Journal of Theoretical and Applied Information Technology
15th December 2018. Vol.96. No 23

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

PROBABILISTIC MODEL OF ALLOCATION LAWS OF EXPERIMENTAL DATA IN INFORMATION SYSTEMS

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ABSTRACT

On the basis of the beta distributions of the 1st and 2nd kind were received probabilistic models of distribution laws, which allow to approximate wider class of distribution laws of experimental data, than the existing Pearson's system of distributions. The method of identification parameters was developed of the generalized beta distribution using power, exponential and logarithmic moments.

Keywords: *Information System, Experimental Data, Probability, Distribution Laws, Approximation.*

1. INTRODUCTION

Instruments for measuring, diagnosing and controlling the parameters of various physical media are widely used in scientific research, industry, agriculture, medicine. In the development of such devices, in most cases, well-known methods of processing measuring signals or primary experimental data are used. However, any procedure for measuring a physical quantity must also include an estimate of the random and systematic errors in the measurement result, which depend not only on the design of the instrument, but also on the various properties of the media under study. Therefore, it is often necessary to solve the problem of estimating the random and systematic errors of the measurement results in the process or after measuring the parameters of the physical medium.

Evaluation of systematic errors in measurement results for specific sensors requires the analysis and identification of the physical and mathematical model of the device based on various methods of processing experimental data. The urgency of solving this metrological task for specific types of sensors seems to be obvious. The problem of identifying the law of distribution of the observed random variable is the task of choosing such a parametric model of the law of probability distribution, which best corresponds to the results of experimental observations. Random measurement errors in most cases are poorly described by the normal-law model [1]. At the heart of measuring instruments and systems are various physical principles, various measurement methods and various transformations of measuring signals. Measurement errors as magnitudes are a consequence of the influence of a variety of factors, random and non-random, acting continuously or sporadically. Because of this, only when certain prerequisites (theoretical and technical) are fulfilled, the measurement errors are sufficiently well described by the model of the normal law.

It should be noted that the true law of distribution, which describes the errors of a specific measuring system, is in most cases unknown and will remain undetected for all attempts to identify it. Based on these measurements and theoretical considerations, it is possible to select a probabilistic model that in some sense best approximates this true law. If the constructed model is adequate, that is, the applied criteria do not give grounds for its rejection, then on the basis of this model it is possible to calculate all probabilistic characteristics of the random component of the measurement system error that differ from the true values only at the expense of non-excluded systematic (unobservable or unregistered) component of the measurement error. Its smallness characterizes the correctness of the measurements. The set of possible probability distribution laws, which can be used to describe observable random magnitudes, is unlimited. It is pointless to set the goal of the identification problem to find the true law of distribution of the observed quantity. We can only solve the problem of choosing the best model from a certain set. For example, from the set of parametric laws and the distribution families that are used in applications, and mention of which can be found in literary sources.

The classical approach to the structural-parametric identification of the distribution law is the algorithm for choosing the distribution law, which is entirely based on the apparatus of mathematical statistics [2]. Such an approach to the identification of the distribution law consists in the consistent realization of the next two-stage procedure for each type of parametric model from the set of laws under consideration. At the first stage of the procedure, based on sample data, a model of a certain type of law is constructed (from the considered set of models), the parameters of this model are estimated. At the second stage, the degree of adequacy of the obtained model to experimental observations is estimated, as a rule, with the use of different agreement criteria.

There are also a number of approaches mentioned in [3], for which an attempt is made to identify the form of the distribution law on the basis of the values of estimates of certain numerical characteristics calculated by selective data. For example, from the estimates of the coefficients of asymmetry and kurtosis, a point in the plane is determined whose position indicates the most preferable distribution law. However, the use of this approach in practice is extremely difficult because of the poor stability of estimates of central moments of high orders. Sample estimates of such moments are not robust. They are very sensitive to minor deviations in selected data from the proposed law, to the presence of emissions. The effectiveness of the approach can only be managed on idealized model data.

Statistical methods are most fully developed in the case of a normal or Gaussian distribution of random variables (RB). But experimental studies show that deviation from the normal distribution law is characteristic for many problems of measuring the parameters of physical media and other physical quantities. For non-Gaussian distributions of RB, the possibilities of solving statistical problems are significantly narrowed, since in many cases there are no proven and effective methods for the analytical or numerical solution of such problems. With reference to measuring tasks, the following main problems can be distinguished:

- the optimal choice of the number of intervals for grouping data in the construction of histograms, depending on the distribution law and the sample size of the RB;
- identification of the histogram form for sampling a sufficiently large volume N in the presence of practically always existing systematic deviations of the empirical probability density from the calculated one;
- determination of confidence intervals for estimating statistical parameters from the available sample of RB values under the unknown distribution law;
- generation of several correlated samples of random numbers with different distribution laws;
- Summation of random measurement errors and analysis of random errors in the results of indirect measurements under various laws of error distribution and correlation between them;
- evaluation of trend and noise parameters in the analysis of a time process with an unknown non-linear trend and noise distribution law;
- estimation of frequency values for short realization of a harmonic signal in the presence of noise with different distribution laws.

The urgency of solving these problems is noted in the monographs of Novitsky P.V. and Zograf I.A. [1], Granovsky V.A. and Syraya T.N. [4], Zemelman M.A. [5], Tyurin Yu.N. and Makarov A.A. [6], Gubarev V.V. [7], Kulaichev A.P. [8], Falkovich S.E. and Khomyakov E.N. [9], Shelukhin O.I. [10] on applied statistics and methods for processing measurement results. Solutions of some problems with non-Gaussian distributions of random variables are considered in the books and papers of Denisov V.I. and Lemeshko B.Yu. [11-13], in the articles of Solopchenko G.N. [14-17], Orlov A.I. [18-19], Kudlayev E.M. and Lagutin M.B. [20,21] and others, as well as in the books of foreign authors Anderson T. [22], Box G. and Jenkins G. [23], Brillinger D. [24], Kendall M. and Stuart A. [25], Cramer H. [26], Bendat J. and Piersol A. [27] and others, but many of these problems remain unresolved to the present day or they are solved only for some particular cases of non-Gaussian distributions.

2. STATEMENT OF THE PROBLEM

There is a view of the law of distribution assumed to be known in classical mathematical statistics and observing the results of its parameters assessed values. But usually pre-form of the distribution law is unknown and theoretical assumptions do not allow it to establish unequivocally. Also, processing of the experimental data does not allow to calculate accurately true distribution law. In this case, you should talk only about the approximation (approximate description) of real law to some others which is consistent with experimental data and in some ways similar to the unknown true law.

Nowadays, for the approximation of the experimental data distribution laws often used Pearson distributions [28-53]. However, the determination of the parameters of the desired distribution from the family of Pearson distributions connected with the decision of the various systems of equations using the method of moments. Besides, the method of moments does not allow to find the parameter estimates those distributions, including those owned by the Pearson family which do not have higher order moments (3rd and 4th). That is why the development of continuous distributions systems wider than the family of Pearson curves, as well as new methods for estimating the parameters has great importance both in theoretical and applied research.

Except of the method of Pearson for this purpose can be used the method based on obtaining a new distribution as a random function argument with the known distribution [25,32,34].

The main objectives of the work:

1) To receive generalized beta distribution based on the beta distribution of the 1st and 2nd kind using method of functional transformation.

2) To consider the possibility of approximating the distribution law of experimental data, taking positive and negative values or only positive values using generalized beta distribution of the 1st and 2nd kind.

3. SOLUTION OF THE PROBLEM

3.1 Unilateral generalized beta distributions of 1st and 2nd kind

Probability density functions (PDF) for the classical beta distributions of 1st and 2nd kind are as follows [25,53]:

$$p(y) = \frac{y^{\alpha - 1}}{B(\alpha, v)}(1 - y)^{v - 1}, \quad 0<y<1; \qquad (1)$$

$$p(y) = \frac{y^{\alpha - 1}}{B(\alpha, v)(1+y)^{\alpha + v}}, \quad 0<y<\infty, \qquad (2)$$

where $\alpha > 0$, $v > 0$ - parameters of the form; B(a, b) - beta function.

After a functional conversion $y = x^c / \chi^c$ or $y = \chi^c / x^c$ PDF (1) respectively, we have

$$p(x) = \frac{c\, x^{\alpha c - 1}}{B(\alpha, v)\chi^{\alpha c}}\left(1 - \frac{x^c}{\chi^c}\right)^{v - 1}, \quad 0<x<\chi; \qquad (3)$$

$$p(x) = \frac{c\, \chi^{\alpha c}}{B(\alpha, v)x^{\alpha c + 1}}\left(1 - \frac{\chi^c}{x^c}\right)^{v - 1}, \quad \chi<x<\infty, \qquad (4)$$

where $\alpha > 0$, $v > 0$, $c > 0$ - parameters of the form; $\chi > 0$ - scale parameter.

Specific cases of distribution (3) there is the power law when $c = 1$ и $v = 1$; Beta distribution when c = 1. The limiting case of (3) is a lognormal distribution when $\alpha\to\infty$, $v\to\infty$ and $c\to 0$.A special case of PDF (4) is a Pareto distribution when c = 1 and v = 1 [54-56].

Using PDF (3) or PDF (4) and the ratio [56]

$$m_s = \int_0^\infty x^s\, p(x)dx, \qquad (5)$$

We can get the initial moments of s-th order for distributions (3)and (4)

$$m_s = \frac{\chi^s\, \Gamma(\alpha + s/c)\Gamma(\alpha + v)}{\Gamma(\alpha)\Gamma(\alpha + v + s/c)};$$
$$m_s = \frac{\chi^s\, \Gamma(\alpha - s/c)\Gamma(\alpha + v)}{\Gamma(\alpha)\Gamma(\alpha + v - s/c)}, \qquad (6)$$

where $\Gamma(z)$ - is the gamma function.

From (6) it follows that for PDF (4), there are only the initial direct points, order s of those satisfies the condition $s<\alpha c$. On the image 1 are presented the regions of existence of PDF(3) and PDF(4) in the plane of variables K_1 and K_2, determined by the expressions (when $c = 1$) [57]

$$K_1 = \frac{m_{1c}^2}{m_{2c}};$$
$$K_2 = \frac{m_{1c}m_{3c} - m_{2c}^2}{m_{2c}\left(m_{2c} - m_{1c}^2\right)}. \qquad (7)$$

On the image 1 the region of distribution's existence (3) and it is special cases left to the curve 3, characterizing a region of existence of the standard logarithmic distribution. Curve 1 characterizes the region of existence of the power law, and the point G - Gaussian distribution. The region of existence of the beta distribution is located to the left of the curve 2.The region of existence of distribution (4) and its special cases is located to the right of the curve 3. The curve 5 characterizes the region of the existence of Pareto distribution.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

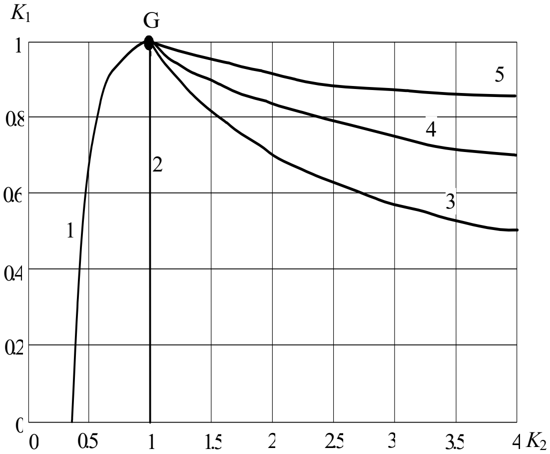

Figure 1. The diagram of unilateral laws of distribution

Performing the functional transformation $y = x^c/\lambda^c$ or $y = \lambda^c/x^c$ of PDF (2), respectively obtain

$$p(x) = \frac{c\,x^{\alpha c-1}}{B(\alpha,v)\lambda^{\alpha c}\left(1+\dfrac{x^c}{\lambda^c}\right)^{\alpha+v}},\ 0<x<\infty, \quad (8)$$

$$p(x) = \frac{c\,\lambda^{\alpha c}}{B(\alpha,v)x^{\alpha c+1}\left(1+\dfrac{\lambda^c}{x^c}\right)^{\alpha+v}},\ 0<x<\infty, \quad (9)$$

where $v> 0$, $0 <\alpha\leq v$, $c> 0$ - parameters of the form; $\lambda> 0$ –scale parameter.

Special cases of PRV (8) are: beta distribution of II sort with c = 1;*F*- distribution when $\alpha = 0{,}5n_1$, $v = 0{,}5n_2$, $c = 1$ and $\lambda = n_2/n_1$ [53].

After substituting the PDF (8) or PDF (9) in (5) and integration we obtain the early moments of s-th order for these distributions [58]

$$m_s = \frac{\lambda^s\Gamma(\alpha+s/c)\Gamma(v-s/c)}{\Gamma(\alpha)\Gamma(v)}. \quad (10)$$

$$m_s = \frac{\lambda^s\Gamma(\alpha-s/c)\Gamma(v+s/c)}{\Gamma(\alpha)\Gamma(v)}. \quad (11)$$

From (10) it follows that for PDF (8), there are only the initial moments, order of which s satisfies the condition $s<vc$. From (11) it follows that there are only the starting points for the PDF (9)order of which s satisfies the conditions$<\alpha c$.

Let's consider the limiting case of PRV (3) and (8), when the parameter $v\to\infty$. In this case distributions (3) and (8) would be transformed into the generalized gamma distribution [55]

$$p(x) = \frac{c\,x^{\alpha c-1}}{\Gamma(\alpha)\beta^{\alpha c}}\exp\left(-\frac{x^c}{\beta^c}\right),\ 0<x<\infty, \quad (12)$$

where $\alpha> 0$, $c> 0$ –parameters of the form; $\beta> 0$ – scale parameter.

Special cases of (12) are: Rayleigh PDF when$\alpha = 1$, $\beta=\sqrt{2}\,\sigma$ and $c = 2$; exponential distribution with α=1 and c=1; gamma distribution when $\alpha=v+1$, c=1; chi-square distribution when $\alpha = 0{,}5n$, $c= 1$ и β=2; Nakagami distribution when α=m, $\beta=\sqrt{\Omega/m}$ и $c = 2$; Weibull distribution when α=1 and $\beta^{-c}= \lambda$. Extreme cases (9) are the power law when $\alpha\to 0$ and $c\to\infty$; lognormal distribution when $\alpha\to\infty$ and $c\to 0$ [58,59].

Substituting the (12) into (5) and integrating [54] we obtain the initial moments of the s-th order

$$m_s = \beta^s\frac{\Gamma(\alpha+s/c)}{\Gamma(\alpha)}. \quad (13)$$

It should be noted, that the property, inherent in the distribution of (12) and presented in the form of equity [53]:

$$\frac{m_{(n+1)c}\,m_{1c} - m_{nc}\,m_{2c}}{(n-1)m_{nc}\left(m_{2c} - m_{1c}^{\ 2}\right)} = 1, \quad (14)$$

when $n\geq 2$. This property is proved by substituting expression (13) into (14) for the corresponding initial moments.

On the image 1the existence region of PRV (12) is located between the curves 1 and 3. Direct line 2 corresponds to the region of existence of the gamma distribution. The region of existence PDF(8) is located between curve 1 and curve 3. It is overlapped considerably with the region of existence of PDF (3) and includes a full region of existence of distribution (12).

Let's consider the limiting case of PRVDF (4) and PDF (9) when the parameter $v\to\infty$. In this case distributions (4) and (9) are converted into distribution

$$p(x) = \frac{c\,\beta^{\alpha c}}{\Gamma(\alpha)x^{\alpha c+1}}\exp\left(-\frac{\beta^c}{x^c}\right),\ 0<x<\infty. \quad (15)$$

where $\alpha> 0$, $c> 0$ – parameters of the form; $\beta> 0$ – scale parameter.

Special cases of (15) when c = 1 V is a type of distribution of the Pearson classification, and when $c\to\infty$ - is a Pareto distribution [53, 54]. Substituting the (15) in (5) and integrating, we obtain the initial moments of s-th order

$$m_s = \beta^s\,\Gamma(\alpha - s/c)/\Gamma(\alpha). \quad (16)$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

From (16) it follows that for the PRV (15), there are only the initial straight times, the order of which satisfies the condition $s<\alpha c$.

On the image 1 the existence region of PDF (15) is located between curves 3 and 5. The curve 4 describes a region of existence V-thtype distribution of the Pearson classification. The region of existence of distribution (9) is located between the third curve and the fifth curve. It is overlapped considerably with the region of the existence of distribution (4) and includes a full region of existence of PDF (15).

For the obtained distributions (3), (8) and (12) are characterized by two properties:

1) the property of moments defined by the equation:

$$\frac{m_{3c}}{m_{4c}}\cdot\frac{3m_{2c}m_{3c}-m_{1c}\left(4m_{1c}m_{3c}-m_{2c}^2\right)}{4m_{2c}^2-m_{1c}\left(3m_{1c}m_{2c}+m_{3c}\right)}=1\,. \quad (17)$$

2) The condition of distribution (3)is $K_2<1$, for distributing (12) - $K_2=1$ and for distributing (8) - $K_2>1$.

These properties are also valid for the distribution (4), (9) and (15), if to substitute in relations (7) and (17) instead of direct power moments inverse points. In this case for distribution (4) is still the condition $K_2<1$, for distributing (15) - $K_2=1$ and for distributing (9) - $K_2>1$.These properties can be used to identify the generalized beta distribution.

Let us to consider now the limiting cases for generalized beta distribution of the 1st and 2nd kind, when the parameter $c\to 0$. In this case distribution (12) and (15) are converted into logarithmic normal distribution. The expression for the PDF has the form

$$p(x)=\frac{1}{\sqrt{2\pi}\,\sigma x}\exp\left(-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right), \quad (18)$$

where $\mu>0$, $\sigma>0$ - distribution options.

For the distribution of (18) there are all direct and inverse power moments. Therefore PDF parameters are defined by the first initial and the second central moments of the logarithmic

$$\hat{\mu}=\hat{l}_1,\ \hat{\sigma}=\sqrt{\hat{L}_2}\,. \quad (19)$$

The region of existence PRV (18) is shown at the image 1 by the curve 3.

Distribution (3) and (8) converted into distribution, when the parameter $c\to 0$

$$p(x)=\frac{\beta^v x^{\beta-1}}{\Gamma(v)\chi^\beta}\left(\ln\left(\frac{\chi}{x}\right)\right)^{v-1},\ 0<x<\chi\,; \quad (20)$$

where $v>0$, $\beta>0$ – parameters of the form; $\chi>0$ – scale parameter.

Substituting the PDV (20) in equation (5) and integrating [58], we get the early moments of the s-th order

$$m_s=\left(\beta/(\beta+s)\right)^v\chi^s\,. \quad (21)$$

On the image 1 the existence region of distribution (20) is located to the left of the curve 3.

When $c\to 0$ distributions (4) and (9) are converted into distribution

$$p(x)=\frac{\beta^v\chi^\beta}{\Gamma(v)\,x^{\beta+1}}\left(\ln\left(\frac{x}{\chi}\right)\right)^{v-1},\ \chi<x<\infty, \quad (22)$$

where $v>0$, $\beta>0$ – parameters of the form; $\chi>0$ – scale parameter.

Substituting the GHD (22) into (5) and integrating [58], We obtain the initial moments of s-th order

$$m_s=\left(\beta/(\beta-s)\right)^v\chi^s\,. \quad (23)$$

From (23) it follows that for distributing (22) there are all inverse initial moments and only those points straight, order of which satisfies $s<\beta$. On the image 1 the region of distribution's existence (22) is located to the right of the curve 3. The distribution parameters (22) are defined by (23) with the help of reverse moments.

Identification of the lognormal distribution (18) is possible only with the use of logarithmic points [53]. Except of PDF (18), this group includes the distribution

$$p(x)=\frac{(-0{,}5\ln(h))^{1-2v}}{B(0{,}5,v)\,x}\left(\ln\left(\frac{x}{h\chi}\right)\ln\left(\frac{\chi}{x}\right)\right)^{v-1},\ h\chi<x<\chi; \quad (24)$$

$$p(x)=\frac{\lambda^{2v}}{B(0{,}5,v)\,x\left[\lambda^2+(\ln(x)-\mu)^2\right]^{v+0{,}5}},\ 0<x<\infty. \quad (25)$$

where $0<h<1$–the form parameter.

They are also limiting distributions for generalized beta distribution. There is only a portion of the logarithmic moments existing for PDF (25). Thus, we get a wide class of models of unilateral laws of distributions based on beta-distributions of the 1st and 2nd kind. When we identifying generalized beta distribution taking into account the consideration of their properties, you can use the forward and reverse power moments (including fractional order), and logarithmic moments [57-60].

3.2 Identification of unilateral generalized beta distributions of the 1st and 2nd kind

Approximation of the experimental distributions with the help of unilateral generalized beta

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

distribution can be carried out using the following algorithm:

1. Initially are determining logarithmic sampling points

$$\widehat{l}_1 = \frac{1}{n}\sum_{i=1}^{n} \ln x_i \ ,$$

$$\widehat{L}_s = \frac{1}{n}\sum_{i=1}^{n}\left(\ln(x_i) - \widehat{l}_1\right)^s \ , \ s = 2,\ 3, \quad (26)$$

and then an estimate the asymmetry coefficient

$$\widehat{L}_a = \widehat{L}_3 \big/ \widehat{L}_2^{1,5} \ .$$

2. If for the coefficient $\widehat{L}_a$ the following condition is $-0{,}1 \le \widehat{L}_a \le 0{,}1$, then further defined selective central point $\widehat{L}_4$ and is a joint evaluation of coefficient skewness and kurtosis

$$\widehat{L}_{ae} = \frac{6\widehat{L}_2^2}{\widehat{L}_4 + 3\widehat{L}_2^2} \ .$$

When the condition $\widehat{L}_{ae}$> 1,04 to approximate of the experimental distribution is used the distribution (24) with parameters

$$\hat{v} = \frac{1{,}5 - \widehat{L}_{ae}}{\widehat{L}_{ae} - 1} \ ;$$

$$\hat{\chi} = \exp\left(\sqrt{\frac{2 - \widehat{L}_{ae}}{\widehat{L}_{ae} - 1}\widehat{L}_2} + \widehat{l}_1\right);$$

$$\hat{h} = \exp\left(-2\sqrt{\frac{2 - \widehat{L}_{ae}}{\widehat{L}_{ae} - 1}\widehat{L}_2}\right) \ .$$

If for the coefficient $\widehat{L}_a$ the following condition is $0{,}96 \le \widehat{L}_{ae} \le 1{,}04$, the lognormal distribution (22) is used to approximate the experimental distribution, whose parameters according to (23) defined by the initial first and second central logarithmic moments ($\hat{\mu} = \widehat{l}_1, \hat{\sigma} = \sqrt{\widehat{L}_2}$).

When the condition $\widehat{L}_{ae}$< 0,96to approximate the experimental distribution is used the distribution (25) with parameters

$$\hat{v} = \frac{2 - 1{,}5\widehat{L}_{ae}}{1 - \widehat{L}_{ae}} \ ;$$

$$\hat{\lambda} = \sqrt{\frac{2 - \widehat{L}_{ae}}{1 - \widehat{L}_{ae}}\widehat{L}_2} \ ; \quad .$$

$$\hat{\mu} = \widehat{l}_1$$

3.If the ratio $\widehat{L}_a$< - 0,1, then for the approximation of the experimental distribution Is used one of the distributions (3), (8) or (12). Type of the distribution and its parameters can be determined as follows: first estimate is the parameter c of the solution of equation

$$\frac{\hat{m}_{3c}}{\hat{m}_{4c}} \cdot \frac{3\hat{m}_{2c}\hat{m}_{3c} - \hat{m}_{1c}\left(4\hat{m}_{1c}\hat{m}_{3c} - \hat{m}_{2c}^2\right)}{4\hat{m}_{2c}^2 - \hat{m}_{1c}\left(3\hat{m}_{1c}\hat{m}_{2c} + \hat{m}_{3c}\right)} = 1 \ , \quad (27)$$

where

$$\hat{m}_{1c} = \frac{1}{n}\sum_{i=1}^{n} x_i^{\hat{c}} \ ,$$

$$\hat{m}_{2c} = \frac{1}{n}\sum_{i=1}^{n} x_i^{2\hat{c}} \ ,$$

$$\hat{m}_{3c} = \frac{1}{n}\sum_{i=1}^{n} x_i^{3\hat{c}} \ , \quad (28)$$

$$\hat{m}_{4c} = \frac{1}{n}\sum_{i=1}^{n} x_i^{4\hat{c}} \ .$$

Then, the coefficient's estimates are determined as K_1 и K_2 with help of the relations (7). If the condition $\hat{K}_2 < 1$, thenfor the approximation of the experimental distribution, use distribution (3) with parameters

$$\hat{\alpha} = \frac{2\hat{K}_1\hat{K}_2}{1 + \hat{K}_2 - 2\hat{K}_1\hat{K}_2};$$

$$\hat{v} = \frac{2\hat{K}_2}{1 - \hat{K}_2} - \hat{\alpha};$$

$$\hat{\chi} = \left(\hat{m}_{1c}\left(1 + \frac{\hat{v}}{\hat{\alpha}}\right)\right)^{1/\hat{c}} . \quad (29)$$

If $\hat{K}_2 = 1$, then to approximate the experimental distribution is used the distribution (12) with parameters

$$\hat{\alpha} = \frac{\hat{K}_1}{1 - \hat{K}_1};$$

$$\hat{\beta} = \left(\frac{\hat{m}_{1c}}{\hat{\alpha}}\right)^{1/\hat{c}} . \quad (30)$$

When the condition is $\hat{K}_2 > 1$ to approximate the experimental distribution is used the distribution (8) with the following parameters

$$\hat{\alpha} = \frac{2\hat{K}_1\hat{K}_2}{1 + \hat{K}_2 - 2\hat{K}_1\hat{K}_2};$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\hat{\nu} = \frac{2\hat{K}_2}{\hat{K}_2 - 1} + 1; \qquad \hat{\lambda} = \left(\hat{m}_{1c} \frac{\hat{\nu} - 1}{\hat{\alpha}} \right)^{1/\hat{c}}. \tag{31}$$

4. If the ratio $\widehat{L}_a >$ 0,1, then for the approximation of the experimental distribution is used one of the distributions (4), (9) or (15).Type of distribution and its parameters can be determined as follows: first evaluate determined parameter c by solving the equation (27), and then determine the coefficient's estimates K_1 и K_2 with help of the relations (7). In this case, relations (7) and (17) are now used selective inverse points

$$\widehat{m}_{1c} = \frac{1}{n}\sum_{i=1}^{n} x_i^{-\hat{c}}, \quad \widehat{m}_{2c} = \frac{1}{n}\sum_{i=1}^{n} x_i^{-2\hat{c}}, \quad \widehat{m}_{3c} = \frac{1}{n}\sum_{i=1}^{n} x_i^{-3\hat{c}}, \quad \widehat{m}_{4c} = \frac{1}{n}\sum_{i=1}^{n} x_i^{-4\hat{c}}. \tag{32}$$

If the condition $\hat{K}_2 < 1$, you should use the distribution (4) for the approximation of the experimental distribution with parameters $\hat{\alpha}$, ν and $\hat{\chi}$, which are defined by (29) with considering the (32).

If $\hat{K}_2 = 1$, then to approximate the experimental distribution the distribution (15)with parameters $\hat{\alpha}$ and $\hat{\beta}$ are used, defining relations (30) with considering the (32).

When the condition $\hat{K}_2 > 1$ is true for approximating the experimental distribution is used the distribution (9) with parameters (31)

5. When the parameter's estimate $\hat{c}$ → 0 (on practice $\hat{c} \leq$ 0,1), then the approximation of the experimental distributions is made with distributions (20) and (22). If the ratio $\widehat{L}_a <$ - 0,1, It is used PDF (20) to approximate the distribution. Its parameters can be determined as follows: first find the estimate of the parameter β is determined by solving the equation

$$\frac{\ln(\hat{m}_1) - \hat{l}_1}{\ln(\hat{m}_2) - 2\ln(\hat{m}_1)} = \frac{\ln(\hat{\beta}) - \ln(\hat{\beta}+1) + 1/\hat{\beta}}{2\ln(\hat{\beta}+1) - \ln(\hat{\beta}) - \ln(\hat{\beta}+2)},$$

where

$$\widehat{m}_1 = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad \widehat{m}_2 = \frac{1}{n}\sum_{i=1}^{n} x_i^2.$$

Then, evaluation parameters are ν and χ determined with the aid of relations

$$\hat{\nu} = \frac{\ln(\hat{m}_1) - \hat{l}_1}{\ln(\hat{\beta}) - \ln(\hat{\beta}+1) + 1/\hat{\beta}}, \quad \hat{\chi} = \exp\left(\hat{l}_1 + \frac{\hat{\nu}}{\hat{\beta}} \right).$$

If the ratios $\widehat{L}_a >$ 0,1and $\hat{c} \leq$ 0,1, then is used PDF (22) for approximating the distribution. It's parameters are defined in a similar distribution of parameters (20). The estimate of parameter β is determined by solving the equation

$$\frac{\ln(\hat{m}_1) + \hat{l}_1}{\ln(\hat{m}_2) - 2\ln(\hat{m}_1)} = \frac{\ln(\hat{\beta}) - \ln(\hat{\beta}+1) + 1/\hat{\beta}}{2\ln(\hat{\beta}+1) - \ln(\hat{\beta}) - \ln(\hat{\beta}+2)},$$

where

$$\widehat{m}_1 = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{x_i}, \quad \widehat{m}_2 = \frac{1}{n}\sum_{i=1}^{n} \frac{1}{x_i^2}.$$

Parameter's estimates ν and χ corresponding expression

$$\hat{\nu} = \frac{\ln(\hat{m}_1) + \hat{l}_1}{\ln(\hat{\beta}) - \ln(\hat{\beta}+1) + 1/\hat{\beta}}, \quad \hat{\chi} = \exp\left(\hat{l}_1 - \frac{\hat{\nu}}{\hat{\beta}} \right).$$

If the resulting parameter estimate $\hat{c} \geq 3$, it can be assumed that $\hat{c} = 3$. The error of approximation of the experimental distribution increases slightly.

Our procedure approximation of the experimental distributions should be used when sample size of $n \geq 1000$.

Similarly it is possible to carry out an approximation of the theoretical distributions, but instead of the sample moments in this case use the appropriate power and logarithmic points of approximating the theoretical distribution.

3.3 Bilateral generalized beta distribution and their identification

Bilateral generalized beta distribution can be obtained by functional transformation $z = \ln(x)$ unilateral generalized beta distribution of the 1st and 2nd kind. As a result, the functional transformation of the distributions (3), (4), (8), (9), (12), (15), (18), (20), (22), (24) and (25) will take correspondingly the following form:

$$p(z)=\frac{c\exp(\alpha c(z-\mu))}{B(\alpha,v)}(1-\exp(c(z-\mu)))^{v-1},-\infty<z<\mu;(33)$$

$$p(z)=\frac{c\exp(-\alpha c(z-\mu))}{B(\alpha,v)}(1-\exp(-c(z-\mu)))^{v-1} \quad (34)$$

$\mu<z<\infty$;

$$p(z)=\frac{c\exp(\alpha c(z-\mu))}{B(\alpha,v)[1+\exp(c(z-\mu))]^{\alpha+v}}, -\infty<z<\infty;(35)$$

$$p(z)=\frac{c\exp(-\alpha c(z-\mu))}{B(\alpha,v)[1+\exp(-c(z-\mu))]^{\alpha+v}}, -\infty<z<\infty;(36)$$

$$p(z)=\frac{c\exp(\alpha c(z-\mu))}{\Gamma(\alpha)}\exp[-\exp(c(z-\mu))],-\infty<z<\infty;(37)$$

$$p(z)=\frac{c\exp(-\alpha c(z-\mu))}{\Gamma(\alpha)}\exp[-\exp(-c(z-\mu))] \quad (38)$$

$-\infty<z<\infty$;

$$p(z)=\frac{1}{\sqrt{2\pi\sigma}}\exp\left[-\frac{(z-\mu)^2}{2\sigma^2}\right], \quad -\infty<z<\infty;(39)$$

$$p(z)=\frac{\lambda^{\alpha}}{\Gamma(\alpha)}(\mu-z)^{\alpha-1}\exp[-\lambda(\mu-z)],-\infty<z<\mu;(40)$$

$$p(z)=\frac{\lambda^{\alpha}}{\Gamma(\alpha)}(z-\mu)^{\alpha-1}\exp[-\lambda(z-\mu)],\mu<z<\infty;(41)$$

$$p(z)=\frac{(z+\chi-\mu)^{v-1}(\chi-z+\mu)^{v-1}}{(\chi)^{2v-1}B(0.5,v)}, -\chi+\mu<x<\chi+\mu;(42)$$

$$p(z)=\frac{\lambda^{2v}}{B(0{,}5,v)\left[\lambda^2+(z-\mu)^2\right]^{v+0{,}5}}, -\infty<z<\infty;(43)$$

where μ - shift parameter. In (35) and (36) satisfies the condition $0<\alpha\leq v$.

Bilateral generalized beta distribution of the 1st and 2nd kind (33)-(43) can be used for the approximation of the experimental distributions NE, taking negative and positive values. When defining their parameters instead of random direct and inverse power moments using direct and inverse exponential moments, and selective direct power points instead of logarithmic moments. This allows you to apply for identification of the parameters of bilateral generalized beta distribution algorithm, discussed above in Section 2. It is found that if the ratio $\widehat{L}_a<0$, then in the law of distribution are dominating direct exponential moments, and if $\widehat{L}_a>0$, the predominant circulating exponential moments. Similarly it is possible to carry out an approximation of the theoretical distributions of bilateral, but instead of sampling points used in this case the corresponding power and exponential moments approximating the theoretical distribution are used.

4. CONCLUSIONS

Thus, there were proposed generalized beta distribution to approximate the laws of unilateral and bilateral distribution of experimental data. This allows to receive wider class of distributions laws, than the existing system of Pearson distributions. Was developed a method for identifying the parameters of generalized beta distribution of the 1st and 2nd kind with the use of power, exponential and logarithmic moments. In this case it is possible in many cases to increase the accuracy of the parameter's estimates of the distributions. A topographic classification of unilateral distribution laws was developed.

The work was supported by the Ministry of Education and Science of the Russian Federation under the Agreement No 14.577.21.0214 (identifier RFMEFI57716X0214).

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