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APPLICATION OF FUZZY AND INTERVAL ANALYSIS TO THE STUDY OF THE PREDICTION AND CONTROL MODEL OF THE EPIDEMIOLOGIC SITUATION

[1]ISSIMOV NURDAULET, [2]MAZAKOV TALGAT, [3]MAMYRBAYEV ORKEN, [4]ZIYATBEKOVA GULZAT

[1,2,3,4] Institute of Information and Computational Technologies CS MES RK, Almaty, Kazakhstan

E-mail: [1]int_nurdaulet@mail.ru, [2]tmazakov@mail.ru, [3]morkeng@mail.ru, [4]ziyatbekova@mail.ru

ABSTRACT

In this article, monitoring and control issues of the social and epidemiological situation have been analyzed. Emergency situations caused by infectious diseases represent the greatest danger in the social and epidemiological sphere. Analysis of infectious morbidity involves determining the quantitative characteristics of the dynamic series, the trend of growth, reducing or stabilizing the incidence, identifying causative factors, in specific areas and for different groups. The criterion of fuzzy controllability was obtained for solving the problem of forecasting and monitoring the epidemiological situation. A new mathematical model and algorithm for solving the task of monitoring and controlling the social and epidemiological situation on the basis of its interval implementation have been described. The social effect will be expressed in increasing the safety of human life. As a consequence, it will be possible to carry out preventive measures in the necessary areas.

Keywords: *Epidemiologic Situation, Controllability, Interval Mathematics, Linguistic Variable.*

1. INTRODUCTION

Methods of predicting an epidemiological situation have been actively developing since the beginning of the XX century. Epidemiological predictions are made for different terms, and depending on those terms they serve different purposes. Basically, three types of prediction are made, they are long-term prediction for a period of several months to several years, medium-term for a period of two months to six months and short-term predictions for several weeks ahead that are used in operational control and in detection of epidemic outbreaks. In recent years, the number of works on this subject has been growing rapidly due to the deployment of information surveillance systems and the emergence of large amounts of statistics available for analysis. The most useful is the medium-term prediction for a period of two months to six months, used in tactical control. Due to various factors, it is less precise than short-term prediction, but gives enough time to prepare for possible emergencies and holding preventive measures. When making strategic decisions, one can not do without long-term predictions for the year ahead or more. In most cases high quality of such predictions is impossible, nevertheless it is required, for example, in estimating necessary volumes of medicines and vaccines production, equipping medical institutions and training personnel. Today the world has found itself in a situation where "old" and "new" infectious diseases have high potential to uncontrolled spread and, with an unprecedentedly high rate. The urbanization, the growing deterioration in the socio-ecological and sanitary and hygienic conditions of the lives of hundred millions of people in the developing and developed countries of the world, the increasing migration flows and the globalization processes of the economy contribute to the rapid spread of infectious diseases. Ironically, today the real threat comes from high biotechnology-genetic engineering and molecular biology. The fact is that modified microorganisms can become the root cause of severe epidemics, for example, as a result of their uncontrolled “exit” from scientific laboratories and industrial enterprises in industrialized countries of the world due to man-made accidents or natural disasters. Mathematical modeling of various situations is used as one of prognosis tools.

Mathematical modeling of the infectious disease risk dynamics is actively used by specialists in solving number of applied problems, such as planning various protective measures, treatment of infectious patients [1, 2, 3]. Upon that, in finding way to the final result different approaches have been implemented.

The foundation of mathematical methods' application in the study of epidemics was laid by Daniil Bernoulli in the middle of the 17th century. He first applied the simplest mathematical apparatus to assess the effectiveness of preventive vaccinations against natural smallpox. It was followed by a considerable break, which culminated in the work of the English scholar William Farr. He studied and simulated the statistical mortality rates of England (Wales) from the smallpox epidemic in 1837-1839. He for the first time obtained mathematical models of smallpox epidemic's "movement" indicators in the form of statistical regularities, which allowed him to make a prognostic model of this epidemic. At the beginning of the XX century, the statistical approach of W. Farr in the study of epidemics was redefined and then developed in the works of John Brownley, in which he analyzed the statistical patterns of epidemiological indicators' "movement" using relatively unknown methods of mathematical statistics. Due to these researchers, at the beginning of the 20th century, the foundations of the modern theory of epidemics mathematical modeling were formulated, the first prognostic models of epidemics (measles, chicken pox, malaria, etc.) were developed, their basic properties were studied, and analytical formulas for predicting epidemics were obtained. In the 1920s, the analytical approach was further developed among British scientists. Theoretical works of these scientists are widely quoted today and are used by scientists of the West in the analysis and prognosis of epidemics (outbreaks) of actual infections. With the advent of the first electronic computing machines in the mid-1950s, the next stage in the development of epidemics mathematical modeling began to take shape, and it was then that the number of scientific papers and publications on mathematical and computer modeling of epidemics began to increase rapidly. In the works of that time the more and more composite mathematical models began to appear, in which random factors of the epidemic process played an essential role, therefore most models of this period had a probabilistic character, and the working apparatus was the theory of probabilities and random processes. In the works of that time the more and more composite mathematical models began to come out, in which random factors of the epidemic process played an essential role, therefore most models of this period had a probabilistic character, and the working apparatus was the theory of probabilities and random processes. This stage in the development of mathematical modeling of epidemics was associated with an "impact" on the epidemiology of "pure" mathematicians, who managed to create many abstract models, but with a very limited epidemiological content. The next stage in the development of mathematical modeling of epidemics, which refers to the second half of the XX century, was associated with rapid progress in the field of computer technology. In the 1960s and 1970s new types of deterministic and stochastic (probabilistic) models of epidemics, targeted at studying the regularities of socially significant viral and bacterial infections' development, were developed in Western countries. However, despite the high complexity of such models and the sophistication of the mathematical apparatus, most of the models continued to have an abstract character, i.e. they were loosely associated with the formulation and solution of practical problems of epidemiology. The fact is that the leading research centers studying epidemics in the United States and in Western Europe at that time were located in universities or in medical schools at universities that were rather far from the real problems of epidemiology and its real practice. Epidemiologists perceived abstract mathematical models of epidemics and outbreaks badly and could not combine them with practical needs. Thus, in the 60s of the XX century, in the West, there was a serious gap between the "pure" theory of mathematical modeling of epidemics and the real practice of applying this theory in epidemiology.

Early studies which had planned paths of overcoming the specified gap were done in the 1960s in the USSR by an academician O.V. Baroyan and the prof. L. A. Rvachev [4-5]. They developed a new methodology for mathematical modeling of epidemics - epidemiological dynamics. The research of Russian scientists is devoted to the problem of monitoring, forecasting the spread of various types of epidemics [6-10].

In Kazakhstan, the problems of monitoring and preventing plague epidemics are dealt by the Kazakh Scientific Center of Quarantinable and Zoogenous Infections named after M. Aikimbayev and territorial plague control stations [11]. The works of Sokolova and her students are devoted to the problems of applying artificial intelligence to forecasting the epidemiological conditions [12-13 A mathematical model was developed in [14], taking into account the dynamics and interrelation of the abiotic and biotic factors characterizing the

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epidemiologic situation in the investigated focus The proposed model [14] was studied by G.Ch. Toikenov. [15], where the problem of controllability was not considered and the main result is the development of an expert system based on the processing of specialists' data.

In this article we suggest to study the controllability of the epidemiological model on the basis of fuzzy and interval analysis, which has allowed to obtain an effective controllability criterion, and also find the optimal control.

2. METHODS

At present, there are various forecasting methods, one of which is the statistical method of forecasting, denoting the observed value of the morbidity rate at time t as yt., we obtain a time series in the form of yt equidistant values sequence, which should be considered as one of the possible implementations of the random process of morbidity. For most infections, as well as for ARVI, seasonal rise is common, and close values of the indicators often repeat from year to year. Severe seasonality is taken into account in any forecast for more than a month. In the simplest case, data for a certain calendar period (week, month of the year) is considered separately. Let T be a period of seasonality, then forecast ˆyt, ˆyt+1, ...,ˆyt−1+T can be calculated for each section of the morbidity process based on the sets of known values {yt−T , yt−2T, yt−3T, ...}, ..., {,yt−1, yt−1−T , yt−1−2T , ...} respectively. So, the common way of getting estimates of the expected morbidity [A practical guide for designing..., 2008] - simple averaging: ˆyt=_nj=1 yt−jT /n, where n is the number of available observations. This approach is widely used in calculating the level of ordinary morbidity. For example, the methodology for calculating epidemic thresholds for flu and ARVI in Kazakhstan assumes averaging the data for calendar weeks of each year with the same number for the last 5-10 years. This method of predicting assumes that yearly incidence is invariable and observations during each year are the following realization of the same random process.

At present time, compartmental models have become widely spread. There are stochastic compartmental models of the epidemic spread when implementing an active detection of the diseased that allow to predict the development of the epidemic process in the population, taking into account the spatial transmission of the disease. In this connection, the community where epidemic takes place is divided into several groups (compartments) based on the values of characteristics that are important from the epidemic point of view [16-17].

3. PROBLEM FORMULATION

Figures should be labeled with "Figure" and tables with "Table" and should be numbered sequentially, for example, Figure 1, Figure 2 and so on (refer to table 1 and figure 1).

The controllability of a nonlinear system described by ordinary differential equations is studied in the work.

$$\dot{x} = f(x,u,t) \quad (1)$$

where $f(x,u,t)$– is n-vector elements of which are continuously differentiable function of its arguments, x is n-dimensional state vector of the system, u is m-dimensional vector control.

The following restrictions are given to the control

$$u(t) \in U = \{u(t): -L_i \le u_i(t) \le L_i,\ i = 1,m, t \in [t_0, t_1]\} \quad (2)$$

In [14] a mathematical model has been proposed describing the epidemiological situation in the studied region. The following abiotic factors are used in the proposed mathematical model:

w_1 – solar activity (the Wolf number),

w_2 – temperature (mean monthly temperature in the study area),

w_3 – the total rainfall for the month,

w_4 – the maximum daily *rainfall* for the month,

w_5 – the level of underground waters,

w_6 – permeability of soil;

biotic factors:

x_1– the total number of carriers of the epidemic (fleas),

x_2 – the number of infectious carriers of the epidemic,

x_3– the total number of vectors of the epidemic (gerbils),

x_4– the number of infectious vector of the epidemic.

The factors $w_i, i = \overline{1,6}$ are independent. The values of the factors $w_i, i = \overline{1,3}$ at time t are determined using time series. The values of the factors $w_i, i = \overline{4,6}$ are determined from geophysical data on the studied area. Dynamics of factors x_i $(i = 1, 2)$ at time t is described by the equations:

$$\dot{x}_1 = f_1(w)x_1 - f_2(w)x_1 - b_1u_1, \qquad (3)$$

$$\dot{x}_2 = \mu_1 x_2\left(1 - \frac{x_2}{x_1}\right) - c_1 x_2 \qquad (4)$$

where the function f_1 determines the birth rate of a population, the function f_2 determines the mortality of a population depending on abiotic environmental factors. The coefficient μ_1 defines the probability of infection of one individual in unit time.

The coefficient c_1 defines the rate of natural enhancement and mortality in patients-carriers. The fertility function f_1 is given as follows :

$$f_1(w) = \sum_{i=1}^{3} f_{1_i}(w_i) \qquad (5)$$

$$f_{1_i}(w_i) = a_i e^{-\frac{(w_i - \widehat{w}_i)^2}{\sigma_i^2}}, i = 1,3 \qquad (6)$$

where $\widehat{w}_i$ determines the most favorable value of the i–th abiotic factor for the life of the carrier, σ_i -the width of the interval centered at the point $\widehat{w}_i$ making possible the activity of the carrier.

Numerical values of the parameters $\widehat{w}_i$ and σ_i are available from the relevant directories. The coefficients a_i determine the degree of influence of the i–th abiotic factor on the fertility of the carrier.

We choose the mortality function f_2 in the following way:

$$f_2(w) = \sum_{i=1}^{3} f_{2_i}(w_i) \qquad (7)$$

$$f_{2_i}(w_i) = \beta_i\left(1 - \varepsilon\, e^{-\frac{\left(w_i - \widehat{w}_i\right)^2}{\sigma_i^2}}\right), i = 1,2 \qquad (8)$$

$$f_{2_3}(w) = \beta_3(1 - \varepsilon e^{-\frac{\left(w_3 - \widehat{w}_3\right)^2}{\sigma_3^2}})/w_4 . \qquad (9)$$

In formulas (8)-(9) ratio ε determines the natural mortality of the carrier. The coefficients β_i determine the efficiency degree of influence of the i–th abiotic factor on the mortality of the carrier.

To predict the values of factors x_i (i = 3,4) at time t the model is suggested:

$$\dot{x}_3 = f_3(w)x_3 - f_4(w)x_3 - b_2u_2, \qquad (10)$$

$$\dot{x}_4 = \frac{x_2}{x_1}x_4\left(1 - \frac{x_4}{x_3}\right) - c_2x_4,$$

where f_3 determines the birth rate, the function f_4 determines the mortality of vectors depending on abiotic environmental factors. The coefficient c_2 determines the mortality of vector patients. Fertility f_3 and mortality f_4 functions respectively similar to the functions f_1 and f_2.

In contrast to the Volterra model in (3)–(10) the controls u_1 and u_2 are given, defining anti-epidemiological actions. The coefficients b_1 and b_2 specify the influence of control on the population dynamics of carriers and vectors.

In [15] for (3-10) model: identification algorithms of parameters a_i, β_i, b_i were developed; the existence and solution of the corresponding Cauchy problem for a fixed control was proved.

As is obvious, the model (3)-(10) is fully immersed in a more general model (1)-(2).

In the classical control theory the problem of controllability [18] is usually studied (Problem 1): is there a control satisfying the constraint (2) and takes the system (1) from the initial state.

$$x(t_0) = x_0 \qquad (11)$$

to the finite given state

$$x(t_1) = x_1 . \qquad (12)$$

for the fixed time $t_1 - t_0$.

The initial values of the state vector x_0 in equation (11) can be set according to actual measurements. At the same time, for the problem of monitoring the epidemiological situation, transfer of the system in a set, allowing to provide a convenient interpretation is relevant, but not a fixed value at a finite time in the formula (12).

In this regard, based on the theory of fuzzy sets, we introduce for the state variables x of system (1) corresponding to the linguistic variables in the following way[19].

Each state variable x_i put in compliance with the linguistic variable $x_{linguistic}$, $i = \overline{1,n}$. As in model (3)-(10) system state variables have a quantitative character and higher value of them

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increases the risk of an epidemic outbreak, the following values of linguistic variables are proposed: TermLin[1]="perfect level", TermLin[2]="optimal level", TermLin[3]="comfort level", TermLin[4]="moderate level", TermLin[5]="permissible level", TermLin[6]="critical level" and TermLin[7]="catastrophic level".

A numerical interval $\left(x_{\min,i,j}, x_{\max,i,j}\right)$ meets each j-th value of the i- th linguistic variable $x_{lingi,j}$, and the set $\bigcup_{j=1}^{7}\left(x_{\min,i,j,}, x_{\max,i,j}\right)$ must cover all possible values of the variable $\left(x_{\min,i,j}, x_{\max,i,j}\right)$. In particular, it is assumed that

$$\bigcup_{j=1}^{7}\left(x_{\min,i,j,}, x_{\max,i,j}\right) = (-\infty, +\infty).$$

We introduce the set of indices $I_{kr} \subseteq [1,..,n]$, defining the list of state variables, on which are imposed terminal constraints. For example, if for the model (3)-(10) the terminal constraints are imposed only on the variable x_2– the number of infectious carriers of the epidemic, then the set of indices $I_{kr} = [2]$ consists of a single element.

Next, the following fuzzy problem of controllability is considered (Problem 2): does there exist a control satisfying the constraint (2) and takes the system (1) from the initial state(11) to the final state

$$x_{lingi}(t_1) = TermLin[i_j] \;, i \in I_{kr} \qquad (13)$$

for a fixed time $t_1 - t_0$.

In (13), the index i_j corresponds to the selected j-th fuzzy linguistic value for the i–th state variable. Problem 1 is a special case of the problem2.

If the problem of controllability (Problem 1) has a positive solution (that is, there is at least one control $u \in U$ ensuring the transfer of system (1) from state (11) to state (12)), then it is expedient to choose a control that, task would deliver a minimum to some criterion $J(u) \to \min_{u \in U}$, (this could be energy expenditure, speed, or other) (Problem 3).

Main results. By the properties imposed on the right part of the system of equations of the Cauchy problem (1), (11) for a fixed control $u(t) \in U$ the conditions of the theorem of existence and uniqueness of the solution are met $x(t), t \in [t_0, t_1]$ [20].

We rewrite the Cauchy problem (1), (11) in the integral recurrent form

$$x_{k+1}(t) = x_0 + \int_{t_0}^{t} f(x_k(\tau), u(\tau), \tau)\, d\tau . \quad (14)$$

By the properties imposed on the right side of the equation (1) and restrictions on the function $u(t)$ in [21] it is proved that the method of successive approximations (5) converges to the solution absolutely and uniformly for any fixed control.

Then the problem of controllability is reduced to studying the following problem: is there at least one control $u(t) \in U$, wherein the solution of integral equation (14) at time t_1 satisfies the condition (13).

We apply the results of interval analysis [22] to solve this problem. Let us denote the interval from $-L$ to L, as $\bar{v}$, via $\bar{f}$ the interval-valued function obtained from the function $f(x_k(t), u(t), t)$.

We get interval integral equation substituting the interval $\bar{v}$ in equation (14) instead of the function $u(t)$

$$\bar{x}_{k+1}(t) = x_0 + \int_{t_0}^{t} \bar{f}(\bar{x}_k(\tau), \bar{v}, \tau)\, d\tau . \quad (15)$$

Theorem 1. So that the studied system shall be controllable it is necessary and sufficient that the given vector, for all $i \in I_{kr}$ the intersection of the set $\left(x_{\min,i,j}, x_{\max,i,j}\right)$ with the set $\bar{x}_{k+1,i}(t_1)$ is non-empty.

The right part of the equation system (1) when defining (3) - (10) can be denoted $f(x,u,t) = g(x,t) + Bu$, B is the constant (n*m) -matrix, $g(x,t)$ is an n-vector elements of which are continuously differentiable functions in their arguments.

We rewrite system (1) in the following form

$$\frac{dx}{dt} = g(x,t) + Bu \qquad (16)$$

The state of the system at the initial instant time t_0 is considered to be known to the state of the system at the initial time t_0 (the initial state)

$$x(t_0) = x_0 . \qquad (17)$$

The desired state at the final time can be described as fixed

$$x(t_1) = x_1 \qquad (18)$$

or mobile (meeting some conditions)

$$\sum_{j=1}^{n} c_{ij} x_j(t_1) \le d_i, i = \overline{1,k} \qquad (19)$$

in this case, the time point t_1 can be given (fixed) or be based on some requirements.

Natural constraints are imposed on quantitative data

$$x_i(t) \ge 0, \quad i = \overline{1,n}, \quad t \in [t_0, t_1]. \qquad (20)$$

The following criteria can be chosen for performance evaluation of the system:

$$J = \int_{t_0}^{t_1} \left[u^*(t)R_0u(t) + (x(t) - g(t))^* R_1(x(t) - g(t))\right]dt \qquad (21)$$

or

$$J = t_1 - t_0 \qquad (22)$$

In the functional (21) R_0 is a positive-definite mxm-matrix, R_1 is a non-negative-definite nxn -matrix.

The problem of optimal control with phase constraints (20), control constraints (2) with fixed (17), (18) or variable endpoints (17), (19) is considered. At present, the solution of such problems contains a number of mathematical difficulties. In this connection, we consider a number of statements of optimal control problems.

1. The problem of optimal control with fixed right endpoint and fixed time.

The problem of minimizing the functional (21) under the constraints (16), (2), (17), (18) is considered. Time t_1 is considered to be fixed.

We formulate the Hamilton function for the problem of optimal control

$$H(x(t),u,\psi(t),\psi_0) = u^*(t)R_0u(t) + (x(t) - g(t))^* R_1(x(t) - g(t)) + \\ + (g(x,t) + Bu(t))^* \psi \qquad (23)$$

We form the conjugate system of differential equations:

$$\frac{d\psi}{dt} = -(\frac{\partial g(t)}{\partial t})^*(t)\psi(t) - 2R_1(x(t) - g(t)), \quad t \in [t_0, t_1] \qquad (24)$$

We define the optimal control from condition (2) and the maximum of the Hamiltonian:

$$u = \begin{cases} -L & if \quad R_0^{-1}B\psi < 0 \\ R_0^{-1}B\psi & if \quad 0 \le R_0^{-1}B\psi \le u_{max} \\ L & if \quad R_0^{-1}B\psi > u_{max} \end{cases} . \qquad (25)$$

Theorem 2. Let the pair $(u(t), x(t)), t \in [t_0, t_1]$ is a solution of the problem above. Then vector function $\psi(t), \quad t \in [t_0, t_1]$ must exist and parameter ψ_0 is such that

1) $\psi_0 \le 0, \ |\psi_0| + |\psi(t)| \ne 0, \quad t \in [t_0, t_1]$

2) here $x(t), \psi(t), \quad t \in [t_0, t_1]$ is the solution of the boundary value problem for the system of differential equations (16) and the corresponding conjugate system of differential equations (20) under the boundary conditions (11) and (12) and control (21).

Proof: Since all conditions of the Pontryagin maximum principle [24] are satisfied for the formulated optimal control problem, this implies correctness of the theorem.

2. The optimal control problem with a movable right endpoint.

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The problem of functional minimization (21), under the constraints (16), (2), (17), (19) is considered. Time t_1 is considered to be fixed.

Optimum control is found by formula (25).

Theorem 3. Let the pair $(u(t), x(t)),\quad t \in [t_0, t_1]$ be a solution of the problem above. Then vector function $\psi(t),\quad t \in [t_0, t_1]$ must exist and parameter ψ_0 is such that

1) $\psi_0 \le 0$, $|\psi_0| + |\psi(t)| \ne 0,\quad t \in [t_0, t_1]$

2) $\psi(t),\quad t \in [t_0, t_1]$ – solution of the adjoint system of differential equations (24), satisfying the condition that there exist numbers $\beta_1, ..., \beta_k$ such that

$$\psi_i(t_1) = \sum_{j=1}^{k} \beta_j c_{ji}, i = \overline{1, n},$$

$$\beta_i \left(\sum_{j=1}^{n} c_{ij} x_j(t_1) - d_i \right) = 0, \beta_i \ge 0, i = \overline{1, k}$$

3) for each $t \in [t_0, t_1]$ function $H(x(t), u, \psi(t), \psi_0)$ (23) with respect to a variable u reaches its upper bound on the set U for $u = u(t)$, i. e.

$$\sup_{u \in U} H(x(t), u, \psi(t), \psi_0) = H(x(t), u(t), \psi(t), \psi_0).$$

Proof: Since for the formulated optimal control problem all the conditions of the Pontryagin maximum principle are satisfied [24], this implies the validity of the theorem.

3. The problem of optimal performance problem with a fixed right end

The problem of minimizing the functional (22) under the constraints (16), (2), (17), (18) is considered. The time t_1 is not specified and is to be determined.

For the problem of optimal control, we formulate the Hamilton function

$$H(x(t), u, \psi(t), \psi_0) = 1 + (g(x,t) + Bu(t))^* \psi \quad (26)$$

We form the conjugate system of differential equations:

$$\frac{d\psi}{dt} = -(\frac{\partial g(t)}{\partial t})^*(t)\psi(t),\quad t \in [t_0, t_1] \quad (27)$$

Theorem 5. Let the pair $(u(t), x(t)),\quad t \in [t_0, t_1]$ be a solution of the problem posed above. Then a vector-valued function $\psi(t),\quad t \in [t_0, t_1]$ must exist and a parameter ψ_0 such that

1) $\psi_0 \le 0$, $|\psi_0| + |\psi(t)| \ne 0,\quad t \in [t_0, t_1]$

2) $\psi(t),\quad t \in [t_0, t_1]$- solution of the adjoint system of differential equations (26), which together with the system (16) satisfies the boundary conditions (17) and (18)

3) for each $t \in [t_0, t_1]$ function $H(x(t), u, \psi(t), \psi_0)$ by variable u reaches its upper bound on the set U when $u = u(t)$, i. e.

$$\sup_{u \in U} H(x(t), u, \psi(t), \psi_0) = H(x(t), u(t), \psi(t), \psi_0)$$

Proof: Since for the formulated optimal control problem all the conditions of the Pontryagin maximum principle are satisfied [24], this implies the validity of the theorem.

4. Numerical algorithm for solving the optimal control problem with fixed ends and phase constraints.

The problem of optimal control with phase constraints (20), with fixed ends (17) - (18) and constraints on control (2) is considered. At present, the solution of such problems contains a number of mathematical difficulties.

In this connection, for the practical solution of the problem of optimal control, the penalty function method and the gradient method are used.

To take into account the phase constraints (20) and the restrictions on the end of the trajectory (18), we introduce the penalty functions

$$F_{1k} = M_{k1} \sum_{i=1}^{n} \int_{t_0}^{t_1} [\max\{-x_i(t); 0\}]^2 dt \quad \text{and}$$

$$F_{2k} = M_{k2} \sum_{i=1}^{n} [x(t_1) - x_1)]^2 \quad ,$$

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where $\{M_{k1}\},\{M_{k2}\}$ -some given positive sequences tend to infinity.

We form a new functional

$$J_k = \int_{t_0}^{t_1} \{u^*(t)R_0u(t) + (x(t)-g(t))^* R_1(x(t)-g(t)) +$$
$$+ M_{k1}[\max\{-x_i(t);0\}]^2\}dt + M_{k2}\sum_{i=1}^{n}[x(t_1)-x_1)]^2$$

We replace the original problem by the following: find the optimal control minimizing the functional J_k under constraints (16), (2), and (17) for the given k. The problem obtained is a problem of optimal control with a free right end and a control constraint. We compose for it the Hamilton function:

$$H_k = u^*(t)R_0u(t) + (x(t)-g(t))^* R_1(x(t)-g(t)) +$$
$$+ M_{k1}[\max\{-x_i(t);0\}]^2 + (g(x,t)+Bu(t))^*\psi_k$$

The following algorithm is proposed.

Step 1. Let $k = 0$.

Step 2. The optimal control for the *k*-th iteration is calculated

$$u_k = \begin{cases} -L & if \quad R_0^{-1}B\psi_k < 0 \\ R_0^{-1}B\psi_k & if \quad 0 \le R_0^{-1}B\psi_k \le u_{\max} \\ L & if \quad R_0^{-1}B\psi_k > u_{\max} \end{cases} \qquad (28)$$

where ψ_k – solution of the adjoint system of differential equations

$$\frac{d\psi_k}{dt} = -(\frac{\partial g}{\partial x})^*\psi_k - 2R_1(x_k(t)-g(t)) + M_{k1}[\max\{-x_{ki}(t);0\}]$$

with the condition at the end

$$\psi_k(t_1) = 2M_{k2}\sum_{i=1}^{n}[x_k(t_1)-x_1)]. \qquad (29)$$

and x_k – solution of the original system (16) under the initial conditions (11).

Step 3. With the found x_k and u_k value of the functional J_k is calculated.

Step 4. If $|J_k - J_{k-1}| \le \varepsilon$ that is the transition to step 5, otherwise $k = k+1$ and the transition to step 2. (Here $\varepsilon > 0$ - the required accuracy of the calculation).

Step 5. The pair (x_k, u_k) found is the optimal solution.

The model problem was considered in the following assumption: the values of all abiotic factors correspond to their optimal values. Thus, the function $(f_i(w), i = 1..4)$ are constant.

The program of numerical calculations for the model problem has been developed on the basis of procedure library of interval computations [23].

The calculations for equations (3),(4),(10),(11)were performed with the following numerical parameter values: alf1=1; alf2=1; bet1=0.5; bet2=0.5; bk1=5; bk2:=1; ck1=0.3; ck2=0.3; mk1=0.2. The integration step was taken equal to 0.05. The point with coordinates (80, 10, 30, 5) is defined as a starting point. Study time is chosen equal to T=2.5 conventional units.

The program developed displays the results of numerical calculations in a table of model parameters' change to a text file and graph of the function values change. In this connection the graph can be saved as an image file or sent to the printer. Table 1 shows a fragment of the text file where the values of the model parameters are presented in the interval form.

Table 1. A fragment of the text file.

N	t	x1	x2	x3	x4
k= 1	t=0,05	= (77,00; 82,00)	= (9,94; 9,94)	= (30,25; 30,75)	= (4,95; 4,95)
k= 2	t=0,10	= (75,45; 82,53)	= (9,87; 9,88)	= (30,66; 31,37)	= (4,90; 4,90)
k= 3	t=0,15	= (74,13; 82,80)	= (9,81; 9,81)	= (31,10; 31,97)	= (4,85; 4,86)
k= 4	t=0,20	= (72,91; 82,94)	= (9,75; 9,75)	= (31,58; 32,58)	= (4,81; 4,81)
k= 5	t=0,25	= (71,76; 82,98)	= (9,69; 9,69)	= (32,07; 33,19)	= (4,76; 4,76)
k= 6	t=0,30	= (70,66; 82,95)	= (9,63; 9,63)	= (32,58; 33,81)	= (4,71; 4,72)
k= 7	t=0,35	= (69,58; 82,87)	= (9,57; 9,57)	= (33,11; 34,44)	= (4,67; 4,67)
k= 8	t=0,40	= (68,52; 82,74)	= (9,51; 9,51)	= (33,66; 35,08)	= (4,62; 4,63)
k= 9	t=0,45	= (67,48; 82,57)	= (9,45; 9,45)	= (34,22; 35,73)	= (4,58; 4,59)
k= 10	t=0,50	= (66,44; 82,36)	= (9,39; 9,39)	= (34,81; 36,40)	= (4,53; 4,54)
……					
k= 49	t=2,45	= (14,76; 51,11)	= (7,26; 7,31)	= (75,25; 78,88)	= (3,26; 3,36)
k= 50	t=2,50	= (12,88; 49,63)	= (7,21; 7,26)	= (76,90; 80,58)	= (3,24; 3,35)

Figure 1 presents a graph of the change of interval variable x1 for the following constraints on control: 0<=u1<=20; 0<=u2<=10. As can be seen from the graph the initial value for the variable x1 corresponds to a catastrophic level. With the resources available on the control for the time T the system on variable x1 can be brought into the state from "moderate" to "perfect."

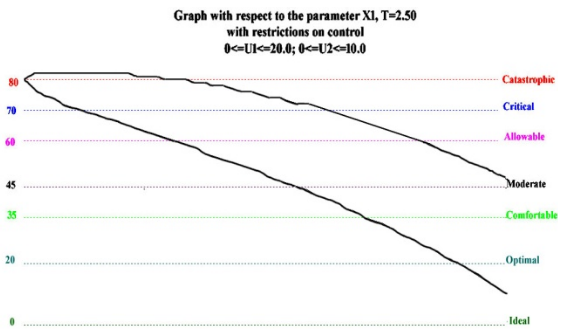

Figure 1: Ttransition schedule

Figure 2 shows a graph of the x1 interval variable change for the following constraints on control: 5<=u1<=15; 0<=u2<=10. Under given resources on control over time T the system on variable x1 can be brought into the state from "comfortable" to "optimal".

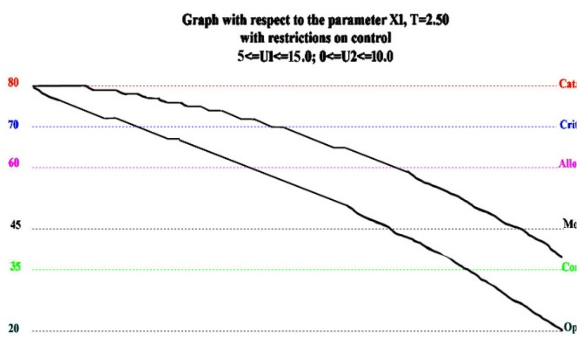

Figure 2: Shows a graph of the change in the variable of the interval x1.

Figure 3 shows a graph of the interval variable change x1 under the following constraints on the control: 7<=u1<=11; 0<=u2<=10. Under given resources to control over time T the system with variable x1 it can result in "moderate" state.

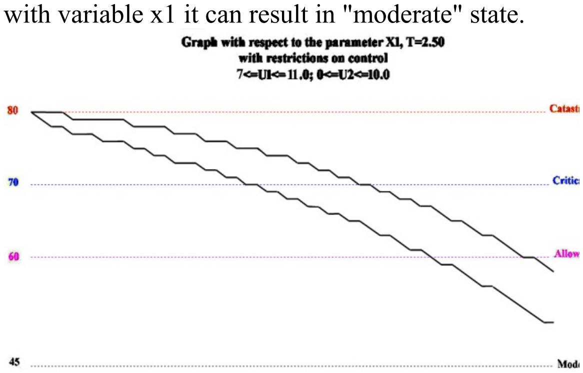

Figure 3: Shows a graph of the interval variable change x1 under the following constraints on the control: 7<=u1<=11; 0<=u2<=10.

Figure 4 shows the graphs of the change in the variable x1 under the following control constraints: 7 <= u1 <= 11; 0 <= u2 <= 10 and a double increase in the penalty coefficient M_{k2} for the next iterations in the functional

$$J_k = \int_{t_0}^{t_1} \{u_1^2(t) + u_2^2(t)\}dt + M_{k2}\sum_{i=1}^{n}[x(t_1) - x_1)]^2$$

With given resources for control over the time T, the system with variable x1 can be brought into a "moderate" state.

Table 2 shows a fragment of the table data for the change in the parameter x1 with an increase in the penalty coefficient M. As can be seen from the table, even at M = 8, the computational process has stabilized and further increase in the penalty does not lead to improved results.

Table 2. Dynamics of trajectory change x1

N	t	M=1	M=2	M=4	M=8	M=16	M=32	M=64
k= 1	t=0,05	= 79,75	= 79,25	= 79,25	= 79,25	= 79,25	= 79,25	= 79,25
k= 2	t=0,10	= 79,49	= 78,48	= 78,48	= 78,48	= 78,48	= 78,48	= 78,48
k= 3	t=0,15	= 79,23	= 77,69	= 77,69	= 77,69	= 77,69	= 77,69	= 77,69
k=4	t=0,20	= 78,96	= 76,89	= 76,89	= 76,89	= 76,89	= 76,89	= 76,89
k=5	t=0,25	= 78,69	= 76,06	= 76,06	= 76,06	= 76,06	= 76,06	= 76,06
k=6	t=0,30	= 78,40	= 75,21	= 75,21	= 75,21	= 75,21	= 75,21	= 75,21
k=7	t=0,35	= 78,11	= 74,34	= 74,34	= 74,34	= 74,34	= 74,34	= 74,34
k=8	t=0,40	= 77,82	= 73,45	= 73,45	= 73,45	= 73,45	= 73,45	= 73,45
k=9	t=0,45	= 77,51	= 72,53	= 72,53	= 72,53	= 72,53	= 72,53	= 72,53
k=10	t=0,50	= 77,20	= 71,60	= 71,60	= 71,60	= 71,60	= 71,60	= 71,60
k=11	t=0,55	= 76,88	= 70,64	= 70,64	= 70,64	= 70,64	= 70,64	= 70,64
k=12	t=0,60	= 76,55	= 69,65	= 69,65	= 69,65	= 69,65	= 69,65	= 69,65
k=13	t=0,65	= 76,21	= 68,64	= 68,64	= 68,64	= 68,64	= 68,64	= 68,64
k=14	t=0,70	= 75,87	= 67,61	= 67,61	= 67,61	= 67,61	= 67,61	= 67,61
k=15	t=0,75	= 75,52	= 66,55	= 66,55	= 66,55	= 66,55	= 66,55	= 66,55
……………………………..								
k=46	t=2,30	= 58,86	= 38,07	= 16,58	= 16,58	= 16,58	= 16,58	= 16,58
k=47	t=2,35	= 58,08	= 37,27	= 14,25	= 14,25	= 14,25	= 14,25	= 14,25
k=48	t=2,40	= 57,29	= 36,45	= 11,86	= 11,86	= 11,86	= 11,86	= 11,86
k=49	t=2,45	= 56,47	= 35,61	= 9,40	= 9,40	= 9,40	= 9,40	= 9,40
k=50	t=2,50	= 55,63	= 34,75	= 6,89	= 6,89	= 6,89	= 6,89	= 6,89

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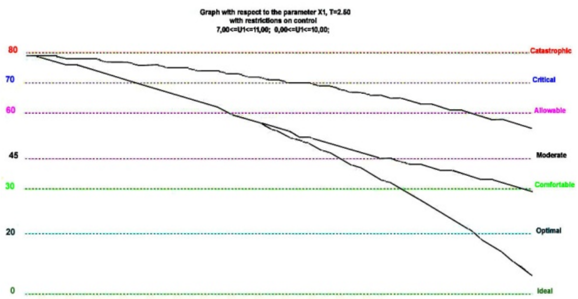

Figure 4: Ttransition schedule

The results of numerical simulation in the graphs (fig.1-3) are consistent with the actually expected data.

The simplicity of software development, algorithmizability the procedure of checking the conditions of the theorem, shows the effectiveness of its application.

4. CONCLUSION

In the article the dynamic model with a restriction on the right end on the basis of linguistic variables has been considered for the first time in theory of controllability.

A criterion of fuzzy control has been obtained for forecasting and controlling of epidemiological situation based on interval mathematics.

On the basis of the method of penalty functions and the gradient method, the problem of optimal control with limited controls and fixed ends.

The effectiveness of the obtained criterion was shown on the basis of solution of the model problem.

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