

A HYBRID ALGORITHM FOR VEHICLE ROUTING PROBLEM WITH TIME WINDOWS AND TARGET TIME

¹ABBASSI ABDERRAHMAN, ²EL BOUYAHYIOUY KARIM, ³EL HILALI ALAOU AHMED, ⁴BELLABDAOUI ADIL

^{1,3}Modeling and Scientific Calculus Laboratory, Department of mathematics, Faculty of Science and Technology Fez, Morocco

^{2,4} Information Technology and Management of Enterprises (TIME), ENSIAS - Mohammed V University in Rabat, Morocco

E-mails: ¹abderrahman.abbassi@usmba.ac.ma, ³elhilali_fstf2002@yahoo.fr, ²karim.bouyahyaoui2015@gmail.com, ⁴adil.bellabdaoui@um5.ac.ma

ABSTRACT

The routing of a fleet of vehicles to service a set of customers is important in the field of goods distribution. Vehicle routing problem with time windows (VRPTW) is a well-known combinatorial problem. This article aims at studying the vehicle routing problem with time windows and target time (VRPTWTT). VRPTWTT involves the routing of a set of vehicles with limited capacity from a central depot to a set of geographically dispersed customers with known demands and predefined time windows as well as a target time. There are penalties associated with servicing either earlier or later than this target servicing time. The goal is to minimize the costs of transport and penalties of delay and ahead of time. Although VRPTWTT is a new variant of the VRP with time windows, the problem is not easy to solve, and it is also NP-hard. To solve the VRPTWTT, we propose a hybrid method combining Neighborhood search with Ant Colony Optimization Algorithm (ACO). Furthermore, when ACO is close to current optimal solution, neighborhood search is used to maintain the diversity of ACO and explore new solutions. First, we present a description of the hybrid method followed by computational results and the conclusion.

Keywords: *Vehicle Routing Problem, Time Window, Target time, Ant Colony Optimization, Neighborhood search.*

1. INTRODUCTION AND RELATED WORKS

The vehicle routing problem (VRP) is a combinatorial optimization problem that has been extensively studied in the literature since it was introduced by Dantzig and Ramser[14]. Its application is very important in many fields such as transportation, logistics, communications, manufacturing, military and relief systems, and so on. In the traditional VRP, client's request consists, generally, of an amount of goods. These goods are delivered to their corresponding clients using a set of homogeneous vehicles based at a single depot. The problem aims to find a set of minimum cost routes to serve the clients respecting vehicle-capacity constraints. The cost of a route can be the total distance or time travelled by the vehicle associated with it. A typical VRP is described as a complete directed graph. Figure1 displays an

example of VRP, where the depot is denoted as 0 and the customers are denoted as 1 to 10. The solution of this VRP includes three routes: 0-6-7-4-5-0, 0-8-9-0 and 0-1-2-3-0.

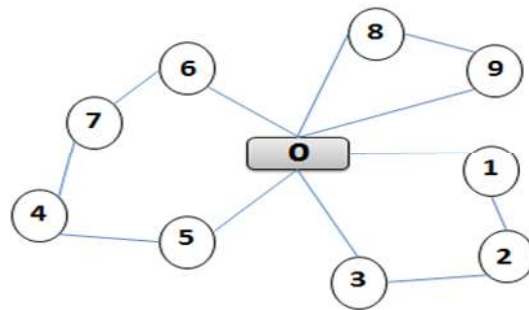


Figure 1: An Example Of Vehicle Routing Problem Network.



There are many different types of vehicle routing problem that have been addressed in the literature according to variants of constraints; for instance the vehicle routing problem with time windows (VRPTW) in which each customer is associated with a time interval and can only be served within this interval [7], the multi-depot vehicle routing problem (MDVRP) in which more than one depot is considered [20], the heterogeneous vehicle routing problem (HVRP) where one can choose among vehicles with different costs and capacities to serve the trips [26],[36], the pickup and delivery vehicle routing problem (PDVRP) in which the customers may both receive and send products [33],[28],[27],[25]. For an overview of the different VRP variants, the reader can consult the paper published in [37] and [19].

All works cited above require to visit every customer only once. Moreover, another important variant studied by researchers is the selective vehicle routing problem SVRP [3], in which visiting each customer would not work. It can be modeled as an integer program with the objective of maximizing the total profit collected in selected nodes.

The major objectives addressed in most studies are to optimize transportation costs, time and travelled distance. But, current trends in the context of sustainable transport are the environment quality taking into account the objective of emissions reduction. This last version is called the green vehicle routing problem (GVRP) [1],[2]. The task is to find the best routes for serving customers while minimizing the CO₂ emissions and not only transportation cost. Such problem is a multi-objective one.

While these problems have been studied extensively by many researchers, the best known problem is the VRPTW.

In VRPTW, a set of vehicles with limited capacity is to be routed from a central depot to a set of geographically dispersed customers with known demands and predefined time windows in order that the fleet size of vehicles and total traveling distance are minimized while capacity and time windows constraints are satisfied. Due to its inherent complexities and usefulness in real life, the VRPTW continues to draw the attention of researchers and has become a well-known problem in network optimization, so a large number of techniques have been proposed in the literature. These techniques can be categorized into exact and heuristic algorithms. There have been many papers

proposing exact solution algorithms to solve the VRPTW which is a well-known NP-hard problem. These algorithms are the type of branch and bound [30], branch and cut [29], column generation [31], constraints programming [32], etc. Since the VRPTW belongs to the class of NP-hard, for which the running time grows exponentially with the problem size, and such methods are inefficient in general. By far [23] is one of the most efficient exact methods for solving 100-customer-size instance. As a result, many researchers have investigated the VRPTW using heuristics and metaheuristics approaches.

In recent years, approximate approaches are used in VRPTW instead of exact methods considering latter's intolerably high cost. Various heuristics algorithms have been proposed for finding good solutions, not necessarily optimal, in reasonable running time. These algorithms include tabu search [34],[10],[12], genetic algorithms [8], and ant colony optimization [17]. Compared with Tabu search and genetic algorithm (GA), The ACO is less applied in VRPTW. However, it has successfully been applied to solve capacitated vehicle routing problems [9].

The main contribution of this paper is to investigate a new variant of the vehicle routing problem with time windows called the vehicle routing problem with time windows and target time (VRPTWTT).

The VRPTWTT involves the routing of a set of vehicles with limited capacity from a central depot to a set of geographically dispersed customers with known demands and predefined time windows as well as a target time. There are penalties associated with servicing either earlier or later than this target servicing time.

The rest of this paper is organized as follows: in the second section we give a formal definition of the VRPTWTT and the mathematical formulation of the problem. In the third section, we give a hybrid method for solving this problem. The 4th section is dedicated to computational results. Finally, a conclusion is given in Section 5.

2. PROBLEM STATEMENT

Many companies prefer to deal with the transportation of their products to customers and to not assign that task to other transport operators. They ensure the quality of services, keep good relationship and confidence towards customers and they manage their entire supply chain operations including transportation.



The VRP is one of the wide studies in transportation problems of combinatorial optimization. As we mention previously, many extensions are developed in the literature such as the VRP with time window. But in many realistic problems, costumers prefer to receive their request at a specific exact time called target time and not only within a time window, especially when products are semi-finished and they will be injected into another production process. In addition, the target time is necessary because in last few years, big costumers manage their inventories using the *juste-à-temps* strategy. Every delay about this target time could make production process unsuccessful and every advance time about it could cause other additional storage costs.

We recall that the purpose of this paper is to study the Vehicle Routing Problem with Time Windows and Target Time (VRPTWTT) as a new variant of VRP. It can be defined as a problem in which many costumers must be served starting from a single depot with known demands, time windows and target time.

The objective is to find routes for serving all costumers such that the following objectives are met and the following conditions are taken into consideration.

Objectives

- Minimizing the total cost traveled by vehicles.
- Minimizing the sum of earliness and tardiness penalties with regard to the servicing deadlines of costumers

Conditions

- Vehicle capacity constraints are respected.
- Time window constraints are met.
- Each customer is served exactly once.
- Each vehicle starts its journey from depot and it ends at the depot.
- The total length of any route may not exceed a preset bound.

We aim to minimize the transportation cost and total penalties of delays and advances.

We consider a distribution network that consists of a set of N nodes which represent costumers, a set of arcs A connecting nodes and a single depot denoted by ‘0’ where a fleet V of vehicles is available with the same capacity Q . The demand of each customer $i \in N$ is denoted by d_i , it must be received within the time window $[E_i, L_i]$, where E_i the earliest arrival is time and L_i is the latest arrival time. Each costumers $i \in N$ imposes request delivery, exactly at a specific time called target time $ta_i \in [E_i, L_i]$. All used notations are given in table 1.

Table 1. Used notations in the mathematical model

Notation	Description
N	Set of nodes (Customers)
A	Set of arcs
V	Set of available vehicles
d_i	Demand of customer i
c_{ij}	Cost incurred on arc from node i to node j
t_{ij}	The travel time between customer i to costumer j
Q	The capacity of a vehicle
E_i	The earliest arrival time at customer i
L_i	The latest arrival time at customer i
ta_i	The target arrival time at customer i
s_i	The service time at customer i
T	The maximum route time allowed for vehicles
Pb_i^k	penalty before target time for vehicle k at customer i
Pa_i^k	penalty after target time for vehicle k at customer i
tr_i^k	The tardiness time of vehicle k in node i
er_i^k	The earliness time of vehicle k in node i
Decision variables :	
u_i^k	The service start time at a node i by vehicle k , if this vehicle doesn't serve the costumers i , then $u_i^k = 0$.
w_i^k	The waiting time for vehicle k at node i
x_{ij}^k	Binary decision variable which is equal to 1 if the link (i, j) is used by vehicle k and 0 otherwise



We give a mathematical formulation of the VRPTWTT based on the classical formulation for VRPTW as follows:

$$\text{Min} \sum_{i \in N} \sum_{j \in N} \sum_{k \in V} c_{ij} \cdot x_{ij}^k \quad (1)$$

$$\text{Min} \sum_{i \in N} \sum_{k \in V} er_i^k \cdot Pb_i^k + tr_i^k \cdot Pa_i^k \quad (2)$$

Subject to

$$\sum_{k \in V} \sum_{j \in N} x_{oj}^k \leq |V| \quad (3)$$

$$\sum_{k \in V} \sum_{i \in N} x_{ij}^k = 1 \quad \forall j \in N \quad (4)$$

$$\sum_{k \in V} \sum_{j \in N} x_{ij}^k = 1 \quad \forall i \in N \quad (5)$$

$$\sum_{i \in U} \sum_{j \in U} x_{ij}^k < |U| - 1 \quad \forall k \in V, 2 \leq |U| \leq n \quad (6)$$

$$\sum_{i \in N} x_{ip}^k = \sum_{j \in N} x_{pj}^k \quad \forall p \in N, \forall k \in V \quad (7)$$

$$\sum_{i \in N} d_i \sum_{j \in N} x_{ij}^k \leq Q \quad \forall k \in V \quad (8)$$

$$\sum_{i \in N} \sum_{j \in N} x_{ij}^k \cdot (t_{ij} + s_i + w_i^k) \leq T_k \quad \forall k \in V \quad (9)$$

$$x_{ij}^k \cdot (u_i^k + t_{ij} + s_i + w_i^k) \leq u_j^k \quad \forall i, j \in N, \forall k \in V \quad (10)$$

$$E_i \cdot \sum_{j \in N} x_{ij}^k \leq u_i^k \leq L_i \cdot \sum_{j \in N} x_{ij}^k \quad \forall i \in N, \forall k \in V \quad (11)$$

$$er_i^k \leq ta_i - E_i \quad \forall i \in N, \forall k \in V \quad (12)$$

$$tr_i^k \leq L_i - ta_i \quad \forall i \in N, \forall k \in V \quad (13)$$

$$x_{ij}^k \in \{0,1\}, \quad u_i^k \geq 0 \quad \forall i \in N, \forall j \in N, \forall k \in V \quad (14)$$

The first objective function (1) seeks to minimize the total cost of transport while the second function (2) minimizes the sum of earliness and tardiness penalties with regard to the servicing deadlines of customers. Note that the delay and the advance

times of a vehicle k , in relation to the nominal date, are defined respectively as follows

$$tr_i^k = \max(0, u_i^k - ta_i) \quad (15)$$

$$er_i^k = \max(0, ta_i - u_i^k) \quad (16)$$

In the same way, we can define the waiting time of vehicle k in the node i using the following formula:

$$w_i^k = \max(0, u_i^k - E_i) \quad (17)$$

That means, there is a waiting time only if the vehicle comes before the earliest arrival time of customer.

Constraints (3) limit the maximum number of routes. Constraints (4) and (5) define that every customer node is visited only once by one vehicle. The next constraints (6) secure the subtours elimination. The flow conservation is ensured by constraints (7), they impose that the number of arcs entering and leaving each node should be the same. Constraints (8) are necessary so as to guarantee that the vehicle's capacity is not exceeded. Temporal constraints are described in constraints (9) for restricting the maximum travel time and constraints (10)-(11) which define the time windows.

All conditions provided right now are associated with the classical VRP with time window. Our new variant contains more additional constraints (12)-(13) for expressing the deviation before and after the target time.

Remark that in the best case, the service starting time is exactly the target time. If they are different, delay time should be added and advance time should be subtracted.

$$u_i^k = ta_i + tr_i^k - er_i^k \quad \forall i \in N, \forall k \in V \quad (18)$$

Finally, conditions (14) are integrity constraints which determine the definition domain of decision variables

This mathematical model is transformed to a problem with a single objective function with proportional parameters.

$$\text{Min } a \sum_{i \in N} \sum_{k \in V} er_i^k \cdot Pb_i^k + tr_i^k \cdot Pa_i^k + b \cdot \sum_{i \in N} \sum_{j \in N} \sum_{k \in V} c_{ij} \cdot x_{ij}^k$$

In this study, we have given the same importance degree to both objective functions by fixing equal weights for parameters $a=b=0.5$.

The aim of the next section is to present a Hybrid Ant Colony optimization (ACOLS) to solve the VRPTW with target time formulated in the previous section.

3. SOLUTION APPROACH

For solving the VRPTWTT, we propose a hybrid method, called ACOLS: Ant Colony Optimization hybrid with local search as an improvement heuristic. Firstly, The ACO is used to find the best initial solution which will be used in a neighborhood heuristic.

3.1 ACO approach for VRPTWTT

Ant Colony Optimization (ACO) is a metaheuristic method based on real ants' behavior in finding a route to food nest. ACO was firstly developed by Dorigo[5] and it has successfully been used for solving different hard combinatorial optimization problems, especially problems can be represented as graphs with relatively few modifications [14],[15],[6],[38]. This solution method has been applied also for solving scheduling problems as aircraft landing problems[16].

3.1.1 Solution construction

The principle of using ACO in VRPTWTT is that each ant starts its route from the central depot and it constructs a path by successively selecting customers under some constraints, of capacity and time length. The process is performed incrementally until a feasible solution can be obtained and all customers are visited.

At every step t , each ant $k \in \{1, \dots, m\}$ located on node i , will select the next node j using the probabilistic formula as follow:

$$P_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}^\alpha(t) \cdot \mu_{ij}^\beta(t)}{\sum_{l \in J_i^k} \tau_{il}^\alpha(t) \cdot \mu_{il}^\beta(t)} & \text{if } j \in J_i^k \\ 0 & \text{Otherwise} \end{cases} \quad (20)$$

Where J_i^k is the set of unvisited feasible nodes.

The parameters α and β define the relative importance of the pheromone trace and the visibility of the ants. $\tau_{iu}(t)$ is the amount of pheromone associated with the edge between the current node i and the next possible node u in the iteration t . The initial pheromone quantity is fixed at a value τ_0 . The value of the heuristic information (the visibility) associated with the edge (i, u) denoted by

$$\mu_{iu}(t) = \frac{1}{c_{iu}} \quad (21).$$

3.1.2 Pheromone updating

This step of pheromone updating is very important. It's a key element of ACO and it allows the improvement of future solutions in the next iteration. The aim is to exchange information of

colonies by pheromone updating; it simulates the real evaporation of pheromone. For this reason, it's necessary to update this information locally for each ant using formula (22) and globally for changing the entire quantity according to formula (23).

$$\tau_{ij}(t+1) = \rho \cdot \tau_{ij}(t) + (1-\rho) \cdot \tau_0 \quad (22)$$

$$\tau_{ij}(t+1) = \rho \cdot \tau_{ij}(t) + (1-\rho) \cdot \Delta\tau_{ij}(t) \quad (23)$$

Where:

- $0 < \rho < 1$ is the coefficient for controlling evaporation, in our study we take $\rho = 0.8$
- $\Delta\tau_{ij}(t) = \begin{cases} \frac{1}{\varphi} & \text{if } (i, j) \in \text{Best Solution} \\ 0 & \text{otherwise} \end{cases} \quad (24)$

And φ is the cost of the best solution in iteration t , that means the path with the a smaller cost in that iteration.

3.1.3 Neighborhood search

Local search heuristics are always considered an efficient method for improving the current solution. That's why a neighborhood search algorithm is introduced to exploit the local search. The local search used in this study for improving the solution is called 2-opt exchange and developed by [9].

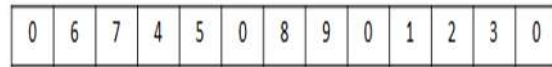


Figure 2: Solution representation from Figure 1

The idea is to change nodes and routes in the current solution. In order to illustrate this heuristic, an example is presented in Figure 2; it's a sequence code which means that the first vehicle visits customer 6,7,4 then 5, the second vehicle serves customer 8 and 9 and finally the third vehicle for customers 1,2 and 3.

The first step of this heuristic is swapping two customers of two different routes. We select randomly two different routes from the current solution, e.g., 0-6-7-4-5-0 and 0-1-2-3-0(Figure 3). Then, we exchange two customers between selected routes satisfying the capacity and the time window constraints, e.g., customer 4 and customer 2. So, a new solution can be obtained.

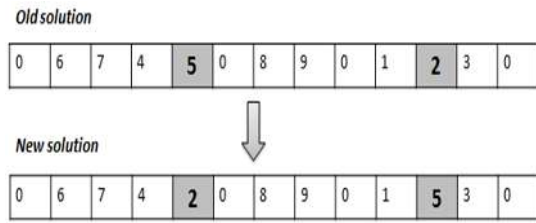


Figure 3. Exchanging two customers between two routes

In addition, we propose to insert customers between two routes (Figure 4) as another step for improving solutions. In other words, we select randomly one route (e.g., 0-6-7-4-5-0) and one customer (e.g. node 4) from the same route and we insert it into another route of the current solution with satisfaction of the capacity and the time window constraints (e.g., 0-1-2-3-0). So also another new solution can be obtained.

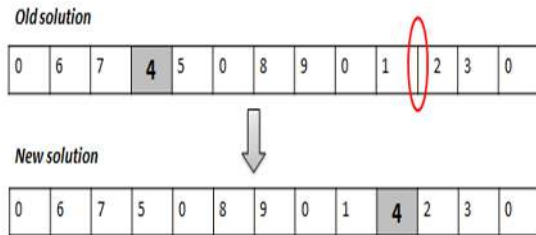


Figure 4. Inserting a customer into another route.

3.2 Hybrid algorithm ACOLS

The hybrid algorithm ACOLS begins with the initialization of parameters and the pheromone. Then each ant starts from the depot and it constructs routes according to the probability transition rule. Local search is based on the neighborhood search heuristic to explore neighbor solutions.

If the result of local search is better than the solution found using ACO, the current solution will be replaced. The pheromone information can be computed according to pheromone quantity on links and the quality of the current solution, it can be updated locally and globally.

The main steps of this solution approach are presented in Algorithm 1.

```

Fixing the number of ants
Fixing the maximum number of iterations
Pheromone initialization:  $\tau_{ij} = \tau_0 \quad \forall i, j \in N$ 
iter ← 0
while(iter < itermax)
    For each ant k
        do
            Random choice of an unused vehicle
            Build route for this vehicle using (20)
            Local updating the pheromone  $\tau_{ij}$  by (22)
        Until Ant k has completed its solution
        Apply the local search for the improvement
            of the solution of the current ant
    End for
    Save the best solution found by the ants
    Update the pheromone  $\tau_{ij}$  globally using (23)
    iter ← iter + 1
End while
    
```

Algorithm 1: ACOLS algorithm for the VRPTWTT

Finally, If a stopping criterion is satisfied, ACOLS is terminated. Otherwise, the algorithm constructs new solutions and repeats the previous steps.

4. COMPUTATIONAL RESULTS

The hybrid algorithm was implemented and run on a PC 2.4 GHz CPUs 4096 MB Memory. Our problem is still new and original as far as VRPTW is concerned. Data are not available to test our algorithm for VRPTWTT. That is why, the ACOLS is tested using the classical set of benchmark problems which can be downloaded from the web page: <http://web.cba.neu.edu/~msolomon/c101.htm> Because there are no previous results to compare it with, we use these benchmarks and we complete them by the data needed. The cost is associated with each arc $(i, j) \in A$ such as the distance d_{ij} and travel time t_{ij} , the times and distances are both simply taken to be the Euclidean distances

$$c_{ij} = t_{ij} = d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$



Table 2: Comparison Between Best Solution With And Without Target Time , Benchmark C107

Benchmark C107 Solution with target time				Benchmark C107 Solution without target time																									
vehicle	Cost	path						vehicle	Cost	path																			
v1	130.9130	1	21	25	26	28	30	31	29	27	24	v1	59.6181	1	6	4	8	9	11	12	10	7	5	3	2	76	1		
v2	107.8894	1	6	4	8	9	11	12	10	7	5	3	v2	64.8075	1	44	43	42	41	45	47	46	49	52	51	53	50	48	1
v3	112.2084	1	44	43	42	41	45	47	46	49	52	v3	50.8036	1	21	25	26	28	30	31	29	27	24	23	22	1			
v4	105.7178	1	68	66	64	63	75	73	62	65	69	v4	59.4031	1	68	66	64	63	75	73	62	65	69	67	70	1			
v5	122.8971	1	91	88	87	84	83	85	86	89	90	v5	76.0696	1	91	88	87	84	83	85	86	89	90	92	1				
v6	142.1348	1	99	97	96	95	93	94	98	101	v6	97.2272	1	33	34	32	36	38	39	40	37	35	1						
v7	174.0096	1	82	79	77	72	71	74	78	80	81	v7	95.9431	1	99	97	96	95	93	94	98	101	100	1					
v8	148.0656	1	58	56	55	54	57	59	61	60	1	v8	101.8826	1	58	56	55	54	57	59	61	60	1						
v9	144.3323	1	33	34	32	36	38	39	40	37	35	v9	127.2975	1	82	79	77	72	71	74	78	80	81	1					
v10	143.3022	1	14	18	19	20	16	17	15	13	1	v10	95.8847	1	14	18	19	20	16	17	15	13	1						
Best cost: 1331.5		TD : 855.0654						Best cost: -		TD : 828.9369																			

Table 3: Comparison Between Total Distances With And Without Target Time

	C101-100	C102-101	C105-100	C107-100	C108-100	C109-100	C201-101	RC204-100
Total distance in VRPTW without Target time	828.94	828.94	828.94	828.94	828.94	828.94	591.56	590.60
Total distance in VRPTW with Target time	828.9369	828.9369	849.6407	862.8783	828.9369	852.9482	591.5566	791.4459
Total cost of VRPTW with Target time (cost and penalty)	967.94	3516.3	1058.1	4267.5	1395.6	1680.9	717.18	6601.7

We consider the target time ta_i the mean of time window (i.e. $ta_i = \frac{E_i + L_i}{2}$) and the penalties of tardiness and earliness are linked by the assumption

$$Pb_i^k = \frac{1}{2} Pa_i^k.$$

This data is new and not tested yet. Taking into account the target time, the routes of vehicles may be changed. As an example, Table 2 describes the effect of the target time on routes of vehicles for the benchmark C 107. We can notice that paths routing obtained for the problem of VRPTW without target time and paths for VRPTW with the target time are totally different. So, setting a specific time for receiving goods by the customer can change the layout and merchandise distribution strategy used by transport operators.

Moreover, the total distances don't remain the same in both cases before and after the inclusion of the target time, for example in instances C101, C102, C105, C107, C108, C109, C201 and RC204, Table 3 shows the remarkable changes in total distance if the customer requires delivery at a specific target time and not only within a time window. The fourth line in this table presents the total objective cost (transportation cost and penalties) for these instances.

So as to prove the efficiency of our solution approach, we test it also on the classical vehicle routing problem with time window.



Table 4 : Comparison Between Best Solutions Of Related Works And Our Solutions

Instance	best *			ACOLS		Gap	Instance	best *			ACOLS		Gap
	NV	TD	Ref	NV	TD			NV	TD	Ref	NV	TD	
C101	10	828.93	[35]	10	828.93	0,00	R112	10	974.73	[35]	11	1001.07	0,03
C102	10	828.937	[35]	10	828.93	0,00	R201	7	1214.22	[35]	5	1567.48	0,23
C103	10	824.06	[21]	10	833.02	0,01	R202	5	1105.2	[35]	3	1201.72	0,08
C104	10	828.2	[35]	10	828.2	0,00	R203	3	942.64	[18]	3	1177.80	0,20
C105	10	828.9	[35]	10	828.93	0,00	R204	4	771.47	[35]	3	1123.85	0,31
C106	10	828.937	[35]	10	828.3	0,00	R205	3	994.42	[22]	4	1321.59	0,25
C107	10	828.937	[35]	10	862.8783	0,04	R206	3	912.97	[21]	3	1206.04	0,24
C108	10	828.94	[21]	10	828.93	0,00	R207	3	870.33	[35]	3	1090.70	0,20
C109	10	828.94	[21]	10	831.8495	0,02	R208	2	713.23	[18]	2	1000.80	0,29
C201	3	591.56	[21]	3	591.56	0,00	R209	3	909.86	[22]	3	1275.89	0,29
C202	3	591.56	[18]	3	591.55	0,00	R210	5	949.02	[35]	3	1302.34	0,27
C203	3	591.17	[21]	3	601.20	0,02	R211	4	877.55	[35]	2	1041.00	0,16
C204	3	590.60	[21]	3	605.55	0,02	RC101	14	1650.14	[35]	17	1619.48	-0,02
C205	3	588.49	[21]	3	593.2053	0,01	RC102	13	1514.85	[35]	15	1531.07	0,01
C206	3	588.29	[21]	3	593.2053	0,01	RC103	11	1262.02	[21]	12	1387.12	0,09
C207	3	588.39	[21]	3	591.38	0,01	RC104	10	1135.48	[11]	12	1105.54	-0,03
C208	3	588.03	[35]	3	589.35	0,00	RC105	15	1617.88	[35]	15	1389.31	-0,16
R101	20	1642.87	[21]	21	1362.98	-0,21	RC106	13	1387.63	[35]	13	1704.13	0,19
R102	17	1486.12	[21]	21	1370.89	-0,08	RC107	11	1230.54	[24]	14	1140.72	-0,08
R103	14	1243.22	[35]	18	1138.87	-0,09	RC108	10	1139.82	[24]	12	1164.32	0,02
R104	10	982.01	[21]	11	1178.55	0,17	RC201	5	1279.65	[35]	5	1794.29	0,29
R105	14	1377.11	[21]	15	1298.51	-0,06	RC202	5	1157.02	[35]	3	1561.60	0,26
R106	12	1252.03	[21]	14	1438.34	0,13	RC203	6	1046.33	[35]	3	1345.72	0,22
R107	11	1100.25	[35]	12	1062.83	-0,04	RC204	3	799.12	[18]	4	791.44	-0,01
R108	9	958.66	[35]	15	809.67	-0,18	RC205	4	1302.42	[18]	4	1599.34	0,19
R109	12	1101.99	[35]	12	1140.19	0,03	RC206	5	1112.2	[35]	3	1580.17	0,30
R110	12	1119.53	[35]	12	1141.77	0,02	RC207	3	1062.05	[11]	4	1163.20	0,09
R111	12	1091.11	[35]	13	1308.25	0,17	RC208	3	829.69	[22]	3	1010.29	0,18

The table 4 below reports the best solution found in terms of the total distance of transport in many papers; we compare them with our solutions.

The column best* shows the best results obtained by a group of studies done between 1995 and 2012.

Where:

- $Gap = \frac{best^* - ACOLS\ result}{best^*}$
- NV : number of vehicles used
- TD : total distance

The ACOLS column presents the solutions found by our solution approach. We can see clearly from table4 that, in some cases, our hybrid algorithm

finds best results. But in the majority of cases, it approaches to best solutions found in literature. While other instances require being more improved.

5. CONCLUSION

In this paper, we have introduced a new extension of Vehicle Routing Problem with Time Windows, namely the Vehicle Routing Problem with Time Windows and Target Time (VRPTWTT). It's an important variant which can be applied in many real fields like distribution problems. We have proposed a mathematical formulation of VRPTWTT and we have adopted a hybrid approach based on ACO and neighborhood search to solve the problem. When



ACO is close to local optimal, the heuristic uses the current best solution as the initial solution to maintain the diversity of ACO.

Our model aims to optimize the costs of transport and the total penalty of delay and ahead of time (i.e, optimize the quality of service offered to the clients in addition to the cost of transport). To evaluate and to compare our approach, we have tested it on several instances of Solomon's 56 VRPTW. The results obtained show that, compared with some meta-heuristic published by a group of studies, the solution approach ACOLS, is also an effective tool for solving the VRPTW.

REFERENCES:

- [1] A. El Bouzekri, A. A.El Hilali, "Evolutionary Algorithm for the Bi-Objective Green Vehicle Routing Problem". *International journal of Scientific & Engineering Research*, Volume 5, Issue 9, September 2014.
- [2] A. El Bouzekri, E. Messaoud, A.A. El Hilali, "A hybrid ant colony system for green capacitated vehicle routing problem in sustainable transport", *Journal of Theoretical and Applied Information Technology*. Vol. 54 No.2. August 2013.
- [3] A. Mahdieh, Y.J.C.Josep and W.RWill. "Selective vehicle routing problems under uncertainty without recourse". *Transportation Research Part E: Logistics and Transportation Review*. Volume 62, February 2014, Pages 68–88.
- [4] B. Ghizlane, J. Boukachour, A.A.Elhilali "A memetic algorithm to solve the dynamic multiple runway aircraft landing problem". *Journal of King Saud University - Computer and information Sciences*. Volume 28, Issue 1, January 2016, Pages 98–109.
- [5] Dorigo M.: Optimization, learning and natural algorithms, Unpublished Doctoral Dissertation Politecnico di Milano, DipartimentodiElettronica, Italy (1992).
- [6] R. Abounacer, J. Boukachour, B. Dkhissi and A.A. El Hilali. "A hybrid Ant Colony Algorithm for the examen timetabling problem". *Revue Africaine de la Recherche en informatique et Mathématiques Appliquées (ARIMA)*, vol. 12, pp. 15-42, 2010.
- [7] M. M.Solomon, "Algorithms for the vehicle routing and scheduling problems with time window constraints". *Operations Research*, 35/2, 254–265, 1987.
- [8] J. Berger M. Barkaoui, "A Parallel hybrid genetic algorithm for the vehicle routing problem with time windows". *Computers and Operations Research*, vol.31, pp. 2037-2053, 2004.
- [9] B. Bullnheimer, R.F. Hartl and C. Strauss. "An improved ant system algorithm for the vehicle routing problem". *Annals of Operations Research*, 89, 319–328, 1999.
- [10] W.C. Chiang and R.A. Rurssel, "A reactive tabu search metaheuristic for the vehicle routing problems with time windows". *INFORMS journal on computing*, vol.9, pp. 417-430, 1997.
- [11] J.F. Cordeau, G. Laporte and A. Mercier, "A unified tabu search heuristic for vehicle routing problems with time windows". *Journal of the Operation Research Society*, vol.52, pp. 928-936, 2001.
- [12] Z.J. Czech and P. Czarns, "A parallel simulated annealing for the vehicle routing problem with time windows". In *10th Euromicro Workshop on Parallel, Distributed and Workshop on parallel, Distributed and Network-based Processing*, pp. 376-383, 2002.
- [13] G.B.Dantzig, D.R. Fulkerson and M.Johnson, "Solution of a large-scale traveling-salesman problem". *Operations Research*, vol.2, pp. 393-410, 1954.
- [14] G.B.Dantzig, J.H. Ramser, "The truck dispatching problem". *Management Science*, 6, pp. 80-91, 1959.
- [15] G. Bencheikh, J. Boukachour, and A. El HilaliAlaoui. "Improved Ant Colony Algorithm to solve the aircraft landing problem". *International Journal of Computer Theory and Engineering*, vol.3, no.2, pp.
- [16] G. Bencheikh, J. Boukachour, A. El HilaliAlaoui, and F. El Khoukhi. "Hybrid method for aircraft landing scheduling based on a Job Shop formulation". *International Journal of Computer Science and Network Security*, vol. 9, no. 8, 2009.
- [17] L.M.Gambardella, E. Taillard and G. AGAZZI, "New ideas in Optimization, chapitre MACS-VRPTW: A Multiple Ant Colony for vehicle routing problem with time windows". pp. 63-76. McGrawHill, 1999.
- [18] J. Homberger and H.Gehring, "Two evolutionary meta-heuristics for the vehicle routing problem with time windows". *INFOR*, vol.37, pp.297-318, 1999.
- [19] P. Toth, D. Vigo, "The vehicle routing



- problem". *SIAM monographs on discrete mathematics and applications*. Vol. 9. Philadelphia: SIAM, 2002.
- [20] J. Renaud, G. Laporte, F.F Boctor, "A tabu search heuristic for the multi-depot vehicle routing problem". *Computers & Operations Research*, 23/3, 229–235, 1996.
- [21] Y. Rochat and E. Taillard, "Probabilistic diversification and intensification in local search for vehicle routing". *Journal of Heuristics*, vol.1, pp. 147-167, 1995.
- [22] Rousseau, L.M, Gendreau, M. Pesant, G. "Using constraint-based operators to solve the vehicle routing problem with time windows". *Journal of Heuristics*, vol.8, pp.43-58, 2002.
- [23] N. Kohl, "Exact methods for time constrained routing and related scheduling problems" [Ph.D. thesis], Department of Mathematical Modeling, Technical University of Denmark, 1995.
- [24] E. Taillard, P. Badeau, M. Gendreau, F. Geurtin and J.Y Potvin, "A tabu search heuristic for the vehicle routing problem with soft time windows". *Transportation Science*, vol.3, pp. 170-186, 1997.
- [25] R. Baldacci, E. Bartolini, A. Mingozzi, "An Exact Algorithm for the Pickup and Delivery Problem with Time Windows". *Operations Research*, 59/2, 414–426. <http://doi.org/10.1287/opre.1100.0881>.
- [26] S. Ceschia, L. Gaspero and A. Schaerf, "Tabu search techniques for the heterogeneous vehicle routing problem with time windows and carrier-dependent costs". *Journal of Scheduling*, 14/6, 601–615. <http://doi.org/10.1007/s10951-010-0213-x>.
- [27] S. N. Parragh, K.F. Doerner, R.F. Hartl, "A survey on pickup and delivery problems. Part II: transportation between pickup and delivery locations". *Journal für Betriebswirtschaft*, 58, 81–117, 2008.
- [28] S.N. Parragh, S. N., Doerner, K. F., Hartl, R. F., (2008a). "A survey on pickup and delivery problems. Part I: transportation between customers and depot". *Journal für Betriebswirtschaft*, 58, 21–51.
- [29] N. Achutan, L. Caccettal, S. Hill, "An improved branch-and-cut algorithm for the capacitated vehicle routing problem". *Transp. Sci.* 37, 153–169 (2003).
- [30] N. Kohl, J. Desrosiers, O.B.G. Madsen, M.M. Solomon and F. Soumis. "2-path cuts for the vehicle routing problem with time windows". *Transportation Science* 33 (1), 101–116.
- [31] I. Dayarian, T. G. Crainic, M. Gendreau, and W. Rei, "A column generation approach for a multi-attribute vehicle routing problem," *European Journal of Operational Research*, vol. 241, no. 3, pp. 888–906, 2015
- [32] L.-M. Rousseau, M. Gendreau, and G. Pesant, "Using constraint-based operators to solve the vehicle routing problem with time windows". *Journal of Heuristics*, vol. 8, no. 1, pp. 43–58, 2002.
- [33] J.F. Cordeau, G. Laporte, S. Ropke, "Recent models and algorithms for one-to-one pickup and delivery problems". In: Golden, B., Raghavan, S., Wasil, E. (Eds.), *The Vehicle Routing Problem: Latest Advances and New Challenges. Operations Research/Computer Science Interfaces Series*, 43. Springer, 327–357, 2008 .
- [34] P. Badeau, F. Guertin, M. Gendreau, J.Y. Potvin and E.D. Taillard, "A Parallel Tabu Search Heuristic for the Vehicle Routing Problem with Time Windows". *Transportation Research C: Emerging Technologies*, 5/2, 109–122, 1997.
- [35] B. Yu, Z. Z. Yang and B. Z. Yao, "A hybrid algorithm for vehicle routing problem with time windows". *Expert Systems with Applications*, 38/1, 435–441, 2011. <http://doi.org/10.1016/j.eswa.2010.06.082>.
- [36] C. Koç, T. Bektaş, O. Jabali, G. Laporte, "Thirty years of heterogeneous vehicle routing". *European Journal of Operational Research*, 000, 1–21, 2015. <http://doi.org/10.1016/j.ejor.2015.07.020>.
- [37] G. Laporte, "Fifty Years of Vehicle Routing". *Transportation Science*, 43/4, 408–416, 2009. <http://doi.org/10.1287/trsc.1090.0301>.