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HEART RATE REGULATION BASED ON T-S FUZZY CONTROLLER

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ABSTRACT

The control of human heart rate (HR) during exercise is an important issue for athletics and assessing physical fitness, weight control, cardiovascular patients and the prevention of heart failure. A T-S type fuzzy model for a nonlinear model of (HR) response that describes the central and peripheral local responses during and after treadmill exercise is constructed and followed by designing a fuzzy controller based on parallel distributed compensation (PDC) method. The state variable is reconstructed using a T-S fuzzy observer. Linear matrix inequality (LMI)-based and Takagi-Sugeno (T-S) model-based fuzzy approach is applied. To relevant simulations are made to verify the effectiveness of this proposed fuzzy controller.

Keywords: *Heart Rate (HR), T-S Fuzzy Model, T-S Fuzzy Controller, T-S Fuzzy Observer, Tracking Error*

1. INTRODUCTION

There exist many studies on how to model (HR) during exercise [1-6]. The modeling of (HR) during exercise are presented in [1-2] where applied feedforward and feedback components to describe their models [7]. The drawback of these works is in their short period of time which is not sufficient for explaining responses in comparison to long period. According to [3-5], the (HR) will increase during exercise. There are some factors contribute to this fact like an increase in body temperature, loss of body fluid and metabolism. A Hammerstein system is applied in [6] to model the (HR) response during exercise. The main drawback of this model is describing (HR) responses for short-duration exercise. Moreover, they studied the regulation of the (HR) response during exercise.

A nonlinear dynamical model of (HR) variation during treadmill exercise is discussed where the emphasis is a second-order dynamics model whereas both the central and local peripheral response effects on (HR) are of major concern with respect to the effects of long duration exercises on the (HR)[8].

To determine exercise schedule for patients, it is important to control the (HR) during exercise. Attempts are made to control the (HR) During exercise in [6], [10] and [11]. PID control in a negative closed loop is defined in [10]. H-infinity controller is designed for approximated linear model to achieve robust tracking performance by [6]. The input nonlinearity is neglected through Hammerstein systems. A model reference adaptive control (MRAC) algorithm is defined in [11]. A feedback control law for the treadmill speed is designed where model input is the treadmill speed. The drawback here is that the controller approximates the nonlinear model through a linear model [9].

The (HR) response to exercise has nonlinear behavior and it gains significant special for cardiovascular patients. All articles before 2011 applied linear approximation to control (HR) response and linear control techniques do not fit in such circumstances.

A nonlinear feedback controller for the (HR) response during treadmill exercise is introduced, where robust techniques and tracking control are applied for nonlinear model [12]. This control design does not rely on linear approximation. The Lyapunov-type stability arguments are applied to design the continuous, model-based, nonlinear feedback laws for treadmill speed to ensure that an ideal (HR) profile is tracked in an exponentially fast manner.

Fuzzy control systems have been used in a wide variety fields in control and bio systems [14-15]. It has also applied in many different aspects of control systems and are being presented as an important tool to control nonlinear systems. The (PDC) method based on T-S model is designed to control nonlinear model of systems where for preparation first, model of system must be changed into a fuzzy model and next, a fuzzy controller would be realized to control the closed loop system. In this method, the nonlinear systems are described as a collection of Linear Time Invariant (LTI) models combined with nonlinear functions named weighting functions [16].

The object of this article is to design a PDC controller based on T-S fuzzy model for (HR) regulation during treadmill exercise under constrained control signal. In addition, a T-S fuzzy observer is designed to estimate the states. To illustrate the advantages of this controller the uncertainties in parameters are implemented, indicating a stable tracking error.

This paper first gives a brief overview of the recent activities of heart rate modeling and controlling. It will then go on to describe T-S fuzzy controller and observer in section 2. Section 3 looks at (HR) nonlinear model and T-S fuzzy model of (HR) and design of fuzzy observer for (HR). To analyze the proposed method we bring the simulation in section 4. Finally the conclusion is in part 5.

2. T-S FUZZY CONTROLLER AND OBSERVER DESIGNING

2.1. General Design of Fuzzy Model

The T-S fuzzy model is described by "$IF - THEN$" rules. Local input-output relations of a nonlinear system are presented by T-S fuzzy model. Using nonlinear sector in fuzzy model construction is the main idea of T-S fuzzy modeling. An exact fuzzy model is made by this method. The closed-loop fuzzy system is defined by "r" plant rules as follow [16]:

$Plant\ Rule\ i:$

$IF\ z_1(t)\ is\ M_{i1}\ and \dots and\ z_\mu(t)\ is\ M_{i\mu}\ THEN$

$$\begin{cases} \dot{\tilde{x}}(t) = A_i \tilde{x}(t) + B_i u(t) \\ y(t) = C_i \tilde{x}(t) \end{cases} \quad (1)$$

where, the terms $z(t) = [z_1(t), \dots, z_\mu(t)]^T$ are the premise variables and $M_{ij}(j = \{1, \dots, \mu\})$ are the membership functions of fuzzy sets. $\tilde{x}(t) \in R^n$ is state, $y(t) \in R^p$ and $u(t) \in R^m$ are the output and input respectively. $A_i \in R_{n\times n}$, $B_i \in R_{n\times m}$ and $C_i \in R_{p\times n}$ are the constant matrices. Logic operator "and" is often chosen as the product. The final output of the fuzzy system is presented as:

$$\begin{cases} \dot{\tilde{x}} = \dfrac{\sum_{i=1}^{r} \omega_i(z(t))\,(A_i \tilde{x}(t) + B_i u(t))}{\sum_{i=1}^{r} \omega_i\,(z(t))} = \\ \displaystyle\sum_{i=1}^{r} h_i(z(t))\,(A_i\tilde{x}(t) + B_i u(t)) \\ y(t) = \displaystyle\sum_{i=1}^{r} h_i\big(z(t)\big) C_i \tilde{x}(t) \end{cases} \quad (2)$$

The term $h_i(z(t))$ specifies the contribution of the local corresponding model to the global model is determined by the following [16]:

$$h_i(z(t)) = \frac{\omega_i\,(z(t))}{\sum_{i=1}^{r} \omega_i\,(z(t))} \quad (3)$$

where, $\omega_i\big(z(t)\big) = \prod_{j=1}^{\mu} M_{ij}(z_j(t))$, $j = 1, \dots, \mu$.

2.2. T-S Fuzzy Model-based Controller

2.2.1. Unconstrained T-S fuzzy controller

In most studies, (PDC) controller is utilized as fuzzy controller design for T-S fuzzy system. The PDC is a model-based controller applied to stabilize T-S fuzzy model. The (PDC) decomposes the whole state space into several fuzzy subspaces and designs a controller for the local fuzzy subsystems. The control of the whole

system is a weighted sum of local control. The PDC controller is presented as follows [16]:
$Controller\ Rule\ i$:

IF

z_1 is M^{i1} and $\cdots$ and z_μ is $M^{i\mu}$

$THEN$

$$u(t) = K_i \tilde{x}(t). \quad (4)$$

where, $u(t)$ is the output of the PDC control in the following form:

$$u(t) = \sum_{i=1}^{r} h_i\big(z(t)\big) K_i \tilde{x}(t) \quad (5)$$

where K_i is the control gain for the i^{th} controller rule.. By determining $G_{ij} = (A_i + B_i K_j)$ and applying control rules to model of closed-loop system's model (2) is in the following form:

$$\dot{\tilde{x}} = \sum_{i=1}^{r} h_i^2\big(z(t)\big) G_{ii} \tilde{x}(t) + \sum_{i=1}^{r} \sum_{i<j}^{r} h_i(z(t)) h_j(z(t)) (G_{ij} + G_{ji}) \tilde{x}(t) \quad (6)$$

where, closed loop stability has been achieved by theorem1.

Theorem1. The equilibrium of a fuzzy system (6) is asymptotically stable if there exists a common positive definite matrix P such that the following set of *Linear Matrix Inequalities* (LMIs) is met [14]:

$$\begin{cases} G_{ii}^T P + P G_{ii} < 0 & i = 1, \dots, r \\ (\frac{G_{ji}+G_{ij}}{2})^T P + P\left(\frac{G_{ji}+G_{ij}}{2}\right) \le 0 & i < j \end{cases} \quad (7)$$

where, $G_{ij} = A_i + B_i K_j$. K_i is calculated with respect to this theorem. (Proven in [14]).

2.2.2. Constrained controller

The subject of this section is at the attendance of control constraint, to design the controller in a manner that the global system becomes asymptotically stable. Consider the following nonlinear system with constrained control that is described by the T–S fuzzy model as detailed in (1). The control signal u is constrained as follow [14]:

$$-q_2 \le u \le q_1;\ q_1, q_2 \in R^m \quad (8)$$

where, $u(t) = Fx(t)$. According to this definition the model of closed-loop system (2) converts into the following form:

$$\dot{\tilde{x}}(t) = (A + BF)\tilde{x}(t) \quad (9)$$

where, the set of F_i is calculated with respect to eq. (10), (11).

$$F(A + BF) = HF\ ;\ H \in R^{m\times m} \quad (10)$$

$$Hq \le 0 \quad q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}. \quad (11)$$

An algorithm to build this controller is the resolution of algebraic equation $XA + XBX = HX$ [17].

2.3. T-S Fuzzy Observer:

An observer is designed to remake the state vector of a system from known inputs, outputs, and its dynamic model. The T-S fuzzy observer rules are defined as below [16]:

$Observer\ Rule\ i$:

If

$z_1(t) is\ M_{i1}\ and \dots, and\ z_\mu(t) is\ M_{i\mu}$

$Then$

$$\dot{\hat{x}} = A_i \hat{x}(t) + B_i u(t) + L_i (y - \hat{y})$$
$$\hat{y} = C_i \hat{x}(t) \quad (12)$$

where, $\hat{x}$ is the T-S fuzzy estimate of state vector and $\hat{y}$ is the T-S fuzzy estimate of output vector and L_i are the gain matrices of fuzzy observer calculated by LMI techniques explained in (14). The final T-S fuzzy observer is defined as follow:

$$\dot{\hat{x}} = \sum_{i=1}^{r} h_i(z(t)) (A_i \hat{x}(t) + B_i u(t) + L_i (y - \hat{y}))$$
$$\hat{y} = \sum_{i=1}^{r} h_i\big(z(t)\big) C_i \hat{x}(t) \quad (13)$$

The stability conditions for closed-loop T-S fuzzy systems and observer gains are achieved by theorem 2.

Theorem2. T–S fuzzy observer (13) is asymptotically stable, provided that there exist symmetric matrix $P > 0$, meeting the following inequalities [16]:

$$\begin{cases} (A_i - L_iC_i)^T P + P(A_i - L_iC_i) + S_{ii} < 0; \quad i = 1,\dots,r \\ \left(\frac{A_i - L_iC_j + A_j - L_jC_i}{2}\right)^T P + S_{ij}^{\ T} + \\ P\left(\frac{A_i - L_iC_j + A_j - L_jC_i}{2}\right) + S_{ij} \le 0; \ i < j \end{cases}$$

$$\begin{bmatrix} S_{11} & \cdots & S_{1r} \\ \vdots & \ddots & \vdots \\ S_{r1} & \cdots & S_{rr} \end{bmatrix} > 0 \qquad (14)$$

where, $S_{ij} = S_{ji}^{\ T}$ and S_{ii} are the symmetric matrices (Proven in [16]).

Furthermore, the controller "u" is based on $\hat{x}$ (the estimate of the state vector) and not x (the vector state of real model) presented as:

$$u(t) = \sum_{i=1}^{r} h_i(z(t))K_i\hat{x}(t) \qquad (15)$$

where, control gains (K_i) are obtained by:

$$G_{ii}^{\ T}P_1 + P_1G_{ii} + Q_{ii} < 0$$

$$((G_{ij} + G_{ji})/2)^T P_1 + P_1\left(\frac{G_{ij}+G_{ji}}{2}\right) + Q_{ij} + Q_{ij}^{\ T} \le 0 \quad i < j$$

$$\begin{bmatrix} Q_{11} & \cdots & Q_{1r} \\ \vdots & \ddots & \vdots \\ Q_{r1} & \cdots & Q_{rr} \end{bmatrix} > 0 \qquad (16)$$

where, $G_{ij} = A_i + B_iK_j$, $P_1 > 0$. Q_{ii} are the symmetric matrices and $Q_{ij} = Q_{ji}^{\ T}$.

3. T-S FUZZY CONTROLLER AND OBSERVER FOR HEART RATE MODEL

The nonlinear model of (HR) is defined as [12]:

$$\begin{cases} \dot{x}_1 = -a_1x_1 + a_2x_2 + a_6{u_s}^2 \\ \dot{x}_2 = -a_3x_2 + a_4\frac{x_1}{1+e^{-(\bar{c}x_1-a_5)}} \end{cases} \qquad (17)$$

where, the deviation of the (HR) from the at-rest (HR) is modeled by the state x_1 and x_2 represents the local peripheral effects on the (HR). The control signal u_s is the treadmill speed. The parameters a_i $i = 1,\dots,6$ and $\bar{c}$ are positive.

The nonlinear reference model of the (HR) here is similar to [12]:

$$\begin{cases} \dot{x}_{1r} = -a_1x_{1r} + a_2x_{2r} + a_6{u_r}^2 \\ \dot{x}_{2r} = -a_3x_{2r} + a_4\frac{x_{1r}}{1+e^{-(\bar{c}x_{1r}-a_5)}} \end{cases} \qquad (18)$$

where, x_{1r}, x_{2r} present reference trajectories for x_1, x_2. u_r is a desired treadmill speed and reference trajectory for u. In figure. (1), u_r is shown. u_r contains three sections: warming up (300 sec), exercise (500 sec) with constant speed, cooling down (100 sec).

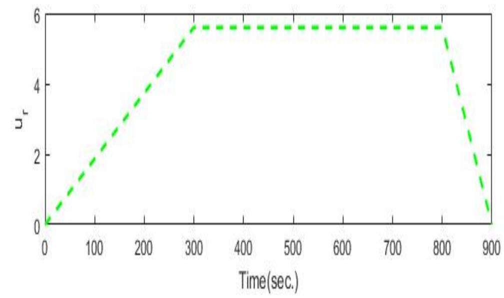

Figure. (1): Reference control input u_r

x_{1r} the reference model of x_1 is shown in figure. (2).

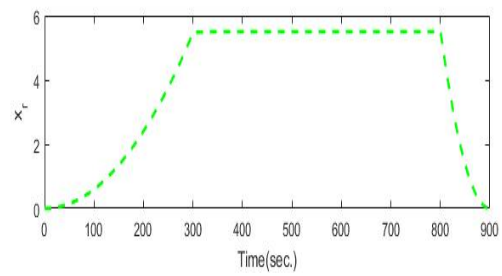

Figure. (2): Reference model (x_{r1})

The tracking error variable $\tilde{x} = (\tilde{x}_1, \tilde{x}_2) = (x_1 - x_{1r}, x_2 - x_{2r})$ are the dynamical model presented as [12]:

$$\begin{cases} \dot{\tilde{x}}_1 = -a_1\tilde{x}_1 + a_2\tilde{x}_2 + a_6({u_s}^2 - {u_r}^2) \\ \dot{\tilde{x}}_2 = -a_3\tilde{x}_2 + a_4\left(\frac{x_1}{1+e^{-(\bar{c}x_1-a_5)}} - \frac{x_{1r}}{1+e^{-(\bar{c}x_{1r}-a_5)}}\right) \\ \quad = -a_3\tilde{x}_2 + a_4R\tilde{x}_1 \end{cases} \qquad (19)$$

where, $R(\tilde{x}_1, t) = \frac{1+be^{-\bar{c}x_{1r}(t)}\,[1+\bar{c}x_{1r}(t)\int_{-1}^{0}\bar{c}\tilde{x}_1 m\, dm]}{(1+be^{(\tilde{x}_1-\bar{c}x_{1r}(t))})(1+be^{-\bar{c}x_{1r}(t)})}$ (20)

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The error dynamical model is rewritten as follow:

$$\begin{bmatrix}\dot{\tilde{x}}_1\\ \dot{\tilde{x}}_2\end{bmatrix}=\begin{bmatrix}-a_1 & a_2\\ a_4R & -a_3\end{bmatrix}\begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2\end{bmatrix}+\begin{bmatrix}1\\0\end{bmatrix}u$$
(21)

where, $u = a_6(u_s{}^2 - u_r{}^2), (u > 0)$.

T-S fuzzy rules are based on the nonlinear part of the dynamical model. The nonlinear part in error dynamical model is defined as $z(t) = a_4R$. The number of the rules based on minimum and maximum of $z(t)$ is determined based on tracking error. Matrices A_1 and A_2 are defined based on $z(t)$ and parameters that are shown as:

$$A_1 = \begin{bmatrix}-a_1 & a_2\\ \min z(t) & -a_3\end{bmatrix}, \quad A_2 = \begin{bmatrix}-a_1 & a_2\\ \max z(t) & -a_3\end{bmatrix}$$
(22)

From equation (2) the T-S fuzzy model of (HR) is written as:

$$\begin{cases}\dot{\tilde{x}} = h_1(z(t))(A_1\tilde{x}+B_1u)+h_2(z(t))(A_2\tilde{x}+B_2u)\\ y = h_1(z(t))(C_1\tilde{x})+h_2(z(t))(C_2\tilde{x})\end{cases} \qquad (23)$$

And according to equation (13), T-S fuzzy observer is written as:

$$\begin{cases}\dot{\hat{x}} = h_1(z(t))(A_1\hat{x}+B_1u+L_1(y-\hat{y})+\\ \quad h_2(z(t))(A_2\hat{x}+B_2u+L_2(y-\hat{y}))\\ \hat{y} = h_1(z(t))(C_1\hat{x})+h_2(z(t))(C_2\hat{x})\end{cases} \qquad (24)$$

where, $h_1(z(t))$ and $h_2(z(t))$ are obtained from (3).

4. SIMULATION AND RESULTS

Here, the model parameters consist of
$a_1 = 2.2,\ a_2 = 19.96,$
$a_3 = 0.083, a_4 = 0.002526$
$a_5 = 8.32, a_6 = 0.38$,
$b = e^{a_5}$, $c = 1$
Initial conditions are $x = [2.0]$ and $x_r = [0,0]$[12].

In figure (3), premise variable $z(t) = a_4R$ is presented:

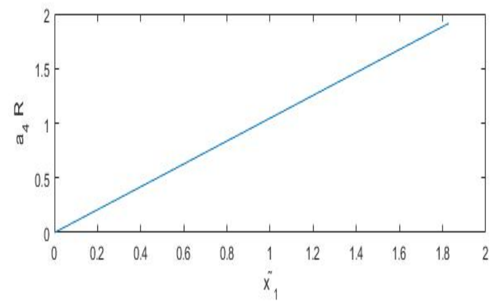

Figure. (3): premise variable $z(t) = a_4R$

In eq. (23 and 24), A_1 and A_2 are defined as follow based on $z(t)$:

$$A_1 = \begin{bmatrix}-2.2 & 19.96\\ \min z(t) & -0.0831\end{bmatrix} = \begin{bmatrix}-2.2 & 19.96\\ 0 & -0.0831\end{bmatrix}$$

and

$$A_2 = \begin{bmatrix}-2.2 & 19.96\\ \max z(t) & -0.0831\end{bmatrix} = \begin{bmatrix}-2.2 & 19.96\\ 1.912 & -0.0831\end{bmatrix}.$$

$B_1 = B_2 = [1 \quad 0]^T$

and $C_1 = C_2 = [1 \quad 0]$.

In figures. (4 and 5), the fuzzy membership function M_1 and M_2 are presented as:

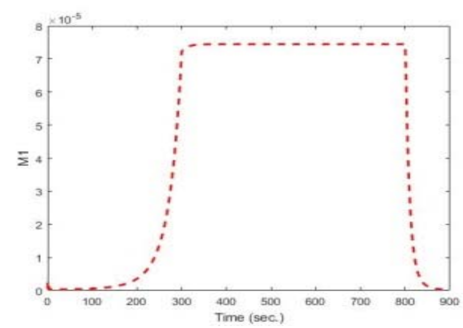

Figure. (4): Membership function "M_1"

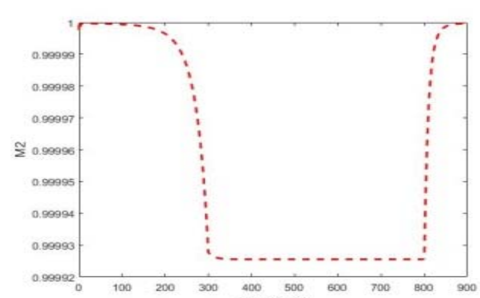

Figure. (5): Membership function "M_2"

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The observer and controller are defined by eq. (24 and 15) respectively. Designing controller and observer gains are calculated by LMI (represented by (16) and (14)) subject to theorem (3). The obtained results are:

$$K_1 = [-4.9266 \quad -31.0970],$$
$$K_2 = [-6.6885 \quad -39.6708]$$
$$L_1 = [26.5061 \quad 7.1978]^T,$$
$$L_2 = [27.3651 \quad 8.6258]^T.$$

Figures. (6 and 7) represent observer variable $(\hat{x}_1)$ and tracking error variable $(\tilde{x}_1)$. As observed the observer variable trajectory follows error variable as if they are the same after 1 sec.

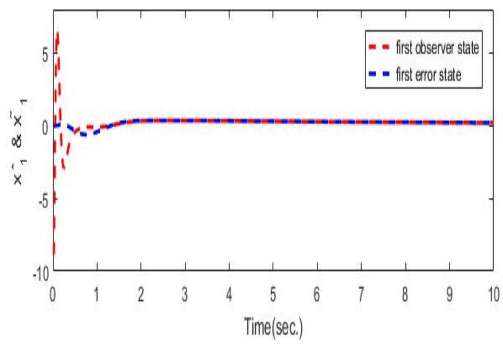

Figure. (6): Observer variable $(\hat{x}_1)$ and tracking error variable $(\tilde{x}_1)$

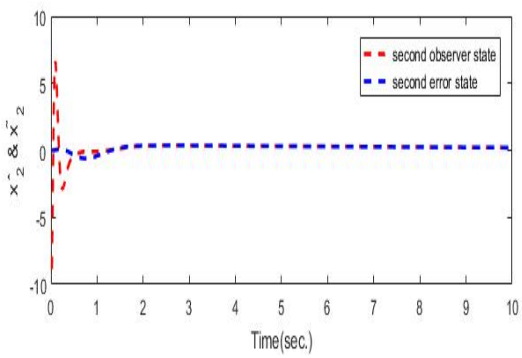

Figure. (7): Observer variable $(\hat{x}_2)$ and tracking error variable $(\tilde{x}_2)$

Tracking error variable is shown in figure. (8), which begins to approach zero in about 6 seconds, indicating that the system is stable due to this controller. This newly introduced control signal u is shown in figure. (9); according to this signal, the real control input (u_s^2) is obtained and shown in figure. (10).

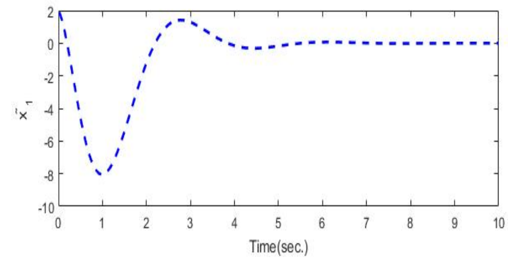

Figure. (8): First state of tracking error variable $(\tilde{x}_1)$

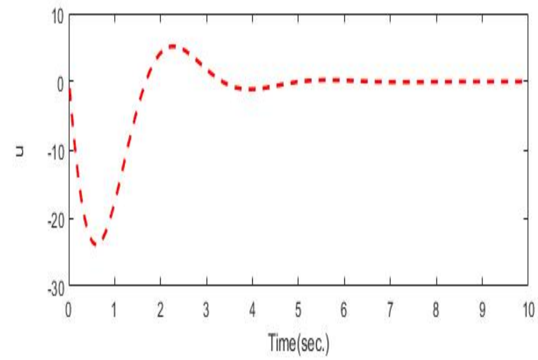

Figure. (9): T-S fuzzy control input (u)

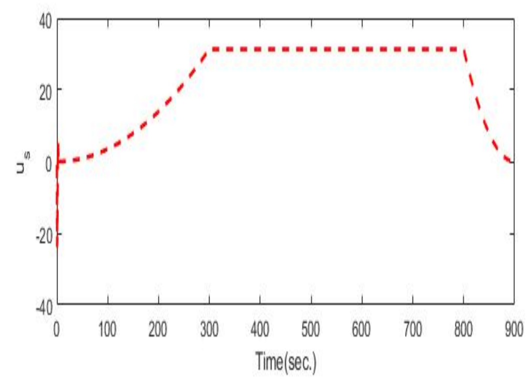

Figure. (10): Control input signal (u_s^2)

u_s^2 must be a positive value but as observed in figure. (10), at (0-10) seconds it is non-positive. This phenomenon can be corrected through two conditions:

1. Using saturated function and 2. Using constrained control signal

1. Using saturated function:

To have positive values for "u_s", equation (25) can be used as a saturation function. This

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equation guaranties that" u_s" is a positive value although the stability condition may be lost.

$$u = \begin{cases} u & u > -u_r^2 \\ -u_r^2 & u < -u_r^2 \end{cases}$$
(25)

Based on eq. (25), fuzzy control input is shown in figure (11) and control input u_s^2 and tracking error variable are shown in figures. (12 and 13) respectively.

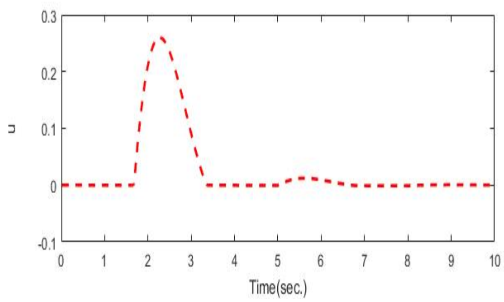

Figure. (11): T-S fuzzy control input (u)

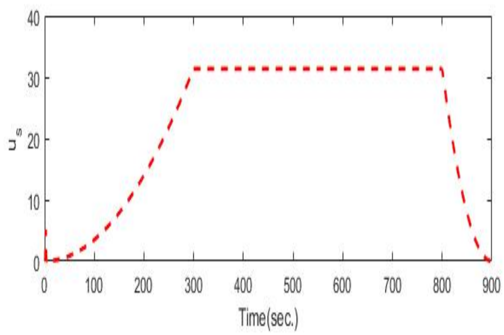

Figure. (12): Control input signal (u_s^2)

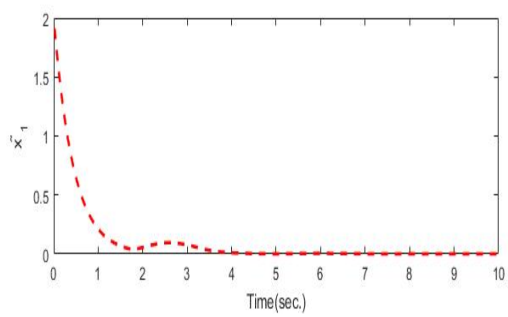

Figure. (13): Tracking error variable ($\tilde{x}_1$)

In this case, tracking error variable is globally exponentially stable even though the system is not stable. In condition (2) error variables approach zero more rapid than condition (1).

2. Using constrained control signal:

Suppose that $q_2 = 0.1, q_1 = 30$ (determined in eq. (11)) have been used for upper and lower limits of constrained control signal. By new controller gains $F_1 = [-0.8000, 7.5405]$ and $F_2 = [-13.1440, -50.3689]$ control signal u_s is a non-negative value (figures. 14 and 15)

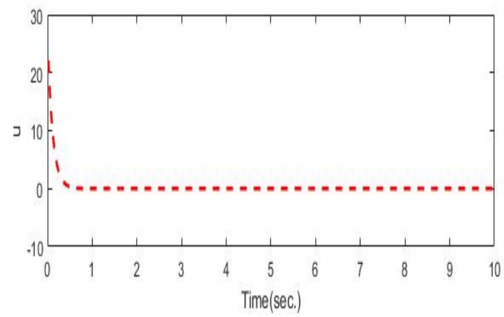

Figure. (14): T-S fuzzy control input (u)

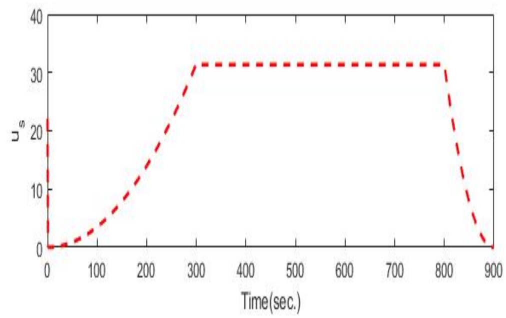

Figure. (15): Control input signal (u_s^2)

Tracking error ($\tilde{x}_1$) is shown in figure. (16). Closed loop system is stable as well.

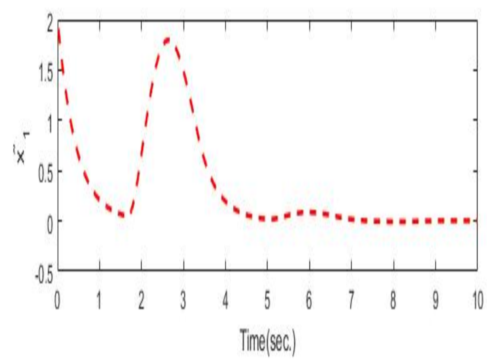

Figure. (16): Tracking error variable ($\tilde{x}_1$)

As shown in figures. (13 and 16) compared with using saturated function, tracking error variables is globally exponentially stable in the shortest time subject to constrained control signal.

In order to show the efficiency of this proposed method, the parameter uncertainties are of concern in this study. The tracking error when the parameters of system are subject to uncertainties is shown in figure. (17-20). Uncertainties are considered as: $(a_i \in [a_i - 0.1a_i, a_i + 0.1a_i]\ i = 1, \dots, 6)$.

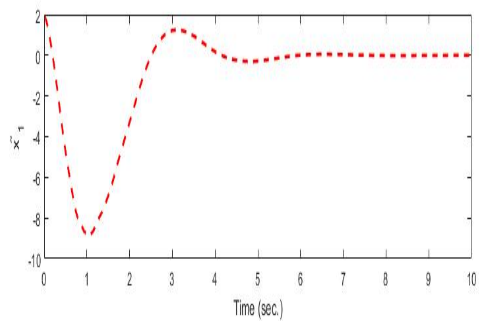

Figure. (17): Tracking error variable ($\tilde{x}_1$) subject to uncertainties

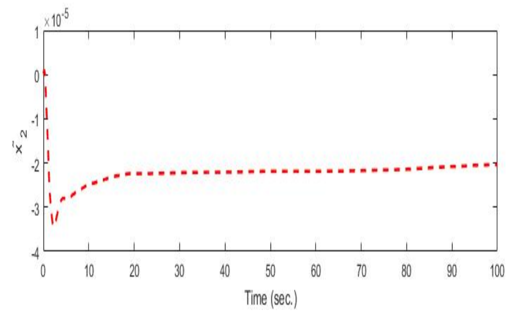

Figure. (18): Tracking error variable ($\tilde{x}_2$) subject to uncertainties

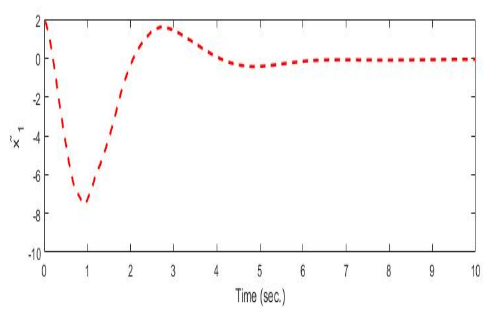

Figure. (19): Tracking error variable ($\tilde{x}_1$) subject to uncertainties

It is observed that tracking error variables trajectory are globally exponentially stable subject to parameter uncertainties.

Tracking error variable signal have the shortest convergence time to zero subject to constrained control, in comparison to nonlinear feedback controller is designed in [12]. The result of fuzzy observer is better than observer design in [12]. It shows the efficiency of PDC controller and its controlling ability for (HR) nonlinear system.

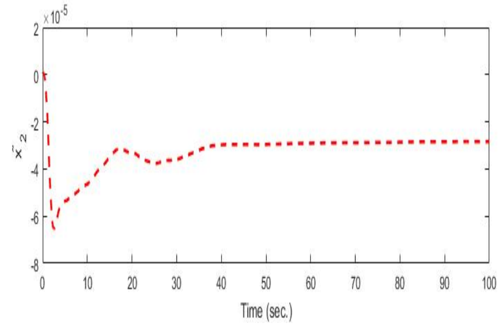

Figure. (20): Tracking Error Variable ($\tilde{x}_2$) Subject To Uncertainties

5. CONCLUSION

In this article, a fuzzy model-based controller is designed for a nonlinear model of human (HR) according to the fuzzy membership function and PDC method. Sufficient stability conditions are presented based on LMI formulation. Further analyses indicated that tracking error is globally exponentially stable. Since (HR) is a nonlinear model, this proposed controller is designed on nonlinear basis. On one hand, T_S fuzzy model-based controller designed here is a nonlinear controller, on the other, tracking error variable trajectory is globally exponentially stable even where there is uncertainty in the parameters. The simulation results indicate the applicability and effectiveness of this propose method.

In the future work, completed heart rate models can be considered. Disturbance observer control (linear or nonlinear) may also considered to reject un-wanted disturbance in the model.

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