

MODELING OF CONCRETE STRENGTH PREDICTION USING FUZZY TYPE-2 TECHNIQUES

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ABSTRACT

The compressive strength of concrete is an important process in constructions arrangement and proportioning new mixtures which demonstrates its level of quality assurance. In this research, a concrete strength prediction model has been developed based on Fuzzy approach. This model will estimate the compressive strength of a concrete based on 1030 sample of ready-mix concrete based on its design mix constituents, namely cement, blast furnace slag, fly ash, water, superplasticizer, coarse aggregate, and age. Four different model are compared with each other using statistical performance. The experimental results show that Type-2 fuzzy model gives minimum errors for predicting compressive strength of concrete.

Keywords: *Type-1 Fuzzy Model, Type-2 Fuzzy Model, Concrete Strength Prediction.*

1. INTRODUCTION

Concrete is the most important material used in building construction. The modeling of concrete material is a difficult task based on its composite nature[1]. The definition of concrete performance are based on several important parameter namely, material used in the concrete mix, the mix process and ratio, the transporting and placing process and the testing procedure for the strength of the concrete [2][3][4]. In practice, the expert commonly used the standard uniaxial compressive test to determine compressive strength. For example, the coring tests that widely used in the construction field [17][18][19]. However, the process is time-consuming and costly. There is a practical limit to decide how many samples should be taken for a consideration. Besides that, only small numbers of samples are used, and it will lead to the difficulty in finding the reliable or meaningful statistical conclusions.

There are various prediction approaches are widely implemented in modeling the concrete material[5][6][7][8][9]. However, these studies do not provide accuracy and predictability where the interactions among the number of variables that are unknown, complex or nonlinear [1][10]. To

overcome the limitation of the existing prediction method, many studies have been done on finding a new approach to predict concrete compressive strength physically and analytically. Regression analysis is one of the traditional methods in predicting model [5][20]. This method is very simple and fast but, the accuracy of the regression analysis is decreased as the number of independent variables increases [21]. In recent years, the used of Artificial Intelligent (AI) as an analytical approach have increasingly been applied to concrete strength prediction [22][23][24][9][25][26]. Here, the AI techniques such as Artificial Neural Network (ANN) use the basic procedure to predict material behavior by using the training dataset [5]. However, ANN is black box learning approach. Therefore, the interpretability of the modeling solution is very low. The relationship between the input and the output data cannot be understand clearly through the modeling result. Besides that, AAN also cannot deal with uncertainties.

Fuzzy system is suitable for solving a non-linear and uncertain problem [11][12]. Moreover, fuzzy can help to improve the interpretability of the solution when underlying physical relationships of the problem are not easy to understand. The constructed fuzzy systems are capable of dealing

with the perceptual uncertainties that involved in real and complex problem [13][14]. Here, the construction of fuzzy system framework is including the two important concepts namely fuzzy rules-based and fuzzy reasoning. Later Type-2 Fuzzy system were introduced by Zadeh in 1975 that can provide better ability to deal with uncertainty problem. Moreover, there are more parameters can be constructed that to provide researcher with more design degrees of freedom than Type-1 Fuzzy system [10][15][16]. The modeling solution produced rules that can be easily interpret the relationship between the input data and output data. In this research, we will developed using Type-2 Fuzzy Model to predict the concrete compressive strength. Then a comparative study of concrete prediction has been done based on the statistical performance measures.

2. FUZZY PREDICTION TECHNIQUES

In this section, we explain the process of constructing the Type-2 Fuzzy Model for concrete strength prediction. The proposed method is composed of three stages: Construction of Fuzzy membership function, development of the Type-2 Fuzzy model, and performance evaluation using RMSE. These steps are explained in details below.

2.1 Stage 1: Construction of fuzzy membership function by using Fuzzy C-mean Clustering

The construction of fuzzy membership function is the main important process in fuzzy system. Important used membership function such as Triangular, Gaussian, and Trapezoidal can be generated using Fuzzy C- Means (FCM) clustering. The process will start with the grouping of patterns from raw data into each fuzzy cluster to perform the membership degree of each data. Then through the fuzzy cluster representation it will be converted into parameterized equation of membership functions. Let $X = \{x^1, x^2, x^3, \dots, x^n\}$ be a set of n variables vectors and each x^i is a point in q-dimensional sample space $\{x_1, x_2, \dots, x_q\}$. Each variable x_j represent important variables of the prediction system. The FCM clustering solution will be assigning fuzzy labels to each of the sample in X. The algorithm will generate c-partitions in the form of matrix $U = [u_{ki}]$ that represent the degree of membership of each sample towards the center of the cluster generated. Finally, the clustered data in matrix U will be used to estimated Triangular /

Gaussian membership function parameter for each cluster [27]. The following is the triangular membership function of fuzzy set F with three parameters a, b, and c where the degree of membership of pattern u;

$$\mu_F(u) = \begin{cases} 0; u \leq a \\ \frac{u-a}{b-a}; a < u \leq b \\ \frac{u-b}{c-b}; b < u < c \\ 0; u \geq c \end{cases} \tag{1}$$

2.2 Stage 2: Developing Fuzzy Model based on Takagi Sugeno approach

The Takagi Sugeno fuzzy approach is also called the Fuzzy Functional Model. Here, the rule base is composed by using fuzzy rules, whose output parts are a function of the input variables. The output rules are represented by either the crisp number or linear functions of the input given. The partition of the input space from clustering result represents the inputs' part by a number of fuzzy regions. The format of the Takagi Sugeno Fuzzy Model represent the non-linear system in an effective processes by bringing together the local functional description with the rule-based description in the form of linearization [28][29]. The process of constructing the model can be divided into a two namely structure and parameter estimation. Here, the structure identification of a model is determined based on the FCM clustering result. The representation of the fuzzy models is,

$$R_i: \text{if } x_1 \text{ is } A_{i,1} \text{ and } \dots x_n \text{ is } A_{i,k} \text{ Then } y_i = \mathbf{a}_i^T \mathbf{x} + a_{i0} \tag{2}$$

where R_i is the i-th rule, $A_{i,k}$ is a fuzzy subset, y_i is the predicted output from the i-th rule, the coefficient \mathbf{a}_i^T of the linear equation is called the output parameter, and a_{i0} is the scalar offset.

The following is the inference formula for Takagi Sugeno Fuzzy Model:

$$y = \frac{\sum_{i=1}^C \lambda_i(x_k) y_i}{\sum_{i=1}^C \lambda_i(x_k)} = \frac{\sum_{i=1}^C \lambda_i(x_k) (a_i^T x + a_{i0})}{\sum_{i=1}^C \lambda_i(x_k)} \tag{3}$$

Where $\lambda_i(x_k)$ is the degree of activation of rule R_i . Figure 1 illustrates the example of each rule that can be represented by a locally linearized model. The model will describe the non-linearity of the given problem, where the universes discourse of the input data is partitioned by using linguistic labels such as LOW, MEDIUM or HIGH. The output is partitioned by using polynomials equation. Here, X is the input variable with six membership functions: A_1, A_2, A_3, A_4, A_5 and A_6 , and y is the output variable that can represent in the form of $y = ax + b$.

- R_1 : If x is A_1 then $y = a_1x + b_1$
- R_2 : If x is A_2 then $y = a_2x + b_2$
- R_3 : If x is A_3 then $y = a_3x + b_3$
- R_4 : If x is A_4 then $y = a_4x + b_4$
- R_5 : If x is A_5 then $y = a_5x + b_5$
- R_6 : If x is A_3 then $y = a_6x + b_6$

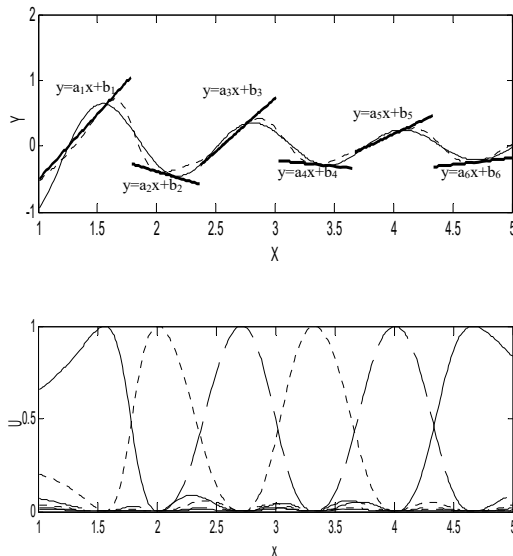


Figure 1: The representation of fuzzy model with 6 rules.

2.3 Stage 3: Development of Type-2 Fuzzy Model

Type-2 fuzzy model is a growing research topic—if the number of publications is taken as a measure. There is much evidence of successful applications in type-2 fuzzy model, so we can expect this growth to

continue. Type-2 fuzzy model allowing fuzzy systems to truly deal with real-world uncertainty because it will provide second order uncertainties representation. In the current technology, more powerful and affordable hardware for Type-2 Fuzzy Model presents an opportunity to explore uncertainties in concrete strength prediction modeling.

The type-2 fuzzy model uses interval type-2 fuzzy sets to represent the inputs and/or outputs of the model [30]. The Type-2 fuzzy model works as follows: the crisp inputs from the input sensors are first fuzzified into input Type-2 fuzzy sets[31]. The input Type-2 Fuzzy sets then activate the inference engine and the rule base to produce output type-2 fuzzy sets. The Type-2 Fuzzy rules will remain the same as in the Type-1 fuzzy model, but the antecedents and/or the consequents will be represented by interval type-2 fuzzy sets. The inference engine combines the fired rules and gives a mapping from input Type-2 fuzzy sets to output Type-2 fuzzy sets. The Type-2 fuzzy outputs of the inference engine are then processed by the type-reducer, which combines the output sets and performs a centroid calculation that leads to type-1 fuzzy sets called the type reduced sets [16][15] [32].

Figure 2 displays the framework for a data driven Type-2 Fuzzy Model. The structure identification represents the problem of constructing the input data X into the number of fuzzy spaces needed to form the Type-2 Fuzzy Model

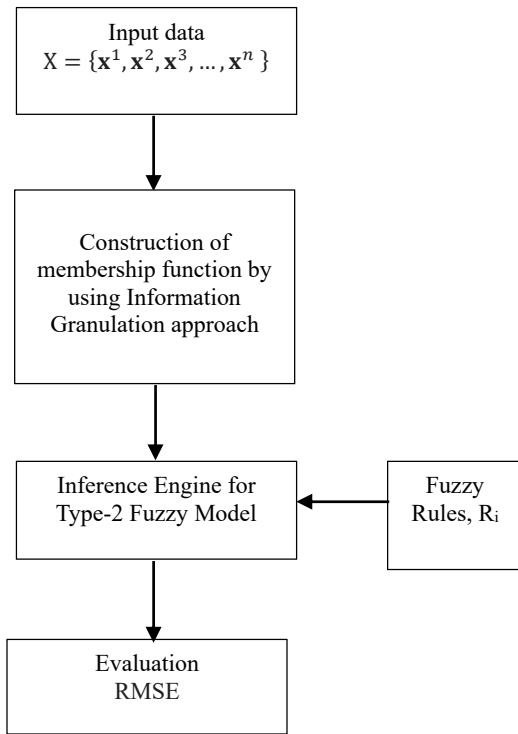


Figure 2: Type-2 Fuzzy Model process

2.4 Stage 4: Performance evaluation

The performance index in constructing the best model parameter for fuzzy system is the root mean square error (RMSE) of the output error. The following is the formula for RMSE;

$$RMSE = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n}} \tag{4}$$

Where n is the number of specimens, y_i is the experimental strength of i th specimen and \hat{y}_i is the calculated compressive strength of i th specimen.

In Figure 3 we show the modeling result based on the actual and model output value for by using two different dimensionality (dim) dataset.

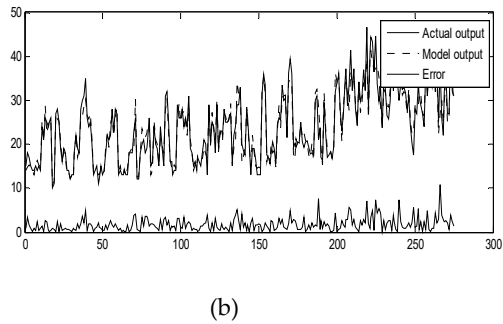
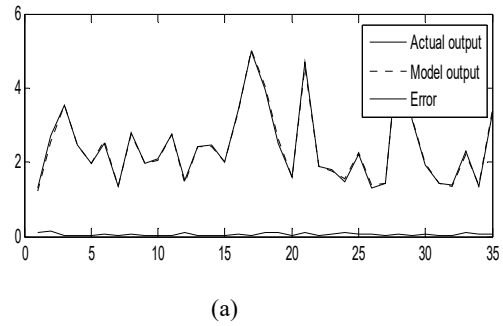


Figure 3: Comparison of modelling result; (a) data with dim=2 and (b) data with dim=7)

3. EXPERIMENTAL RESULT

In this section, we explain the dataset used for constructing the Type-2 Fuzzy prediction model. For developing the model, different concrete specimens are constructed with different mix designs were determined in laboratory. There are 1030 sample of mix designs consist of cement, blast furnace slag, fly ash, water, superplasticizer, coarse aggregate, and age. The range of the concrete mix constitute is presented in Table I. In this experiment we divided the data into training and testing data. The training data are used to construct the fuzzy models while the testing data are used to evaluate the accuracy of the fuzzy models constructed for unseen data.

Table 1: The range of training data

Input / Output variables	Minimum data	Maximum data
Cement	0	900.9
Water	118	238
Sand	208	879
Gravel	389	1285
Superplasticizer	0	3.5
Fly ash	0	275
Silica fume	0	90
Slag	0	500
Compressive strength	6.3	107.7

In prediction modeling, an important step is the input variables representing the system to be models. The input variables of a concrete data should comprise all relevant information on the target output. This study used all input variables in concrete dataset as the main factor affecting the compressive strength of concrete. The models were developed by using eight input variables and one output variable (compressive strength). Equation 5 illustrate the multiple linear regression model representation using the input variables in concrete dataset. From the equation we can see the important variables for the linear model based on the coefficient values for each variable. The highest coefficient means that the variable will give more impact to the calculation of the output, \hat{y}_i . In the multiple linear regression model, 60% of total data that comprise of 618 sample of data were used as the training data and remaining data were employed as the testing data.

$$\hat{y} = 0.103 \times \text{Cement} + 0.082 \times \text{Water} + 0.074 \times \text{Sand} - 0.191 \times \text{Gravel} + 0.375 \times \text{Superplasticizer} + 0.106 \times \text{Fly ash} + 0.121 \times \text{Slag} + 7.936 \quad (5)$$

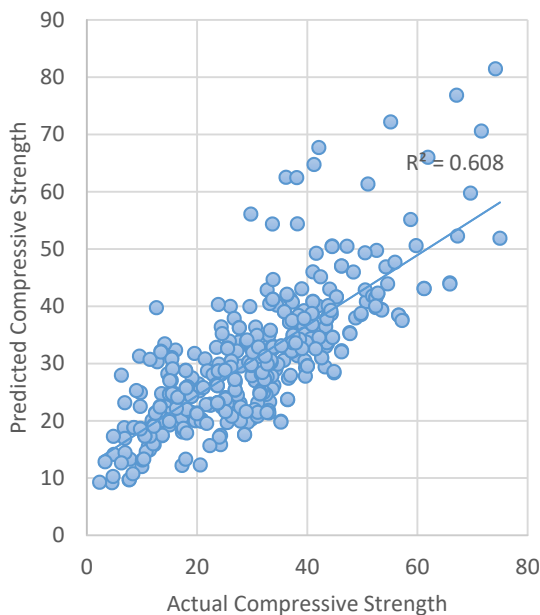


Figure 4: Comparison between the Actual and Predicted Compressive strength for testing data.

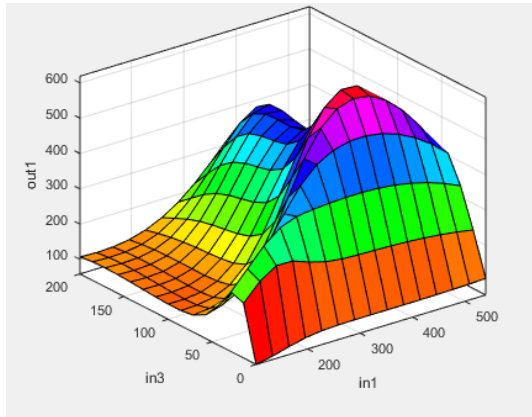
Figure 4 shows the comparison between the actual compressive strength and predicted

compressive strength for the testing data in Multi Linear Regression Model. Equation 5 show the coefficients of each variable used in forming the concrete. The most important variable is superplasticizer which represent 35% of the total variables. Second important variable is Gravel that have 18% of negative relation with compressive strength. The least important variable is sand and water which is 7% and 8 % respectively. The Multi Linear Regression model generated from the training data is not perform strong enough because the value of R^2 is just 0.608. This is because we use all the 412 testing data to calculate the predictive compression strength.

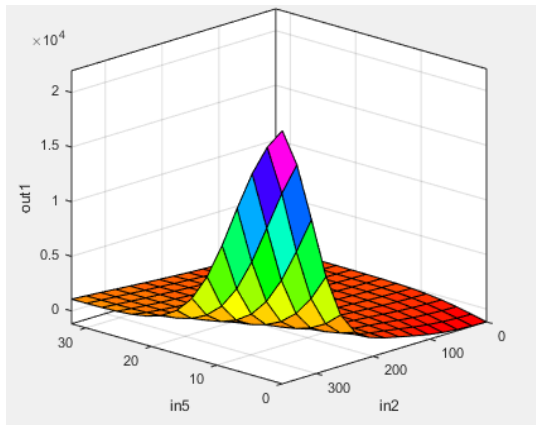
Next, we implement the modeling of concrete strength prediction using fuzzy approach. Here, we implement 3 different fuzzy models approach called fuzzy Type-1 Model, Adaptive Neuro-Fuzzy Modeling (ANFIS) and Type-2 Fuzzy Model for concrete strength prediction modeling. The K-fold cross-validation techniques is used to increase the generalization capability of the fuzzy model approach and to overcome the over-fitting problem. In the experiment, we implement $k = 10$ where we divide all the training data at random into 10 distinct subsets, train the model using 9 subsets, and test the model on the remaining subset. The process of training and testing is then repeated for each of the K possible choices of the subset omitted from the training. Finally, the average performance on the 10 omitted subsets is then estimate the generalization performance.

Figure 5 presents the 3D surface viewer from the compressive strenght with its various input variables. The surface plots display both the connecting lines and aces of the surface in color. The 3D surface viewer obtained from Fuzzy Model explains the relation between the output and two inputs. Figure 5(a), 5(b), 5(c), 5(d), show surface plots for six surface viewer relating inputs to compressive strenght of concrete. It concludes from the surface viewer that the contribution of interdependent parameters toward obtaining the output can easily provide through the Fuzzy System algorithm and can be hardly obtained otherwise without employing massive computations. All the surface viewer plots show that the total surface is covered by rule base. In Figure 5(c) show the strong

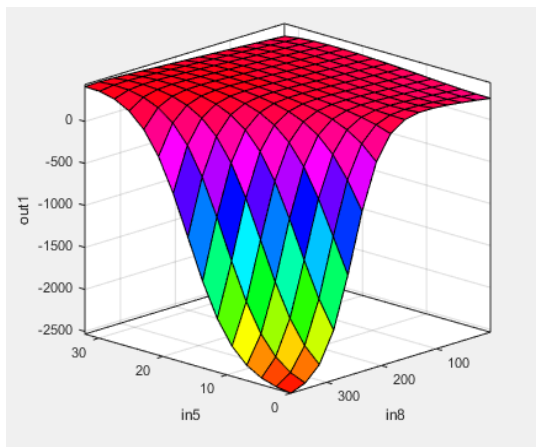
relationship between the two input variable namely Superplasticizer and Cements. The same pattern also show in the coefficient of Multiple Linear regression model in equation (5).



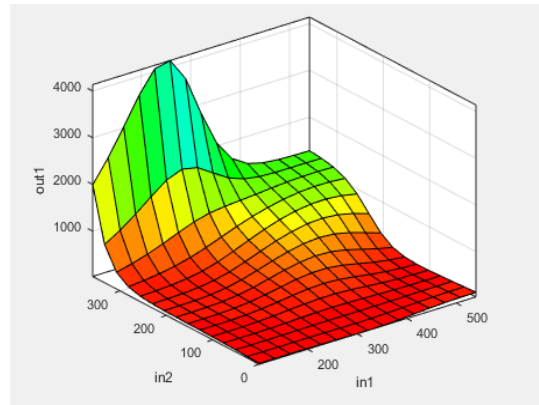
(a)



(b)



(c)



(d)

Figure 5 (a, b, c, d): Surface Viewer compressive strength and several input data.

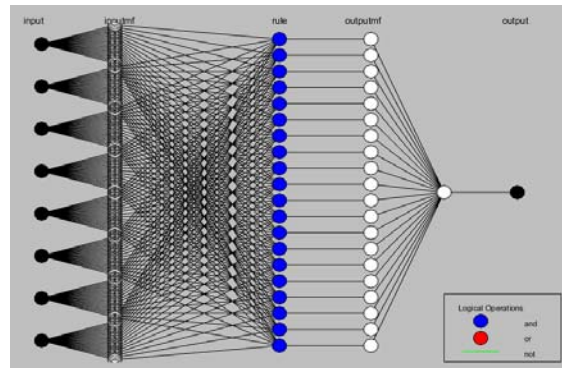


Figure 6: Schematic of ANFIS architecture.

ANFIS is a fuzzy system whose membership function parameters have been tuned using neuro-adaptive learning methods similar to methods used in training neural networks. The schematic structure and general properties of ANFIS shown in Figure 6. The architecture of ANFIS are composed of eight input data, one output data, hidden layers, neurons, fuzzy rules and fuzzy membership function. In the first layer, ANFIS generates the membership grades of input variable. Then in the next layer, each node calculates the firing strength of each rule using the min or prod operator. The final layer is the output. In ANFIS, the training involves the adjustments of weights of each parameter, such that the variation between actual and predicted values are minimized. The optimization method used is hybrid method, which uses a combination of backpropagation to compute input membership function parameters, and least squares estimation to compute output membership function parameters.

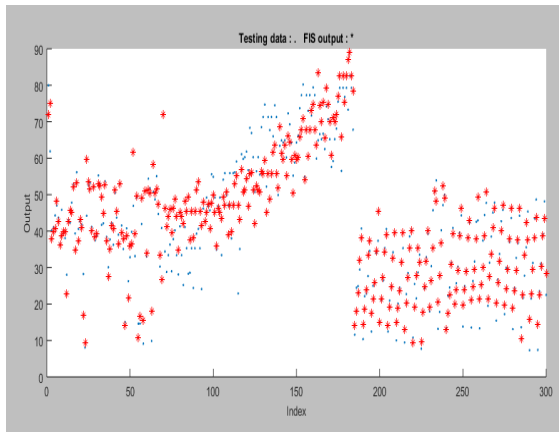
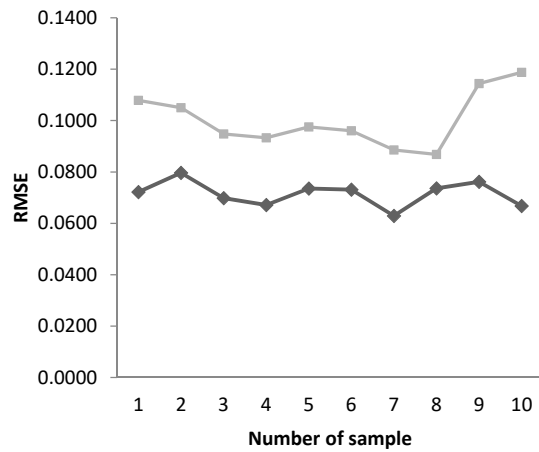


Figure 7. Comparison between the “Predicted” and “Output” parameters for “Testing” data in ANFIS model

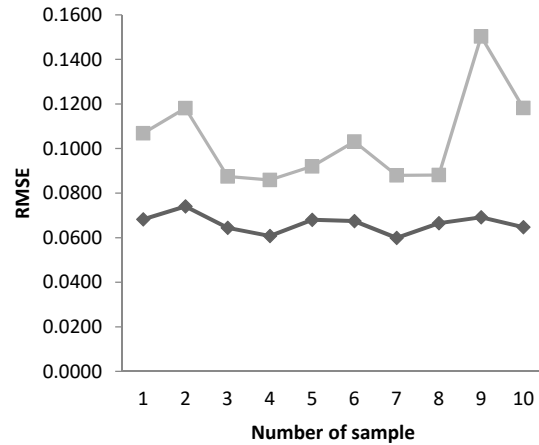
Figure 7 presents the predicted output in blue nodes and actual output in red nodes for testing data. The correlation between the predicted and the actual output is acceptable and good. Table 2 shows the performance of compressive strength prediction of the three different fuzzy model. It can be seen from the table, the ANFIS result outperform the other two model in the training result. However, the Type-2 Fuzzy model is the best RMSE for the testing data. From the result, we can conclude that Fuzzy Type-2 model is good in prediction unseen data and the model constructed is more generic compared to the other two models.

Table 2: Comparison of Prediction Models

Model	Root Means Square Error (RMSE)	
	Traning	Testing
Type-1 Fuzzy	0.0715	0.1003
ANFIS	0.0663	0.1038
Type-2 Fuzzy	0.1327	0.0427



(a)



(b)

Figure 8: The performance for training and testing data (a) fuzzy type-1 model and (b) ANFIS

Figure 8 shows the performance of the other two fuzzy models for training and testing data in 10-fold cross-validation approach. From the figure, we can see that the result for each subset of training and testing data is consistent and always the training data outperform the testing data. Figure 9 shows the RMSE for testing data using all three models. Here, the Fuzzy Type-2 Model outperforms

the other two approaches. The superiority of the Type-2 Fuzzy model may be attributed to the uncertainty involved between the input variables which can be solved better by Fuzzy Type-2 Model. This result has shown that the ability of Type-2 Fuzzy Model to model and consequently handle large amounts of uncertainty better than Type-1 Fuzzy Models [33]. It is important to model input uncertainties using input fuzzy sets that can be captured from sensor data. Then the input fuzzy set will be combined with antecedent fuzzy sets which reflect the uncertainty of the linguistic variables such as “LOW”, “MEDIUM” and “HIGH”. Figure 10 plot the predicted compressive strength values vs. actual data in Fuzzy Model for 10-fold cross validation result for the testing data.

Type-2 Fuzzy Model shows significant improvement for predicting the compressive strength of the concrete. The model helps to improve the interpretability of the solution when underlying physical relationships of the problem are not easy to understand. The constructed fuzzy systems are capable of dealing with the perceptual uncertainties that involved in compressive strength prediction problem.

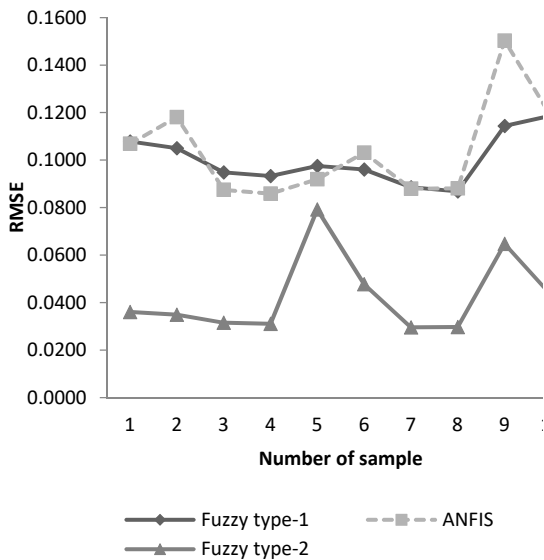
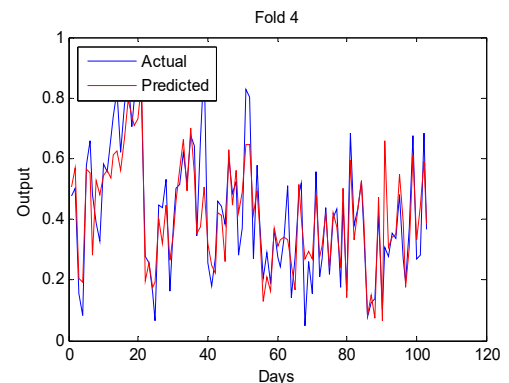
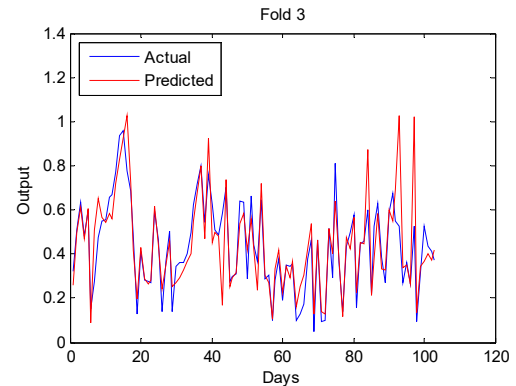
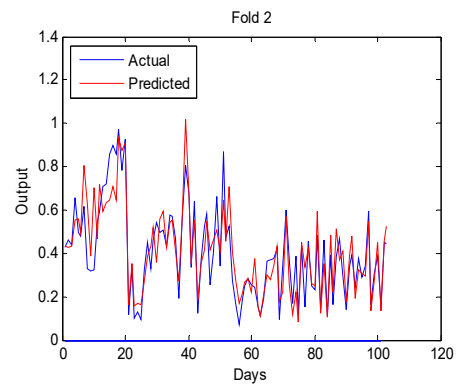
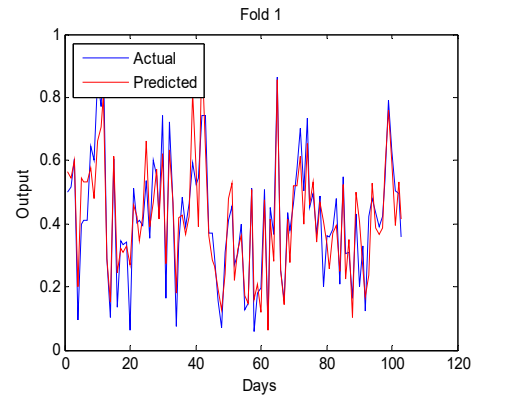


Figure 9: The RMSE for testing data



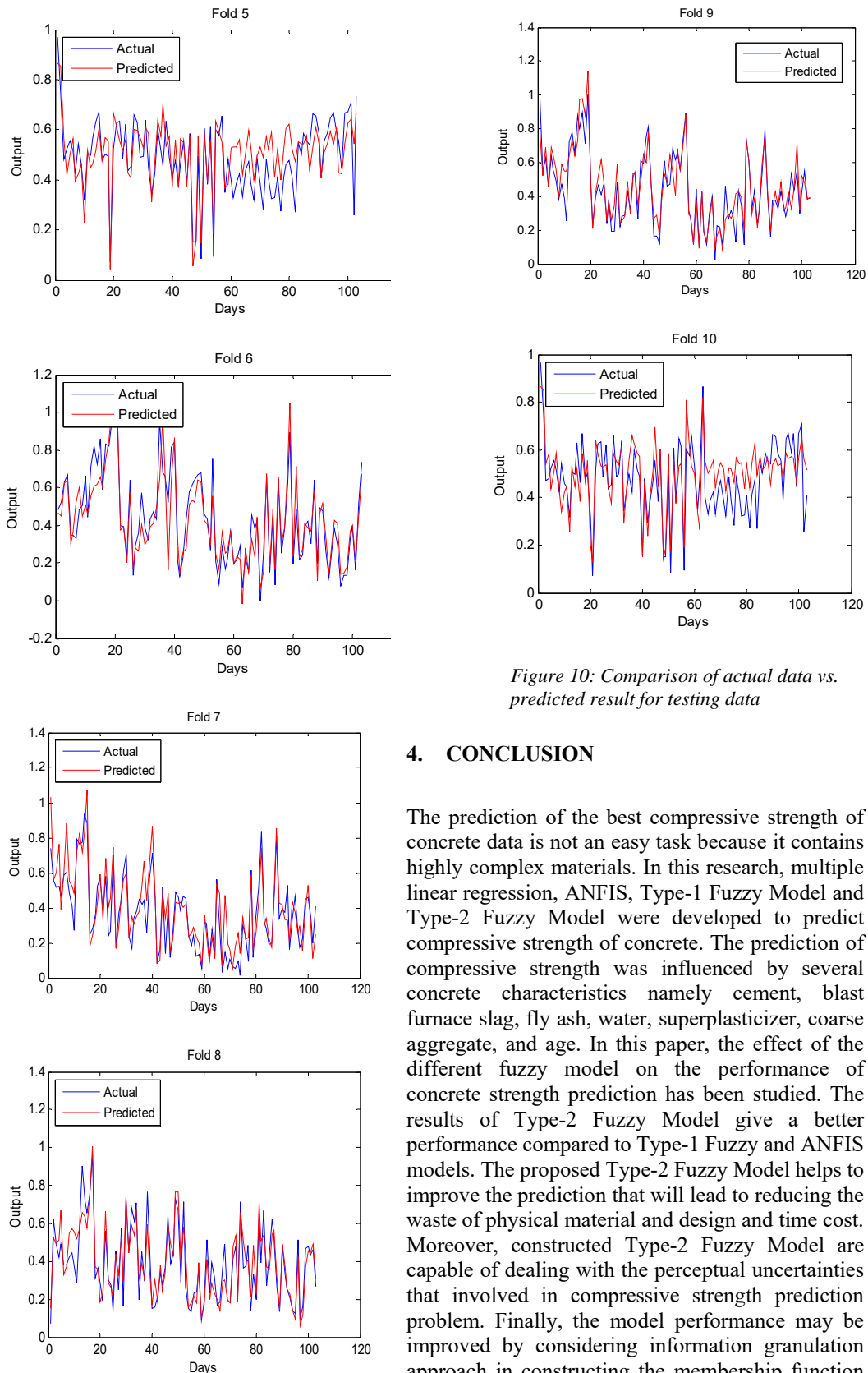


Figure 10: Comparison of actual data vs. predicted result for testing data

4. CONCLUSION

The prediction of the best compressive strength of concrete data is not an easy task because it contains highly complex materials. In this research, multiple linear regression, ANFIS, Type-1 Fuzzy Model and Type-2 Fuzzy Model were developed to predict compressive strength of concrete. The prediction of compressive strength was influenced by several concrete characteristics namely cement, blast furnace slag, fly ash, water, superplasticizer, coarse aggregate, and age. In this paper, the effect of the different fuzzy model on the performance of concrete strength prediction has been studied. The results of Type-2 Fuzzy Model give a better performance compared to Type-1 Fuzzy and ANFIS models. The proposed Type-2 Fuzzy Model helps to improve the prediction that will lead to reducing the waste of physical material and design and time cost. Moreover, constructed Type-2 Fuzzy Model are capable of dealing with the perceptual uncertainties that involved in compressive strength prediction problem. Finally, the model performance may be improved by considering information granulation approach in constructing the membership function

for Type-2 Fuzzy Model for predicting the concrete compressive strength estimation.

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