# SWITCHING BOND GRAPH APPROACH FOR STRUCTURAL CONTROLLABILITY OF SWITCHED LINEAR SINGULAR SYSTEMS

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**ABSTRACT**

This paper investigates the structural controllability of switched linear singular systems (SLSS). Graphical methods are proposed in order to determine different conditions for the structural controllability of SLSS systems. These methods are based on simple causal paths and causal manipulations on the switching bond graph model. Our approach can be implemented in software such as Symbol2000 or 20sim, in order to control the systems in real time.

**Keywords:** *Singular System, Switched Systems, Bond Graph, Structural R-Controllability, Structural I-Controllability, Structural C-Controllability.*

## 1. INTRODUCTION:

Switched systems are frequently encountered in practice, for example (hydraulic systems with valves, electric systems with diodes, relays, mechanical systems with clutches...). It is for this reason that various researchers have approached the study of controllability/observability for this systems, and a lot of results have emerged during the twenty last years with different approaches (algebraic, graphical...) [1]–[6].

Some sufficient conditions and necessary conditions for controllability of hybrid system were presented in [3], where the system operating period within each mode was assumed to be fixed and known. Complete geometric criteria for controllability and reachability are established in [1], [2]. Some necessary and sufficient conditions for controllability are derived in [4], [5]. The observability of the continuous and discrete states of hybrid systems are studied in [6], [7].

The switched linear singular systems are an important class of switched systems. Due to the existence of switching, discontinuity phenomenon appears in the state variables at the switching moments. Physically, some problems such as sparks and short circuits can occur. Therefore the stability, controllability and observability of switched singular systems are important research topics in the area of switching control. Little works have been done on the controllability of switched linear singular systems. In [8] and [9], the solvability and controllability of periodically switched singular systems were studied. By using the geometric approach, a necessary condition and a sufficient condition on complete reachability are presented in [10].

Up to now, all previous work mentioned above has been based on the traditional controllability concept, for example in [10] the conditions proposed require a lot of matrix calculates to check controllability, Hence it is desirable to investigate controllability and observability by structural properties and not by the parameter numerical values, this properties are independent of the numerical value of the system and depending only on the architecture of the system.

The analysis of structural properties of linear multivariable time invariant systems has received great attention. Different approaches have been used. The first one is the graph approach introduced in [11], and extended for the design of multivariable control systems in [12]–[14]. The bond graph approach has also been exploited to analyses the structural properties. Some recent works permit to highlight structural properties of these systems [15]–[22], based on simple causal manipulations on the bond graph model.

The controllability and observability study of linear time invariant system (LTI) is based on two fundamental notions: attainability and structural rank, the latter is determined directly from the bond graph model [20], [21].

When switched linear system has just one mode, it can be considered as a LTI system. So we can therefore apply the same results obtained for the LTI systems. In this context some necessary and sufficient conditions for controllability and observability for switched systems are derived in [18], [19], with the aid of simple causal manipulations on the switching bond graph (SBG).

On the other hand, the controllability property is decomposed into R-controllability, impulse controllability and complete controllability [23]. For R-controllability of switched singular systems, we proposed some conditions using simple causal manipulations on the bond graph model in [24].

In this paper, we investigate the structural controllability problem for switched linear singular systems modeled by switching bond graph. Unlike the other approaches (algebraic) [8]–[10], the results obtained in this work are more applicable since the conditions developed in this paper are based on simple causal manipulations on the bond graph model, which not only avoids lot of matrix calculates but can also check controllability without knowing the system parameters.

This paper is organized as follows: the second section formulates algebraic results related to the analysis of controllability. Section three recalls some background about bond graph modelling of switched systems. The modelling is done using the structure junction equation, leading to an implicit model. In section four, graphical methods for structural R-controllability, I-controllability and C-controllability of these systems are proposed. This procedure is based on simple causal manipulations on the bond graph model. Finally, a simple example illustrating the previous results is proposed.

## 2. ALGEBRAIC ANALYSIS OF THE CONTROLLABILITY

### 2.1. System description and preliminaries

Considering a switched linear singular system, given by equation (1):

$$\begin{cases} E(\sigma(t))\dot{x}(t) = A(\sigma(t))x(t) + B(\sigma(t))u(t) \\ y(t) = C(\sigma(t))x(t) \end{cases} \quad (1)$$

Where $x(t) \in \Re^n$, $u \in \Re^p$, and $y(t) \in \Re^m$ are respectively the state, input and output vectors.

If we consider this system in a particular mode $j$, the equation (1) can be written as:

$$\begin{cases} E_j\dot{x}(t) = A_jx(t) + B_ju(t) \\ y_j(t) = C_jx(t) \end{cases} \quad (2)$$

With $E_j = E\left(\sigma_j(t)\right), A_j = A(\sigma_j(t))$ , $B_j = B(\sigma_j(t)), C_j = C\left(\sigma_j(t)\right)$, $j \in \{1, \dots, q\}$ and $q$ is the number of modes.

### 2.2. Decomposition of the singular system

It is usual, when analyzing the properties of (2), define equivalent forms by pre-multiplying it by a non-singular matrix $P$, and by operating a variable change $Q_j$ in order to obtain a new equivalent implicit state equation [23]:

$$P_jE_jQ_j\left(Q_j^{-1}\dot{x}(t)\right) = P_jA_jQ_j\left(Q_j^{-1}x(t)\right) + P_jB_ju(t) \quad (3)$$

Such that $P_jE_jQ_j = \begin{pmatrix} I_{n_1} & 0 \\ 0 & N_j \end{pmatrix}$,

$P_jA_jQ_j = \begin{pmatrix} G_j & 0 \\ 0 & I_{n-n_1} \end{pmatrix}$ and $P_jB_j = \begin{pmatrix} H_j \\ J_j \end{pmatrix}$

Introducing the state transformation:

$$\bar{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = Q_j^{-1}x(t) \quad (4)$$

Using (4), the equivalent canonical form of equation (2) can be defined as:

$$\begin{cases} \dot{x}_1(t) = G_jx_1(t) + H_ju(t) & \text{(a)} \\ N_j\dot{x}_2(t) = x_2(t) + J_ju(t) & \text{(b)} \\ y_j(t) = C_1^jx_1(t) + C_2^jx_2(t) & \text{(c)} \end{cases} \quad (5)$$

The equation (5) usually called Kronecker form. The subsystems (5.a) and (5.b) are called slow and fast subsystems respectively.

$x_1(t) \in \Re^{n_1}$ and $x_2(t) \in \Re^{n-n_1}$ are the slow and fast substates respectively, and $N_j$ is a nilpotent matrix of index $h$.

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The Kronecker form separates the finite dynamic modes from pulse and non-dynamic modes, and solves each of the subsystems separately.

### 2.3. R-controllability of SLSS system in a particular mode *j*

When system (1) has just one mode, it can be considered as a singular LTI system, and we consider the equivalent form of Kronecker-Weierstrass [23].

Thus, the slow subsystem (5.a) is an ordinary differential equation. It has a unique solution for any piecewise continuous input $u(t)$ and an initial condition $x_1(0)$, given by (6).

$$x_1 = e^{G_j t} x_1(0) + \int_0^t e^{G_j(t-\tau)} H_j u(\tau) d(\tau) \quad (6)$$

The slow subsystem is controllable if $rank(W_j) = n_1$ and the controllability matrix defined as $W_j = [H_j \; G_j H_j \ldots G_j^{n_1-1} H_j]$.

**Definition 1** [23]: R-controllability is related to the ability to control the finite dynamic modes (classical controllability of exponential modes for regular system). It is associated with the differential part composing the state space.

The R-controllability guarantees our controllability for the system from any admissible initial condition $x_1(0)$ to any reachable state and this process will be finished in any given time period if the control $u(t)$ is suitably chosen.

**Theorem 1**[23]: The system (2) in a particular mode j is called R-controllable, if the slow subsystem (5.a) is controllable.

### 2.4. R-controllability of SLSS system with q modes

We can define a combined matrix $W_{RC}$ of SLSS system as:

$$W_{RC} = [W_1 \; W_2 \ldots W_j \ldots W_q] \quad (7)$$

With $W_j$ is the controllability matrix in mode $j$.

**Theorem 2**(Extension of Yang's Theorem): The SLSS with $q$ modes is R-controllable; if the controllability matrix $W_{RC}$ defined in (7) is of full row rank, i.e. $rank(W_{RC}) = n_1$.

**Proof of theorem 2**: See the Appendix. □

**Remark 1**: From theorem 2, we can deduce that:

- The system (1) can be R-controllable, if the system (2) in a particular mode $j$ is R-controllable.

- However, it is possible that no mode is R-controllable but the system (1) is R-controllable.

### 2.5. Impulse controllability

There exist impulse terms that is set out either by the initial condition or by the possible jump behavior in control input $u(t)$ and its derivatives. Therefore, it is necessary to analyze the control effect on impulse terms in the stale response.

**Definition 2**: Impulse controllability is important for the necessity to eliminate the impulse portions in a system in which impulse terms are generally not expected to appear.

**Theorem 3** [23] : The system (2) in a particular mode j is called I-controllable, if the fast subsystem (5.b) is controllable.

### 2.6. C-controllability

**Definition 3** [23]: The system (2) in a particular mode $j$ is called C-controllable, if both its slow and fast subsystems are controllable.

## 3. REPRESENTATION OF A LINEAR SINGULAR SYSTEM FROM A SWITCHING BOND GRAPH

The structure junction of a switching bond graph (SBG) can be represented by figure 1.

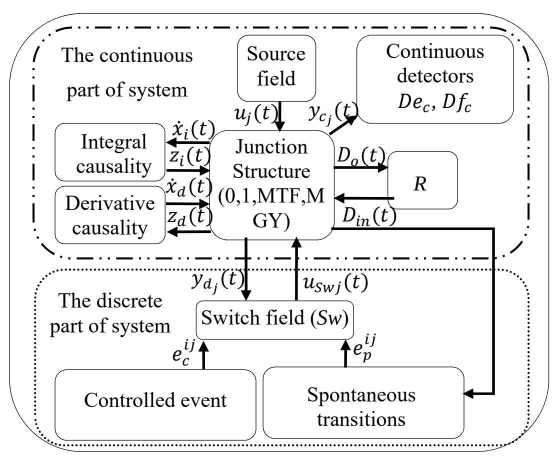


*Figure 1: Junction structure of a switching bond graph*

Five fields model the components behavior: - source field which produces energy, - R field which dissipates it, - *I* and *C* field which can store it, - $De$ and $Df$ continuous detectors fields, and the *Sw* field is the switching component. These elements are linked directly to the control system discrete. $x_i(t)$

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is the state vector. It contains the variables $p$ on $I$ elements and the variables $q$ on $C$ elements when these elements are in integral causality. $x_d(t)$ is the pseudo state vector: it contains the variables $p$ on $I$ elements and the variables $q$ on $C$ elements when these elements are in derivative causality. $z_i(t)$ and $z_d(t)$ are vectors that contain the coenergy variables associated to $x_i(t)$ and $x_d(t)$. $D_{in}(t)$ and $D_o(t)$ contain the effort and flow variables respectively entering and exiting from resistive ports. $u_{Swj}(t)$ and $y_{d_j}(t)$ are vectors that contain the variables respectively imposed and exiting from switches in mode $j$.

### 3.1. Bond graph models of switch and switched sources elements

The switch elements can be modelled using two main bond graph approaches: non ideal switches [16] or ideal switches [15]. In the second case, the sources standing for ideal switches have two states (figure 2): the first state denoted by ON, when they behave like zero effort sources ($Se_d$: 0) and a second state denoted by OFF when they behave like zero flow sources ($Sf_d$: 0). These two sources represent the discrete inputs and are noted $u_{Swj}$, they can be efforts or flows entering in structure of junction. For a system contain N switchs, we need to define the discrete sources $u_{Sw}(t)$ by: $u_{Sw}(t) = \left\{ \left(u_{Sw_j}\right)_l, j \in \{1, \cdots, q\},\ l \in \{1, \cdots, N\} \right\}$ with $q = 2^N$ those define the set of discrete inputs, and are presented in Figure 2.

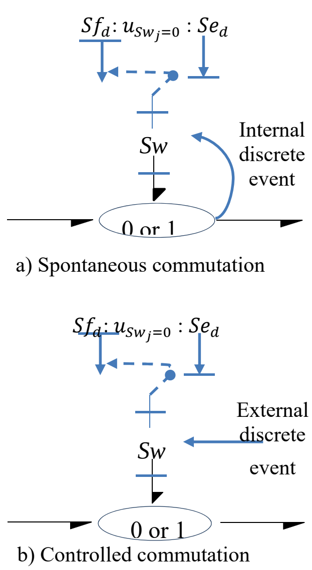


*Figure 2: Bond graph models of switched sources $Se_d$ and $Sf_d$*

### 3.2. State representation from switching bond graph

Each output of the junction structure ($\dot{x}_i(t)$, $z_d(t), D_o(t)$, $y_{d_j}(t)$ and $y_{c_j}(t)$) can be expressed as function of all its inputs ($z_i(t)$, $\dot{x}_d(t)$, $D_{in}(t), u_{Sw_j}(t)$, and $u_j(t)$):

$$\begin{bmatrix} \dot{x}_i(t) \\ z_d(t) \\ D_o(t) \\ y_{d_j}(t) \\ y_{c_j}(t) \end{bmatrix} =$$

$$\begin{bmatrix} S_{11}^j & S_{12}^j & S_{13}^j & S_{14}^j & S_{15}^j \\ -S_{12}^{tj} & 0 & 0 & S_{24}^j & S_{25}^j \\ S_{13}^{tj} & 0 & S_{33}^j & S_{34}^j & S_{35}^j \\ -S_{14}^{tj} & -S_{24}^j & -S_{34}^j & S_{44}^j & S_{45}^j \\ S_{51}^{tj} & S_{52}^j & S_{53}^j & S_{54}^j & S_{55}^j \end{bmatrix} \begin{bmatrix} z_i(t) \\ \dot{x}_d(t) \\ D_{in}(t) \\ u_{Sw_j}(t) \\ u_j(t) \end{bmatrix} \quad (8)$$

This linear relation can be written as an implicit equation that is called in the following the standard implicit form:

$$\begin{bmatrix} I_{n_i} & S_{12}^j \\ 0 & 0 \\ 0 & 0 \\ 0 & S_{24}^{tj} \\ 0 & -S_{52}^j \end{bmatrix} \begin{bmatrix} \dot{x}_i(t) \\ \dot{x}_d(t) \end{bmatrix} = [S] \begin{bmatrix} z_i(t) \\ z_d(t) \\ D_{in}(t) \\ D_o(t) \\ u_{Sw_j}(t) \\ y_{d_j}(t) \\ u_j(t) \\ y_{c_j}(t) \end{bmatrix} \quad (9)$$

Where

$$[S] = \begin{bmatrix} S_{12}^j & 0 & S_{13}^j & 0 & S_{14}^j & 0 & S_{15}^j & 0 \\ -S_{12}^{tj} & I_{n_{dj}} & 0 & 0 & S_{24}^j & 0 & S_{25}^j & 0 \\ S_{13}^{tj} & 0 & S_{33}^j & I_{R_j} & S_{34}^j & 0 & S_{35}^j & 0 \\ -S_{14}^{tj} & 0 & -S_{34}^j & 0 & S_{44}^j & I_{Sw_j} & S_{45}^j & 0 \\ S_{15}^{tj} & 0 & S_{53}^j & 0 & S_{54}^j & 0 & S_{55}^j & -I_{y_j} \end{bmatrix}$$

Matrices $S_{11}^j$, $\mathrm{S}_{33}^{\mathrm{j}}$, $S_{44}^j$ and $S_{55}^j$ are skew symmetric due to energy considerations.

Let the constitutive law of the $R$ field be:

$$D_{in}(t) = L_j D_o(t).$$

$L_j$ is a positive matrix, with $L_j = \begin{bmatrix} [R] & 0 \\ 0 & [1/R] \end{bmatrix}$.

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If $H_j = L_j\left(I - S^j_{33}L_j\right)^{-1}$ exists, which is particularly true when $L_j$ is a symmetric and positive definite matrix, then third row of (8) leads to:

$$D_{in}(t) = H_j\left(-S^{tj}_{13}z_i(t) + S^j_{34}u_{sw_j}(t) + S^j_{35}u_j(t)\right) \qquad (10)$$

By eliminating $D_{in}(t)$ and $D_o(t)$ from (8) we obtain the equation (11):

$$\begin{cases} \begin{bmatrix} I_{n_i} & -S^j_{12} \\ 0 & 0 \end{bmatrix}\dot{x}(t) = \begin{bmatrix} K & 0 \\ -S^{tj}_{12} & I_{n_d} \end{bmatrix}\begin{bmatrix} z_i(t) \\ z_d(t) \end{bmatrix} + \begin{bmatrix} K_1 & K_2 \\ S^j_{25} & S^j_{24} \end{bmatrix}\begin{bmatrix} u_j(t) \\ u_{sw_j}(t) \end{bmatrix} \\ y_{c_j}(t) - S^j_{52}\dot{x}_d(t) = [K'' \quad 0]\begin{bmatrix} z_i(t) \\ z_d(t) \end{bmatrix} + [K''_1 \quad K''_2]\begin{bmatrix} u_j(t) \\ u_{sw_j}(t) \end{bmatrix} \\ y_{d_j}(t) + S^{tj}_{24}\dot{x}_d(t) = [K' \quad 0]\begin{bmatrix} z_i(t) \\ z_d(t) \end{bmatrix} + [K'_1 \quad K'_2]\begin{bmatrix} u_j(t) \\ u_{sw_j}(t) \end{bmatrix} \end{cases} \qquad (11)$$

Where $x(t) = \begin{bmatrix} x_i(t) \\ x_d(t) \end{bmatrix}$, $K = S^j_{11} - S^j_{13}H_jS^{tj}_{13}$,

$K' = -S^{tj}_{14} + S^{tj}_{34}H_jS^{tj}_{13}$, $K_1 = S^j_{15} + S^j_{13}H_jS^j_{35}$,

$K_2 = S^j_{14} + S^j_{13}H_jS^j_{34}$, $K'_1 = S^j_{45} - S^{tj}_{34}H_jS^j_{35}$,

$K'_2 = S^j_{44} - S^{tj}_{34}H_jS^j_{34}$, $K''_1 = S^j_{55} + S^j_{53}H_jS^j_{35}$,

$K''_2 = S^j_{54} + S^j_{53}H_jS^j_{34}$ and $K'' = -S^j_{51} - S^j_{53}H_jS^{tj}_{13}$.

In a linear case, the law constitutive for the fields of storage $I$ and $C$ can be written as:

$$\begin{bmatrix} z_i(t) \\ z_d(t) \end{bmatrix} = \underbrace{\begin{bmatrix} F^j_i & 0 \\ 0 & F^j_d \end{bmatrix}}_{M}\begin{bmatrix} x_i(t) \\ x_d(t) \end{bmatrix}$$

Where $F^j_i = \begin{bmatrix} 1/I & 0 \\ 0 & 1/C \end{bmatrix}$ and $\left(F^j_d\right)^{-1} = \begin{bmatrix} I & 0 \\ 0 & C \end{bmatrix}$.

In mode $j$, we have $u_{sw_j}(t) = 0$, so for $t \in [t_{j-1}, t_j)$ the state representation is given by:

$$\begin{cases} E_j\dot{x}(t) = A_jx(t) + B_{cj}u_j(t) & (a) \\ y_{c_j}(t) + L'_j\dot{x}_d(t) = C_jx(t) + D''_{cj}u_j(t) & (b) \\ y_{d_j}(t) + L_j\dot{x}_d(t) = C_{dj}x(t) + D_{cj}u_j(t) & (c) \end{cases} \qquad (12)$$

Where

$$\begin{cases} x(t) = \begin{pmatrix} x_i(t) \\ x_d(t) \end{pmatrix}, E_j = \begin{bmatrix} E^j_{ii} & E^j_{id} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} I_{n_i} & -S^j_{12} \\ 0 & 0 \end{bmatrix}, \\ A_j = \begin{bmatrix} A^j_{ii} & 0 \\ A^j_{di} & A^j_{dd} \end{bmatrix} = \begin{bmatrix} KF^j_i & 0 \\ -S^{tj}_{12}F^j_i & -F^j_d \end{bmatrix}, B_{cj} = \begin{bmatrix} B^j_i = K_1 \\ B^j_d = S^j_{25} \end{bmatrix}, \\ L'_j = -S^j_{52}, C_j = [K''F^j_i \quad 0] = [C^j_c \quad 0] \\ D''_{cj} = K''_1, \quad D_{cj} = S^j_{45} - S^{tj}_{34}H_jS^j_{35}, \\ L_j = S^{tj}_{24}, C_{dj} = [K'F^j_i \quad 0] = [C^j_d \quad 0] \end{cases}$$

Where $x_i \in R^{n_i}$ and $x_d \in R^{n_d}$

Thus, for a system with $N$ switches, the number of modes is given by $2^N = q$. The hybrid system evolves by according to the following dynamical:

$$\begin{cases} \begin{cases} E_1\dot{x}(t) = A_1x(t) + B_{c1}u_1(t) & (a) \\ y_{c_1}(t) + L'_1\dot{x}_d(t) = C_1x(t) + D''_{c1}u_1(t) & (b) \quad t \in [t_0, t_1) \\ y_{d_1}(t) + L_1\dot{x}_d(t) = C_{d1}x(t) + D_{c1}u_1(t) & (c) \end{cases} \\ \vdots \\ \begin{cases} E_j\dot{x}(t) = A_jx(t) + B_{cj}u_j(t) & (a) \\ y_{c_j}(t) + L'_j\dot{x}_d(t) = C_jx(t) + D''_{cj}u_j(t) & (b) \quad t \in [t_{j-1}, t_j) \\ y_{d_j}(t) + L_j\dot{x}_d(t) = C_{dj}x(t) + D_{cj}u_j(t) & (c) \end{cases} \\ \vdots \\ \begin{cases} E_q\dot{x}(t) = A_qx(t) + B_{cq}u_q(t) & (a) \\ y_{c_q}(t) + L'_q\dot{x}_d(t) = C_qx(t) + D''_{cq}u_q(t) & (b) \quad t \in [t_{q-1}, t_q) \\ y_{d_q}(t) + L_q\dot{x}_d(t) = C_{dq}x(t) + D_{cq}u_q(t) & (c) \end{cases} \end{cases} \qquad (13)$$

### 3.3. Decomposition of the singular system

To go further in the analysis of the implicit equation (12), it is pre-multiplied by the nonsingular matrix:

$$P^j = \begin{pmatrix} I & -A^j_{ii}S^j_{12} \\ 0 & I \end{pmatrix}\begin{pmatrix} I & 0 \\ 0 & -R^j \end{pmatrix} \qquad (14)$$

Where $R^j = \left(A^j_{di}S^j_{12} + A^j_{dd}\right)^{-1}$

Defining also the variable change:

$$Q^j\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} x_i(t) \\ x_d(t) \end{pmatrix} \qquad (15)$$

Where

$$Q^j = \begin{pmatrix} I & S^j_{12} \\ 0 & I \end{pmatrix}\begin{pmatrix} I & 0 \\ -R^jA^j_{di} & I \end{pmatrix} \qquad (16)$$

Leads to the following explicit state representation:

$$\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{pmatrix} A^j_{ii}[I - S^j_{12}R^jA^j_{di}] & 0 \\ 0 & I \end{pmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{pmatrix} B_i{}^j + A_{ii}{}^jS_{12}{}^jR^jB_d{}^j \\ -R^jB_d{}^j \end{pmatrix}u_j(t) \qquad (17)$$

Equation (17) is equivalent to an ordinary state representation:

$$\dot{x}_1(t) = G_jx_1(t) + H_ju_j(t) \qquad (18)$$

Where the state is continuous at the origin, associated to an algebraic equation:

$$x_2(t) = J_ju_j(t) = R^jB_d{}^ju_j(t) \qquad (19)$$

Where $G_j = A^j_{ii}[I - S^j_{12}R^jA^j_{di}]$

and $H_j = B_i{}^j + A_{ii}{}^jS_{12}{}^jR^jB_d{}^j$

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### 3.4. Determination of the equivalent bond graph of slow subsystem

The following procedure shows the symbolic calculation of the equivalent ordinary state representation (18) for linear singular systems directly from their bond graph model.

$x_1(t)$ components of vector are calculated so as to reduce the components of the vector $x_d(t)$ (which are causally connected to an element of the vector $x_i(t)$) to the components of the vector $x_i(t)$ extracted by equation (15) :

$$x_1(t) = x_i(t) - S_{12}x_d(t) \quad (20)$$

Changing the $x_i(t)$ elements by $x_1(t)$ element in bond graph model. To explain that, we propose the following example:

**Example 1**: We consider the bond graph model given by Figure 3.a and its equivalent bond graph (EBG) model given by Figure 3.b.

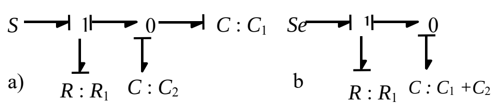



*Figure 3 a) BGI, b) Equivalent bond graph model (EBGI)*

Where

$$x_1(t) = q_{C2}(t) + q_{C1}(t) = C_2e_2(t) + C_1e_1(t) \quad (21)$$

Note that

$$e_1(t) = e_2(t) \quad (22)$$

Then, we obtain the relations:

$$x_1(t) = (C_1+C_2)e_1(t) = Ce_1(t)$$

and $C = C_1 + C_2$ (23)

So the EBG of slow subsystem is found by changing the value of $C_1$ through $C = C_1 + C_2$ and by removing the element in derivative causality. Physically, we can explain that by the existence of an equivalent C-element, which groups the two elements in parallel.

$G_j$ and $H_j$ are obtained by causal manipulations on the EBGI$_j$ model of slow subsystems and, they are given by the following propositions:

**Proposition 1**[24]**:** In the $G_j$-matrix, the $(g_j)_{kh}$ -term is obtained by expression (24).

$$(g_j)_{kh} = \sum_{p\in P}\left(\tilde{G}_1((\dot{x_1})_h, (x_1)_k)\right)_p \times \tilde{g}(x_1)_h \quad (24)$$

Where $h \in \{1, \dots, n_1\}$, $k \in \{1, \dots, n_1\}$, $j \in \{1, \dots, q\}$.

$\tilde{G}_1((\dot{x_1})_h, (x_1)_k)$ is the causal path gain of length $L_1 = 1$ from $\dot{x_1}$ *to* $x_1$.

$\tilde{g}(x_1)$ is the gain of the I or C element in integral causality associated with $x_1$*:* $\tilde{g}(I) = \frac{1}{I}$ and $\tilde{g}(C) = \frac{1}{C}$.

**Proposition 2**[24]**:** In the $H_j$-matrix, the $(h_j)_{kl}$ term is obtained by expression (25):

$$(h_j)_{kl} = \sum_{p\in P}\left(\tilde{G}_1((u)_k, (x_1)_l)\right)_p \quad (25)$$

Where $l \in \{1, \dots, n_1\}$, $k \in \{1, \dots, m\}$ and $j \in \{1, \dots, q\}$.

$\tilde{G}_1((u)_k, (x_1)_l)$ is the constant term of the gain of the causal path of generalized length from the (*Se* or *Sf* ) associated with $u_k$ to dynamical element *(I,C)* in integral causality associated with $x_1$.

## 4. STRUCTURAL CONTROLLABILITY

The objective of this part is to present graphical methods using the bond graph methodology to derive information on structural controllability. For the SLSS, the controllability property is decomposed into R-controllability, impulse controllability and complete controllability.

In the following, EBGI and EBGD denote respectively the equivalent bond graph model of slow subsystem when the preferential integral (respectively derivative) causality is affected.

### 4.1. R-controllability

**- Graphical sufficient condition 1**

To study structural R-controllability of switched singular system modeled by switching bond graph, we must for each mode, transform it to an equivalent bond graph of the slow subsystem. Therefore, we can therefore apply the same results obtained for the LTI systems.

**Proposition 3***:* In a particular mode $j$, the slow subsystem (18) is structurally controllable if:

1- All dynamic elements in integral causality are causally connected with a continuous input control.

2- $EBG - rank[G_j\ H_j] = n_1$, with $j \in \{1, \dots, q\}$.

**Proof of proposition 3***:* This result is derived from digraph theory [25]. ☐

**Property 1***:* $EBG - rank[G_j\ H_j] = n_1 - t_s^j$.

Where $t_s^j$ is the number of elements remaining in integral causality in EBGD$_j$, when a dualism of the maximum number of continuous input sources is applied (in order to eliminate elements in integral causalities). And $n_1$ is the number of element in integral causality in EBGI$_j$.

***Proof of Property 1*** *:* After transformation of switched singular system modeled by switching bond graph, to an equivalent bond graph of the slow subsystem, the proof of this property is equivalent to the one proposed in the case of switched systems [21]. □

On the other hand, the switched linear singular system (12) in a particular mode is called R-controllable, if the slow subsystem (18) is controllable. Hence to study the R-controllability of system (1), it is necessary to apply this result to all modes; if one controllable mode exists, the procedure is stopped. The case where no mode is R-controllable, but the system (1) is R-controllable, Therefore the sufficient condition 1 cannot be applied in this case, where the interest of the proposed condition below.

- **Graphical sufficient condition 2**

After transformation of switched singular system modeled by switching bond graph, to an equivalent bond graph of the slow subsystem, we can apply the proposed procedure in [22], in order to calculate the subspace of structural controllability of each mode $j$, noted $R_0^j$.

On the EBGD$_j$ when a dualization of the maximum number of continuous input sources is applied (in order to eliminate elements in integral causalities), we can write for each element *I* and *C* remaining in integral causality $t_s^j$ algebraic equations:

$$g_k^j - \sum_r \alpha_r^{jk} g_r^j = 0 \qquad (26)$$

$g_k^j$ is either an effort variable $e_r$ for *I*-element in integral causality or a flow variable $f_r$ for *C*-element in integral causality.

$g_r^j$ is either an effort variable $e_r$ for *I*-element in derivative causality or a flow variable $f_r$ for *C*-element in derivative causality.

$\alpha_r^{jk}$ is the gain of the causal path between the $K^{th}$ *I* or *C*-elements in integral causality and the $r^{th}$ *I* or *C*-elements in derivative causality.

Let us consider the $t_s^j$ row vectors $z_k^j(k = 1, \ldots, t_s^j)$ whose components are the coefficients of the variables $g_l^j(l = k, r)$ in the equation (26).

***Property 2****:* The $t_s^j$ row vectors $z_k^j(k = 1, \ldots, t_s^j)$ are orthogonal to the structural controllability subspace vectors of the $l^{th}$ mode. We write $Z_j = (z_k^j)_{k=1,\ldots,t_s^i}$ and $R_0^{j\perp} = Im(Z_j)$. With $R_0^{j\perp}$ is uncontrollable subspace in mode $j$, used to check orthogonality.

In the same way, from the EBGD$_j$ (with dualization of inputs sources), we can write for each element *I* and *C* remaining in derivative causality $n_1 - t_s^j$ algebraic equations:

$$g_r^j - \sum_k \gamma_K^{jr} g_k^j = 0 \qquad (27)$$

$g_r^j$ is either a flow variable $f_r$ for *I*-element in derivative causality or an effort variable $e_r$ for *C*-element in derivative causality.

$g_k^j$ is either a flow variable $f_r$ for *I*-element in integral causality or an effort variable $e_r$ for *C*-element in integral causality.

$\gamma_K^{jr}$ is the gain of the causal path between the $r^{th}$ element in derivative causality and the $K^{th}$ element in integral causality.

Now, we consider the $n_1 - t_s^j$ column vectors $w^{jr}$ whose components are the coefficients of $g_r^j$ and $g_k^j$ variables in equation (27).

**Property 3:** $n_1 - t_s^j$ column vectors $w^{jr}(r = 1, \ldots, n_1 - t_s^j)$ compose a basis for the structural R-controllability subspace of $j^{th}$ mode. With $W^j = {w^{jr}}_{r=1,\ldots,n_1-t_s^j}$ and $R_0^j = Im(W^j)$.

**Proof of Property 3**: Equations (26) and (27) provide dual algebraic relations. So we can easily verify that $z_k^j w^{jr} = 0$. □

Now, we can define a combined matrix $W^{RC}$ of SLSS system as: $W^{RC} = [W^1\, W^2 \ldots W^j \ldots W^q]$. With $W^j = {w^{jr}}_{r=1,\ldots,n_1-t_s^j}$ is the controllability matrix of $j^{th}$ mode.

Using the graphical calculation of structural controllability subspaces and theorem 2, the following theorem is proposed.

**Theorem 4:** If $rank[W^{RC}] = n_1$, the switched linear singular system (1) is structurally R-controllable.

**Proof of theorem4:**

We have shown for a given mode $j$ that the $EBGD_j$ (with dualization of inputs sources) is characterized by an algebraic equation (27). From this equation we build a basis $W^j$ for the structural R-controllability subspace of j$^{th}$ mode. With $W^j = w^{jr}{}_{r=1,\dots,n_1-t_s^j}$ and $R_0^j = Im(W^j)$. In the same way, we build $W^{RC} = [W^1\, W^2 \dots W^j \dots W^q]$, with $W^j = w^{jr}{}_{r=1,\dots,n_1-t_s^j}$ for all mode.

However, the condition of Theorem 2 is sufficient for the controllability of the system, which implies that the condition $rank[W^{RC}] = n_1$ is also sufficient. □

### 4.2. Impulse controllability

**Proposition 4** [19]: A SLSS system is impulse controllable if and only if the number of impulse modes is equal to the number of disjoint causal paths between input sources and switches passing through (*I*,*C*) elements in derivative causality in the $BGI_j$.

$$bg\_rank[S_{24}^{tj} B_d^j] = bg\_rank[S_{24}^{tj}] \quad (28)$$

- $S_{24}^j$ represents causal paths between *(I,C)* elements in derivative causality and switches elements and $bg\_rank[S_{24}^{tj}]$ is equal to the number of impulse modes.
- $B_d^j$ is the input sub-matrix connecting input sources and *(I,C)* elements in derivative causality in the $BGI_j$,
- $S_{24}^{tj} B_d^j$ is composed by causal paths between input sources and switches passing through elements in derivative causality in the $BGI_j$. So $bg\_rank[S_{24}^{tj} B_d^j]$ corresponds to the number of disjoint causal paths between input sources and switches passing through elements in derivative causality.

## 5. EXAMPLE

We consider the following acausal switching bond graph model (Figure 4):

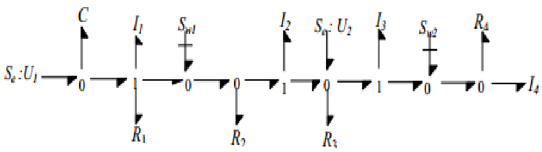



*Figure 4: Acausal switching bond graph model*

This switching bond graph model contains two switches (*Sw1* and *Sw2*), so four modes are possible, but only three are considered: Mode 1 (*Sw1* open, *Sw2* closed), Mode 2 (*Sw1* closed, *Sw2* closed), Mode 3 (*Sw1* closed, *Sw2* open).

There are five state variables ($q_c$, $P_{I1}, P_{I2}, P_{I3}, P_{I4}$), one element in derivative causality ($q_c$) and four element in integral causality ($P_{I1}, P_{I2}\ P_{I3}, P_{I4}$). The switching bond graph models in integral causality for these three modes are given in figure 5.

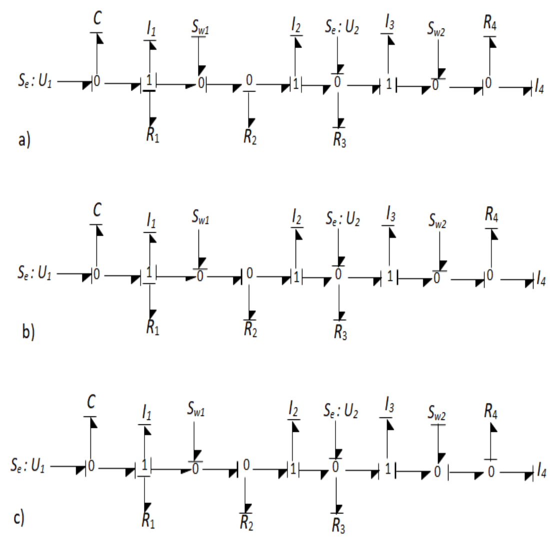
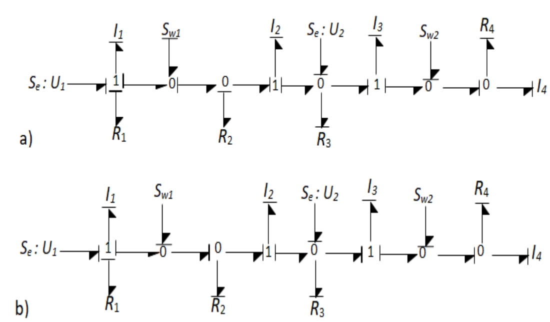



*Figure 5: Switching bond graph model in integral causality for a) mode 1, b) mode 2, c) mode3*

The switching bond graph models in integral causality for slow subsystems are found by ignoring elements in derivative causality ($q_c$). The three bond graph models in integral causalities for slow subsystems $EBGI_1$, $EBGI_2$ and $EBGI_3$ are associated respectively to mode 1, mode 2 and mode 3 (Figure 6).



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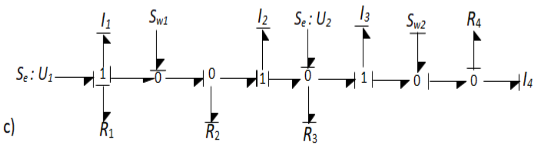


*Figure 6 : EBGIj for a) mode 1, b) mode 2, c) mode 3*

The application of the derivative causality and dualisation of these modes are given in Figure 7:

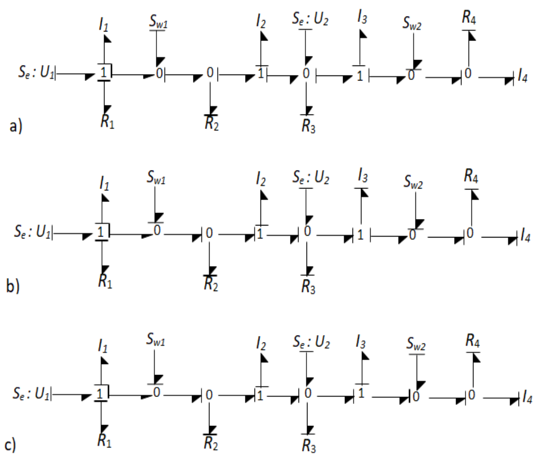


*Figure 7 : EBGDj+dualisation a) mode 1, b) mode 2, c) mode 3*

**Application of graphical sufficient condition 1**

i. On the $\text{EBGI}_1$, all the elements in integral causality are connected with a continuous input control, and on the $\text{EBGD}_1$, one element stays in integral causality $P_{I4}$ (Figure 6.a), and the dualization of inputs sources does not change its causality. So this mode is not R-controllable.

ii. On the $\text{EBGI}_2$, all the elements in integral causality are connected with a continuous input control, and on the $\text{EBGD}_2$, two elements stays in integral causality $P_{I4}$ and $P_{I3}$ (Figure 6.b), and the dualization of inputs sources does not change its causality. So this mode is not R-controllable.

iii. On the $\text{EBGI}_3$, all the elements in integral causality are connected with a continuous input control, and on the $\text{EBGD}_3$, one element stays in integral causality $P_{I4}$ (Figure 6.c), and the dualization of inputs sources does not change its causality. So this mode is not R-controllable.

Since no mode is controllable, we apply the second graphical sufficient condition:

**Application of graphical sufficient condition 2**

i. In the $\text{EBGD}_1$ (Figure 7.a), $I_4$ remain in integral causality, we can write $e_{I4} = 0$, thus $z_1^1 = (0\ 0\ 0\ 1)$. The dynamic elements $I_1$, $I_2$, and $I_3$ are not causally connected with $I_4$. So $\text{f}_{\text{I1}} = \text{f}_{\text{I2}} = \text{f}_{\text{I3}} = 0$. The three corresponding vectors are $w^{11} = (1\ 0\ 0\ 0)^t$, $w^{12} = (0\ 1\ 0\ 0)^t$ and $w^{13} = (0\ 0\ 1\ 0)^t$.

We have $Z_1 W^1 = 0$, then $R_0^1 = Im(w^{11}\, w^{12}\, w^{13})$ and $W^1 = [w^{11}\, w^{12}\, w^{13}]$, with $rank(W^1) = 3$

ii. In the $\text{EBGD}_2$ (Figure 7.b), $I_4$ and $I_3$ remain in integral causality, we can write $e_{I4} = 0$, thus $z_1^2 = (0\ 0\ 0\ 1)$. And $e_{I3} + e_{I2} = 0$ thus $z_2^2 = (0\ 1\ 1\ 0)$. The algebraic equations corresponding to $I_1$ and $I_2$ are given by: $f_{I1} = 0$ and $f_{I2} - f_{I3} = 0$. then $w^{21} = (1\ 0\ 0\ 0)^t$ and $w^{22} = (0\ 1 - 1\ 0)^t$.

We have $Z_2 W^2 = 0$, then $R_0^2 = Im(w^{21}\, w^{22})$ and $W^2 = [w^{21}\, w^{22}\,]$, with $rank(W^2) = 2$ .

iii. In the $\text{EBGD}_3$ (Figure 7.c), The element $I_4$ is in integral causality and causally connected with $I_3$ and $I_2$, we can write $e_{I4} - e_{I3} + e_{I2} = 0$, thus $z_1^3 = (0\ 1 - 1\ 1)$. The algebraic equations corresponding to $I_1, I_2$ and $I_3$ are given by : $f_{I1} = 0$, $f_{I2} - f_{I4} = 0$ and $f_{I3} + f_{I4} = 0$. The three corresponding vectors are $\text{w}^{31} = (1\ 0\ 0\ 0)^{\text{t}}$, $\text{w}^{32} = (0\ 1\ 0 - 1)^{\text{t}}$ and $w^{33} = (0\ 0\ 1\ 1)^t$.

We have $Z_3 W^3 = 0$, then $R_0^3 = Im(w^{31}\, w^{32}\, w^{33})$ and $W^3 = [w^{31}\, w^{32}\, w^{33}]$, with $rank(W^3) = 3$

From theorem 4 we have:

$$rank(W^1\, W^2\, W^3]) = rank\begin{bmatrix} 1 & 0 & 0 \vdots & 1 & 0 \vdots & 1 & 0 & 0 \\ 0 & 1 & 0 \vdots & 0 & 1 \vdots & 0 & 1 & 0 \\ 0 & 0 & 1 \vdots & 0 & -1 \vdots & 0 & 0 & 1 \\ 0 & 0 & 0 \vdots & 0 & 0 \vdots & 0 & -1 & 1 \end{bmatrix} = 4$$

Then the system is R-controllable.

**Impulse controllability**

Four all mode, element in derivative causality ($q_c$) is not causally connected with the switches *Sw1* and *Sw2* so, the number of impulse modes equal to *0*.Then according to proposition 4 the system is structurally Impulse controllable.

**C-controllability**

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The system is structurally R- controllable and Impulse controllable then the system is structurally C- controllable.

## 6. CONCLUSION

In this paper, we have shown a very simple graphical method, based on the manipulation of the causal path leading to the determination of the equivalent explicit state equation of the singular state equation. From this result we have been able to extend the procedures of controllability analysis of switching systems to SLSS. On the other hand the controllability property is decomposed into R-controllability, impulse controllability and complete controllability. For that we have proposed two sufficient graphical conditions for R-controllability, and a procedure that allows an easy determination of impulse modes from a bond graph model. This procedure is based on simple causal manipulations on the equivalent bond graph model in integral and derivative causality. Finally, we have proposed a simple example illustrating our results.

The bond graph model appears to be an excellent tool for structural analysis, through its graphic character and its causal structure. It provides directly to the user information about controllability and observability, sometimes difficult to obtain by other routes.

The second graphical sufficient condition that we have proposed in this paper is limited to the case when all storage elements keep their initial causality during the commutation. Therefore, in our next work, we will take into consideration the changes of causality in the storage elements at the commutation time.

Other aspects which remain to be investigated are: -proposition of a feedback control of finite mode (slow subsystem) -compensation of infinite modes (fast subsystem), in the case where the system is impulse controllable.

## Appendix A. Proof of Theorem 2

The proof is similar to that of Yang's theorem in [4]

We consider the slow subsystem (5.a) and we assume that the $rank(x_1(t))$ is invariable for all mode, i.e. $n_1^{j+1} = n_1^j$ with $j \in \{1, \dots, q-1\}$.

Going through all the modes with $t_0 < t_1 < \cdots t_{q-1} < t_f$ the continuous state at $t_f$ can be expressed as:

$$x(t_f) = e^{G_q(t_f-t_{q-1})}e^{G_{q-1}(t_{q-1}-t_{q-2})} \dots e^{G_1(t_1-t_0)}x(t_0) + \int_{t_0}^{t_1} e^{G_q(t_f-t_{q-1})}e^{G_{q-1}(t_{q-1}-t_{q-2})} \dots e^{G_1(t_1-\tau)}H_1u(\tau)d\tau + \cdots + \int_{t_{q-1}}^{t_f} e^{G_q(t_f-\tau)}H_qu(\tau)d\tau \quad (A.1)$$

Note that $t_f = t_q$. Then, from (A.1) we can obtain the relation:

$$\bar{x}_f \triangleq x(t_f) - e^{G_q(t_f-t_{q-1})}e^{G_{q-1}(t_{q-1}-t_{q-2})} \dots e^{G_1(t_1-t_0)}x(t_0) \quad (A.2)$$

For $j = 1, \dots, q-1$ we define :

$$T_j \triangleq e^{G_q(t_q-t_{q-1})}e^{G_{q-1}(t_{q-1}-t_{q-2})} \dots e^{G_{j+1}(t_{j+1}-t_j)} \quad (A.3)$$

Such that $T_q = I$, where $I$ denotes the unitary matrix.

So the equation (A.1) can be transferred into:

$$\bar{x}_f = T_1 \int_{t_0}^{t_1} e^{G_1(t_1-\tau)}H_1u(\tau)d\tau + T_2\int_{t_1}^{t_2} e^{G_2(t_2-\tau)}H_2u(\tau)d\tau + \dots + T_q\int_{t_{q-1}}^{t_q} e^{G_q(t_q-\tau)}H_qu(\tau)d\tau \quad (A.4)$$

For $j^{th}$ term of (A.4) we define:

$$X_j \triangleq T_j\int_{t_{j-1}}^{t_j} e^{G_j(t_j-\tau)}H_ju(\tau)d\tau \quad (A.5)$$

On the other hand, the exponential matrix $e^{Gt}$ can be expressed as [26]:

$$e^{Gt} \triangleq b_0(t)I + \cdots + b_{n_1-1}(t)G^{n_1-1} = \sum_{i=0}^{n_1-1} b_i(t)G^i \quad (A.6)$$

Divide interval $[t_{j-1}, t_j)$ into $n_1$ subintervals, with property $t_{j-1,0} < t_{j-1,1} < \cdots < t_{j-1,n_1}$. It is noted that $t_{j-1,0} = t_{j-1}$ and $t_{j-1,n_1} = t_j$. And we can define the piecewise continuous input $u(t)$ as a piecewise constant function, denoted as $u(t) = U_{j-1,i}$ for $t \in [t_{j-1,i-1}, t_{j-1,i})$ where $U_{j-1,i} \in \Re^m$ for $i = 1,2, \dots, n_1$. Substitute the above-defined input $u(t)$ and equation (A.6) into (A.5), then we have:

$$X_j \triangleq T_jW_jF_jU_j \quad (A.7)$$

Where $W_j = [H_j \; G_jH_j \dots G_j^{n_1-1}H_j]$,

$$F_j = \begin{bmatrix} \int_{t_{j-1,0}}^{t_{j-1,1}} b_0^j(t_j-\tau)d\tau & \cdots & \int_{t_{j-1,n_1-1}}^{t_{j-1,n_1}} b_0^j(t_j-\tau)d\tau \\ \vdots & \ddots & \vdots \\ \int_{t_{j-1,0}}^{t_{j-1,1}} b_{n_1-1}^j(t_j-\tau)d\tau & \cdots & \int_{t_{j-1,n_1-1}}^{t_{j-1,n_1}} b_{n_1-1}^j(t_j-\tau)d\tau \end{bmatrix},$$

and $U_j = \begin{bmatrix} U_{j-1,1} \\ \vdots \\ U_{j-1,n_1} \end{bmatrix}$.

Where $b_k^j(t)$ for $k = 0,1,\dots,n_1 - 1$ are the expansion coefficients of $e^{Gt}$ as shown in (A.6).

By following the same process, equation (A.4) can be expressed as:

$$\bar{x}_f = T_1W_1F_1U_1 + T_2W_2F_2U_2 + \cdots + T_qW_qF_qU_q$$

$$= [T_1W_1 \quad \cdots \quad T_qW_q]\begin{bmatrix} F_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & F_q \end{bmatrix}\begin{bmatrix} U_1 \\ \vdots \\ U_N \end{bmatrix} \quad \text{(A.8)}$$

Denote $\gamma_j \triangleq t_j - t_{j-1}$ for $j = 1,\dots,q$ then with respect to (A.3), we have $\lim_{\gamma_{j+1},\dots,\gamma_q \to 0} T_j = I$.

When $\gamma_j$ for $j = 1,\dots,q$ are small enough, we can get:

$$rank([T_1W_1 \quad \cdots \quad T_qW_q]) \geq rank([W_1 \quad \cdots \quad W_q]) \quad \text{(A.9)}$$

Considering the assumption that the controllability matrix $W_{RC}$ is of full row rank, then we have $rank[T_1W_1 \quad \cdots \quad T_qW_q] = n_1$.

Summing up the above analysis, we can see that there exists a timed mode-switching sequence $\{\sigma_j, t_{j,n_1}, \sigma_{j+1}\}_{j=1}^{q-1}$, and a corresponding piecewise continuous input signal $u(t) = U_{j-1,i}$ for $t \in [t_{j-1,i-1}, t_{j-1,i})$ with $j = 1,\dots,q$ and $i = 1,\dots,n_1$, they make hybrid state $(\sigma_f; x_f)$ reachable from $(\sigma_0; x_0)$ within period $[t_0; t_f]$. Here, the proper selection of $t_{j-1,i}$ for $i = 1,\dots,n_1 - 1$ makes the corresponding $F_j$ nonsingular; the proper selection of $t_{j-1,n_1}$ for $j = 1,\dots,q$ makes condition (A.9) satisfied and the assignment of $U_{j-1,i}$ is the corresponding solution of (A.8) when $x(t_f) = x_f$ and $x(t_0) = x_0$.

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