda^{\sigma}}})$ is a permutation subspace induced by soft topology Γ, where $t_m^{\gamma^{\lambda^{\sigma}}} = t_4^{\gamma^{\lambda^{\sigma}}} = \{\Omega_1', \phi, \{2\}, \{1,3,4\}\}$. Moreover, lemma 3.4 is not hold for $\lambda^{\sigma} = \{1,4\}$. Since $\sum_{T_i^{\sigma} \in \Re} |T_i| = s$
$\Rightarrow |(1\ 3)| + |(2)| + |(4)| + |(1\ 4)| = 2+1+1+2 = 6$ and hence $m - s = 4 - 6 = -2$. But $t = 0$ (since $B = \phi$), thus $t \ne m - s$. That means, lemma 3.4 is not hold for $\lambda^{\sigma} = \{1,4\}$ (since there are two different members $\{1,3\}, \{1,4\} \in \Re$ are not separate). Now to find the permutation subspace for $\pi^{\sigma} = \{12\}$,

we consider that:

Journal of Theoretical and Applied Information Technology
15th May 2018. Vol.96. No 09

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$T_1^{\sigma} = \{1,3\}$, $T_2^{\sigma} = \{2\}$, $T_3^{\sigma} = \{4\}$, $T_4^{\sigma} = \{5\}$, $T_5^{\sigma} = \{6\}$, $T_6^{\sigma} = \{7\}$, $T_7^{\sigma} = \{8\}$, $T_8^{\sigma} = \{9\}$, $T_9^{\sigma} = \{10\}$, $T_{10}^{\sigma} = \{12\} \Rightarrow$
$\mathfrak{R} = \{\{1,3\},\{2\},\{4\},\{5\},\{6\},\{7\},\{8\},\{9\},\{10\},\{12\}\}$
$\Rightarrow m = 12 \Rightarrow \Omega'_2 = \Omega$, $\sum_{T_i^{\sigma}\in\mathfrak{R}} |T_i| = s = 11$ and $B = \bigcap_{T_i^{\sigma}\in\mathfrak{R}} (\Omega'_2 - T_i^{\sigma}) = \{11\}$. This implies that $t = |B| = 1$, also $m - s = 12 - 11 = 1$. That means $t = m - s$. Further,
$\gamma^{\pi^{\sigma}} = (1\ \ 3)(2)(4)(5)(6)(7)(8)(9)(10)(11)(12)$
is a permutation in symmetric group S_{12} induced by $\pi^{\sigma} = \{12\}$ and $(\Omega'_2, t_{12}^{\gamma^{\lambda^{\beta}}})_{\Gamma}$ is a permutation subspace induced by soft topology Γ, where $t_m^{\gamma^{\pi^{\sigma}}} = t_{12}^{\gamma^{\pi^{\sigma}}} = \{\Omega'_2, \phi, \{1,3\}, \{2\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}, \{10\}, \{11\}, \{12\}\}$. Moreover, (lemma 3.4) is hold for $\pi^{\sigma} = \{12\}$ (since each pair of different members in $\mathfrak{R}$ are separate).

4. Some Notions of Permutation Spaces

Definition 4.1:

Assume (Ω, t_n^{β}) is a (PTS), we say (Ω, t_n^{β}) is even (odd) permutation space if its permutation β is even (odd) in S_n.

Example 4.2:

Assume (Ω, t_9^{β}) is a (PTS), where $\beta = (4\ 1\ 6\ 2)\ (7\ 3\ 8\ 9)\ (5)$. Then (Ω, t_9^{β}) is an even permutation space since $\beta \in S_9$ is an even.

Definition 4.3:

Assume (Ω, t_n^{β}) is a (PTS), we say (Ω, t_n^{β}) is splittable permutation space if its permutation β satisfies $\beta \in H_n$.

Example 4.4:

Assume (Ω, t_{10}^{β}) is a (PTS), where $\beta = (5\ 9\ 6\ 4\ 3\ 7\ 1\ 8\ 10)\ (2)$. Therefore, we have (Ω, t_{10}^{β}) is splittable permutation space since $\beta \in H_n$.

Definition 4.5:

Assume (Ω, t_n^{β}) is a (PTS), we say (Ω, t_n^{β}) is ambivalent permutation space if (Ω, t_n^{β}) is splittable permutation space and for each part α_i of $\alpha(\beta)$ satisfies $4 \mid (\alpha_i - 1)$.

Example 4.6:

Assume (Ω, t_6^{β}) be a (PTS), where $\beta = (6\ 1\ 5\ 3\ 2)\ (4)$. Then (Ω, t_6^{β}) is ambivalent permutation space since (Ω, t_6^{β}) is splittable permutation space and for each part α_i of $\alpha(\beta)$ satisfies $4 \mid (\alpha_i - 1)$.

The Maps induced by soft topologies 4.7

Suppose that (X, E, τ_1), (X, E, τ_2) and (X, E, τ_3) are three (STSs) over the common universe X and the parameter set E with their permutations β, μ and δ in symmetric group S_n where $n = (|X|) \times (|E|)$. Hence, we consider that $\delta : (\Omega, t_n^{\beta})_{\tau_1} \to (\Omega, t_n^{\mu})_{\tau_2}$ is a map, and $\forall$ $\lambda^{\beta} = \{b_1, b_2, ..., b_k\}$ $\beta -$set, $\delta(\lambda^{\beta})$ is said to be $\mu -$set and it is defined as $\delta(\lambda^{\beta}) = \{\delta(b_1), \delta(b_2), ..., \delta(b_k)\}$. We say δ is a permutation map induced by soft topology τ_3.

Definition 4.8

Suppose that (X, E, τ_1), (X, E, τ_2) and (X, E, τ_3) are three (STSs) over the common universe X and the parameter set E with their permutations β, μ and δ in symmetric group S_n where $n=(|X|)(|E|)$. We say $\delta:(\Omega,t_n^{\beta})_{\tau_1}\to(\Omega,t_n^{\mu})_{\tau_2}$ is a permutation continuous induced by soft topology τ_3, if $\delta^{-1}(\lambda^{\mu})\in t_n^{\beta}$ whenever $\lambda^{\mu}\in t_n^{\mu}$.

Theorem: 4.9

Let $(X=\{x_j\}_{j=1}^{k},E=\{e_r\}_{r=1}^{n},\Gamma)$ be a (STS). If for any pair $\omega_{i_1}\neq\omega_{i_2}\in T_i'$ $(1\le i\le n)$ such that $\omega_{i_1}\cap\omega_{i_2}=\phi$ and $\delta_i(x_j)\in\bigcup_{\psi\in T_i^{/*}-\{X_i\}}\psi$, for any $(1\le i\le n)$ and $(1\le j\le k)$. Then $c(\beta)=\sum_{i=1}^{n}|T_i'|$, where $(\Omega,t_{nk}^{\beta})_\Gamma$ is a permutation space induced by soft topology Γ.

Proof:

Let $\omega_{i_1}=\{x_{g_1},x_{g_2},\ldots,x_{g_{\bar{\omega}_{i_1}}}\}$, $\omega_{i_2}=\{x_{h_1},x_{h_2},\ldots,x_{h_{\bar{\omega}_{i_2}}}\}\in T_i'$, $(1\le i\le n)$ and $\bar{\omega}i_j=|\omega i_j|$, $j=1,2$. Since any pair $\omega_{i_1}\neq\omega_{i_2}\in T_i'$ $(1\le i\le n)$ such that $\omega_{i_1}\cap\omega_{i_2}=\phi$. Then $(\delta_i(x_{g_1})\ \delta_i(x_{g_2})\ldots\delta_i(x_{g_{\bar{\omega}_{i_1}}}))$ $(\delta_i(x_{g_1})\ \delta_i(x_{g_2})\ldots\delta_i(x_{g_{\bar{\omega}_{i_1}}}))$ and $(\delta_i(x_{h_1})\ \delta_i(x_{h_2})\ldots\delta_i(x_{h_{\bar{\omega}_{i_2}}}))$ are separate cycles in symmetric group S_{nk} [since $\delta_i(x_{g_t})\neq\delta_i(x_{h_r})$, for any $x_{g_t}\neq x_{h_r}\in X$].

Also, for any $\omega_i=\{x_{g_1},x_{g_2},\ldots,x_{g_{\bar{\omega}i}}\}\in T_i'$ and $\omega_l=\{x_{h_1},x_{h_2},\ldots,x_{h_{\bar{\omega}_l}}\}\in T_l'$, $(1\le i\neq l\le n)$. Hence, we consider that $(\delta_i(x_{g_1})\ \delta_i(x_{g_2})\ \ldots\ \delta_i(x_{g_{\bar{\omega}i}}))$ and $(\delta_l(x_{h_1})\ \delta_l(x_{h_2})\ldots\ \delta_l(x_{h_{\bar{\omega}l}}))$ are separate cycles in symmetric group S_{nk} [since $\delta_i(x_j)\neq\delta_l(x_f)$, for any $1\le i\neq l\le n$ and $x_j,x_f\in X$]. In other side,

$$\beta=\prod_{i=1}^{n}\Big(\prod_{j=1}^{|T_i'|}(\delta_i(x_{g_1})\ \delta_i(x_{g_2})\ \ldots\delta_i(x_{g_{\bar{\omega}i_j}}))\Big)\in S_{nk}$$

is a permutation for the permutation space $(\Omega,t_{nk}^{\beta})_\Gamma$. Thus, we consider that $\sum_{i=1}^{n}|T_i'|$ is the number of the product of separate cycles of $\prod_{i=1}^{n}(\prod_{j=1}^{|T_i'|}(\delta_i(x_{g_1})\ \delta_i(x_{g_2})\ \ldots\delta_i(x_{g_{\bar{\omega}i_j}})))$. Further, $c(\beta)$ is the number of the product of separate cycles with the 1-cycles of β. Thus $c(\beta)\ge\sum_{i=1}^{n}|T_i'|$ in general. Therefore either $c(\beta)=\sum_{i=1}^{n}|T_i'|$ or $c(\beta)>\sum_{i=1}^{n}|T_i'|$. If $c(\beta)>\sum_{i=1}^{n}|T_i'|$, then there is at least 1-cycle say $(\delta_i(x_j))$ for some ($x_j\in X$, $1\le i\le n$) with $\delta_i(x_j)\notin\bigcup_{\psi\in T_i^{/*}-\{X_i\}}\psi$ and this contradiction with our hypothesis. Hence $c(\beta)=\sum_{i=1}^{n}|T_i'|$.

Theorem: 4.10

Let $(X=\{x_j\}_{j=1}^{k},E=\{e_r\}_{r=1}^{n},\Gamma)$ be a (STS) and for any pair $\omega_{i_1}\neq\omega_{i_2}\in T_i'$ $(1\le i\le n)$ such that $\omega_{i_1}\cap\omega_{i_2}=\phi$ and $\delta_i(x_j)\in\bigcup_{\psi\in T_i^{/*}-\{X_i\}}\psi$, for any $(1\le i\le n)$ and $(1\le j\le k)$. Then $(\Omega,t_{nk}^{\beta})_\Gamma$ is an even permutation space, if $2\,|\,(nk-\sum_{i=1}^{n}|T_i'|)$

Proof:

By [Theorem (4.9)], we consider that $(\Omega, t_{nk}^{\beta})_{\Gamma}$ is a permutation space induced by soft topology Γ and its permutation β in symmetric group S_{nk} satisfies $c(\beta) = \sum_{i=1}^{n} \left|T_i^{'}\right|$. However, $2 \mid (nk - \sum_{i=1}^{n}\left|T_i^{'}\right|)$ and hence $nk - c(\beta)$ is even. Then $(\Omega, t_n^{\beta})_{\Gamma}$ is an even permutation space.

Lemma: 4.11

Assume (X, E, Γ) is a (STS). Then $(\Omega, t_n^{\beta})_{\Gamma}$ is odd permutation space, if (X, E, Γ) is a (SITS) and $2 \mid n$.

Proof:

Assume (X, E, Γ) is a (SITS) and let $(\Omega, t_n^{\beta})_{\Gamma}$ be a permutation space induced by soft topology Γ. Hence $\beta = (1\ 2\ 3 \ldots n)$ [since (X, E, Γ) is (SITS)], thus $c(\beta) = 1$. Also, since $2 \mid n$, Then there is a positive integer number q such that $n = 2q$ and hence $n - c(\beta)$ =(even) - (odd)=(odd). Hence β is odd permutation in S_n. Then $(\Omega, t_n^{\beta})_{\Gamma}$ is odd permutation space.

Lemma: 4.12

Assume $(X = \{x_j\}_{j=1}^{k}, E = \{e_r\}_{r=1}^{n}, \Gamma)$ is a soft discrete topological space. Then $\sum_{i=1}^{n}\left|T_i^{'}\right| = n\left|T_j^{'}\right|$ for any $(1 \le j \le n)$.

Proof:

Assume (X, E, Γ) is a soft discrete topological space. Then, we consider that $T_i^{'} = T_j^{'}$, $\forall (1 \le i, j \le n)$. So $\left|T_i^{'}\right| = \left|T_j^{'}\right|$, $\forall (1 \le i, j \le n)$ and hence $\sum_{i=1}^{n}\left|T_i^{'}\right| = \left|T_1^{'}\right| + \left|T_2^{'}\right| + \ldots + \left|T_n^{'}\right| = \left|T_j^{'}\right| + \left|T_j^{'}\right| + \ldots + \left|T_j^{'}\right| = n\left|T_j^{'}\right|$, $\forall (1 \le j \le n)$.

Lemma: 4.13

Let $(X = \{x_j\}_{j=1}^{k}, E = \{e_r\}_{r=1}^{n}, \Gamma)$ be a (STS). Then $(\Omega, t_n^{\beta})_{\Gamma}$ is (PSS), if $\left|\omega_i\right| = 1, \forall \omega_i \in T_i^{'}$, where $(1 \le i \le n)$.

Proof:

Let (X, E, Γ) be a (STS). Then $(\Omega, t_n^{\beta})_{\Gamma}$ is (PSS), if $\left|\omega_i\right| = 1, \forall \omega_i \in T_i^{'}$, and $(\Omega, t_n^{\beta})_{\Gamma}$ be a permutation space induced by soft topology Γ. Since $\left|\omega_i\right| = 1, \forall \omega_i \in T_i^{'}$, this implies that $\omega_i = \{x_g\}$ for some $x_g \in X$. Therefore $\beta = \prod_{i=1}^{n}(\prod_{j=1}^{\left|T_i^{'}\right|}(\delta_i(x_{g_j})))$ for some $x_{g_j} \in X$. Then $(\Omega, t_n^{\beta})_{\Gamma}$ is (PSS) [since each proper open $\beta -$ set is a singleton].

Lemma: 4.14

Let $(X = \{x_j\}_{j=1}^{k}, E = \{e_r\}_{r=1}^{n}, \Gamma)$ be a (STS). Then $(\Omega, t_n^{\beta})_{\Gamma}$ is an even permutation

space, if $\left|\omega_i\right| = 1, \forall \omega_i \in T_i^{'}$, where $(1 \le i \le n)$.

Proof:

By [Lemma, (4.13)] we get $(\Omega, t_n^{\beta})_{\Gamma}$ is (PSS). Let $\{\lambda_i\}_{i=1}^{c(\beta)}$ be the family of the product of separate cycles with the 1-cycles of β

where $\lambda_i = (\delta_i(x_{g_j}))$, for some $x_{g_j} \in X$ and $1 \le i \le c(\beta)$ [since each proper open β – set is a singleton]. However,

$\beta = \prod_{i=1}^{n}(\prod_{j=1}^{|T_i'|}(\delta_i(x_{g_j}))) = e \in S_{nk}$. But e is an identity element in S_{nk}. Thus $\beta = e = (1)\,(2)\,(3)\ldots(nk)$ and hence $c(\beta) = nk$. This implies $nk - c(\beta) = 0$ (even). Hence $(\Omega, t_n^{\beta})_\Gamma$ is an even permutation space.

Theorem: 4.15

Let $(X = \{x_j\}_{j=1}^{k}, E = \{e_r\}_{r=1}^{n}, \Gamma)$ be a (STS) and for any pair $\omega_{i_1} \neq \omega_{i_2} \in T_i'$ $(1 \le i \le n)$ such that $\omega_{i_1} \cap \omega_{i_2} = \phi$ and $\delta_i(x_j) \in \bigcup_{\psi \in T_i^{*} - \{X_i\}} \psi$, for any $(1 \le i \le n)$ and $(1 \le j \le k)$. Then $(\Omega, t_{nk}^{\beta})_\Gamma$ is a splittable permutation space, if the following are hold.

(1) $2 \mid (|\omega_i| - 1), \forall \omega_i \in T_i'$ $(1 \le i \le n)$,

(2) If $\omega_i \neq \omega_j$, then $|\omega_i| \neq |\omega_j|$, where $\omega_i \in T_i'$ and $\omega_j \in T_j'$ $(1 \le j, i \le n)$.

Proof:

By [Theorem (4.9)], we consider that $(\Omega, t_{nk}^{\beta})_\Gamma$ is a permutation space induced by soft topology Γ and its permutation β in symmetric group S_{nk} satisfies $c(\beta) = \sum_{i=1}^{n}|T_i'|$. Moreover, $\beta = \prod_{i=1}^{n}\sigma_i$ where for all $1 \le i \le n$, $\sigma_i = \prod_{L=1}^{|T_i'|}\sigma_{i_L} = \prod_{L=1}^{|T_i'|}(\delta_i(x_{g_1})\,\delta_i(x_{g_2})\ldots\delta_i(x_{g_{|\omega_{i_L}|}}))$ is permutation which is product of $|T_i'|$ cyclic factors of the length $|\omega_{i_L}|$, where $\omega_{i_L} \in T_i'$ and $1 \le L \le |T_i'|$.

For each $\beta \in S_n$ can be written as uniquely product of separate cycles. Thus $\beta = \lambda_1\lambda_2 \ldots \lambda_{c(\beta)}$, where $|\lambda_i| = \alpha_i$, $\{\lambda_i\}_{i=1}^{c(\beta)}$ are separate cycles and $c(\beta)$ is the number of the product of separate cycles with the 1-cycles of β. Hence $\alpha = \alpha(\beta) = (\alpha_1, \alpha_2, \ldots, \alpha_{c(\beta)})$ is the "cycle type" of β. Since $\beta = \prod_{i=1}^{n}\prod_{L=1}^{|T_i'|}\sigma_{i_L}$ and $c(\beta) = \sum_{i=1}^{n}|T_i'|$. Then for any λ_i there exists σ_{i_L} satisfies $\lambda_i = \sigma_{i_L}$, where $(1 \le i \le n)$. The length of any cycle $\sigma_{i_L} = (\delta_i(x_{g_1})\,\delta_i(x_{g_2})\ldots\delta_i(x_{g_{|\omega_{i_L}|}}))$ is $|\omega_{i_L}|$ and this implies that

$(\alpha_1, \alpha_2, \ldots, \alpha_{c(\beta)}) = (|\omega_{1_1}|, |\omega_{1_2}|, \ldots, |\omega_{1_{|T_1'|}}|, |\omega_{2_1}|, |\omega_{2_2}|, \ldots, |\omega_{2_{|T_2'|}}|, \ldots, |\omega_{n_1}|, |\omega_{n_2}|, \ldots, |\omega_{n_{|T_n'|}}|)$ Now, if $|\omega_{i_L}|$ is even number for some $(1 \le i \le n)$ and $(1 \le L \le |T_i'|)$, thus $(|\omega_{i_L}| - 1)$ is odd and this contradiction with (1) of our hypothesis which state that $2 \mid (|\omega_i| - 1)$, $\forall \omega_i \in T_i'$ $(1 \le i \le n)$. Then $|\omega_{i_L}|$ are odd numbers for all $(1 \le i \le n)$ and $(1 \le L \le |T_i'|)$. Also, for any $(i \neq j)$ or $(g \neq h)$ we have $\omega_{i_g} \neq \omega_{j_h}$ where $(1 \le i, j \le n)$, $(1 \le g \le |T_i'|)$ and $(1 \le h \le |T_j'|)$. Then $\{\omega_{i_L} \mid 1 \le i \le n; 1 \le L \le |T_i'|\}$ are different sets and by (2) of our hypothesis we consider that $\{|\omega_{i_L}| \mid 1 \le i \le n; 1 \le L \le |T_i'|\}$ are different too. Therefore $\beta \in H_n$ and hence $(\Omega, t_{nk}^{\beta})_\Gamma$ is a splittable permutation space.

Theorem: 4.16

Let $(X=\{x_j\}_{j=1}^{k}, E=\{e_r\}_{r=1}^{n}, \Gamma)$ be a (STS) and for any pair $\omega_{i_1} \neq \omega_{i_2} \in T_i^{'}$ $(1 \le i \le n)$ such that $\omega_{i_1} \cap \omega_{i_2} = \phi$ and $\delta_i(x_j) \in \underset{\psi \in T_i - \{X_i\}}{\overset{*}{\cup}} \psi$, for any $(1 \le i \le n)$ and $(1 \le j \le k)$. Then $(\Omega, t_{nk}^{\beta})_\Gamma$ is an ambivalent permutation space, if the following are hold.

(1) $2 \mid (|\omega_i| - 1), \forall \omega_i \in T_i^{'}$ $(1 \le i \le n)$,

(2) If $\omega_i \neq \omega_j$, then $|\omega_i| \neq |\omega_j|$, where $\omega_i \in T_i^{'}$ and $\omega_j \in T_j^{'}$ $(1 \le j, i \le n)$,

(3) If $4 \mid (|\omega_i| - 1), \forall \omega_i \in T_i^{'}$ $(1 \le i \le n)$.

Proof:

From (1) and (2) we consider that permutation space $(\Omega, t_{nk}^{\beta})_\Gamma$ is a splittable [By Lemma, (4.13)]. Also, from (1) and (2) that easy to show that $(\alpha_1, \alpha_2, ..., \alpha_{c(\beta)}) = (|\omega_{1_1}|, |\omega_{1_2}|, \ldots, |\omega_{1_{|T_1^{'}|}}|, |\omega_{2_1}|, |\omega_{2_2}|, \ldots, |\omega_{2_{|T_2^{'}|}}|, \ldots, |\omega_{n_1}|, |\omega_{n_2}|, \ldots, |\omega_{n_{|T_n^{'}|}}|)$. That means for any part α_i of $\alpha(\beta)$, there exists $\omega_{i_L} \in T_i^{'}$ for some $(1 \le i \le n)$ and $(1 \le L \le |T_i^{'}|)$ satisfies $\alpha_i = |\omega_{i_L}|$ and hence from (3) we have $4 \mid (\alpha_i - 1), \forall (1 \le i \le c(\beta))$. Then $(\Omega, t_{nk}^{\beta})_\Gamma$ is an ambivalent permutation space.

5. PROPOSED IMPROVEMENTS

Due to the difficulties of finding link between two different topological spaces in different strictures, the present algorithm is the first link between soft space (X, E, Γ) and permutation space (Ω, t_n^{β}), where they are two different topological spaces in different strictures. This algorithm will help us to study all of the notions in (STSs) that are given in past work in permutation topological spaces when they are induced by some soft topologies. This algorithm is highly recommended.

6. CONCLUSIONS AND OPEN PROBLEMS

In this work, an algorithm has been introduced to find the permutation in symmetric group using soft topological space to structure permutation topological space. Further, suppose that (X, E, τ_1), (X, E, τ_2) and (X, E, τ_3) are three (STSs) over the common universe X the parameter set E with their permutations β, μ and δ in symmetric group S_n where $n = (|X|) \times (|E|)$. Hence the questions can be summarized as follows:

1) Is necessarily true, if $(\Omega, t_n^{\beta})_{\tau_1}$ is a even (res. odd, splittable, ambivalent) permutation space and $\delta : (\Omega, t_n^{\beta})_{\tau_1} \to (\Omega, t_n^{\mu})_{\tau_2}$ is continuous permutation map. Then the image under δ is even (res. odd, splittable, ambivalent) permutation space, too.
2) Is necessarily true, if (X, E, τ_1) and (X, E, τ_2) are two (STSs). Then $(\Omega, t_n^{\beta})_T$ is a permutation space induced by soft topology T, where $T = \tau_1 \times \tau_2$.
3) Is necessarily true, if (X, E, T) is a soft connected. Then $(\Omega, t_n^{\beta})_T$ is a permutation connected induced by soft topology T.
4) Is necessarily true, if (X, E, T) is a soft compact space. Then $(\Omega, t_n^{\beta})_T$ is a permutation compact space induced by soft topology T.

ACKNOWLEDGEMENT

"This paper is part of master research work in the College of science, University of Basrah, Iraq."

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

REFERENCES

[1] D. Molodtsov, "Soft set theory−First results", *Comput. Math. Appl*. Vol. 37, No. (4−5) (1999), pp. 19–31.
[2] M. Shabir and M. Naz," On Soft topological spaces", *Comp. And Math. With applications*, Vol. 61, No. 7, 2011, pp.1786-1799.
[3] P. Dutta and B. Limboo, "Bell-shaped Fuzzy Soft Sets and Their Application in Medical Diagnosis", *Fuzzy Information and Engineering*, Vol. 9, No.(1), (2017), pp. 67-91.
[4] S. Mahmood, "Soft Regular Generalized b-Closed Sets in Soft Topological Spaces", *Journal of Linear and Topological Algebra,* Vol.3, No.(4), (2014), pp. 195- 204.
[5] S. Mahmood, "Tychonoff Spaces in Soft Setting and their Basic Properties", *International Journal of, Applications of Fuzzy Sets and Artificial Intelligence*, Vol. 7, (2017), pp.93-112.
[6] S. Mahmood and Z. Al-Batat, "Intuitionistic Fuzzy Soft LA- Semigroups and Intuitionistic Fuzzy Soft Ideals", *International Journal of Applications of Fuzzy Sets and Artificial Intelligence*, Vol. 6, (2016), pp. 119 – 132.
[7] S. Mahmood, "Soft Sequentially Absolutely Closed Spaces", *International Journal of Applications of Fuzzy Sets and Artificial Intelligence*, Vol. 7, (2017), pp.73-92.
[8] S. Mahmood, "Dissimilarity Fuzzy Soft Points and their Applications", *Fuzzy Information and Engineering*, Vol. 8, (2016), pp. 281-294.
[9] C. Yang, "A not on soft set theory", *Comput. Math. Appl*. Vol. 56, (2008), pp.1899-1900.
[10] S. Mahmood, "The Permutation Topological Spaces and their Bases", *Basrah Journal of Science*, Vol. 32, No. (1), (2014), pp. 28-42.
[11] J. D. Dixon, "Problems in group theory", *University of New South Wales, Printing in the United State of America,* (1967).
[12] S. Mahmood, "On intuitionistic fuzzy soft b-closed sets in intuitionistic fuzzy soft topological spaces", *Annals of Fuzzy Mathematics and Informatics,* Vol. 10, No. (2), (2015), pp. 221-233.
[13] S. Mahmood and A. Rajah, "Solving the Class Equation $x^d = \beta$ in an Alternating Group for each $\beta \in H_n \cap C^\alpha$ and $n \notin \theta$", *Journal of the Association of Arab Universities for Basic and* Applied Sciences Vol. 10, (2011), pp. 42–50.
[14] S. Mahmood and A. Rajah, "The Ambivalent Conjugacy Classes of Alternating Groups", *Pioneer Journal of Algebra, Number Theory and its Applications*, Vol.1, No. (2), (2011), pp. 67-72.
[15] S. Mahmood, M. Abud Alradha, "Extension Permutation Spaces with Separation Axioms in Topological Groups", *American Journal of Mathematics and Statistics*, Vol. 7, No. (5), (2017), pp.183-198.
[16] P. K. Maji, A. R. Roy and R. Biswas, "An application of soft sets in a decision making problem", *Comput. Math. Appl*. Vol .44, (2002), pp.1077-1083.
[17] D. Bump, "Lie groups", *Springer-Verlag, New York*, (2004).
[18] V. Runde, "A taste of topology", *Universitext, Springer, Printing in the United State of America*, (2005).