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# ALGEBRA FOR TRANSFORMING THE STRUCTURE OF THE SEARCH SPACES ASSOCIATED WITH THE COMBINATORIAL OPTIMIZATION PROBLEMS

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#### ABSTRACT

This article proposes the classification of the search spaces associated with the combinatorial optimization problems based on the type of their constituent solutions. The spaces belonging to each identified class are accompanied by the corresponding graph models. Against this background, the article introduces the original algebra allowing the representation of the search spaces in the unified homogeneous form. The proposed algebra consists of a set of transformations given in an analytical form and illustrated by the modifications of the graph models constructed for the search spaces.

Keywords: Combinatorial Optimization Problem, Search Space, Bipartite Graph, Multigraph, Graph.

## 1. INTRODUCTION

Solving the combinatorial optimization problems constitutes the keystone of modeling and designing the computer-based information systems. For example, the problem of the VLSI circuits partitioning could be reduced to the problem of finding the cuts in hypergraphs [1]. In turn, the quality of its solving significantly affects the properties of the manufactured devices such as the energy consumption, delays, cost of producing, etc.

Similarly, the construction of the large-scale GRID and cloud computing infrastructures is inextricably linked with solving the tasks scheduling and load balancing problems in order to ensure the optimal usage of the available resources [2]. Another prime example is associated with scheduling packets in the wireless networks equipped with the relay nodes, which represents the variation of the multiple-choice multi-dimensional knapsack problem [3, 4].

At the same time, a number of the known combinatorial optimization problems formulated in the decision form are *NP*-complete [5, 6]. Due to the combinatorial explosion effect, the algorithms returning their exact solutions have the exponential time complexity, which underlies the inefficiency of their application to the large-scale problem instances. As a result, the efforts to solve such problems are intrinsically linked with the development of the metaheuristic algorithms aimed

to produce the approximate solutions in a reasonable time. The fundamental strategy followed by such algorithms lies in manipulating by the solutions in order to shrink the explored search space. Looking into more detail, the search process implemented by the metaheuristics is based on performing the iterative improvements of the single current solution (e.g. simulated annealing, guided local search, tabu search) or population of solutions (e.g. evolutionary algorithms) [7].

The implementations of these algorithms are extremely sensitive to the representation of the solutions comprising the search spaces associated with the instances of the problems. However, the natural form of solutions encoding is problemspecific, which sufficiently complicates processing and analyzing the corresponding search spaces [8]. For example, the solutions of the Boolean satisfiability problem could be easily encoded by the bit vectors indicating the values taken by the variables. On the contrary, the solutions of the traveling salesman problem naturallv are represented by the permutations of the graph's vertices [7, 9]. Notice that the form of solutions encoding is particularly acute when designing the adaptations of the metaheuristics for the specialized computer architectures such as the graphical processing units due to the need for managing the memory allocation [10].

Clearly, the sets of the equal-length strings defined over the binary alphabet constitute the most

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canonical way for representing the search spaces. However, for a number of problems such encoding requires performing the non-trivial transformations of the search spaces. Therefore, *the objective of this article* is to propose the algebra for transforming the structure of the search spaces to the unified homogeneous form.

#### 2. CLASSIFICATION OF THE SEARCH SPACES ASSOCIATED WITH THE COMBINATORIAL OPTIMIZATION PROBLEMS

The primary challenge in achieving the intended objective is that the well-known definitions of the combinatorial optimization problem [11, 12] are formulated without detailing the structure of the search spaces. Consequently, due to such insensitivity, these definitions could not be used as a basis for classifying the search spaces and their constituent solutions. This causes the need for introducing the new more detailed definition of the combinatorial optimization problem, which constitutes the first significant contribution of this article.

**Definition 1.** The combinatorial optimization *problem* is represented by a set of instances given in the form of pentuples  $I = \langle \mathcal{G}, P, \varphi, \Phi, \Psi \rangle$ , where  $\mathcal{G}$  denotes the finite (or countably infinite) search space associated with the particular instance. This space encompasses the combinatorial configurations X and is constructed by applying the generating combinatorial operator  $\mathcal{F}$  to the finite (or countably infinite) base set  $P = \{p_1, ..., p_n\}$ . Moreover, each instance I is equipped with the indicator function  $\varphi: \mathcal{G} \to \{0,1\}$  taking the value of 1 for all configurations  $X' \in \mathcal{G}$  satisfying the constraints of the problem. The optimality of all elements  $X \in \mathcal{G}$  is reflected by the family  $\Phi = \{f_1, ..., f_m\}$  of the objective functions  $f_i: \mathcal{G} \to \mathbf{R}$  defined over the entire configuration space  $\mathcal{G}$  and the collection  $\Psi = \{extr_1, ..., extr_m\}$  of the optimization criterions  $extr_i = \{\min, \max\}$  for all functions  $f_i$ .

In the context of such formulation, the configurations X comprising the search space  $\mathcal{G}$  represent all possible solutions of the problem instance I. Notice that according to Definition 1, the elements of both set P and space  $\mathcal{G}$  are isolated from each other. Obviously, there always exists the uncountable infinite set  $U \supset P$  in which any

element  $p \in P$  could be surrounded by the nonempty punctured neighborhood  $N(p) \subseteq U$  such that  $N(p) \bigcap P = \emptyset$ . The isolation of all configurations  $X \in \mathcal{G}$  could be demonstrated in a similar manner. This clearly shows that Definition 1 does not cover the continuous optimization problems.

Remark that by virtue of introducing the families of the functions  $\Phi$  and criterions  $\Psi$ , Definition 1 is sufficiently flexible to cover the multi-objective optimization problems. At the same time, in the simplest case of the single-objective problems, the families  $\Phi$  and  $\Psi$  contain respectively just one function f and criterion *extr*, i.e. m = 1. As a consequence, the instances of such problems could be represented in the simplified form  $I = \langle \mathcal{G}, P, \varphi, f, extr \rangle$ .

Let us denote by  $\mathcal{G}' \subseteq \mathcal{G}$  the set of all feasible solutions satisfying the constraints of the problem instance *I*, i.e.  $\varphi(\mathcal{G}') = 1$ . Under such approach, the exact solution for the instance *I* of the singleobjective problem takes the form of the configuration  $X'_0 \in \mathcal{G}'$  such that the value  $f(X'_0)$ constitutes the global minimum or maximum (depending on the criterion *extr*) of the objective function *f* over the set  $\mathcal{G}'$ . Notice that the instance of the combinatorial optimization problem might be deprived of the feasible solutions in the case of  $\mathcal{G}' = \emptyset$ .

The generating combinatorial operator  $\mathcal{F}$ ensures filling the configuration space G with the structures implementing different forms of the relationship between the elements of the base set P(such as the combinations, permutations, selections, etc.). Accordingly, the instances of the combinatorial optimization problems could be equipped with the configuration spaces  $\mathcal{G}$  of various types, which substantially differ in representing the elements  $X \in \mathcal{G}$ . In spite of the huge variety of possible concrete implementations of the  $\mathcal{F}$  operator, we can indicate four fundamental classes of the search spaces G. Let us introduce the notations  $\mathcal{G}^{(i)}$  and  $X^{(i)}$  for the spaces belonging to the *i*-th class and their constituent configurations. Remark that the search space  $\mathcal{G}^{\prime(i)}$  composed of the feasible solutions  $X'^{(i)}$  belongs to the same class as the complete space  $\mathcal{G}^{(i)} \supset \mathcal{G}^{\prime(i)}$ .



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Looking into detail, the classification proposed in this article distinguishes the classes whose representatives are:

- 1. The spaces  $\mathcal{G}^{(1)}$  whose elements are represented simply by the unordered sets  $X^{(1)} = \{x_1, ..., x_t\} \subseteq P$  deprived of the repeated elements.
- 2. The search spaces  $\mathcal{G}^{(2)}$  composed of the multisets that allow the multiple occurrences of the constituent elements but are indistinguishable from the perspective of their order. The solutions contained in such spaces have the form of pairs  $X^{(2)} = \langle X_U, \theta \rangle$  including of the underlying ordinary set of elements  $X_U \subseteq P$  associated with the multiplicity function  $\theta : X_U \to \mathbf{N}_{\geq 1}$ . The image of any element  $p \in X_U$  under the function  $\theta$  reflects the number of all its occurrences in the configuration  $X^{(2)}$ .
- 3. The search spaces  $\mathcal{G}^{(3)}$  comprised of the chains that are sensitive to the order of the elements but do not presuppose their duplications. In this case, each configuration is given by the set  $X_U \subseteq P$  paired with the function  $\delta : X_U \rightarrow \{1, 2, ..., |X_U|\}$  reflecting the order of elements, i.e.  $X^{(3)} = \langle X_U, \delta \rangle$ . In particular,  $\delta(p) = t$  shows that the element  $p \in X_U$  occupies the *t*-th position in the solution  $X^{(3)}$ .
- 4. The spaces  $\mathcal{G}^{(4)}$  whose elements are represented in the most sophisticated form of the "ordered multisets" encapsulating the information about the order of the elements that could be repeated. In turn, the solutions comprising such spaces could be represented by quadruples  $X^{(4)} = \langle H, X_U, \theta, \lambda \rangle$ , where Hdenotes the auxiliary set equipped with the function  $\lambda : H \rightarrow \{1, 2, ..., |H|\}$  specifying the order of its elements. On the other hand, the set H is constructed from the underlying set  $X_U$  and function  $\theta : X_U \rightarrow \mathbf{N}_{\geq 1}$  in the following way:

$$H = \bigcup_{p_i \in X_U} \left\{ p_i^1, ..., p_i^{\theta(p_i)} \right\}.$$

For example, the instances of the 0-1 knapsack problem are generated from the base sets Pcomposed of n items associated with the prescribed values of the weight w(p) and cost c(p). These instances are equipped with the configuration spaces  $\mathcal{G}^{(1)}$  that include  $2^n$  unordered sets  $X^{(1)} \subseteq P$  reflecting all possible variants of packing the knapsack represented as the combinations of the items  $p \in P$  produced by the operator  $\mathcal{F}$ . Obviously, such spaces belong to the first class according to the proposed classification. The limitation of the maximum load capacity of the packaged knapsack Q is taken into account by the function  $\varphi$  that outputs the value of 1 only for the

configurations  $X'^{(1)}$  satisfying the following condition:

$$\sum_{p\in X'^{(1)}} w(p) \leq Q.$$

In turn, the pursuit of packing the knapsack with the most valuable items leads to introducing the criterion extr = max and specifying the next objective function:

$$f(X'^{(1)}) = \sum_{p \in X'^{(1)}} c(p).$$

For contrast, let us consider the modified version of this problem that allows adding the multiple copies of any item to the knapsack. Its instances are equipped with the search spaces  $\mathcal{G}^{(2)}$  of the second class, while their constituent configurations  $X^{(2)}$ are the multisets representing the combinations of the elements  $p \in P$  with repetitions.

It is noteworthy that some combinatorial optimization problems exhibit several natural schemes of generating the instances equipped with the search spaces of different classes. A prime example is the traveling salesman problem for the graph G = (V, E) whose instances could be associated with the third class space  $\mathcal{G}^{(3)}$  composed of the configurations  $X^{(3)}$  reflecting the permutations of the vertex set V that serves as the base set. However, this problem could be characterized by the alternative set of instances having the first class spaces  $\mathcal{G}^{(1)}$  comprising the |V|-combinations of the edges  $e \in E$ . In this case, the base set is represented by the collection of edges E.

Finally, let us take a look at the modification of the traveling salesman problem allowing the

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multiple visits of each node  $v \in V$ . The instances of such problem could be equipped with the fourth class spaces  $\mathcal{G}^{(4)}$  whose elements  $X^{(4)}$  represent the permutations with repetitions of the set V. The alternative way involves the formation of the instances accompanied by the second class spaces  $\mathcal{G}^{(2)}$  including the combinations with repetitions of the edges  $e \in E$ . These examples clearly insinuate the possibility of transforming the search spaces, which is discussed more fully in the remainder of this article.

## 3. REPRESENTATION OF THE SEARCH SPACES BY THE GRAPH STRUCTURES

Since according to Definition 1 all solutions are constructed from the elements of the base set, the structural organization of the search space  $\mathcal{G}$  could be described in the form of the undirected bipartite graph  $S(\mathcal{G}) = (V_P, V_X, E_{PX})$ . The vertices of its first part  $v^P \in V_P$  are in line with the elements of the base set P, while the nodes of the second part  $v^X \in V_X$  embody the configurations  $X \in \mathcal{G}$ . Notice that the forbidden solutions belonging to  $\mathcal{G} \setminus \mathcal{G}'$  are ignored in the structure of the graphs  $S(\mathcal{G})$ , which ensures considering the problem constraints. Moreover, each vertex  $v^X \in V_X$  is equipped with m weight coefficients reflecting the values of the weight functions  $\{f_1, ..., f_m\}$  for the corresponding configuration X. In turn, the collection (set or multiset depending on the class of the modeled space  $\mathcal{G}$ )  $E_{PX}$  contains the edges connecting the nodes that belong to the different parts of the graph  $S(\mathcal{G})$ . In particular, each edge  $(v_k^P, v_i^X) \in E_{PX}$  reflects the inclusion of the base set element  $p_k$  in the configuration  $X_i$ .

Taking into account the classification of the configuration spaces proposed in the previous section, we should emphasize that the type of the graph  $S(\mathcal{G})$  significantly depends on the class of both spaces  $\mathcal{G}$  and  $\mathcal{G}'$ . In particular, the first class spaces  $\mathcal{G}^{(1)}$  are represented by the simple (i.e. deprived of the parallel edges) graphs  $S(\mathcal{G}^{(1)})$ . On the contrary, the search spaces  $\mathcal{G}^{(2)}$  of the second class are described in the form of the multigraphs

 $S(\mathcal{G}^{(2)})$ . Their multisets  $E_{PX}$  include the parallel edges that are needed for reflecting the repeated entries of the base set elements in the configurations. For the third class spaces  $\mathcal{G}^{(3)}$ , the graphs  $S(\mathcal{G}^{(3)})$  are simple but additionally equipped with the weight function defined over the set of edges  $E_{PX}$ . The value of this function for each edge  $(v_k^P, v_j^X)$  reflects the position of the base set element  $p_k$  in the solution  $X_j^{(3)}$  and equals  $\delta(p_k)$ . The fourth class spaces  $\mathcal{G}^{(4)}$  constitute the most complicated case for representing and are reflected by the multigraphs  $S(\mathcal{G}^{(4)})$  with the weight function defined over the multisets of edges  $E_{PX}$ .

#### 4. OPERATIONS FOR UNIFICATION AND HOMOGENIZATION OF THE SEARCH SPACES STRUCTURE

In summing up the previous section, the search spaces of the first class are the most convenient for the analysis and representation in terms of the graph structures. This underlines the desirability of converting the spaces  $\mathcal{G}^{(2)}$ ,  $\mathcal{G}^{(3)}$ , and  $\mathcal{G}^{(4)}$  into the equivalent first class spaces.

**Definition 2.** The operation of *the search space* unification involves representing the problem  $I = \left\langle \mathcal{G}^{(i)}, P, \varphi, \Phi, \Psi \right\rangle$ instances for which  $i \in \{2, 3, 4\}$ in the form new  $I^* = \langle \mathcal{G}^{*(1)}, P^*, \phi^*, \Phi^*, \Psi \rangle$ . Accordingly, all search spaces belonging to the first class are referred as having the unified structure. This operation could be reduced to implementing the transformations  $\langle \mathcal{T}_i^P, \mathcal{T}_i^X \rangle$  such that

 $\mathcal{T}_{i}^{P}(P) = P^{*}$  and  $\mathcal{T}_{i}^{X}(X^{(i)}) = X^{*(1)}$ . Here the transformation  $\mathcal{T}_{i}^{P}$  ensures translating the base set, while  $\mathcal{T}_{i}^{X}$  converts all possible configurations.

Let us demonstrate that the specification of the transformations  $\mathcal{T}_i^P$  and  $\mathcal{T}_i^X$  is sufficient for constructing all components comprising the tuple of the resulting instance  $I^*$ . In particular, the space



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 $\mathcal{G}^{*(1)}$  is obtained simply by substituting all configurations  $X^{(i)} \in \mathcal{G}^{(i)}$  with the corresponding outputs  $\mathcal{T}_i^X (X^{(i)})$ . The family of criterions  $\Psi$ remains unchanged under the unification, while the functions  $f^* \in \Phi^*$  and  $\varphi^*$  are defined on the new domain  $\mathcal{G}^{*(1)}$ . Let us represent the function  $\varphi$  by the set of pairs  $(X^{(i)}, \varphi(X^{(i)}))$ . Under this representation, the function  $\varphi^*$  could be constructed by replacing these pairs with the modified ones  $(\mathcal{T}_i^X (X^{(i)}), \varphi(X^{(i)}))$ . Moreover, the objective functions  $f^* \in \Phi^*$  could be formed in the similar manner.

Obviously, the unification does not affect the cardinality of the search space, i.e.  $|\mathcal{G}^{(i)}| = |\mathcal{G}^{*(1)}|$ . Notice that the transformations  $\mathcal{T}_i^P$  and  $\mathcal{T}_i^X$  are applied only to the base sets and configurations of problem instances equipped with the search spaces belonging to the *i*-th class.

The transformation  $\mathcal{T}_2^P$  translates the initial base set *P* into the following one:

$$\mathcal{T}_{2}^{P}(P) = \bigcup_{i=1}^{n} \left\{ p_{i}^{1}, ..., p_{i}^{m_{i}^{2}} \right\},$$

where *n* denotes the cardinality of the set *P*, while  $m_i^2$  is the maximum value of  $\theta(p_i)$  among all configurations  $X^{(2)} \in \mathcal{G}^{(2)}$ . Notice that the set  $\mathcal{T}_2^P(P)$  holds  $\sum_{i=1}^n (m_i^2 - 1)$  more elements than the initial set *P*. In turn, the transformation  $\mathcal{T}_2^X$  ensures representing each concrete configuration  $X^{(2)} = \langle X_U, \theta \rangle$  in the next form:

$$\mathcal{T}_2^X(X^{(2)}) = \bigcup_{p_i \in X_U} \left\{ p_i^1, \dots, p_i^{\theta(p_i)} \right\}.$$

In simple terms, any element  $p_i^k \in \mathcal{T}_2^P(P)$ corresponds to the *k*-th copy of the element  $p_i \in P$ appearing in the solutions  $X^{(2)} \in \mathcal{G}^{(2)}$ . Fig. 1a shows the simplified example of the second class search space  $\mathcal{G}^{(2)}$  represented by the multigraph structure  $S(\mathcal{G}^{(2)})$ . In this case, the base set includes the elements  $p_1$ ,  $p_2$ , and  $p_3$ , while the space  $\mathcal{G}^{(2)}$  encompasses two solutions  $X_1^{(2)}$  and  $X_2^{(2)}$ . For example, the configuration  $X_1^{(2)}$  is composed of the element  $p_2$  and three copies of the element  $p_1$ . As is evident from Fig. 1a, the existence of *n* copies of the base set element in one configuration is reflected in the corresponding multigraph by the "dipole" structure composed of *n* parallel edges.

The result of performing the unification of this space is presented in Fig. 1b. Notice that the vertices  $v_{i,k}^{P^*}$  correspond to the elements  $p_i^k$  of the base set  $\mathcal{T}_2^P(P)$ . The representation of all copies of the elements  $p_i \in P$  by the separate elements (distinguished by the index *k*) in the set  $\mathcal{T}_2^P(P)$  leads to the elimination of all parallel edges in the corresponding graph structure. Therefore, the resulting graph given in Fig. 1b is simple.





On the contrary, the transformation  $\mathcal{T}_3^P$  produces the following output set:

$$\mathcal{T}_{3}^{P}(P) = \bigcup_{i=1}^{n} \left\{ p_{i}^{1}, ..., p_{i}^{m_{i}^{3}} \right\},$$

where  $m_i^3$  is the maximum cardinality of the underlying set  $X_U$  such that  $p_i \in X_U$  among all solutions  $X^{(3)} \in \mathcal{G}^{(3)}$ . Similarly to the previous case, the cardinality of the resulting set  $\mathcal{T}_3^P(P)$  is larger by  $\sum_{i=1}^n (m_i^3 - 1)$  compared to the initial set *P*. At the same time, the appropriate transformation  $\mathcal{T}_3^X$  translates the argument configurations  $X^{(3)} = \langle X_U, \delta \rangle$  into the next ones:

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$$\mathcal{T}_3^X\left(X^{(3)}\right) = \bigcup_{p_i \in X_U} \left\{ p_i^{\delta(p_i)} \right\}$$

In short, each element  $p_i^k \in \mathcal{T}_3^P(P)$  corresponds to the element  $p_i \in P$  contained in the configurations  $X^{(3)} \in \mathcal{G}^{(3)}$  at the *k*-th position. Fig. 2a depicts the example of the third class search space  $\mathcal{G}^{(3)}$ . Its structure is illustrated by the graph  $S(\mathcal{G}^{(3)})$  whose edges are accompanied with the weight coefficients reflecting the values of the function  $\delta$ . In particular, the configuration  $X_1^{(3)}$ includes the elements  $p_1$  and  $p_2$  located respectively at the first and second positions. Fig. 2b presents the structure of the search space obtained after implementing the operation of the unification. The more close inspection shows that the information encoded by the weight coefficients in Fig. 2a is expressed in Fig. 2b by the index *k* of the vertices  $v_i^{P_i^*}$  representing the elements

the vertices  $v_{i,k}^{P^*}$  representing the elements  $p_i^k \in \mathcal{T}_3^P(P)$ .

At the same time, the formation of the base sets representing the outputs of the transformation  $\mathcal{T}_4^P$  involves the appropriate consideration of both ordering and repeating of the elements in the configurations  $X^{(4)} \in \mathcal{G}^{(4)}$ . Thus, the expression for the sets  $\mathcal{T}_4^P(P)$  takes the following complicated form:

$$\mathcal{T}_{4}^{P}(P) = \bigcup_{p_{i}^{k} \in P_{H}^{*}} \left\{ p_{i}^{k,1}, ..., p_{i}^{k,m_{i}^{4'}} \right\},$$

where  $m_i^{4'}$  is the maximum value of  $\sum_{p \in X_U} \theta(p)$ among all configurations  $X^{(4)}$  such that  $p_i \in X_U$ . In turn, the set  $P_H^*$  is defined as follows:

$$P_{H}^{*} = \bigcup_{i=1}^{n} \left\{ p_{i}^{1}, ..., p_{i}^{m_{i}^{4}} \right\},$$

where  $m_i^4$  denotes the maximum value of  $\theta(p_i)$ among all configurations  $X^{(4)} \in \mathcal{G}^{(4)}$ .

The presence of the auxiliary set *H* in the structure of the tuples  $X^{(4)} = \langle H, X_U, \theta, \lambda \rangle$  provides the opportunity to present the expressions for the outputs produced by the transformation  $\mathcal{T}_4^X$  in the next simple form:

$$\mathcal{T}_4^X(X^{(4)}) = \bigcup_{p_i^k \in H} \left\{ p_i^{k, \lambda\left(p_i^k\right)} \right\}.$$

Thus, the unification of the fourth class search spaces requires the combination of the techniques used for unifying the second and third class spaces. The example of implementing such operation is shown in Fig. 3. Remark that the nodes  $v_{i,k,l}^{P^*}$  of the resulting graph reflect the elements  $p_i^{k,l}$  of the base set  $\mathcal{T}_4^P(P)$ .



Figure 2: Structure of the Sample Third Class Search Space (a) and Its Modification in the Process of the Unification (b)





Another challenging issue that dramatically complicates the analysis of the search spaces structure is associated with unequal cardinality of the configurations  $X^{(1)}$  comprising the search spaces  $\mathcal{G}^{(1)}$  of some real problems. For example, the problem of finding the longest path between the vertices  $v_s, v_t \in V$  in the graph G = (V, E) has a number of possible solutions constructed from the

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different number of edges  $e \in E$ . In order to deal with such situations, this article introduces the following operation:

**Definition 3.** The operation of *the search space homogenization* is applied to the problem instances  $I = \langle \mathcal{G}^{(1)}, P, \varphi, \Phi, \Psi \rangle$  having at least two solutions  $X_k^{(1)}, X_j^{(1)} \in \mathcal{G}^{(1)}$  such that  $|X_k^{(1)}| \neq |X_j^{(1)}|$  and consists in their transition to the new form  $\hat{I} = \langle \hat{\mathcal{G}}^{(1)}, \hat{P}, \hat{\varphi}, \hat{\Phi}, \Psi \rangle$  in which all solutions  $\hat{X}^{(1)} \in \hat{\mathcal{G}}^{(1)}$  have the equal cardinality. The search spaces are referred as having *the homogeneous structure* if they have the unified structure and encompass only the configurations having the equal cardinality.

Similarly to the previous case, the implementation of this operation comes down to specifying the pair of transformations  $\langle \hat{\mathcal{T}}^P, \hat{\mathcal{T}}^X \rangle$  such that  $\hat{\mathcal{T}}^P(P) = \hat{P}$  and  $\hat{\mathcal{T}}^X(X^{(1)}) = \hat{X}^{(1)}$ . These transformations are given by the following expressions:

$$\hat{\mathcal{T}}^{P}(P) = \bigcup_{i=1}^{n} \left\{ p_{i}^{0}, p_{i}^{1} \right\},$$
$$\hat{\mathcal{T}}^{X}(X^{(1)}) = \left\{ p_{i}^{1} | p_{i} \in X^{(1)} \right\} \bigcup \left\{ p_{i}^{0} | p_{i} \notin X^{(1)} \right\}.$$

In simple terms, the transformation  $\hat{\mathcal{T}}^{P}(P)$  performs "splitting" each element  $p_i \in P$  into the pair of the new elements  $\{p_i^0, p_i^1\}$ . In turn, the configurations  $\hat{\mathcal{T}}^{X}(X^{(1)})$  are required to include exactly one element from each such pair, which clearly shows the equalization of their cardinality.



Figure 4: Graph Models Demonstrating the Structure of the Sample First Class Search Space Before (a) and After (b) Performing Its Homogenization





Fig. 4 depicts the example of the search space homogenization. In particular, the graph model constructed for the initial non-homogeneous space (Fig. 4a) includes the vertices  $v_1^{X^{(1)}}$  and  $v_2^{X^{(1)}}$ reflecting the solutions  $X_1^{(1)}, X_2^{(1)} \in \mathcal{G}^{(1)}$  and having the degrees of 3 and 1, respectively. On the contrary, the nodes  $v_1^{\hat{\chi}^{(1)}}$  and  $v_2^{\hat{\chi}^{(1)}}$  of the graph given in Fig. 4b represent the configurations formed after performing the homogenization of the search space. Note that both these nodes have four adjacent vertices  $v_{i,k}^{\hat{P}}$  corresponding to the elements  $p_i^k$  of the base set  $\hat{T}^P(P)$ , which clearly illustrates the effect of the homogenization.

Obviously, the application of the transformation  $\hat{\mathcal{T}}^P(P)$  results in doubling the cardinality of the base set. Notice that according to Definition 3, the operation of the search space homogenization is performed only for the problem instances equipped with the first class spaces  $\mathcal{G}^{(1)}$ . Such limitation is associated with the absence of the clear and unequivocal concept of the "cardinality" for the solutions contained in the search spaces belonging to the other classes. On the same grounds, the homogeneous structure of the search space is also unified, while the converse is not always true.

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#### 5. CONCLUSIONS

As shown in Fig. 5, the fundamental idea of the proposed algebra lies in performing two operations modifying the structure of the search spaces associated with the problem instances. In particular, the operation of the unification results in blurring the class-based differentiation of the search spaces. It is followed by the operation of the homogenization intended to equalize the cardinality of all possible solutions.

As a result, the search spaces passed through both stages of transformation are eligible for encoding by the sets of the equal-length strings drawn from the binary alphabet. Under such encoding, the elements of the base set included in any solution could be indicated by ones in the corresponding string. At the same time, the opportunity to present the structure of the search spaces in the unified homogeneous form afforded by the proposed algebra has the drawback expressed by increasing the cardinality of the base sets.

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