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SPECTRAL H_2 FAULT ESTIMATION OBSERVER DESIGN BASED ON ALLOCATION OF THE CORRECTION EFFECT

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ABSTRACT

The paper is devoted to problem of additive fault estimation observer design for LTI plants with scalar measurement and external disturbance with the known spectral structure. The filter with the calculated parameters should enhance such performances as rapidity of the fault estimation and its insensitivity to the polyharmonical disturbance signal with the certain central frequency, provided by the special filter, generating the corrective signal. The specific spectral approach of H_2 optimization in frequency domain, based on the polynomial factorization, is applied with the aim to improve computational effectiveness of the synthesis. Some theoretical aspects are discussed and numerical algorithm for practical implementation is formulated. Their effectiveness is demonstrated by the numerical example with implementation of MATLAB package.

Keywords: *Linear-quadratic control, H_2 optimal control, stability, fault estimation.*

1. INTRODUCTION

Increasing complexity of the controlled plants results a high probability of system failures, which makes problems of safety and reliability crucial. Fault tolerant control (FTC) techniques can be used to achieve these properties and it has been paid serious attention for the past two decades [1]. Nowadays, fault tolerant control techniques can be classified into two types: passive FTC [2,3], using robust with respect to possible faults controllers, and active one [4-7], i.e. design of additional control laws, depending on real-time fault estimation, provided by the special module (fault estimation observer).

As it follows from the above, fast and accurate estimation of the fault is necessary to design effective active fault tolerant controller. As a result, various effective approaches have been developed and there are a lot of papers devoted to this problem (about 100 of them are cited in [5]). Most of the described solutions (e.g. multiconstrained full-order fault estimation observer (FFEO) design) are based on Lyapunov stability theory and linear matrix inequalities. Such approaches usually include a regional pole placement and H_∞ performance level to provide the certain degree of stability and suppression of the external disturbance effect.

However, such problems with its initially known structure are not fully studied until now. There are a lot of situations (e.g. marine ship motion process), where the external disturbance has known structure, e.g. can be described as a random Gaussian process with the given spectral power density. This property should be taken into account to improve effectiveness of the adaptive observers.

The circumstances, mentioned above, motivate us to research devoted to fault estimation observer design for controlled plants affected by polyharmonical external disturbances. Algorithm of parameters computation, characterizing the allocation of the correction effect (i. e. accuracy of the state vector estimation) and a special filter, providing insensitivity to the signal with the certain frequency, is proposed. The presented method is based on H_2 optimization ideology.

One of the key features of this paper is implementation of specific spectral approach [8] in frequency domain, based on polynomial model representation instead of well-known methods, based on Riccati equations or on linear matrix inequalities (LMI). This technique is used with the aim to increase computational effectiveness: its complexity is not very high that is crucial for systems with real-time regime of operating, e.g. for onboard control systems.

This paper has the following structure. The next section demonstrates the equations of controlled plant, structure of the observer-filter and initial problem statement. Section 3 is devoted to alternative problem statement, used as basis for the investigations, demonstrated in the rest of paper, description of the proposed approach with implementation of mean-square optimization ideology and formulation of the observer design algorithm. In Section 4, we present illustrative numerical example of synthesis. Finally, in Section 5, we describe overall results the investigation and mention some directions of the future research.

2. PROBLEM STATEMENT

Consider the following linear time invariant system:

$$\begin{aligned} \dot{x} &= \mathbf{A}x + \mathbf{b}u + \mathbf{E}f + \mathbf{P}d, \\ y &= \mathbf{c}x, \end{aligned} \tag{1}$$

where $x \in R^{n_0}$ is the state space vector, $u(t)$ is the scalar control, $d(t)$ is the external disturbance, $f(t)$ is the additive fault action and $y(t)$ is the output measured signal. All components of the matrices $\mathbf{A}, \mathbf{b}, \mathbf{c}, \mathbf{E}, \mathbf{P}$, are given constants.

The adaptive fault estimation observer is constructed as

$$\begin{aligned} \dot{\hat{x}} &= \mathbf{A}\hat{x} + \mathbf{b}u + \mathbf{E}\hat{f} + \mathbf{L}_x \nu, \\ \hat{y} &= \mathbf{c}\hat{x} + l_0 \nu, \\ \dot{\hat{f}} &= l_f \nu, \\ \nu(s) &= W(s)(y - \hat{y}), \end{aligned} \tag{2}$$

where ν is the correcting term, vector $\mathbf{L}_x$, values l_f, l_0 and the transfer function $W(s) \equiv W_1(s)/W_2(s)$ are to be calculated. The designed observer-filter must generate fast and accurate fault estimation signal $\hat{f}(t)$, despite the presence of the external disturbance. The fault signal is slow varying within the framework of this paper, i.e., $\dot{f} \approx 0$. Denote the following notations

$$e_x = x - \hat{x}, \; e_f = f - \hat{f}, \; e_y = y - \hat{y},$$

where e_x is the state estimation error vector, e_f is error of the fault estimation and e_y is output estimation error, and consider errors dynamics for the state and fault estimation process with the estimator (2),

$$\begin{aligned} \dot{e}_x &= \mathbf{A}e_x + \mathbf{E}e_f - \mathbf{L}_x \nu + \mathbf{P}d, \\ \dot{e}_f &= -l_f \nu, \\ \nu(s) &= W(s)e_y, \end{aligned} \tag{3}$$

Let us use the following notations

$$z = \begin{pmatrix} \hat{x}^T & \hat{f} \end{pmatrix}^T, z \in R^n, \bar{e} = \begin{pmatrix} e_x^T & e_f \end{pmatrix}^T, \bar{e} \in R^n,$$

$$\overline{\mathbf{L}} = \begin{pmatrix} \mathbf{L}_x^T & l_f \end{pmatrix}^T, \overline{\mathbf{A}} = \begin{pmatrix} \mathbf{A} & \mathbf{E} \\ 0 & 0 \end{pmatrix}, \overline{\mathbf{b}} = \begin{pmatrix} \mathbf{b} \\ 0 \end{pmatrix},$$

$$\overline{\mathbf{P}} = \begin{pmatrix} \mathbf{P} \\ 0 \end{pmatrix}, \overline{\mathbf{c}} = \begin{pmatrix} \mathbf{c} & 0 \end{pmatrix},$$

and rewrite the systems (2) and (3) as

$$\begin{aligned} \dot{z} &= \overline{\mathbf{A}}z + \overline{\mathbf{b}}u + \overline{\mathbf{L}}\nu + \overline{\mathbf{P}}d, \\ \hat{y} &= \overline{\mathbf{c}}z + l_0 \nu, \\ \nu(s) &= W(s)(y - \hat{y}), \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{\bar{e}} &= \overline{\mathbf{A}}\bar{e} - \overline{\mathbf{L}}\nu + \overline{\mathbf{P}}d, \\ e_y &= \overline{\mathbf{c}}e - l_0 \nu, \\ \nu &= W(s)e_y. \end{aligned} \tag{5}$$

We can rewrite the equations (3) in frequency domain

$$\begin{aligned} sA(s)e_y(s) &= -sL(s)\nu(s) - \\ &- E(s)l_f \nu(s) + sP(s)d(s), \\ \nu &= W(s)e_y, \text{ where} \end{aligned} \tag{6}$$

$$A(s) = \det(\mathbf{A}), L(s) = A(s)\mathbf{c}(s\mathbf{I} - A)^{-1}\mathbf{L}_x - l_0 A(s),$$
$$E(s) = A(s)\mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{E}, P(s) = A(s)\mathbf{c}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{P},$$

respectively, (5), (6) could be rewritten as

$$\begin{aligned} \tilde{A}(s)e_y(s) &= -\tilde{L}(s)\nu(s) + \tilde{P}(s)d(s), \\ \nu(s) &= W(s)e_y, \text{ where} \end{aligned} \tag{7}$$

$$\tilde{A}(s) = \det(\overline{A}) = sA(s),$$
$$\tilde{L}(s) = \tilde{A}(s)\left(\overline{\mathbf{c}}(s\mathbf{I} - \overline{\mathbf{A}})^{-1}\overline{\mathbf{L}} - l_0\right) = sL(s) + l_f E(s),$$
$$\tilde{P}(s) = \tilde{A}(s)\overline{\mathbf{c}}(s\mathbf{I} - \overline{\mathbf{A}})^{-1}\overline{\mathbf{P}} = sP(s).$$

Consider the possible case, when $\deg \widetilde{L}(s) = n$, i.e. $l_0 \neq 0$. We can rewrite expression of the corrective term $\nu(s)$ as

$$\nu(s) = W(s)e_y = W(s)(y - \bar{\mathbf{c}}z - l_0\nu) = \\ = \frac{1}{1 + l_0 W(s)} W(s)(y - \bar{\mathbf{c}}z),$$

and using the new transfer function $\widetilde{W}(s) = \widetilde{W}_1(s)/\widetilde{W}_2(s)$

$$\nu(s) = \frac{\widetilde{W}_1(s)}{\widetilde{W}_2(s)}(y - \bar{\mathbf{c}}z), \\ \widetilde{W}_1(s) = W_1(s), \widetilde{W}_2(s) = W_2(s) + l_0 W_1(s). \tag{8}$$

Consider transfer function from the external disturbance $d(t)$ to the error of the fault estimation

$$e_f(s) = F_{e_f d}(s)d(s) = -\frac{\mathbf{l}_f P(s)W_1(s)}{\Delta_2(s)} d(s), \tag{9}$$

where $\Delta_2(s)$ is characteristic polynomial of the closed-loop system (3)

$$\Delta_2(s) = sA(s)W_2(s) + sL(s)W_1(s) + \\ + l_f E(s)W_1(s). \tag{10}$$

Stability of the polynomial (10) guarantees asymptotic convergence of estimation errors, i.e. all its roots must be located in the open left half plane.

External disturbance $d(t)$ for the system (1) can be considered as a random stationary Gaussian process with zero mathematical expectation and with the following spectral power density:

$$S_d(\omega) = S_1(s)S_1(-s)|_{s=j\omega}, \\ S_1(s) = N_d(s)/T(s),$$

where the polynomials $N_d(s)$ and $T(s)$ are Hurwitz. A sea disturbance, considered in the Section 4, has the following structure

$$S_1(s) \equiv \frac{N_d(s)}{T(s)} = \frac{\sqrt{4D_r\alpha(\alpha^2 + \beta^2)}}{s^2 + 2\alpha s + \alpha^s + \beta^2}, \tag{11}$$

where β is the central frequency and $\alpha = s_t\beta$, where s_t presents the spectrum blurriness. Here we note that the constant factor could be neglected for a case of scalar disturbance signal. We can also represent it in the simpler polyharmonical form

$$d(t) = \sum_{i=1}^{n_h} A_{di}\sin(\sigma_i t + \varphi_i), \tag{12}$$

where A_{di}, σ_i, φ_i are amplitudes, frequencies and phases of the corresponding harmonics. Let us define values, characterizing effectiveness of the designed observer: fault estimation process settling time T_p and J_ω

$$J_\omega = \max_i \{A_{di}|F_{e_f d}(j\sigma_i)|\}, \tag{13}$$

expressing influence of the disturbance $d(t)$ to the fault estimation process. Note that T_p and J_ω are functions of $\bar{\mathbf{L}}$, $W(s)$ and, in such a way, we should design such items $\bar{\mathbf{L}}_0$, $W_0(s)$ that

$$J_\omega(\bar{\mathbf{L}}_0, W_0(s)) \leq J_\omega^0, \; T_p(\bar{\mathbf{L}}_0, W_0(s)) \leq T_p^0,$$

where J_ω^0, T_p^0 are given desired values of J_ω and T_p. Solution of this problem can be considered as minimization of the functional

$$J = T_p - T_p^0 + |T_p - T_p^0| + \ldots \\ \ldots + J_\omega - J_\omega^0 + |J_\omega - J_\omega^0|. \tag{14}$$

3. OPTIMIZATION PROBLEM WITH ALLOCATION OF THE CONTROL ACTION

It can be seen that the error dynamics, presented by equations (6) (and effectiveness of the observer-filter (2), respectively), depend on the vector $\bar{\mathbf{L}}$, characterizing the correction effect, and the transfer function $W(s)$, which must guarantee suppression of harmonic external disturbance signal with the certain central frequency. One of the ways is to solve this problem quasi-optimally, calculating these parameters consequently, e.g. using vector $\bar{\mathbf{L}}$ computed by any method [9], but computational complexity significantly increases in this case. Let us propose the alternative algorithm of their simultaneous search. Here we note that the considered task is close to the problem of mean-square synthesis with allocation of the control action [10] (i.e. the parameters, characterizing a control effect to coordinates of the state space vector, are not fixed a priory and are to be chosen to increase effectiveness of the control process) in some details. Denote new corrective term

$$\widetilde{\nu}(s) = V(s)e_y = V_1(s)/V_2(s)e_y, \\ V(s) = L(s)W(s),$$

and rewrite the expression (7) in the frequency domain as

$$\tilde{A}(s)e_y(s) = -\tilde{v}(s) + \tilde{P}(s)d(s),\quad \tilde{v} = V(s)e_y. \qquad (15)$$

Error dynamics of the closed-loop system (15) depends only on the transfer function $V(s)$ and the aforementioned problem can be solved with implementation of H - optimization ideology. One of the ways is to state mean-square optimization problem. Let introduce the mean square functional of a form

$$\tilde{J} = \tilde{J}(V) = \lim_{T\to\infty}\frac{1}{T}\int_0^T [e_y^2(t) + \ldots \ldots + k^2\tilde{v}^2(t)]dt = \langle e_y^2\rangle + \ldots \ldots + k^2\langle\tilde{v}^2\rangle \to \min_V, \qquad (16)$$

where the second summand can be treated as intensity of the estimation process. The parameter k characterizes the tradeoff between sensitivity to external disturbance and degree of stability of the closed-loop loop system (i.e. response speed of the fault estimation). Note that if the functional (16) is designed for "economical" mode (with enough large value of k), the numerators of the transfer function $W(s)$ must have pair of complex-conjugated roots close to the $\pm\beta j$ to guarantee frequency properties.

Let us note that most of cited papers describe algorithms of fault estimation observer design based on linear matrix inequalities. However, much easier method, described in [8], can be applied for the system with the scalar external disturbance and the measurement signal. The special approach in frequency domain, based on polynomial factorization and on a special parameterization of a set of stabilizing controllers, has significant advantages. First, it takes into account spectral structure of the signal $d(t)$. Second, it has high computational effectiveness that is crucial for onboard control systems, e.g. marine autopilots. Finally, the spectral presentation is convenient to explore behavior of the closed-loop system (3). Also we remark, that the polynomial $N(s)$, which is determined by identity

$$N(s)N(-s) \equiv -s^2P(s)P(-s)N_d(s)N_d(-s),$$

characterizing external disturbance effect, is divisor of the characteristic polynomial (10) of the closed-loop system (5) [8], i.e. it must be Hurwitz to provide its stability. One of the ways of this problem solving is to replace $N(s)$ to the close Hurwitz polynomial $\tilde{N}(s)$:

$$\tilde{N}(s)\tilde{N}(-s) \equiv (s+p)(-s+p)P(s)P(-s)\times \times N_d(s)N_d(-s), \qquad (17)$$

where the parameter $p > 0$ can be used for optimization too. Finally, we formulate the following algorithm.

1. Calculate the polynomials

$$\tilde{A}(s) = \det(\mathbf{I}s - \overline{\mathbf{A}}),\ P(s) = A(s)\mathbf{c}(\mathbf{I}s - \mathbf{A})^{-1}\mathbf{P},$$
$$C_i(s),\ i = \overline{1,n},\ \text{where}$$
$$(C_1(s)\quad C_2(s)\quad \ldots\quad C_n(s)) \equiv A(s)\overline{\mathbf{c}}(\mathbf{I}s - \mathbf{A})^{-1}.$$

2. Set the parameters k and p in (14), (5). Execute factorization of the polynomials

$$k^2\tilde{A}(s)\tilde{A}(-s) + 1 \equiv G(s)G(-s),\quad P(s)P(-s) \equiv P_1(s)P_1(-s), \qquad (18)$$

where $G(s)$ and $P_1(s)$ are Hurwitz polynomials. Receive the polynomial $\tilde{N}(s)$ (17):

$$\tilde{N}(s) = (s+p)P_1(s)N_d(s). \qquad (19)$$

3. Calculate the polynomial

$$R(s) = -\sum_{i=1}^{n}\frac{G(-s)}{g_i - s}\frac{\tilde{N}(g_i)}{\tilde{A}(g_i)T(g_i)G'(-g_i)}. \qquad (20)$$

4. Construct the auxiliary transfer function

$$V = V(s) = V_1^0(s)/V_2^0(s),$$
$$V_1^0(s) = [\tilde{A}(s)T(s)R(s) - \tilde{N}(s)]/G(-s), \qquad (21)$$
$$V_2^0(s) = [-T(s)R(s) - k^2\tilde{A}(-s)\tilde{N}(s)]/G(-s),$$

where a division to $G(-s)$ is done totally.

5. Choose n (or less, if necessary) roots ξ_i of the polynomial $V_1(s)$. Calculate the following polynomial

$$L(s) = \prod_{i=1}^{n}(s - \xi_i) = l_{0n}s^n + l_{0(n-1)}s^{n-1} + \ldots \ldots + l_{0(n-2)}s^{n-2} + \ldots + l_{01}s + l_{00}, \qquad (22)$$
$$W_{01}(s) = V_{01}(s)/L_0(s)$$

Let us note that complex conjugate roots $\xi_{i,i+1}$ close to $\pm\beta j$ must be the roots of the polynomial $W_{10}(s)$ to provide frequency properties of the designed observer-filter.

6. Construct two vectors

$$l_{s0} = (l_{0(n-1)} \quad l_{0(n-2)} \quad \ldots \quad l_{01} \quad l_{00})^{\mathrm{T}},$$

$$\mathbf{a}_s = (a_{n-1} \quad a_{n-2} \quad \ldots \quad a_1 \quad a_0)^{\mathrm{T}},$$

of the coefficients of the polynomials

$$L_0(s) = l_{0n}s^n + l_{0(n-1)}s^{n-1} + l_{0(n-2)}s^{n-2} + \ldots + l_{01}s + l_{00},$$

$$A(s) = s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \ldots + a_1 s + a_0,$$

respectively, and the matrix

$$\mathbf{C}_s = \begin{pmatrix} c_{1(n-1)} & c_{2(n-1)} & \cdots & c_{n(n-1)} \\ c_{1(n-2)} & c_{2(n-2)} & \cdots & c_{n(n-2)} \\ \cdots & \cdots & \cdots & \cdots \\ c_{11} & c_{21} & \cdots & c_{n1} \\ c_{10} & c_{20} & \cdots & c_{n0} \end{pmatrix},$$

consisting of the coefficients of the following ones

$$C_i(s) = c_{i(n-1)}s^{n-1} + c_{i(n-2)}s^{n-2} + \ldots + c_{i1}s + c_{i0}, \; i = \overline{1,n}.$$

7. Set $l_0^* = l_{0n}$ and compute the solution $\overline{\mathbf{L}} = \overline{\mathbf{L}}_{\mathbf{0}}$ of the set of linear equations

$$\mathbf{C}_s \overline{L} = l_{s0} - l_0^* \mathbf{a}_s, \tag{23}$$

8. Receive the transfer function of the optimal filter $W(s)$ from (3) $W_0(s) = W_{01}(s)/W_{02}(s)$, where $W_{02}(s) \equiv V_{02}(s)$, the optimal vector $\overline{\mathbf{L}} = \overline{\mathbf{L}}_{\mathbf{0}}$, and optimal $l_0 = l_0^*$. Receive the optimal $\widetilde{W}_0(s)$ (8)

$$\widetilde{W}_0(s) = \frac{\widetilde{W}_{01}(s)}{\widetilde{W}_{02}(s)} = \frac{W_{01}(s)}{W_{02}(s) + l_0^* W_{01}(s)}. \tag{24}$$

9. Let consider state space realization of the transfer function $\widetilde{W}_0(s)$, computed above:

$$\begin{aligned} \dot{\xi} &= \mathbf{A}_W \xi + \mathbf{B}_W (y - \mathbf{c}z), \\ \nu &= \mathbf{C}_W \xi + \mathbf{D}_W (y - \mathbf{c}z), \end{aligned} \tag{25}$$

The fault estimation observer-filter (4) with the computed parameters can be considered in state-space form as the following plant

$$\begin{pmatrix} \dot{\overline{z}} \\ \dot{\xi} \end{pmatrix} = \begin{pmatrix} \overline{\mathbf{A}} - \overline{\mathbf{L}}\mathbf{D}_W \overline{\mathbf{c}} & \overline{\mathbf{L}}\mathbf{D}_W \\ -\mathbf{B}_W \overline{\mathbf{c}} & \mathbf{A}_W \end{pmatrix} \begin{pmatrix} \overline{z} \\ \xi \end{pmatrix} + \begin{pmatrix} \mathbf{b} \\ 0 \end{pmatrix} u + \begin{pmatrix} \overline{\mathbf{L}}\mathbf{D}_W \\ \mathbf{B}_W \end{pmatrix} y$$

$$\hat{f} = \Big(\underbrace{0\,0\ldots0\,1}_{n_0} \Big) \overline{z}.$$

10. Evaluate the functional (14) $J = J(\overline{\mathbf{L}}_{\mathbf{0}}, W_0)$. If its value is not close to zero, then minimize J, repeating steps 2-10 with new parameters k, p, searched with any numerical method or with enumeration. Receive the optimal parameters $k = k^*$, $p = p^*$.

11. Calculate optimal $\overline{\mathbf{L}} = \overline{\mathbf{L}}_0$, $W(s) = W_0(s)$, using optimal k^*, p^*.

4. NUMERICAL EXAMPLE

Let us illustrate the practical implementation of the proposed approach by the example of a marine ship moving on the horizontal plane with constant longitudinal speed [11]. Assume that we have mathematical model (1) of the ship motion with the constant speed under sea wave action and with the following parameters:

$$\mathbf{A} = \begin{pmatrix} -0.0936 & 0.634 & 0 \\ 0.048 & -0.717 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\mathbf{b} = \mathbf{E} = \begin{pmatrix} 0.0196 \\ 0.0160 \\ 0 \end{pmatrix}, \mathbf{P} = \begin{pmatrix} 0.41 \\ 0.0076 \\ 0 \end{pmatrix},$$

$$\mathbf{c} = (0 \quad 0 \quad 1).$$

Vector $\mathbf{x} \in E^3$ consists of three components: drift angle, angular velocity and yaw angle (measured one). External disturbance $d(t)$ can de expressed by the formula (11) with the parameters

$$D_r = 1.52 \cdot 10^{-4}, \beta = \omega_0 = 0.45, s_t = 0.01.$$

Besides, the polyharmonical representation

$$d(t) = \sin(\omega_0 t) + 0.1\sin(0.9\omega_0 t) + 0.1\sin(1.1\omega_0 t)$$

also can be used. Now we define $J_\omega^0 = 0.01$, $T_p^0 = 22$ and apply the algorithm, described in the previous section, using the initial $k = 50$, $p = 0.1$. Optimal $p^* = 0.1$, $k^* = 100$ are received by the enumeration. Let us consequently execute all steps of the optimal observer-filter design.

1. Initial polynomials

$\widetilde{A}(s) = s^4 + 0.81s^3 + .037s^2$,
$P(s) = 0.0076s + 0.0204$,

$C_1(s) = 0.0480s,\ C_2(s) = s^2 + 0.0936s,$
$C_3(s) = s^3 + 0.811s^2 + 0.0367s,$
$C_4(s) = 0.016s + 0.00241.$

2. Receive outputs of the factorizations (18)

$$G(s) = 100s^4 + 123.41s^3 + 46.97s^2 + 9.69s + 1,$$
$$P_1(s) = 0.0076s + 0.0204.$$

and the polynomial $\widetilde{N}(s)$ (19)

$$\widetilde{N}(s) = 10^{-5}(0.566s^2 + 1.576s + 0.152).$$

3. Calculate the polynomial (20)

$$R(s) = -0.103s^3 + 0.112s^2 - \ldots$$
$$\ldots - 0.301s + 0.004.$$

4. The auxiliary transfer function $V_0(s)$ (21)

$$V_{01}(s) = 10^{-5}(-102.74s^5 - 98.65s^4 - \ldots$$
$$\ldots - 37.28s^3 - 20.35s^2 - 3.05s - 0.15),$$
$$V_{02}(s) = 10^{-4}(-5.6s^2 - 7.88s - 7.17).$$

5. The roots of the polynomial $V_{01}(s)$ are

$$\xi_1 = -0.7623,$$
$$\xi_{2,3} = -0.008 \pm 0.446j,$$
$$\xi_{4,5} = -0.0909 \pm 0.0365j.$$

We use ξ_1, ξ_4, ξ_5 to construct the polynomial $L_0(s)$ from (22) and then $W_{01}(s)$

$$L_0(s) = s^3 + 0.943s^2 + 0.148s + 0.007,$$
$$W_{01}(s) = 10^{-4}(-10.27s^2 - 0.17s - 2.05).$$

6. Construct $\mathbf{C}_s$ and b_{s0}:

$$\mathbf{C}_s = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0.8106 & 0 \\ 0.0480 & 0.0936 & 0.0367 & 0.0160 \\ 0 & 0 & 0 & 0.0024 \end{pmatrix},$$
$$b_{s0} = (1 \quad 0.944 \quad 0.148 \quad 0.007)^{\mathrm{T}}.$$

7. Solve (23) and receive

$$\overline{\mathbf{L}} = (1.037 \quad 0.133 \quad 1 \quad 3.078)^T,\ l_0^* = 0.$$

8. Calculate the transfer function $\widetilde{W}_0(s)$ (24)

$$\widetilde{W}_0(s) = \frac{\widetilde{W}_{01}(s)}{\widetilde{W}_{02}(s)} = \frac{(1.81s^2 + 0.03s + .036)}{(s^2 + 0.139s + 1.27)}.$$

9. Receive state space realization (25) of the $\widetilde{W}_0(s)$

$$\mathbf{A}_W = \begin{pmatrix} -1.39 & -1.27 \\ 1 & 0 \end{pmatrix},\ \mathbf{B}_W = \begin{pmatrix} 2 \\ 0 \end{pmatrix},$$
$$\mathbf{C}_W = (-1.25 \quad -0.97),\ \mathbf{D}_W = 1.81.$$

Fig. 1 represents magnitude of the transfer function (9) of the closed loop system with parameters computed in this section. The curve A_ω has a pronounced dip in the area of the central frequency ω_0, i. e. effect of the harmonical external disturbance is successfully suppressed.

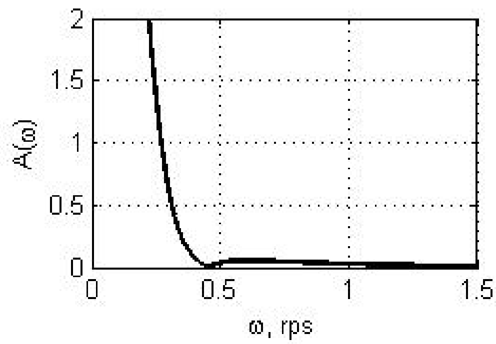

Figure 1: Frequency response of the optimal closed-loop system.

Assume that the constant fault signal $f(t)$ is created as

$$f(t) = \begin{cases} \pi/6,\ 300 \le t \le 500, \\ 0,\ \text{else}. \end{cases}$$

and Fig. 2 represents process of its estimation.

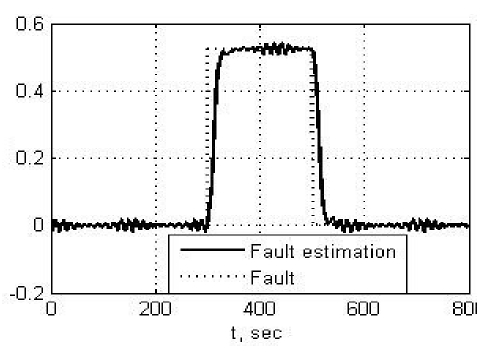

Figure 2: Fault estimation process

From the above simulation results it can be concluded that the observer, designed in this section, successfully estimates the fault. Let note that rapidity of the estimation can be improved by varying the parameter l_f , but its significant increasing can upset stability of the closed loop system.

5. CONCLUSIONS

A fault estimation strategy for a class of SISO LTI systems has been proposed in this paper. The specific algorithm of adaptive fault estimation observer analytical synthesis, based on polynomial factorization, is presented that is the main goal of this paper. Effectiveness of this scheme is demonstrated by application to the plane motion model of the marine ship.

Let us consider main merits of the proposed approach. First, the observer with calculated parameters can estimate additive faults (e.g. actuator ones) with satisfactory accuracy and rapidity. Second, problem of suppression of polyharmonical disturbance signals with the certain central frequency is successfully solved as well. On the other hand, there are some serious demerits. First, the proposed approach does not take into account dynamics of the fault. Second, it cannot be applied necessary to deal with more general plants then SISO ones, i.e. multidimensional output, external disturbance and control signals.

Overcoming of the mentioned demerits is the object of the future research. Also robust features or various delays should be taken into account in the sequel.

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