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MONITORING MEAN SHIFT OF SKEWED DISTRIBUTION USING MODIFIED ONE-STEP *M*-ESTIMATOR WITH EWMA CONTROL STRUCTURE

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ABSTRACT

A generalized form of Shewhart chart, known as Exponentially Weighted Moving Average (EWMA) control chart is frequently exercised to monitor small shift in the process mean. Aptly tune, it is claimed to be robust to slight deviation in normality. For that to be successful, the weighting constant ($\boldsymbol{\lambda}$) shall be set quite small. However, too small of the value may reduce the effectiveness of the chart in shift detection, a phenomenon known as the inertia effect. Thus, meticulous approach ought to be exerted to tune the traditional EWMA chart under non-normality. Recurrent use of robust control charts is now seen in quality control literature as one of the few solutions to cope with non-normality. In line with this, a novel EWMA control chart was proposed in this paper. The proposed chart was constructed using a highly robust breakdown point location estimator, known as modified one-step *M*-estimator (*MOM*). Monte Carlo simulation approach was used to model and evaluate performance of the proposed chart when process data was subjected to non-normality using skewed distributions. Two separate cases were considered: (i) when both mean and standard deviation of the process were known and (ii) when the mean was unknown and estimated from an in-control Phase I sample. While demonstrating a mediocre power to detect shift in the first case, an outcome on simultaneous effect of parameter estimation and non-normality for the proposed chart indicated a reversal. Besides equipped to regulate false alarm rate following an increase in the level of skewness of the distribution, the proposed chart also possessed the best-shift detecting ability in extreme non-normal cases as observed in this paper. This was demonstrated using average run length (*ARL*) when the underlying distribution of Phase I and Phase II data were matched.

Keywords:- *Average Run Length (ARL), EWMA Control Chart, Skewed Distribution, MOM, Robust Process Location.*

1. INTRODUCTION

Parametric control charts are generally designed based on an assumption of a specific underlying process distribution, such as normality. Nonetheless, parametric EWMA control chart is not quite restricted by this principal [1]. Aptly tuned (designed), EWMA control chart could be robust to non-normality, in a sense that the in-control performance of the chart remains stable even when the underlying distributional assumption is not met [2]. Whilst other researchers may have claimed otherwise on a broader scope of analysis (see, e.g. [3] and [4]), the robustness of EWMA control chart to non-normality is allegedly true when process data follow either gamma distribution or student-*t* distribution [2]. For this to take effect, however, value of the weighting constant λ ought to be set quite small, as low as 0.05. As the value increases, EWMA control chart may no longer retain the advantage. Yet, EWMA control chart designed with a small value of λ is vulnerable to the inertia effect; potentially delaying EWMA reaction to a shift. This may happen when EWMA plotting statistic is

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at odds with the direction of the shift [5].

The ongoing discussion on some conceivable ways to maintain EWMA robustness without much loss in its shift detection properties called forth the use of robust techniques. In many areas of statistical approach, robust estimation is particularly useful when some degree of deviation in the distributional assumption is observed. This is due to the good properties possessed by robust statistics under this condition. A good case in point is their high resistance to outlying values, which is defined by the breakdown point (*BP*).

To date, numerous works on robust control charts have been proposed; aiming to restore charts' ability to some degree when they are used for monitoring mean of non-normal processes. Langenberg and Iglewicz [6] and Rocke [7] proposed using trimmed mean on Shewhart control limits. Abu-Shawiesh and Abdullah [8] recommended plotting Hodges-Lehmann (*HL*) instead of $\bar{X}$ and had shown that it is far superior than the traditional Shewhart chart under mixed normal and skewed distributions. Another dominant robust location estimator that has been applied in many works related to quality control is the sample median. Highly robust, the median estimator has been used almost exhaustively to amplify performance of the traditional control charts when data is heavily skewed or contaminated. The median chart based on EWMA control structure was first introduced by Castagliola [9]. On Shewhart structure, the effect was investigated by Khoo [10]. Following that, several papers have been published, concentrating on a duo merit provided by the median chart; i.e. its simplicity and its robustness against deviation from normality [11], [12].

The charting structure of a usual sample median, however, has some drawback. Because median control charts are more outlier-resistant than the mean charts, in general, they yield less efficiency than the latter [12], [13], [14]. The current study adds to the existing literature in the form of following contribution. We have suggested a highly robust location estimator known as modified one-step *M*-estimator (*MOM*) to be applied on EWMA charting structure. The aim is to provide a robust structure of control charting under severe non-normality without much loss in responsiveness to actual mean shift. If achieved, the proposed chart shall be a good substitute to a median chart when small mean shifts are of interest under limitation to fulfill the normality assumption. It will be shown that when sampling from a normal or moderately skewed distribution, the *MOM* estimator has a smaller standard error than the sample median. This in turn, shall help to maintain sensitivity of the EWMA control chart in ideal circumstances.

When testing hypotheses, method based on a 20% trimmed mean is recommended as a compromise between the mean and median if the goal is to maintain good power under normality, and yet, still achieve high power under non-normality [15]. Close relationship between Phase II control charting and hypothesis testing has been debated for a while. See Woodall [16] for a review. Thus, an attempt to monitor Phase II process through a control chart based on MOM is an expedient strategy since the MOM estimator, which is also based on a trimming approach was suggested by Wilcox and Keselmen [17] to overcome major drawbacks of any trimmed means.

To assess the impact of MOM on EMWA control charting when the chart is used to monitor Phase II processes, simulation procedure is conducted to evaluate the average run length (ARL) of various process mean shifts under normal and skewed distributions. The performance of the proposed chart is then compared to the three existing EWMA control charts. These existing charts are constructed based on mean, mid-range and median estimators.

The proposed method in this article concentrates on two separate cases: (i) when the in-control mean and standard deviation of the process are known and (ii) when the in-control mean is unknown and estimated from Phase I sample. It is noted that the effect of parameter estimation on EWMA chart based on mid-range and median estimators had been investigated by Nazir et al. [11] in a recently published paper. Yet, rather than limiting Phase I samples to normal data as adhered by the previous study, we let the underlying distribution in Phase I varies according to the distribution of subsequently monitored Phase II samples. Whilst this approach allows us to monitor location shifts in the out-of-control situation, it also enables us to keep tab on the robustness of all charts in the presence of no shift.

The outline of the paper is structured as follows. In Section 2, the structure of EWMA control chart is presented. Explanation of the appointed location estimators may be referred in Section 3. The set-up for the simulation procedure for Case I, together with the results are delineated in Section 4. Following that, Section 5 is dedicated for Case II study. Illustrative examples are provided in Section 6. The final section, Section 7, summarizes the conclusion for this study.

2. DESCRIPTION OF EWMA CONTROL STRUCTURE

Roberts [18] proposed a generalized form of Shewhart chart, in which the plotting statistic is now composed of two components; i.e. the current information and the past information. The structure is known as the EWMA chart and it is classed under the memory-type chart. The plotting statistic and the control limits for the chart are given as:

$$E_i = \lambda\hat{\theta}_i + (1-\lambda)E_{i-1}\,; \quad \text{for } i = 1, 2, \dots, m \tag{1}$$

$$UCL_{EWMA} = \theta_0 + K\sigma_{\hat{\theta}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \tag{2a}$$

$$LCL_{EWMA} = \theta_0 - K\sigma_{\hat{\theta}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \tag{2b}$$

where i defines the subgroup number, $\lambda \in (0,1)$ is the weighting factor, and together with K, they determine the in-control performance of the chart. Here, K is a positive coefficient. The location estimator, $\hat{\theta}$, is used to monitor a shift from the assumed in-control process parameter (θ_0) and $\sigma_{\hat{\theta}}$ is the standard deviation of $\hat{\theta}$. The initial value of the plotting statistic in (1) is usually taken as $E_0 = \theta_0$. Under this setting: $\hat{\theta} = \bar{X}$ and $(\lambda, K) = (1, 3)$, the control structure is reduced to the usual Shewhart control chart with 3σ limit.

When θ_0 is unknown and requires to be estimated from Phase I samples, then, the EWMA charting statistic is given by:

$$E_i = \lambda\hat{\theta}_i + (1-\lambda)E_{i-1}\,; \text{ for } E_0 = \hat{\theta}_0 \tag{3}$$

The estimated control limits for Phase II applications are now defined as in equation (4a) and (4b).

$$\widehat{UCL}_{EWMA} = \hat{\theta}_0 + K\sigma_{\hat{\theta}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \tag{4a}$$

$$\widehat{LCL}_{EWMA} = \hat{\theta}_0 - K\sigma_{\hat{\theta}}\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \tag{4b}$$

It cannot be ignored that an expression for the asymptotic standard error for $\hat{\theta} = MOM$ is not available. Thus, the control limits for the proposed chart is developed empirically through two series of Monte Carlo simulations which will be explained later in Section 4. For fair comparison, control limits for the existing charts are developed via the same way. For a review on the asymptotic standard error for mean, median and mid-range, the reader is referred to Nazir et al. [11].

3. LOCATION ESTIMATORS

Suppose the above-mentioned EWMA chart is used to monitor a population location parameter signified by θ. Thus, $\hat{\theta}$ denotes an estimator that could be attained from a subgroup of size *n*. In this article, $\hat{\theta}$ may come from one of these four location estimators; modified one-step *M*-estimator (*MOM*), median, mid-range or mean. Explanation of each estimator is presented as follows.

3.1 Modified One-Step M-estimator (MOM)

Proposed by Wilcox and Keselman [17], this median based estimator is defined as:

$$\hat{\theta} = \frac{\sum_{i=i_1+1}^{n-i_2} X_{(i)}}{n-i_1-i_2} \tag{5}$$

where

$X_{(i)}$ = the i^{th} ordered observation
i_1 = number of observations X_i such that $(X_i - \text{Median}) < -2.24(MAD_n)$
i_2 = number of observations X_i such that $(X_i - \text{Median}) > 2.24(MAD_n)$ (6)

MAD_n is set to $MAD/0.6745$, where $1/0.6745$ is the normalizing constant for the parameter of interest, σ. The constant in the outlier detection criteria is fixed at 2.24 prompted by the desire to achieve greater efficiency under normality when subjected to small sample size [16]. This estimator achieves the highest possible *BP* at 50% ensuing the use of MAD_n in the rules [17].

Unlike the classical trimmed mean, *MOM* engages asymmetric trimming. Via this approach, amount of trimming is not fixed prior to the procedure. Rather, the trimming is done accordingly based on the shape of the data distribution. On a skewed data, more trimming will be carried out on the tails, whereas for symmetric heavy-tailed distribution, the trimming will be conducted equally on both sides, and no trimming is needed on a normal distribution.

3.2 Median

The estimate provided by median separates the lower half of the data to its upper half. The usual sample median $\tilde{X}$ is computed as:

$$\hat{\theta} = \begin{cases} \frac{1}{2}\left[X_{\left(\frac{n}{2}-1\right)} + X_{\left(\frac{n}{2}+1\right)}\right], & if\ n\ is\ even \\ X_{\left(\frac{n}{2}+\frac{1}{2}\right)}, & if\ n\ is\ odd \end{cases} \tag{7}$$

As robust as *MOM*, efficiency of $\tilde{X}$ also rivals the sample mean when tails of the distribution

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becomes heavier, irrespective of the sample size [19]. On normal data, the efficiency is set at 64% [20]. However, a finding from simulation study by Özdemir [21] reveals that the squared standard error of $\tilde{X}$ is relatively greater than *MOM* on normal data as well as contaminated data.

3.3 Mid-Range

This estimator is characterized by the minimum and maximum order of statistics of a random sample with size *n*. The mid-range (*MR*) of a sample is the average between the lowest and the highest order of statistics:

$$\hat{\theta} = \frac{X_{(1)}+X_{(n)}}{2}; \quad (8)$$

where $X_{(1)}$ and $X_{(n)}$ are the respective order of the statistics. Owing to this arrangement, as little as one outlying value would render the estimator useless. Highly influenced by outliers, the mid-range estimator has 0% *BP*. Yet, its pronounced efficiency in the case of platykurtic distribution commended the estimator as an alternative to the sample mean.

3.4 Mean

For normally distributed data, it is regarded as the most efficient estimator. Yet, sample mean $\bar{X}$ is very receptive to outlying values, with 0% *BP* and its standard error is extremely sensitive to the changes in the tails of the distribution [20]. The estimator is defined as a linear function of data and the formula is given as follows:

$$\hat{\theta} = \frac{\sum_i^n x_i}{n} \quad (9)$$

Based on these estimators; *MOM*, median, mid-range and mean, there are four EWMA control charts to be designed, evaluated and compared using the ARL. Accordingly, these charts are identified as EWMA-*MOM*, EWMA-$\tilde{X}$, EWMA-*MR*, and EWMA-$\bar{X}$ throughout this article. The appraisal is first conducted for Case I, i.e. when the process parameters are known. Later, the analysis is extended to cover performance of the charts when the in-control mean of the process is estimated in Phase I (Case II).

4. CASE I (KNOWN PARAMETERS): PERFORMANCE EVALUATION AND SIMULATION PROCEDURE

Performance of the charts are based on the average run length (ARL), which is defined as the average number of points plotted on the chart before going off the limits. This criterion is used to assess in-control and out-of-control state of the process, where ARL_0 denotes the in-control ARL and ARL_1 refers to the out-of-control ARL. A high value of ARL_0, accompanied by low values of ARL_1 is always desirable; as they signify a good control chart.

When several charts are being evaluated, common approach is to fix ARL_0 for a desired shift size δ at the ideal condition. Then, ARL_1 of the charts are compared. Chart with smaller ARL_1 outweighs its competitors. Value of λ, however, shall be selected carefully as it may influence the calculation of ARL, which further determined the robustness and sensitivity of the EWMA chart under the non-ideal condition. The relationship between λ and the usual EWMA chart performance was expressed succinctly in the beginning of this paper. Considering its impact toward EWMA chart performance, a moderate value of $\lambda = 0.13$ is selected in this study. This value was recommended by Crowder [22] when a shift size 1.0 is of interest. Accordingly, *K* is taken as 2.88 based on Lee, Khoo and Yap [23]. For EWMA-$\bar{X}$ chart, the designated parameters give $ARL_0 \cong 500$, for certain when process is ideal, i.e. normally distributed. We note that the pre-specified ARL_0 value of 500 is a widely used choice in literatures.

The ARL are estimated by means of simulation using SAS Version 9.4 software. The procedures are taken as follows. First, we generate 1,000,000 samples of size $n \in \{5,10\}$ and simulate the mean (θ_0) and the standard deviation ($\sigma_{\hat{\theta}}$) of the chosen distribution (as described later) for the estimators. We further generate 15,000 Phase II samples of size n from the selected distributions under in-control state and apply them on the charts. Note that, in this study, we assume data are independent and identically distributed. The chart statistic, E_i is computed and noted when it falls outside the limit ($E_i > UCL_{EWMA}$ or $E_i < LCL_{EWMA}$). When this occurs, a signal is triggered and the corresponding in-control run length, i.e. time *i*, is recorded. The process is repeated for 10,000 simulation runs. With that, ARL_0 is attained by averaging the value. Ultimately, a shift in the form of δ is introduced in the process, in which δ is set for {0.1, 0.15, 0.2, 0.25, 0.5, 0.75, 1, 1.5, 2.0, 2.5, 3.0}. Analogous to ARL_0, identical steps are taken to procure ARL_1 with respect to the δ.

4.1 Simulation Outcomes

This section discusses the performance of the EWMA chart for non-normal data using skewed

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distributions, when mean and standard deviation of the process are known. Specifically, Weibull distribution with various shape β parameters is considered. The designated βs correspond to a specific skewness coefficient of $\alpha_3 \in \{0.5, 1, 2, 3\}$. Meanwhile, scale parameter of the Weibull is fixed at 1. To get more insight on in-control performance of the EWMA chart under asymmetric distribution, two additional distributions are considered. They are lognormal and Gamma distributions.

Table 1 and 2 give in-control performance of the four charts for the skewed processes. For fair comparison, ARL_0 under the ideal condition (i.e. normally distributed data) is also presented. The tabulated values show that the EWMA-$\tilde{X}$ chart performance is almost identical to the EWMA-$\bar{X}$ chart. The distribution of the sample median $\tilde{X}$ is very close with the $N(\theta_0, \tilde{\sigma})$ where $\tilde{\sigma}$ is the standard deviation of the $\tilde{X}_i$ [24]. As such, the designated optimal parameters for the EWMA-$\bar{X}$ chart would yield a comparable simulation result for the EWMA-$\tilde{X}$ when process is ideal.

In general, Table 1 and 2 shows that ARL_0 for all charts digress from the nominal value as degree of skewness increases. Performance of the EWMA-*MOM* chart is akin to the EWMA-$\tilde{X}$ chart when α_3 is quite small but outweighs the latter as data becomes heavily skewed. Meanwhile, performance of the EWMA-*MR* chart is visibly worse than its counterpart as level of skewness rises. Increasing *n* from 5 to 10, however, helps to boost the performance of all charts.

While changes in in-control performance of the EWMA-$\bar{X}$ chart is consistent throughout Weibull, lognormal and Gamma distributions for the same level of skewness and sample sizes, a more distinct pattern is exhibited by the proposed method (*MOM*). The EWMA-*MOM* chart is highly robust when underlying distribution of the data assumes lognormal. Moreover, the chart provides slight advantage over the median chart when α_3 is set between 1.0 to 2. This is evident in each case of the distribution when we consider both sample sizes.

Table 1: In-control ARL for n = 5 for the known parameter case

	Distributions	α_3	EWMA-*MOM*	EWMA-$\bar{X}$	EWMA-*MR*	EWMA-$\tilde{X}$
	Normal	**0.0**	484.75	504.92	488.22	507.13
	Weibull					
	2.2266	**0.5**	487.41	512.62	501.67	518.30
β	**1.5688**	**1.0**	473.99	487.03	455.45	463.86
	1.2123	**1.5**	440.98	454.45	406.77	311.41
	0.9987	**2**	385.80	415.49	349.27	361.55
	0.7637	**3**	298.21	337.40	273.70	284.59
	Lognormal					
	0.1656	**0.5**	483.00	519.97	492.10	445.42
ω	**0.3170**	**1.0**	467.63	477.02	422.60	478.66
	0.4484	**1.5**	441.37	436.30	353.18	426.45
	0.5593	**2**	415.07	397.03	313.40	396.48
	0.7315	**3**	358.23	332.32	256.29	343.64
	Gamma					
	15.4	**0.5**	471.38	499.05	472.92	488.66
α	**3.913**	**1.0**	466.59	481.89	447.05	463.37
	1.788	**1.5**	445.84	454.06	397.18	410.97
	0.983	**2**	385.27	415.42	355.15	354.21
	0.442	**3**	265.06	347.23	298.19	256.68

Table 2: In-control ARL for n = 10 for the known parameter case

	Distribution	α_3	EWMA-*MOM*	EWMA-$\bar{X}$	EWMA-*MR*	EWMA-$\tilde{X}$
	Normal	**0.0**	488.85	508.78	487.43	508.56
	Weibull					
	2.2266	**0.5**	494.17	503.42	485.32	508.10
β	**1.5688**	**1.0**	504.08	490.60	445.57	482.82
	1.2123	**1.5**	496.44	475.35	396.50	451.32
	0.9987	**2**	452.25	452.30	352.99	408.61
	0.7637	**3**	364.37	399.36	289.06	347.46
	Lognormal					
	0.1656	**0.5**	468.49	472.33	468.31	512.02
ω	**0.3170**	**1.0**	488.85	482.21	399.91	469.92
	0.4484	**1.5**	480.06	475.23	341.83	467.30
	0.5593	**2**	454.00	439.76	298.80	444.19
	0.7315	**3**	421.72	390.41	257.57	405.95
	Gamma					
	15.4	**0.5**	487.31	502.95	459.95	499.02
α	**3.913**	**1.0**	492.89	500.26	427.31	479.07
	1.788	**1.5**	490.06	481.62	376.48	461.86
	0.983	**2**	441.66	450.75	354.16	417.34
	0.442	**3**	316.65	400.68	312.12	315.37

Table 3 and 4 provide the out-of-control performance of the four EWMA charts when underlying distribution of the data is set for Weibull. The results show that ARL_1 of all four charts decreases as level of skewness increases.

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Note also that ARL_1 decreases as n increases. In general, the use of different estimators dictates EWMA ability to perform on small range of δ and bears little to no importance when $\delta > 1$. The EWMA-*MR* and EWMA-$\bar{X}$ chart tied up when process is not heavily skewed. Both charts possess smaller ARL_1 than the robust methods (*MOM* and $\tilde{X}$), except on a few occasions when the robust methods overtake the duo. The EWMA-$\bar{X}$ chart retains its advantage in detecting small shifts as level of skewness increases, but the EWMA-*MOM* chart is doing a fairly good job in catching up behind it.

Table 3: Out-of-control ARL for n = 5 for the known parameter case

β	α_3	δ	EWMA -*MOM*	EWMA -$\bar{X}$	EWMA -*MR*	EWMA -$\tilde{X}$
2.2266	**0.5**	**0.1**	139.24	134.89	**129.21**	134.00
		0.15	69.77	69.08	**68.40**	68.68
		0.2	41.73	41.25	**41.08**	41.35
		0.25	27.69	**27.14**	27.55	27.83
		0.5	8.72	8.75	8.71	8.70
		0.75	5.06	5.06	5.08	5.06
		1	3.64	3.62	3.62	3.63
		1.5	2.38	2.40	2.40	2.39
		2	1.94	1.93	1.93	1.93
		2.5	1.61	1.61	1.62	1.62
		3	1.19	1.20	1.20	1.20
1.5688	**1.0**	**0.1**	125.80	127.43	**120.41**	120.20
		0.15	67.91	67.19	**66.22**	66.67
		0.2	40.65	41.47	**40.47**	39.95
		0.25	**27.35**	27.37	27.80	27.47
		0.5	8.78	8.81	8.90	**8.74**
		0.75	**5.07**	5.08	5.11	5.12
		1	3.65	3.63	3.62	3.64
		1.5	2.39	2.39	2.42	2.40
		2	1.92	1.93	1.92	1.91
		2.5	1.61	1.63	1.64	1.64
		3	1.21	1.20	1.20	1.21
1.2123	**1.5**	**0.1**	119.82	119.03	110.37	**86.22**
		0.15	66.07	65.78	63.78	**51.41**
		0.2	40.34	40.19	40.57	**34.22**
		0.25	27.55	27.80	27.42	**24.07**
		0.5	8.84	8.87	8.89	**8.40**
		0.75	5.13	5.13	5.11	**4.96**
		1	3.65	3.63	3.64	**3.55**
		1.5	2.39	2.39	2.38	2.37
		2	1.91	1.92	1.91	1.91
		2.5	1.65	1.64	1.65	1.63
		3	1.21	1.20	1.19	1.18
0.9987	**2**	**0.1**	109.39	112.34	**104.49**	105.70
		0.15	63.91	64.72	**62.59**	63.62
		0.2	**40.23**	40.59	40.49	41.07
		0.25	28.03	**27.55**	27.86	27.92
		0.5	8.97	**8.89**	9.10	9.08
		0.75	5.18	**5.14**	5.17	5.19
		1	3.64	3.65	3.63	3.65
		1.5	2.40	2.39	2.41	2.40
		2	1.90	1.91	1.91	1.90
		2.5	1.66	1.65	1.67	1.67
		3	1.20	1.20	1.19	1.19
0.7637	**3**	**0.1**	97.89	103.35	**96.22**	96.99
		0.15	**60.51**	61.78	60.65	60.82
		0.2	40.94	40.79	40.77	**40.75**
		0.25	28.80	**28.09**	28.89	28.89
		0.5	9.16	**9.14**	9.28	9.29
		0.75	5.23	**5.17**	5.18	5.21
		1	3.65	3.65	3.63	3.63
		1.5	2.42	2.40	2.41	2.41
		2	1.91	1.90	1.91	1.91
		2.5	1.70	1.69	1.70	1.69
		3	1.16	1.18	1.16	1.15

Note:
The bold values represent the best performance at specific δ and α_3

Table 4: Out-of-control ARL for n = 10 for the known parameter case

β	α_3	δ	EWMA -*MOM*	EWMA -$\bar{X}$	EWMA -*MR*	EWMA -$\tilde{X}$
2.2266	**0.5**	**0.1**	81.34	79.47	**76.54**	77.83
		0.15	37.53	37.04	**36.98**	37.01
		0.2	22.23	**22.00**	22.17	21.83
		0.25	15.00	**14.85**	15.13	14.97
		0.5	5.43	5.47	5.47	5.**42**
		0.75	3.41	**3.38**	3.41	3.40
		1	2.54	2.54	2.54	2.53
		1.5	1.88	1.87	1.88	1.87
		2	1.31	1.32	1.33	1.33
		2.5	1.02	1.01	1.01	1.02
		3	1.00	1.00	1.00	1.00
1.5688	**1.0**	**0.1**	78.15	77.06	**72.61**	75.16
		0.15	**36.93**	**36.92**	37.24	37.57
		0.2	22.07	**21.89**	21.97	22.13
		0.25	**14.89**	14.99	15.26	15.06
		0.5	5.49	**5.46**	5.52	5.51
		0.75	**3.38**	3.42	3.39	3.40
		1	2.53	2.52	2.54	2.54
		1.5	1.87	1.86	1.87	1.86
		2	1.32	1.33	1.34	1.33
		2.5	1.01	1.01	1.00	1.01

		3	1.00	1.00	1.00	1.00
1.2123	**1.5**	**0.1**	75.58	75.53	**69.21**	71.61
		0.15	37.04	37.19	37.29	**36.83**
		0.2	22.10	**22.06**	22.72	22.42
		0.25	15.11	**14.98**	15.57	15.15
		0.5	5.53	**5.48**	5.53	5.50
		0.75	3.40	3.41	**3.39**	3.40
		1	2.54	2.53	2.55	2.53
		1.5	1.86	1.87	1.87	1.86
		2	1.34	1.34	1.35	1.34
		2.5	1.00	1.01	1.00	1.00
		3	1.00	1.00	1.00	1.00
0.9987	**2**	**0.1**	70.95	73.24	**68.55**	70.22
		0.15	36.64	37.07	36.86	**36.10**
		0.2	22.47	**22.36**	22.73	22.41
		0.25	15.25	**15.12**	15.47	15.38
		0.5	5.51	**5.48**	5.56	5.51
		0.75	3.41	3.42	3.42	3.42
		1	2.54	2.53	2.53	2.55
		1.5	1.87	1.86	1.87	1.87
		2	1.35	1.34	1.36	1.36
		2.5	1.00	1.00	1.00	1.00
		3	1.00	1.00	1.00	1.00
0.7637	**3**	**0.1**	68.33	69.78	**66.94**	68.05
		0.15	**35.98**	36.81	37.48	36.70
		0.2	22.66	**22.56**	23.50	22.79
		0.25	15.60	15.61	15.93	**15.58**
		0.5	5.58	**5.50**	5.58	5.56
		0.75	3.41	3.42	3.41	3.42
		1	2.54	2.54	2.54	2.54
		1.5	1.87	1.87	1.88	1.87
		2	1.38	1.35	1.37	1.37
		2.5	1.00	1.00	1.00	1.00
		3	1.00	1.00	1.00	1.00

Note:
The bold values represent the best performance at specific δ and α_3

5. CASE II (UNKNOWN PARAMETER): PERFORMANCE EVALUATION AND SIMULATION PROCEDURE

Because the estimate is substituted for the unknown process parameter θ_0, it is of interest to examine the impact of estimation on the Phase II run length distribution and hence, the robustness of the proposed charts. To achieve the target, both Phase I and Phase II underlying distributions are assumed to be the same, whilst shifts are introduced to monitor the charts' reaction in the out-of-control state.

Similarly, as in Case I, performance of the EWMA-*MOM*, EWMA-$\bar{X}$, EWMA-*MR* and EWMA-$\tilde{X}$ charts are evaluated and compared using the ARL by means of simulation. All charts are constructed using design parameters of (λ, K) = (0.13, 2.88), which supposedly give the ARL_0 of 500. A total subgroup of m = 50 with a fixed sample size of n = 10 is chosen to monitor performance of the charts, conditioned on the Weibull distribution. While scale parameter of Weibull is fixed at 1, several coefficients of skewness α_3, ranging from 0 to 3 are selected to capture the charts' behavior in the in-control state as well as the out-of-control state.

The following simulation algorithm is used to obtain the unconditional run-length distribution for the EWMA-$\hat{\theta}$ chart based on a specific Weibull of α_3. Divided into two series of Monte Carlo simulations, the first series is used to determine the standard deviation of the sampling distribution of the location estimator (i.e. the standard error of the $\hat{\theta}$, denoted as $\sigma_{\hat{\theta}}$). The value is determined based on 1,000,000 simulation runs for n = 10. The second series involves 10,000 trials of *50* in-control Phase I samples of size 10. First, the average value of $\hat{\theta}$ is determined and equated as $\hat{\theta}_0$. Paired with its standard error $\sigma_{\hat{\theta}}$, they are used to estimate the Phase II limits as defined by equation (4a) and (4b). Next, 15,000 samples from Phase II distribution (which is the same as the Phase I distribution) are then generated. From each sample, $\hat{\theta}_i$ is computed and then applied to the EWMA plotting statistic, E_i as defined by equation (3). The starting values are taken as $E_0 = \hat{\theta}_0$. The respective sample numbers (when either $E_i > \widehat{UCL}_{EWMA}$ or $E_i < \widehat{LCL}_{EWMA}$) are signaled as out-of-control. The corresponding run length, i.e. the time i, is then recorded. The calculations are made for δ = 0, 0.1, 0.15, 0.2, 0.25, 0.5, 0.75, 1, 1.5, 2.0, 2.5, 3.0. Once the procedures are completed for 10,000 times, the *ARL* is attained by averaging over the values. The outcomes are presented in Table 5 and Table 6.

5.1 Simulation Outcomes

The simulation findings are divided into two parts. The first part covers the charts' behavior under the in-control state, whilst the second part of the analysis detailed out the performance of all charts when shift occurs.

Table 5: In-control ARL for n = 10 for the unknown parameter case

Distribution	α_3	EWMA-*MOM*	EWMA-$\bar{X}$	EWMA-*MR*	EWMA-$\tilde{X}$
Normal	**0.0**	326.71	342.32	328.52	336.36

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Weibull					
3.6286	**0**	326.58	331.52	342.12	331.59
2.2266	**0.5**	329.66	331.65	330.77	336.52
1.5688	**1.0**	333.80	331.09	322.47	329.81
1.2123	**1.5**	332.73	329.44	306.58	326.88
0.9987	**2**	331.27	324.26	298.46	312.75
0.7637	**3**	307.10	312.41	266.89	294.88

The tabulated results in Table 5 indicate that in-control *ARL* is less than the nominal value of 500, regardless of our choice of estimators. Note that, when process mean is unknown and estimated from the in-control Phase I samples, the decline in in-control performance happens as early as when $\alpha_3 = 0$. We observe the reverse in the previous case study as referred in Table 1 wherein value of α_3 is spotted to be quite large before ARL_0 value digresses from 500. Thus, in general, we would expect more false alarms when mean of the process is estimated from the underlying Weibull distribution, except for some cases when skewness coefficient is set at 3. While the other three charts display a sure decrement in the *ARL* values as level of skewness increases, the EWMA-*MOM* chart, on the contrary, proffers the upper hand in this stipulated non-normal situations. We notice some improvement in the chart performance (increase in the ARL_0) when α_3 changes to 1 from .5. From then onwards, the EWMA-*MOM* chart outperforms the rest.

Meanwhile, Table 6 displays results for out-of-control measurements. All four charts share the same pattern; ARL_1 decreases as α_3 increases, if same magnitude of shift is considered. Overall, the EWMA-*MR* chart is highly efficient when δ is set between 0.1 and 0.2. That is, the chart produces the smallest ARL_1 in more than a single instance. The exception is when the process becomes heavily skewed, i.e. when α_3 shifts to 2 and above. The proposed chart (EWMA-*MOM*) wins in this case. While showing a mediocre performance on slight non-normality, the chart provides a quick detection for a more heavily skewed process, especially when $0.2 \le \delta \le 0.75$. At the same time, the $\bar{X}$ and $\tilde{X}$ charts are only better than the other two charts on a few rare occasions.

Table 6: Out-of-control ARL for n = 10 for the unknown parameter case

β	α_3	δ	EWMA -*MOM*	EWMA -$\bar{X}$	EWMA -*MR*	EWMA -$\tilde{X}$
3.6286	**0.0**	**0.1**	118.51	118.68	**115.74**	120.64
		0.15	50.92	52.36	**50.72**	51.88
		0.2	26.86	26.65	**26.25**	26.73
		0.25	**16.25**	16.36	16.62	16.32
		0.5	5.54	5.52	**5.49**	5.52
		0.75	3.41	3.41	3.42	3.41
		1	2.53	2.53	2.54	2.54
		1.5	1.88	1.87	1.87	1.88
		2	1.31	1.32	1.32	1.31
		2.5	1.02	1.02	1.02	1.02
		3	1.00	1.00	1.00	1.00
2.2266	**0.5**	**0.1**	115.80	115.90	**108.49**	113.50
		0.15	51.24	50.41	**49.30**	51.11
		0.2	26.91	26.51	**25.82**	26.75
		0.25	**16.34**	**16.34**	16.78	16.45
		0.5	**5.49**	5.53	5.57	5.54
		0.75	**3.39**	3.41	3.41	3.43
		1	**2.54**	2.55	2.54	2.53
		1.5	1.88	1.87	1.87	1.87
		2	1.31	1.33	1.34	1.33
		2.5	1.02	1.02	1.01	1.01
		3	1.00	1.00	1.00	1.00
1.5688	**1.0**	**0.1**	111.64	110.59	**99.61**	107.63
		0.15	49.98	48.86	**48.60**	48.77
		0.2	26.42	26.22	**25.89**	26.61
		0.25	16.46	16.76	16.63	16.34
		0.5	**5.52**	5.55	5.63	5.58
		0.75	3.43	3.43	3.42	3.43
		1	2.54	2.54	2.55	2.54
		1.5	1.87	1.87	1.87	1.87
		2	1.32	1.33	1.34	1.34
		2.5	1.01	1.01	1.01	1.01
		3	1.00	1.00	1.00	1.00
1.2123	**1.5**	**0.1**	106.74	106.37	**94.96**	101.35
		0.15	49.34	48.47	**46.48**	47.24
		0.2	26.31	26.08	**25.96**	26.59
		0.25	**16.54**	16.69	16.93	16.71
		0.5	5.58	5.57	5.64	5.58
		0.75	3.42	3.43	3.44	3.44
		1	2.55	2.55	2.56	2.55
		1.5	1.86	1.86	1.87	1.87
		2	1.34	1.34	1.36	1.34
		2.5	1.01	1.01	1.00	1.00
		3	1.00	1.00	1.00	1.00
0.9987	**2**	**0.1**	97.89	103.89	**91.22**	96.51
		0.15	47.29	47.97	**46.35**	46.21
		0.2	**25.99**	26.22	26.25	26.58
		0.25	**16.78**	16.77	17.16	16.83
		0.5	5.62	**5.59**	5.67	**5.59**
		0.75	3.43	3.42	3.43	3.43
		1	2.54	2.55	2.55	2.54

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		1.5	1.86	1.87	1.87	1.86
		2	1.35	1.34	1.36	1.35
		2.5	1.00	1.00	1.00	1.00
		3	1.00	1.00	1.00	1.00
0.7637	**3**	**0.1**	89.63	95.21	**86.32**	89.37
		0.15	45.02	45.75	45.14	**44.95**
		0.2	**25.98**	26.26	26.49	26.76
		0.25	17.26	**16.97**	17.67	17.32
		0.5	5.66	**5.64**	5.70	5.67
		0.75	3.44	3.42	3.43	3.43
		1	2.54	2.55	2.55	2.54
		1.5	1.86	1.87	1.88	1.87
		2	1.37	1.36	1.38	1.36
		2.5	1.00	1.00	1.00	1.00
		3	1.00	1.00	1.00	1.00

Note:
The bold values represent the best performance at specific δ and α_3

6. ILLUSTRATIVE EXAMPLES

Graphical examples are given to portray performance of the EWMA-*MOM* and EWMA-$\bar{X}$ charts when mean of the process is estimated. For the said purpose, forty samples of size five (n = 5) are generated via Monte-Carlo simulation from two distributions, which gives us a total of 200 observations for each data set. The first twenty samples are set to be in-control, whilst the last twenty are the out-of-control samples. $\hat{\theta}_0$ is estimated from the in-control samples that may come from either the normal distribution or the Weibull distributions. Both mean and standard deviation of the normal distribution are allotted to 5 and 16, respectively. Meanwhile, the scale and shape (β) parameters of the Weibull distribution are set at 2 and 0.9887, accordingly. Table 7 – 12 give the data for these processes.

The limits for both charts are constructed using design parameters of (λ, K) = (0.13, 2.88). Therefore, the upper and lower limits for the charts can be determined using equation (4a) and (4b), respectively. The values are attained as follow:

$$\widehat{UCL}_{MOM,normal} = 5.0559;$$
$$\widehat{LCL}_{MOM,normal} = -7.7255.$$

$$\widehat{UCL}_{\bar{X},normal} = 3.9703;$$
$$\widehat{LCL}_{\bar{X},normal} = -6.8960.$$

$$\widehat{UCL}_{MOM,Weibull} = 2.2668;$$
$$\widehat{LCL}_{MOM,Weibull} = 0.8241.$$

$$\widehat{UCL}_{\bar{X},Weibull} = 2.5463;$$
$$\widehat{LCL}_{\bar{X},Weibull} = 1.1871.$$

Figure 1 and 2 display the EWMA-*MOM* and EWMA-$\bar{X}$ charts, together with their respective control limits (UCL and LCL) for the normal distribution. General observation shows that the two methods give comparable in-control and out-of-control performance. Even so, the first out-of-control sample is detected at sample 22 by the EWMA-*MOM* control charts, which is one unit less than perceived by the EWMA-$\bar{X}$ chart. The pattern persists in for the Weibull. Figure 3 and 4 presented the performance of the two charts under this scenario. Both charts are equally delayed in signaling out-of-control situation, unlike when the underlying process distribution is normal. Yet, the EWMA-*MOM* control chart still takes the lead at sample 26. Meanwhile, the EWMA-$\bar{X}$ chart is one step behind as a signal is triggered only at sample 27. It is worth to note that this example is provided to illustrate the increase in the shift of the process, wherein the magnitude of the shift depends on the in-control standard deviation of the process, i.e. $\delta\sigma_{\hat{\theta}}$.

7. CONCLUSION

While charts implemented based on a normal distribution is efficient for normally distributed outputs, they may not be effective for other distributions. Worse, if the charts lead to erroneous conclusion in real data application on non-normally distributed outputs. Yet, in many situations, non-normality is frequently encountered. For example, distribution measurement from chemical process and pharmaceutical data are often skewed. Similarly, output from service processes such as waiting or failure time is also suitably modeled by a certain right-skewed distribution. Scores of studies, concentrating on robust statistical controls, have been organized in the past to monitor small mean shifts under this term. Whilst findings have shown that outliers-resistant chart, specifically a median chart, typically possesses a higher ARL_0 than the standard chart in severe non-normality cases, its shift detection ability is secondary to the standard charts, thus, leaving a room for improvement for future discussion. Taking a leaf out from this context, we have proposed an alternative outlier-resistant chart based on EWMA design structure. Constructed using modified one-step *M*-estimator (*MOM*), this new EWMA chart performance is studied against mean, mid-range and median chart.

Simulation studies are conducted to analyze

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these four EWMA charts for skewed distributions when mean and standard deviation of the process are known. Later, we have extended the analysis by assuming that information on the process mean is not readily available and thus, the parameter is estimated from the in-control Phase I samples.

Although it appears that all four charts are fairly robust to a violation of normality when they are tuned with design parameters of (λ, K) = (0.13, 2.88), this statement is merely true in the first case study, i.e. when none of the process parameters are estimated. When a closer look is taken at the performance of charts with estimated parameter, all in-control performances are adversely affected (more false alarms). Nonetheless, the proposed EWMA-*MOM* chart is seen to gain control (higher ARL_0) over the other as underlying process distribution becomes heavily skewed. Whilst it is apparent that the mid-range and mean charts, overall, are more efficient than the proposed method in the first case study, the extended simulation outcomes reveal the opposite. In addition to providing protection against rise in the false alarm rates, the EWMA-*MOM* chart also offers the best shift-detecting ability, considering various levels of skewness. These duo features are generally desirable for any control charts to be applied in the industry, thus making it the charting procedure we recommend to researchers. It cannot be ignored that all the results in this article are based on the assumption of independent data. If this assumption does not hold, the results may have to be re-examined.

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REFERENCES:

[1] Montgomery DC. Introduction to Statistical Quality Control. 6th edition. New York: Wiley; 2009.
[2] Borror CM, Montgomery DC, Runger GC. Robustness of the EWMA Control Chart to Non-normality. Journal of Quality Technology. 1999;31(3):309-16.
[3] Human SW, Kritzinger P, Chakraborti S. Robustness of the EWMA control chart for individual observations. J Appl Stat. 2011;38(10): 2071–87.
[4] Noorossana R, Fathizadan S, Nayebpour MR. EWMA Control Chart Performance with Estimated Parameters under Non-normality. Qual Reliab Eng Int. 2015.
[5] Woodall WH, Mahmoud MA. The Inertial Properties of Quality Control Charts. Technometrics. 2005;47(4):425–36.
[6] Langenberg P, Iglewicz B. Trimmed Mean $\bar{X}$ and R Charts. Journal of Quality Technology. 1986;18(3):152–61.
[7] Rocke DM. Robust Control Charts. Technometrics.1989;31(2):173–84.
[8] Abu-Shawiesh MO, Abdullah MB. New Robust Statistical Process Control Chart for Location. Qual Eng. 1999;12(2):149–59.
[9] Castagliola P. An $(\bar{X}/R)$ Ewma Control Chart for Monitoring the Process Sample Median. Int J Reliab Qual Saf Eng. 2001;8(2):123–35
[10] Khoo MBC. A Control Chart Based on Sample Median for the Detection of a Permanent Shift in the Process Mean. Qual Eng. 2005;17(2):243–57.
[11] Nazir HZ, Abbas N, Riaz M, Does RJMM. A comparative study of memory-type control charts under normal and contaminated normal environments. Qual Reliab Eng Int. 2016;32(4):1347–56.
[12] Sheu SH, Yang L. The Generally Weighted Moving Average Control Chart for Monitoring the Process Median. Qual Eng. 2006;18(3):333–44.
[13] Ahmad S, Riaz M, Abbasi SA, Lin Z. On efficient median control charting. J Chinese Inst Eng. 2014;37(3):358–75.
[14] Yang L, Pai S, Wang Y. A Novel CUSUM Median Control Chart. Paper presented at: the International Multiconference of Engineers and Computer Scientists; 2010 March 17-19; Hong Kong.
[15] Wilcox RR, Keselman HJ. Modern robust data analysis methods: measures of central tendency. Psychol Methods. 2003;8(3):254–74.
[16] Woodall WH. Controversies and contradictions in statistical process control. J Qual Technol. 2000;32(4):341–50.
[17] Wilcox RR, Keselman HJ. Repeated measures one-way ANOVA based on a modified one-step M-estimator. British J Math Stat Psychology. 2003;56(1):15–25.
[18] Roberts SW. Control Chart Tests Based on Geometric Moving Averages. Technometrics. 2000;42(1):239–50.
[19] Figueiredo F, Gomes MI. The total median in

statistical quality control. Applied Stochastic Models in Business and Industry. 2004;20(4):339–53.
[20] Rousseeuw PJ, Croux C. Alternatives to the Median Absolute Deviation. Journal of the American Statistical Association. 1993;88(424):1273–83.
[21] Özdemir AF. Comparing measures of location when the underlying distribution has heavier tails than normal. _Istatistikçiler Derg. 2010;3:8–16.
[22] Crowder SV. Design of Exponentially Weighted Moving Average Schemes. Journal of Quality Technology. 1989;21(3):155–62.
[23] Lee L, Khoo MB, Yap Y. A Comparison Between the Standard Deviation Of The Run Length (SDRL) Performance of Optimal Ewma and Optimal Cusum Charts. Journal of Quality Measurement and Analysis. 2013; 9(1): 1–8.
[24] Castagliola P. Approximation of the normal sample median distribution using symmetrical johnson Su distributions: application to quality control - Communications in Statistics - Simulation and Com. 1998;27(2):289–301.

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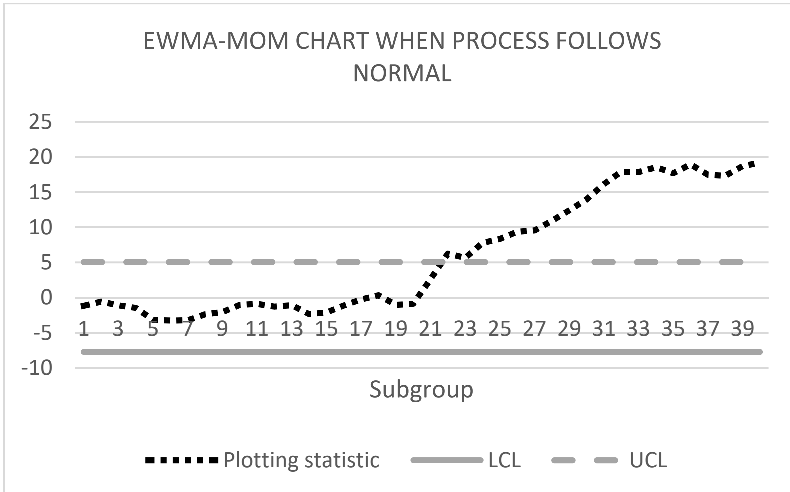

Figure 1: The EWMA-MOM Chart for the Normal Data

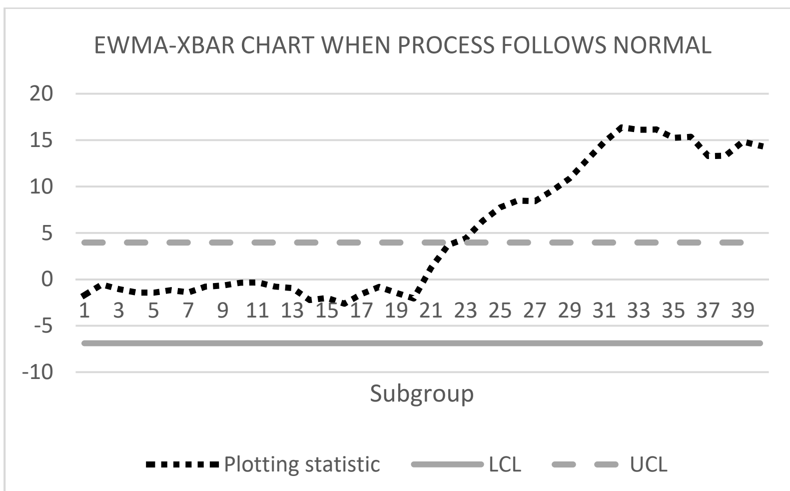

Figure 2: The EWMA-$\bar{X}$ Chart for the Normal Data

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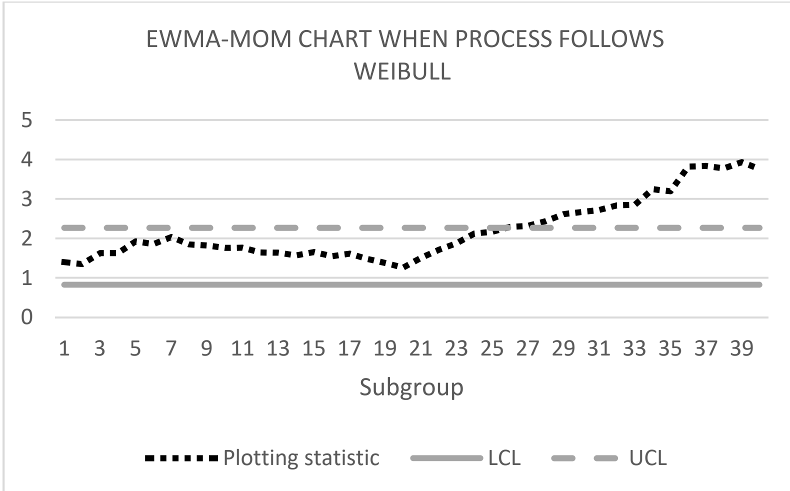

Figure 3: The EWMA-MOM Chart for the Weibull Data

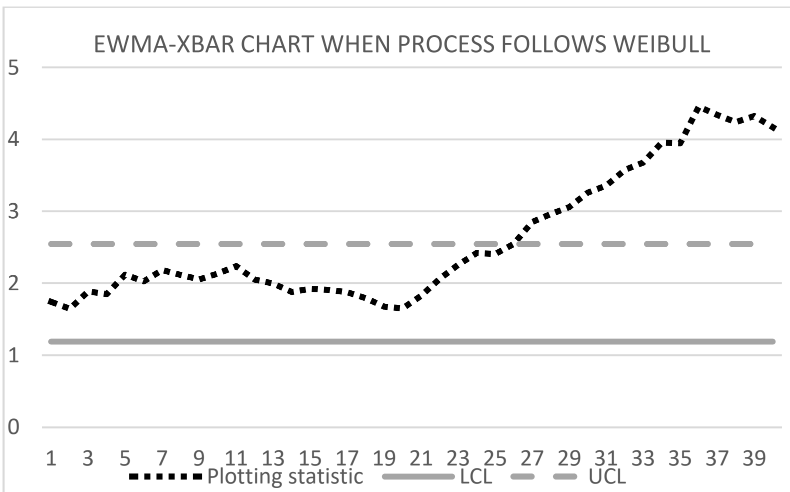

Figure 4: The EWMA-$\bar{X}$ Chart for the Weibull Data

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Table 7: In-control data from the normal distribution

	Observations								
Subgroup *j*	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	MOM_j	$\bar{X}_j$	$E_{i,MOM}$	$E_{i,\bar{X}}$
1	6.4688	0.3919	-0.2387	-22.4635	-0.2541	-0.0337	-3.2191	-1.1657	-1.6912
2	6.8216	9.2187	2.3880	21.8599	-4.9471	3.3703	7.0682	-0.5760	-0.5524
3	36.6873	-33.0936	-4.9633	0.6696	-21.6743	-4.4748	-4.4748	-1.0828	-1.0623
4	-22.5703	25.5206	-28.6430	0.9099	5.4710	-3.8624	-3.8624	-1.4442	-1.4264
5	18.2757	-17.3621	-13.0970	18.5504	-13.6579	-14.7057	-1.4582	-3.1682	-1.4305
6	26.9394	-12.4875	-2.8781	-5.2519	-3.2357	-3.7886	0.6172	-3.2488	-1.1643
7	23.3670	-26.3294	-11.8767	-7.5094	7.7253	-2.9246	-2.9246	-3.2067	-1.3931
8	10.1727	-17.7378	23.1720	5.2045	-4.1488	3.3325	3.3325	-2.3566	-0.7788
9	-14.6129	-2.4844	24.3634	4.8609	-11.6282	0.0998	0.0998	-2.0373	-0.6646
10	-15.1205	6.3878	2.0385	12.4521	2.5201	5.8496	1.6556	-1.0120	-0.3630
11	-6.4127	-3.3150	12.7357	6.4391	-9.8706	-0.0847	-0.0847	-0.8914	-0.3268
12	5.8414	3.8128	-7.7217	-4.6844	-16.1950	-3.7894	-3.7894	-1.2682	-0.7769
13	1.6330	2.2610	-7.0891	4.6254	-11.4912	0.3576	-2.0122	-1.0568	-0.9375
14	6.9484	-9.4385	-36.4163	1.7275	-17.1490	-10.8656	-10.8656	-2.3320	-2.2282
15	-19.3951	15.2706	-9.6233	17.2124	-5.7058	-0.4482	-0.4482	-2.0871	-1.9968
16	3.1880	-17.8138	9.1165	3.9458	-32.9072	5.4168	-6.8941	-1.1116	-2.6334
17	-0.2541	-5.9009	-14.6669	24.1289	24.5997	5.5813	5.5813	-0.2415	-1.5655
18	2.9858	-13.6403	-40.7867	14.6450	57.8583	4.2124	4.2124	0.3375	-0.8144
19	-12.8425	-12.0422	-16.3992	0.5706	12.1068	-10.1784	-5.7213	-1.0295	-1.4523
20	-31.3035	5.6993	-2.1563	0.5289	-3.1148	0.2393	-6.0693	-0.8646	-2.0525
						$\overline{MOM}$ = **-1.3348**	$\bar{\bar{X}}$ = **-1.4628**		

Table 8: In-control data from the Weibull distribution

	Observations								
Subgroup *j*	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	MOM_j	$\bar{X}_j$	$E_{i,MOM}$	$E_{i,\bar{X}}$
1	0.4190	0.3073	0.8826	2.8073	0.1077	0.4292	0.9048	1.4003	1.7417
2	0.5938	2.1892	1.6014	0.0199	0.6228	1.0054	1.0054	1.3490	1.6459
3	10.6099	2.9488	0.4542	0.1816	3.2654	3.4920	3.4920	1.6276	1.8859
4	2.8821	1.8805	0.0276	0.9378	2.3973	1.6251	1.6251	1.6272	1.8520
5	5.8554	5.9436	4.6166	1.2193	1.9948	3.9259	3.9259	1.9261	2.1216
6	0.6707	0.2647	2.8803	1.8432	1.3432	1.4004	1.4004	1.8577	2.0279
7	2.1346	1.2246	5.4085	4.0901	3.2919	3.2299	3.2299	2.0361	2.1841
8	1.2854	0.4074	0.5370	0.1297	5.9369	0.5898	1.6593	1.8481	2.1159
9	1.6990	0.1663	3.2806	2.0868	0.9917	1.6449	1.6449	1.8217	2.0547
10	1.8151	1.3613	8.0269	0.7407	1.4722	1.3473	2.6833	1.7600	2.1364
11	2.9071	7.3944	1.0495	0.9804	2.2728	1.8025	2.9209	1.7655	2.2384
12	0.6258	0.1094	1.4754	1.1522	0.6565	0.8039	0.8039	1.6405	2.0519
13	2.1341	0.0070	3.8527	1.0488	1.0780	1.6241	1.6241	1.6384	1.9963
14	1.9525	2.2845	0.7131	0.3702	0.2197	1.1080	1.1080	1.5694	1.8808
15	2.2326	3.4315	1.3994	1.6271	2.4298	2.2241	2.2241	1.6545	1.9254
16	5.6264	0.7445	0.4169	0.1726	2.0597	0.8484	1.8040	1.5497	1.9096
17	2.3023	2.0838	0.0260	1.2127	2.6283	2.0568	1.6506	1.6157	1.8760
18	0.1142	0.2294	3.9237	0.9761	0.9599	0.5699	1.2407	1.4797	1.7934
19	1.0512	1.6577	0.5667	0.3506	0.8184	0.6967	0.8889	1.3779	1.6758
20	0.5154	3.8420	0.4083	2.1957	0.5298	0.4845	1.4982	1.2618	1.6527
						$\overline{MOM}$ = **1.5454**	$\bar{\bar{X}}$ = **1.8667**		

Table 9: Out-of-control data from the normal distribution when $\hat{\theta} = MOM$

	Observations						
Subgroup j	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	MOM_j	$E_{i,MOM}$
21	16.2144	21.2283	37.9647	41.3282	13.3801	26.0231	2.6308
22	31.1199	12.0312	32.4198	11.7281	28.0565	30.5321	6.2580
23	-8.6042	-16.4254	-1.5895	33.4571	53.1830	1.7095	5.6667
24	23.8180	20.0656	26.7865	15.4001	23.7572	21.9655	7.7855
25	52.0354	11.2260	23.0254	15.8857	-1.6388	12.1246	8.3496
26	18.0018	-2.9704	13.0877	53.7231	-0.3087	16.3067	9.3840
27	-1.7105	26.0118	4.4044	14.0197	11.2169	10.7885	9.5666
28	14.9793	35.7797	-3.4225	25.7350	26.6643	19.9472	10.9161
29	53.7497	3.0087	21.9658	36.9720	-3.3189	22.4755	12.4188
30	29.3995	24.2664	24.7054	19.3402	47.3989	24.4279	13.9800
31	23.0598	40.7420	28.0525	27.4545	33.9283	30.6474	16.1467
32	27.8764	50.7082	49.8517	2.6368	17.0639	29.6274	17.8992
33	15.4433	1.8431	19.2496	31.8909	17.9497	17.5475	17.8535
34	15.4433	1.8431	19.2496	31.8909	17.9497	23.1494	18.5420
35	15.4433	1.8431	19.2496	31.8909	17.9497	12.0607	17.6994
36	3.2691	16.4694	27.0401	23.9817	25.1063	27.3325	18.9517
37	5.3278	31.3975	34.6828	2.1508	-13.2555	7.6324	17.4802
38	23.0576	46.7636	11.9829	-14.8240	27.5259	16.3762	17.3367
39	48.6139	0.2416	42.4952	29.6925	17.9365	27.7960	18.6964
40	23.6293	22.4215	-9.9167	21.9173	13.2920	22.6561	19.2111

Table 10: Out-of-control data from the normal distribution when $\hat{\theta} = \bar{X}$

	Observations						
Subgroup j	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	$\bar{X}_j$	$E_{i,\bar{X}}$
21	13.3957	18.4096	35.1459	38.5094	10.5614	23.2057	1.2311
22	28.3012	9.2124	29.6010	8.9094	25.2377	20.2537	3.7040
23	-11.4230	-19.2442	-4.4082	30.6383	50.3642	9.1867	4.4168
24	20.9993	17.2469	23.9677	12.5814	20.9384	19.1480	6.3318
25	49.2166	8.4072	20.2067	13.0670	-4.4576	17.2893	7.7563
26	15.1831	-5.7892	10.2690	50.9044	-3.1274	13.4893	8.5016
27	-4.5292	23.1930	1.5856	11.2009	8.3982	7.9710	8.4326
28	12.1605	32.9610	-6.2412	22.9162	23.8455	17.1297	9.5632
29	50.9310	0.1899	19.1471	34.1533	-6.1376	19.6580	10.8756
30	26.5808	21.4476	21.8866	16.5215	44.5801	26.2046	12.8683
31	20.2410	37.9233	25.2338	24.6357	31.1096	27.8300	14.8134
32	25.0577	47.8895	47.0330	-0.1820	14.2451	26.8100	16.3729
33	12.6246	-0.9757	16.4309	29.0721	15.1310	14.4579	16.1240
34	0.4503	13.6506	24.2213	21.1630	22.2876	16.3559	16.1541
35	2.5091	28.5788	31.8641	-0.6680	-16.0743	9.2432	15.2557
36	20.2388	43.9449	9.1642	-17.6428	24.7072	16.0838	15.3633
37	-1.0616	-22.1999	7.5841	9.4387	3.2935	-0.5877	13.2897
38	-5.5001	17.1709	7.7739	23.2510	25.0916	13.5588	13.3247
39	45.7952	-2.5771	39.6765	26.8738	15.1178	24.9785	14.8397
40	20.8106	19.6028	-12.7354	19.0985	10.4733	11.4513	14.3992

Table 11: Out-of-control data from the Weibull distribution when $\hat{\theta} = MOM$

	Observations						
Subgroup *j*	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	MOM_j	$E_{i,MOM}$
21	2.4461	4.0877	2.1688	3.6458	3.3105	3.1318	1.5049
22	2.3784	2.4081	6.4728	3.2901	4.2427	3.0798	1.7096
23	2.8869	2.5237	2.2239	6.4685	4.3474	2.9955	1.8768
24	4.5164	2.2132	3.6435	3.7597	3.9802	3.7945	2.1261
25	2.1533	2.4440	2.6830	2.6197	2.3430	2.4486	2.1680
26	3.3100	2.2805	6.3733	2.8995	3.1506	3.1201	2.2918
27	2.5039	2.3922	2.6576	3.9593	13.5661	2.5179	2.3212
28	3.0399	6.3140	2.6764	3.9681	3.1277	3.2030	2.4358
29	4.7690	4.0389	4.4745	2.4322	3.4105	3.8250	2.6164
30	2.9511	3.3606	8.5286	5.9948	2.7776	3.0298	2.6701
31	2.2206	2.4236	8.5199	4.3872	3.1930	3.0561	2.7203
32	2.4654	2.1851	11.3036	6.4257	3.4316	3.6269	2.8382
33	2.1245	3.6642	2.2679	3.6682	10.7174	2.9312	2.8503
34	2.8386	8.3983	5.9610	5.0913	7.3283	5.9235	3.2498
35	2.4707	2.5665	2.8572	3.4479	8.6840	2.8356	3.1959
36	3.5672	3.0087	8.7163	16.0299	8.6508	7.9946	3.8198
37	3.9140	4.2103	2.6949	3.4503	4.1734	3.9370	3.8350
38	3.0698	2.9526	4.9486	4.9262	2.5090	3.3644	3.7738
39	5.9705	3.4702	4.0483	5.2601	6.3110	5.0120	3.9348
40	2.7269	4.0604	2.6886	2.3821	4.4508	2.5992	3.7612

Table 12: Out-of-control data from the Weibull distribution when $\hat{\theta} = \bar{X}$

	Observations						
Subgroup *j*	**Y1**	**Y2**	**Y3**	**Y4**	**Y5**	$\bar{X}_j$	$E_{i,\bar{X}}$
21	2.3231	3.9647	2.0459	3.5228	3.1875	3.0088	1.8290
22	2.2554	2.2851	6.3498	3.1671	4.1197	3.6354	2.0638
23	2.7639	2.4008	2.1009	6.3456	4.2244	3.5671	2.2593
24	4.3934	2.0903	3.5205	3.6367	3.8572	3.4996	2.4205
25	2.0303	2.3210	2.5600	2.4967	2.2200	2.3256	2.4082
26	3.1870	2.1575	6.2503	2.7765	3.0277	3.4798	2.5475
27	2.3810	2.2692	2.5347	3.8363	13.4431	4.8929	2.8524
28	2.9169	6.1910	2.5534	3.8452	3.0047	3.7022	2.9629
29	4.6460	3.9159	4.3515	2.3092	3.2875	3.7020	3.0590
30	2.8281	3.2376	8.4056	5.8718	2.6546	4.5995	3.2592
31	2.0976	2.3006	8.3969	4.2642	3.0700	4.0259	3.3589
32	2.3424	2.0621	11.1806	6.3027	3.3086	5.0393	3.5773
33	2.0015	3.5412	2.1449	3.5452	10.5944	4.3654	3.6798
34	2.7156	8.2753	5.8380	4.9683	7.2053	5.8005	3.9555
35	2.3477	2.4435	2.7343	3.3249	8.5610	3.8823	3.9460
36	3.4442	2.8857	8.5934	15.9069	8.5278	7.8716	4.4563
37	3.7910	4.0873	2.5719	3.3274	4.0504	3.5656	4.3405
38	2.9468	2.8296	4.8256	4.8032	2.3860	3.5583	4.2388
39	5.8476	3.3472	3.9253	5.1372	6.1880	4.8890	4.3233
40	2.6039	3.9375	2.5657	2.2592	4.3279	3.1388	4.1694