

# THE IDENTIFICATION OF DATA ANOMALIES FROM INFORMATION SENSORS BASED ON THE ESTIMATION OF THE CORRELATION DIMENSION OF THE TIME SERIES ATTRACTOR IN SITUATIONAL MANAGEMENT SYSTEMS

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## ABSTRACT

**Purpose:** The goal is the timely detection of uncharacteristic behavior of the observed processes in the systems of situational management, leading to the development or occurrence of emergency situations.

**Methodological approach:** In the article, it is proposed to analyze the change dynamics in the correlation dimension of the attractor in order to detect anomalies in the behavior of the observed process. A sharp change in the correlation dimension is a reflection of the uncharacteristic (anomalous) nature of the data of the observed processes. This anomaly is a consequence of external influences on the generating system and requires an analysis of the causes of its occurrence.

**Uniqueness/value:** The uniqueness of the proposed approach consists in the fact that an abrupt change in the correlation dimension of the attractor is the information about the occurrence of uncharacteristic behavior of the observed system. The value of the study is determined by the relevance of the problem of modeling the development and occurrence of emergency situations in situational management systems based on the analysis of the time series of observed processes.

**Summary:** The proposed approach is designed to identify the critical states of the generating dynamical systems by their time series. Timely response to the transition of the monitored system to a critical state will allow preventing any critical consequences.

**Keywords:** *Correlation dimension, Attractor, Anomalous behavior, Data of information sensors.*

## 1. INTRODUCTION

The identification of anomalies of the data received from information sensors is an integral part of the situational centers' activity. In most processes observed in them, the measured parameters are of oscillatory nature with no obvious trends. Therefore, it is difficult to identify the fact of the information parameter's change in such

processes and takes a long time, substantially exceeding the period of the parameter change. In this article, the anomalous data mean a qualitative change in the behavior of the observed processes. Such an unusual behavior, in most cases, reflects the objective development of the process, but differs significantly from the general trend.

The purpose of this study is the timely detection of the unusual behavior of the observed processes in situational management systems, leading to the development or occurrence of emergency situations.

To achieve the goal, we need to solve two problems:

1) To substantiate the choice of the approach to revealing anomalous data based on the calculation of the correlation dimension of the time series attractor;

2) To determine the conditions for treating the given time series as anomalous based on an analysis of the dynamics of the change in the correlation dimension of the attractor.

## 2. LITERATURE REVIEW

Situational management systems are currently intensively implemented in various application areas. For example, publication [1] presents the concept of a management system for earthquake consequences. Publication [2] deals with an on-board situational awareness system for the control of unmanned aerial vehicles. Publication [3] describes a system for the traffic situation monitoring, enabling travelers and road traffic controllers to make decisions. Publication [4] deals with solving problems arising in the system for livestock facility management based on monitoring harmful environmental factors.

It is obvious that there is no universal technology for data processing in situational management systems in various fields. Methods, ways, and technologies of comprehensive processing of monitoring data depend on the purpose of such situational management systems and the physical meaning of the observed processes. The existing methods for processing the data of the observed processes can be divided into two classes: with continuous time and with discrete time. Publications [5-7] suggest methods of identification and prediction of stochastic systems and processes with continuous time: [5] describes a method for determining the Bayesian identification of stochastic nonlinear systems using the Kalman filter; [6] suggests a method for predicting the performance of large-scale stochastic linear hybrid systems with a small probability of failure, [7] offers a method for predicting the state of nonlinear stochastic differential systems with multiplicative noises.

A promising direction for studying the behavior of a potential emergency situation is the analysis of a data anomaly based on the calculation of the metric characteristics of the time series

attractor. Non-linear dynamics methods, in particular methods for analyzing the characteristics of the time series attractor, enable the detection of qualitative changes in the behavior of the observed process. Such changes are anomalous, because at this point in time the system changes to a new qualitative state. The applications of nonlinear dynamics methods based on the analysis of the attractor's characteristics can be found in publications [8-10]. Publication [8] proposes to use an attractor-based approach, estimate the attractors' duration, and decide whether to use the attractor (if it is single) or the last of the attractors (if there are several attractors) as a learning data sequence. Publication [9] adds the white and color noises to a chaotic time series to verify the change in the topology and structure of the attractor. Article [10] considers a strange attractor with an infinite set of embedded invariant tori in the state space. Articles [11-13] study the influence of the parameters of the attractors of oscillatory processes on the stability of the stochastic regime in nonlinear dynamical systems.

According to the classification of Bill M. Williams [14], there are four types of attractors: point, cyclic, strange attractors and the Torus attractor. The first attractor is the attractor of first dimension; the second one is in the second dimension of the plane; the third one starts a complex circulation, which reiterates as it moves forward; the fourth attractor is perceived at a superficial glance as the absolute chaos. In article [15], a system of three coupled nonlinear ordinary differential equations, whose dynamics, with parameter variation, support the cyclic and strange attractors, is studied. In article [16], the possibility of mutual fusion of evolutionary algorithms and methods of deterministic chaos is discussed. And in publication [17], two approaches, the nonlinear deterministic and linear stochastic approaches, are combined. This combination allows considering the detailed geometric structure of the strange attractor. It is possible to determine the state, in which the system under consideration is, based on the analysis of the stochastic characteristics of the attractor of the observed processes' time series.

Scientific publications dealing with the calculation of these stochastic characteristics of the time series attractor provide improved approaches to the calculation and justification of the obtained results. Article [18] proposes a new approach to estimating the correlation dimension in the frequency domain using the power spectrum of the fractal set under study. And article [19] describes the developed model method for calculating the

correlation dimension, Lyapunov exponents, and synchronization by the depth signals. Article [20], using the example of a boiling water reactor, shows how to detect the stability (instability) of a system based on the analysis of orbits and radii of the attractor of the time series of the processes occurring within this system. Publication [21] analyzes the relationship between the correlation dimension, the theoretical entropy of a nonlinear system, and the entropy of an experimental sample. An improved algorithm for estimating the correlation dimensions of the time series of a hyper-chaotic structure based on cell counting was proposed in [22]. Publication [23] offers an algorithm for efficient and reliable calculation of the correlation dimension in cases when the correlation integral is set up using all pairs of the points. Publication [24] compares the correlation and Lyapunov dimensions and studies the relationship between the correlation dimension and the Kaplan-York dimension of three-dimensional chaotic flows.

We present an overview of the most significant results of the data anomaly detection based on the analysis of the stochastic characteristics of the attractor. Publication [25] studies the threat of flooding of the terrain from the river runoff based on the analysis of the dynamics in the correlation dimension and Lyapunov exponents' change. Publication [26] considers the characteristics of nuclear reactors at nuclear power plants. An analysis of the dynamics of the Lyapunov exponents and their prediction makes it possible to predict possible failures and anomalies. Similar results are presented in [27], which suggests measuring the Lyapunov exponents of ultrasonic waves in order to detect small defects in steel pipes. In addition, the successful application of the analysis of stochastic characteristics of the time series attractor to detect data anomalies can be seen in communication systems and medicine. For example, article [28] proposes a method for detecting a weak signal with high detection accuracy and a low signal-to-noise ratio. Article [29] studies the noise effect on the correlation dimension of a chaotic attractor and suggests the following conclusion: the presence of noise leads to an increase in the correlation dimension. Article [30] provides an analysis of the Doppler signals using the largest Lyapunov exponent and the correlation dimension in healthy and stenotic carotid artery patients. And publication [31] proposes a method for calculating the correlation dimension of the human electroencephalogram and

estimating the variability of the human brain functioning.

An important step in the process of timely response of a person to a possible anomalous state of the observed dynamic system is the prediction of its behavior. A reconstruction of the attractor of the phase path allows predicting the states of the dynamic system. Publication [32] suggests methods, by which the delay and implementation measurement can be selected for a typical reconstruction of the delay coordinate. The use of the autocorrelation function and mutual information in the quantitative determination of the delay are compared.

### 3. MATERIALS AND METHODS

#### 3.1. Estimation of the time delay of the phase path of the attractor

Denote the time series in the general form as:

$$Z = \langle z_i \rangle, \quad i = 1, 2, \dots, m. \quad (1)$$

The phase path of time series (1) is a sequence of points in the  $n$ -dimensional space

$$\Phi(Z) = (z_i, z_{i+\tau}, \dots, z_{i+(n-1)\tau}), \quad (2)$$

where  $\tau$  is the time delay;  $n$  is the embedding dimension.

When analyzing the phase path attractor, the correct choice of the time delay  $\tau$  plays an important role. It is necessary to try to choose  $\tau$

so that the correlation between  $z_i$  and  $z_{i+\tau}$  was as small as possible. The traditional way to choose the time delay consists in calculating the autocorrelation function of the time series [32]

$$r(\tau) = \frac{1}{m} \sum_{t=0}^{m-1} (z_t - \bar{z}) \cdot (z_{t+\tau} - \bar{z}), \quad (3)$$

$$m = M - \tau$$

Delay  $\tau$  is chosen equal to the time of the first zero crossing of the autocorrelation function. In addition to the time delay in plotting the phase path, an important step is to choose the embedding dimension. The embedding dimension determines the number of variables that objectively describe the behavior of the dynamic system. The upper bound of the embedding dimension is chosen using the Takens' theorem [32], the meaning of which is as follows. Let the state of the system be completely described by  $m$  variables:  $x_1(t), x_2(t), \dots, x_m(t)$ . With interval  $T$ , any one of them is measured, for example  $x_1(t)$ . According to the theorem, instead of a sequence consisting of

$m$  variables  $x_1(t), x_2(t), \dots, x_m(t)$ , we can consider sequence  $x_1(t+T), x_1(t+2T), \dots, x_1(t+(m-1)T)$ , i.e. at any time point the state of the system can be described with  $m$  values of one variable, taken with shift  $T$ . This article uses the standard embedding dimension  $n = 3$ .

**3.2. Estimation of the attractor's correlation dimension**

The attractor's correlation dimension is defined through the correlation integral

$$C(r) = \frac{1}{n^2} \sum_{i=1}^{n-1} \sum_{j=i+1}^n \theta(r - \|z_i - z_j\|), \quad (4)$$

where  $\theta$  is the Heaviside function:  $\theta(z) = 0$  at  $z < 0$  and  $\theta(z) = 1$  at  $z > 0$ ;  $r$  is the radius of the ball, circumscribed alternately around each of the path points.

Function  $C(r)$  is the measure of the connection of close points of the path. Therefore,  $C(r)$  is the integral correlation function of the attractor. With comparatively small  $r$ , function  $C(r)$  changes as follows:

$$C(r) \approx r^d. \quad (5)$$

The dimension of the attractor  $d$  for the given phase space is calculated as the limiting tangent of the inclination angle of dependence  $\ln C(r)$  from  $\ln r$  within determined range  $r$ :

$$\ln C(r) \approx d \ln r. \quad (6)$$

If the system has an attractor, then starting with certain  $d$ , the angle of inclination of the graphs ceases to increase. Value  $D$ , i.e. the tangent of the dependence inclination angle that has ceased to

increase, was considered as the correlation dimension of the attractor.

**3.3. Conditions for considering the data anomalous based on the estimate of the correlation dimension**

It is proposed to consider the estimate of correlation dimension  $D_j, j = \overline{1, m}$  over time. If the correlation dimension of the attractor at certain time point  $j$  sharply decreases, and then at time point  $j + 1$  again increases, it should be assumed that the generated system at time point  $j$  generates anomalous values.

The presence of anomalous data in the time series can be confirmed with the help of a statistical indicator such as standard deviation  $\sigma$ : when there is a data anomaly, either sharp convergence or sharp discrepancy of the line of its change with respect to the line of change in the mathematical expectation  $M_j$  occurs.

**4. RESULTS**

Consider the time series of experimental data

$$X = \langle x_i \rangle, i = \overline{1, 2, \dots, m}, m = 60. \quad (7)$$

Figure 1 shows a graph of this series and Table 1 provides its values. Next, we demonstrate the proposed approach to the detection of anomalous data on the example of time series  $X$  (7). To do this, we should consider the correlation dimension of series  $D$  over time. Take the initial segment of the time series of length 30:  $X^* = \langle x_1, \dots, x_{30} \rangle$ .

Calculate correlation dimension  $D(X^*)$  of the attractor of the phase path of this time series segment.

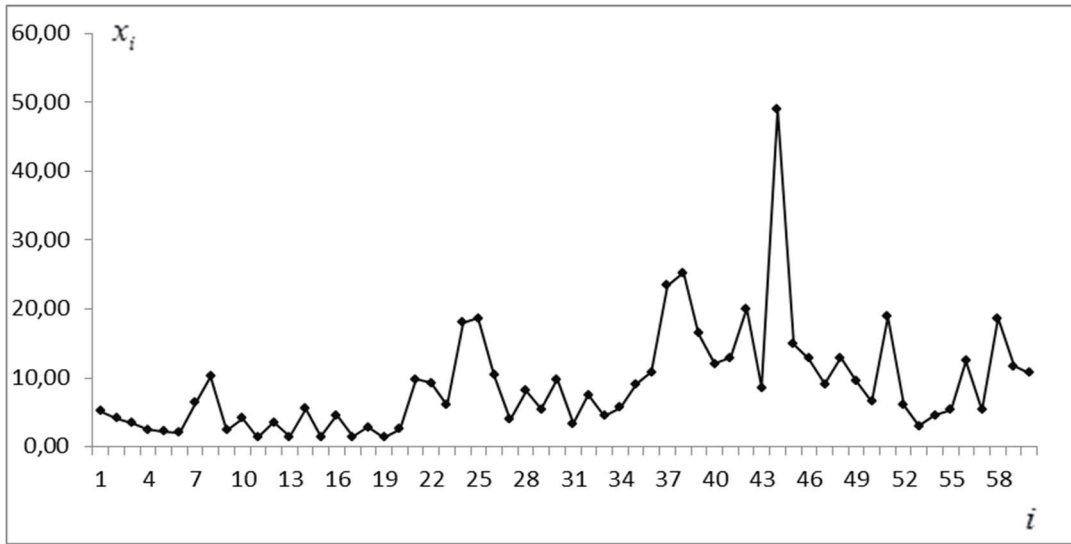


Figure 1 – Time series  $X$

Table 1. Values of time series  $X$ .

Sq. No.	$x_i$	Sq. No.	$x_i$	Sq. No.	$x_i$
1	5.2254	21	9.8079	41	12.9323
2	4.1846	22	9.2878	42	20.0149
3	3.4549	23	6.0586	43	8.5581
4	2.5182	24	18.0361	44	48.9311
5	2.2051	25	18.6659	45	14.9117
6	2.1016	26	10.4328	46	12.8288
7	6.3710	27	3.9763	47	9.0789
8	10.2251	28	8.1415	48	12.9329
9	2.4134	29	5.4343	49	9.5996
10	4.1846	30	9.8085	50	6.5799
11	1.3719	31	3.4074	51	18.8688
12	3.5597	32	7.5166	52	6.0592
13	1.4767	33	4.4970	53	3.0383
14	5.5378	34	5.7461	54	4.5884
15	1.4761	35	9.0795	55	5.3302

Sq. No.	$x_i$	Sq. No.	$x_i$	Sq. No.	$x_i$
16	4.6011	36	10.8494	56	12.4857
17	1.4767	37	23.4518	57	5.3302
18	2.8300	38	25.2218	58	18.6605
19	1.3719	39	16.4739	59	11.6832
20	2.6223	40	12.0126	60	10.7452

To set up phase path  $\Phi(X^*)$ , it is necessary to define two parameters: embedding dimension  $n$  and time delay  $\tau$ . We suggest the first parameter to be taken equal to  $n = 3$ ; the second parameter to be estimated by formula (3). The results of calculations of selective correlation coefficients  $r(\tau)$  for time series  $X^*$  are provided in Table 2.

Table 2. Calculation of sample correlation coefficients for the segment of time series  $X^*$

Delay, $\tau$	$\sum_{i=1}^{30} (x_i - \bar{x}) \cdot (x_{i+\tau} - \bar{x}) \cdot (x_{i+2\tau} - \bar{x})$	$r(\tau)$
1	504.48	18.02
2	205.27	7.89
3	-297.80	-12.41

As can be seen in Table 2, autocorrelation coefficient  $r(\tau)$  becomes negative with delay  $\tau = 3$ . This means that the correlation between elements  $x_i^*$  and  $x_{i+3}^*$  is minimal. With embedding dimension  $n = 3$  and time delay  $\tau = 3$ , the phase path can be represented as a sequence of the point pairs

$$\Phi(X^*) = \{(x_i^*, x_{i+3}^*, x_{i+6}^*)\}. \quad (8)$$

Now estimate correlation integral  $C(r)$  (4) of phase path  $\Phi(X^*)$ . Column 2 of Table 3 shows the calculated values of correlation integral  $C(r)$  with the radius values  $r = \overline{1,10}$ . Columns 3 and 4 contain the corresponding values of the logarithmic function of  $r$  and  $C(r)$ .

Table 3. Calculation of correlation integral  $C(r)$  of the segment of time series  $X^*$  at delay  $\tau = 3$

$r$	$C(r)$	$\ln(r)$	$\ln(C(r))$
1	0.0000	0.0000	-
2	0.0004	0.6931	-7.9149
3	0.0015	1.0986	-6.5182
4	0.0032	1.3863	-5.7582
5	0.0105	1.6094	-4.5573
6	0.0236	1.7918	-3.7459
7	0.0389	1.9459	-3.2474
8	0.0546	2.0794	-2.9077
9	0.0730	2.1972	-2.6168
10	0.0949	2.3026	-2.3549

Correlation dimension  $D(X^*)$  is the value of the tangent of the inclination angle of function  $\ln(C(r))$  from  $\ln(r)$ . Figure 2 shows line graph  $\ln(C(r))$ , the linear trend of which is found by function

$$y = 3.599x - 10.442. \quad (9)$$

The coefficient for variable  $x$  is the magnitude of the tangent of the inclination angle, so the correlation dimension will be

$$D(X^*) \approx 3.60. \quad (10)$$

Estimate the correlation dimension of the segments of time series  $X$  of length 30 when shifted by one to the right. In this paper, we estimate the correlation dimension for the shifts of initial segment  $X^*$  successively by 21 (denote the shifts of variable  $q = \overline{0, 21}$ , where 0 means that there is no shift).

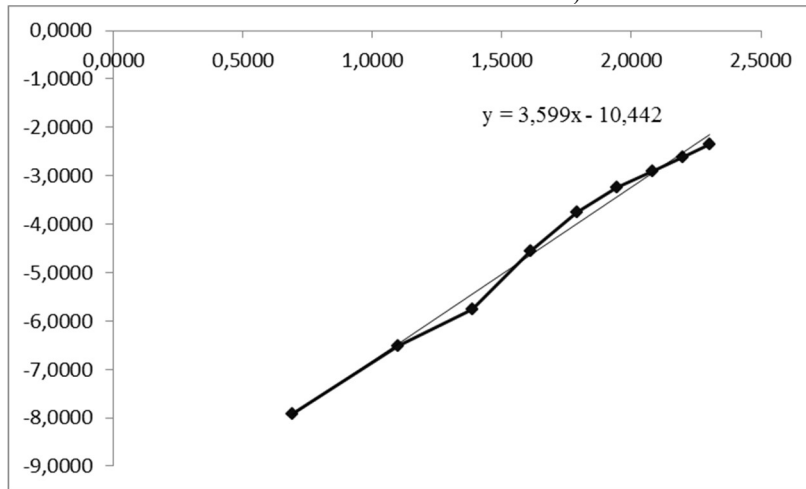


Figure 2. Dependence of  $\ln(C(r))$  from  $\ln(r)$  in phase path  $\Phi(X^*)$

For segments  $X_q^*$ , sample correlation coefficients were calculated and optimal time delays  $\tau$  were determined (See Table 4). In Table 4, the values, for which autocorrelation function  $r(\tau)$  (3) intersects zero for the first time, are put in bold. As a result, the following delays were

obtained:  $\tau = 3$  at shifts  $q = 1 - 4, 13 - 15, 19 - 21$ ;  $\tau = 5$  at shift  $q = 7$ ;  $\tau = 2$  at shifts  $q = 5, 6, 8 - 12, 16 - 18$ .

Table 4. Results of the calculation of delays  $\tau$  for segments  $X_q^*$ ,  $q = \overline{1, 21}$

The value of the shift of segment	Sample coefficients	Delay, $\tau$

$X^*, q$	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$	
1	19.84	9.05	<b>-13.24</b>	-	-	3
2	16.46	8.47	<b>-19.66</b>	-	-	3
3	16.59	7.63	<b>-19.20</b>	-	-	3
4	15.33	3.65	<b>-19.75</b>	-	-	3
5	9.49	<b>-0.25</b>	-	-	-	2
6	2.14	<b>-6.87</b>	-	-	-	2
7	-7.94	-20.53	-29.62	-15.60	<b>16.85</b>	5
8	9.32	<b>-35.47</b>	-	-	-	2
9	73.91	<b>-31.21</b>	-	-	-	2
10	80.41	<b>-29.41</b>	-	-	-	2
11	65.34	<b>-7.46</b>	-	-	-	2
12	35.25	<b>-3.03</b>	-	-	-	2
13	25.22	4.60	<b>-2.69</b>	-	-	3
14	-95.67	-23.72	<b>6.29</b>	-	-	3
15	-122.66	-12.67	<b>48.40</b>	-	-	3
16	-115.83	<b>12.13</b>	-	-	-	2
17	-101.20	<b>7.96</b>	-	-	-	2
18	-89.24	<b>2.89</b>	-	-	-	2
19	-93.79	-1.06	<b>38.12</b>	-	-	3
20	-96.68	-12.70	<b>35.78</b>	-	-	3
21	-103.38	-25.94	<b>64.42</b>	-	-	3

Table 5 shows the results obtained for segments  $X_q^*, q = \overline{1, 21}$  analytical dependencies  $\ln(C(r))$  from  $\ln(r)$  in phase paths  $\Phi(X_q^*)$ .

Table 5. Correlation dimensions  $D(X_q^*)$  of segments  $X_q^*, q = \overline{1, 21}$ .

The value of the shift of segment $X^*, q$	Functions of dependency $\ln(C(r))$ from $\ln(r)$	Correlation dimension, $D$
1	$y = 3,58x - 10,03$	3.58
2	$y = 3,57x - 10,01$	3.57
3	$y = 3,51x - 9,94$	3.51
4	$y = 3,45x - 9,89$	3.45
5	$y = 3,64x - 10,59$	3.64
6	$y = 3,71x - 10,65$	3.71
7	$y = 3,42x - 9,47$	3.42
8	$y = 3,46x - 10,04$	3.46
9	$y = 3,38x - 9,95$	3.38
10	$y = 3,38x - 9,95$	3.38
11	$y = 3,31x - 9,86$	3.31
12	$y = 3,25x - 9,79$	3.25
13	$y = 3,15x - 9,53$	3.15
14	$y = 3,13x - 9,51$	3.13
15	$y = 3,08x - 9,57$	3.08
16	$y = 2,96x - 9,51$	2.96

17	$y = 4,23x - 12,48$	4.23
18	$y = 4,20x - 12,44$	4.20
19	$y = 4,09x - 12,18$	4.09
20	$y = 4,09x - 12,18$	4.09
21	$y = 4,58x - 13,49$	4.58

Figure 3 shows the dynamics of the change in correlation dimensions  $D$  for segments  $X_q^*$ ,  $q = \overline{1,21}$ . It is obvious that starting from point 18, there is a sharp increase in the correlation dimension (it becomes more than 4). A sharp increase in the correlation dimension indicates that in these time intervals, the behavior of the generating system has changed significantly: previously uncharacteristic anomalous values have appeared. Consider the dynamics of the change in mathematical expectation  $M$  and standard deviation  $\sigma$  of segments  $X_q^*$ ,  $q = \overline{1,21}$ . Figure 4 shows graphs  $M(X_q^*)$  and  $\sigma(X_q^*)$  for shifts  $q = \overline{1,21}$ .

Figure 4 shows that the line of mathematical expectation  $M(X_q^*)$  grows smoothly and is not affected by the anomalous data. And the line of standard deviation  $\sigma(X_q^*)$ , starting from point

$q = 15$ , demonstrates an increasing divergence from line  $M(X_q^*)$ . This discrepancy indirectly indicates a qualitative (anomalous) change in the behavior of the dynamic system.

In Figure 5, highlighted in black are the bars that determine the values of time series  $X$  for critical values  $D(X_{18}^*) - D(X_{22}^*)$ ; the entire time series  $X$  is empirically divided into five periods of oscillations  $Q_1, Q_2, Q_3, Q_4, Q_5$ .

As can be seen from Figure 5, the values of the time series in periods  $Q_3, Q_4$  significantly exceed the values in periods  $Q_1, Q_2$ . This proves the fact that the behavior of the generating system changes from the reference time point  $i = 18$ . In other words, according to the sharply increased correlation dimension of the time series attractor, it was possible to detect the moment of transition of the generating system from the usual mode to the sharp mode.

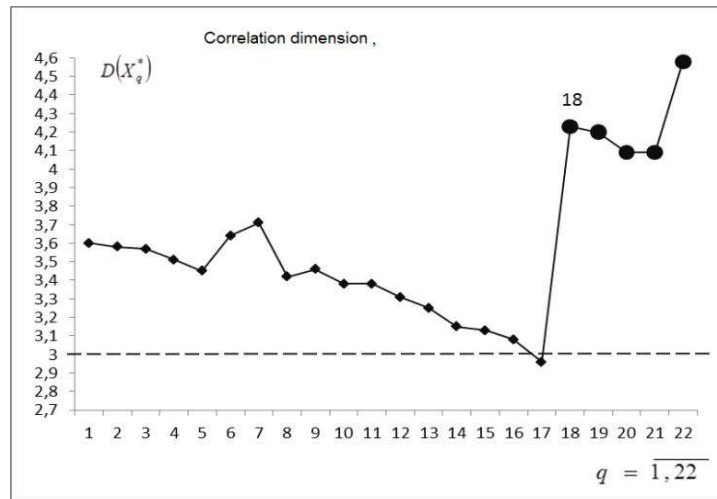


Figure 3. Change in correlation dimension  $D(X_q^*)$ ,  $q = \overline{1,21}$ .



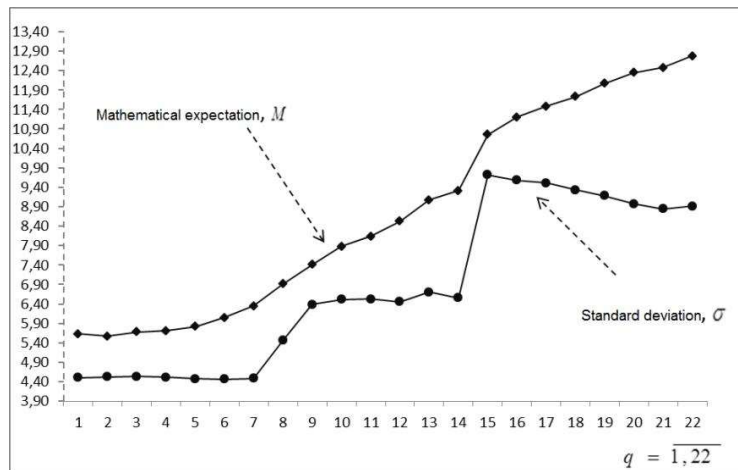


Figure 4. Dynamics of the change in mathematical expectation  $M(X_q^*)$  and standard deviation  $\sigma(X_q^*)$  for shifts  $q = \overline{1, 21}$ .

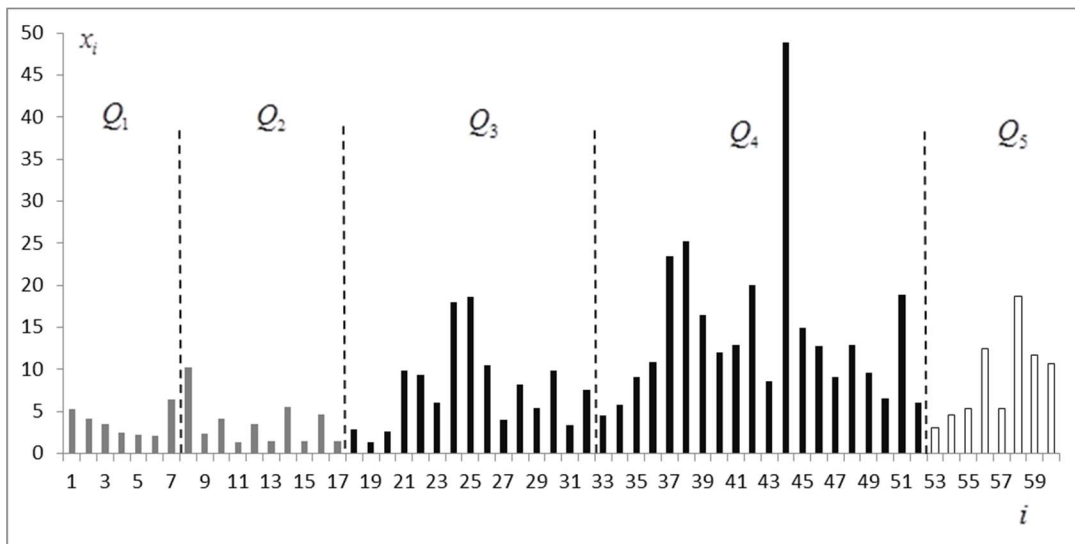


Figure 5. Oscillation periods of time series  $X$ .

**5. DISCUSSION**

This article suggests an approach for detecting data anomalies from information sensors based on estimating the correlation dimension of the time series attractor in situational management systems. The application of the proposed approach will make it possible to unambiguously detect the changes in oscillatory processes that result in the occurrence of anomalous data. Such anomalousness is not a measurement error, but reflects the objective reality. Therefore, it is necessary to analyze the causes that have led to such a qualitative change in the behavior of the generating system.

**6. CONCLUSION**

The results obtained within this article can be interpreted as follows. The correlation dimension of the attractor is traditionally considered as a measure of stochasticity of the process: the smaller it is, the more this process is determined. Sharp drops and rises in the values of the correlation dimension are an indication of the instability of the generating system and the change in its characteristic states to uncharacteristic ones.

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