

DISCRETE RISK MODELS OF THE PROCESS OF VIRAL EPIDEMICS DEVELOPMENT IN HOMOGENOUS INFORMATION AND TELECOMMUNICATION NETWORKS

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ABSTRACT

The object of this research is homogeneous networks, i.e., networks with small fluctuations of the vertices degree. In other words, in such homogeneous structures assume that $k \cong \langle k \rangle$, where parenthesis mean averaging over the degree of distribution. The structures also have a very specific application in practice. Typically, such networks require stringent communication within any corporation. This network organization is particularly relevant in information and telecommunication networks (ITN) of critically important objects. In this paper, the models such as SI, SIS, SEIS, SIR, SEIR, MSEIR are distinguished, when a malware attack is performed by network virus, which uses vulnerability in the work of network services of the operating system for its spread. The model synthesis is performed under the conditions that ITN contacts can be represented by a complete graph, and the epidemic occurs in a closed ITN. The features of the process of malware program transmission from one computer to another, as well as the internal features of malware program execution on the computer, are ignored. For each model, the analytical expressions of epidemic resistance were obtained. The prospects of using the proposed models in the process of network warfare were distinguished.

Keywords: *Epidemic, Risk, Epidemic Resistance, Mathematical Expectation, Mode, Standard Deviation (SD), Information And Telecommunication Network (ITN).*

1. INTRODUCTION

A virus attack should be considered as one of the most common tools for network warfare; their purpose is the occurrence of epidemics in networks [9-24]. In this context, objects of destructive impact are two core network classes: heterogeneous [9-23] and homogenous [24]. The epidemic resistance of the first of them (heterogeneous) was investigated [9-3]. On the basis of the analog paradigmatic models, the claim about a relatively high security of this class of networks compared to homogeneous networks has been made. Based on this, the study of epidemic processes in homogeneous networks has particular relevance; the present work is devoted to this.

At the basis of the study, there are discrete, but not analog models. The reason is that discrete models are offered as the most promising because of their greater adequacy (the process of infection spread is objectively discrete, both from the point of view of the discreteness of the affected network elements, and of the discontinuity of their states) and also because of the opportunity to make step-by-step analysis of the process (in case of virus mutation,

etc.) and to implement management of protection (when switching immunization tools and treatment of network elements). This is especially important for tightly-organized fragments of corporate networks with homogeneous elements (vertices of the network graph with a constant and equal degree).

2. PRINCIPLES OF EPIDEMICS FORMALIZATION

It should be noted that the SIR, SI, and SIS models [9-23] are continuous. However, the transitions of ITN processes (elements) from one state to another (S, I, R) is clearly discrete.

In this context, it would be rational to construct and investigate a discrete model of the process of information epidemic development.

To describe the models of viral epidemics implementation, let us consider the approach according to which the spread of the virus in ITN is estimated using the mathematical expectation $M(X)$ (average evaluation), mode $Mo(X)$ (peak evaluation) or the mathematical expectation $M(X)$

with the variance $D(X)$ (range evaluation) of the number of infected elements X of the system depending on the analysis needs. For this approach, it is necessary to consider that ITN is closed, i.e. there is no immigration or emigration of objects in the system. In addition, given the time frame of information epidemic, let us assume that elements are not eliminated from the system in the process of the infection spread.

These estimates allow to evaluate ITN epidemic resistance.

Instant epidemic resistance – the ratio of uninfected network elements to the total number of susceptible to infection elements of analyzed ITN at a given time moment.

Band epidemic resistance – the ratio of uninfected network elements to the total number of susceptible to infection elements of analyzed ITN in a given time interval.

Let us consider the models of viral epidemic development, in which the spread begins with the single element injection. In this case, let us assume the worst variant when only uninfected and unrestored elements will be exposed to the impact at the each stage of the process.

3. RISK ASSESSMENT OF THE PROCESS OF INFORMATION INFECTION DISSEMINATION FOR SI MODEL

Method of discrete SI model constructing

Let us consider the scenario of a malware ITN attack with a *network virus* that uses for its spread vulnerabilities in the work of network services of the operating systems providing access to the network [1-2, 4, 5]. This model does not reflect the work of virus protection tools.

Network viruses penetrate into the computer memory from a computer network, evaluate network addresses of other computers and send their copies to these addresses.

Previously in the work, it was noted that models construction is conducted under the conditions when contacts in the ITN can be represented by a complete graph, and the epidemic occurs in a closed ITN in the case of immigration or emigration processes absence. The features of the process of malware information transmission from one computer to another, as well as the internal features of malware program execution in the computer, are ignored.

For this scenario, SI model is applicable, according to which the elements of ITN can be a part of one of the following subsets:

1. Susceptible (S) is a set of elements that are susceptible to malware information receiving. As soon as they become infected, they move to the category of the infected elements. $S[i]$ is the number of susceptible elements at the i^{th} stage of the infection process;

2. Infected (I) – elements that can spread malware information to susceptible objects. $I[i]$ is the number of infected elements at the i^{th} stage of the epidemic.

Let us describe the parameters of information epidemic development in the following way:

N is the total number of system elements, it is a specified parameter, not changeable in the epidemic process, and does not have a probabilistic nature;

$(1+n)$ is the average number of elements that are in direct contact with each element, it is a probabilistic parameter for some extent that depends on the network topology;

Q_i is the evaluation of elements infecting expectation at the i^{th} stage of the epidemic, according to the relevant probability distribution.

Note: It should be noted that n averaging is not always valid. For example, for scale-free networks and exponential graphs it should be considered that it is incrementally changed.

In the general case, the number of contacting elements with an infected element n may be different. This property specifies the distribution law of single viral exposure success, i.e., determines the possibility (probability) of occurrence of Q_i infected elements among n attacked. Such distributions could be binomial, Poisson, or others, for which mathematical expectation and modes are known. Therefore, it is appropriate to take into account this chance through possible Q_i , which can be set in different ways (depending on the distribution type).

At the initial stage of the epidemic, the first element became infected, from which begins a viral epidemic in the system. Thus:

$$S[0] = 1, \tag{1}$$

$$Q_0 = 1, \tag{2}$$

$$I[0] = 1. \tag{3}$$

The number of susceptible and infected elements at the first stage can be represented as:

$$S[1] = 1 + n, \tag{4}$$

$$I[1] = 1 + Q_1. \tag{5}$$



At the next stage each of Q_1 infected elements interacts with n adjacent elements, so Q_2 become infected. Thus:

$$S[2] = 1 + n + Q_1 n, \tag{6}$$

$$I[2] = 1 + Q_1 + Q_2. \tag{7}$$

Similarly, for the k^{th} stage of the epidemic, we get the following expressions:

$$S[k] = 1 + n + Q_1 n + Q_2 n + \dots + Q_{k-1} n = 1 + \sum_{i=0}^{k-1} n Q_i, \tag{8}$$

$$I[k] = 1 + Q_1 + Q_2 + \dots + Q_k = \sum_{i=0}^k Q_i. \tag{9}$$

Let us modify the formulas (8) and (9) for different evaluations. For the average evaluation we get:

$$S[k] = 1 + \sum_{i=0}^{k-1} n M_i(X), \tag{10}$$

$$I[k] = \sum_{i=0}^k M_i(X), \tag{11}$$

where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic.

For peak evaluation we obtain:

$$S[k] = 1 + \sum_{i=0}^{k-1} n M_{0i}(X), \tag{12}$$

$$I[k] = \sum_{i=0}^k M_{0i}(X), \tag{13}$$

where $M_{0i}(X)$ is the mode of infected elements number at the i^{th} stage of the epidemic.

In the case of interval evaluation we have:

$$S[k] = 1 + \sum_{i=0}^{k-1} n (M_i(X) \mp D_i(X)), \tag{14}$$

$$S_1[k] = 1 + \sum_{i=0}^{k-1} n (M_i(X) - D_i(X)), \tag{15}$$

$$S_2[k] = 1 + \sum_{i=0}^{k-1} n (M_i(X) + D_i(X)), \tag{16}$$

$$I[k] = \sum_{i=0}^k (M_i(X) \mp D_i(X)), \tag{17}$$

$$I_1[k] = \sum_{i=0}^k (M_i(X) - D_i(X)), \tag{18}$$

$$I_2[k] = \sum_{i=0}^k (M_i(X) + D_i(X)), \tag{19}$$

where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic;

$D_i(X)$ is the standard deviation (SD) of infected elements number at the i^{th} stage of the epidemic.

Here the numbers of infected and susceptible elements are in the intervals $(I_1[k], I_2[k])$ and $(S_1[k], S_2[k])$, respectively.

Risk analysis and evaluation of network epidemic resistance in conditions of epidemic spread by SI model

Overall risk $Risk[k]$ at the k^{th} stage of the epidemic is equal to:

$$Risk[k] = \frac{I[k]}{S[k]} = \frac{\sum_{i=0}^k Q_i}{1 + \sum_{i=0}^{k-1} n Q_i}. \tag{20}$$

The epidemic resistance of $L[k]$ system can be evaluated as the ratio of the expected number of uninfected nodes to the total number of nodes, involved in the process. As a result, we have:

$$L[k] = \frac{S[k]-I[k]}{S[k]} = 1 - Risk[k] = 1 - \frac{\sum_{i=0}^k Q_i}{1 + \sum_{i=0}^{k-1} n Q_i}. \tag{21}$$

Consider the formulas (20) and (21) for different evaluations. For the average evaluation of risk and epidemic resistance of system, the formulas are as follows:

$$Risk[k] = \frac{I[k]}{S[k]} = \frac{\sum_{i=0}^k M_i(X)}{1 + \sum_{i=0}^{k-1} n M_i(X)}, \tag{22}$$

$$L[k] = \frac{S[k]-I[k]}{S[k]} = 1 - Risk[k] = 1 - \frac{\sum_{i=0}^k M_i(X)}{1 + \sum_{i=0}^{k-1} n M_i(X)}, \tag{23}$$

where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic.

In the case of peak evaluation, risk and epidemic resistance of the system can be evaluated by the following formulas:

$$Risk[k] = \frac{I[k]}{S[k]} = \frac{\sum_{i=0}^k M_{0i}(X)}{1 + \sum_{i=0}^{k-1} n M_{0i}(X)}, \tag{24}$$

$$L[k] = \frac{S[k]-I[k]}{S[k]} = 1 - Risk(k) = 1 - \frac{\sum_{i=0}^k M_{0i}(X)}{1 + \sum_{i=0}^{k-1} n M_{0i}(X)}, \tag{25}$$

where $M_{0i}(X)$ is the mode of infected elements number at the i^{th} stage of the epidemic.

Similarly, we obtain the following formulas for interval estimates:

$$Risk[k] = \frac{I[k]}{S[k]} = \frac{\sum_{i=0}^k (M_i(X) \mp D_i(X))}{1 + \sum_{i=0}^{k-1} n (M_i(X) \pm D_i(X))}, \tag{26}$$

$$Risk_1[k] = \frac{I[k]}{S[k]} = \frac{\sum_{i=0}^k (M_i(X) - D_i(X))}{1 + \sum_{i=0}^{k-1} n (M_i(X) + D_i(X))}, \tag{27}$$

$$Risk_2[k] = \frac{I[k]}{S[k]} = \frac{\sum_{i=0}^k (M_i(X) + D_i(X))}{1 + \sum_{i=0}^{k-1} n (M_i(X) - D_i(X))}, \tag{28}$$

$$L[k] = \frac{S[k]-I[k]}{S[k]} = 1 - Risk(k) = 1 - \frac{\sum_{i=0}^k (M_i(X) \pm D_i(X))}{1 + \sum_{i=0}^{k-1} n (M_i(X) \mp D_i(X))}, \tag{29}$$

$$L_1[k] = 1 - \frac{\sum_{i=0}^k (M_i(X) + D_i(X))}{1 + \sum_{i=1}^k n (M_i(X) - D_i(X))}, \tag{30}$$



$$L_2[k] = 1 - \frac{\sum_{i=0}^k (M_i(X) - D_i(X))}{1 + \sum_{i=1}^k n(M_i(X) + D_i(X))} \quad (31)$$

where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic;

$D_i(X)$ is the SD of infected elements number at the i^{th} stage of the epidemic.

In such type of evaluation, the overall risk of the system and epidemic resistance of the system are evaluated in intervals ($Risk_1[k], Risk_2[k]$) and ($L_1[k], L_2[k]$), respectively.

Based on the methodology proposed in the present section, the same description of other varieties of the processes of ITN infection by viruses is possible, including elements recovering.

4. RISK ANALYSIS OF THE PROCESS OF INFORMATION INFECTION DISSEMINATION BY SIS MODEL
Method of discrete SIS model constructing

Let us consider the previous scenario of the ITN attack by *network virus* and reflect the work of antiviral tools. Let us describe the case when the antivirus is able to detect malware virus and restore the infected ITN elements [3].

The SIS model is applicable to this scenario, according to which the elements of ITN can be a part of one of the following subsets:

1. Susceptible (S) – a set of elements that are susceptible to receiving malware information. As soon as they become infected, they go to the category of the infected elements. $S[i]$ is the number of susceptible elements at the i^{th} stage of the infecting process;
2. Infected (I) – elements that can spread malware information to susceptible objects. $I[i]$ is the number of infected elements at the i^{th} stage of the epidemic.

Parameters of information epidemic development we will describe in the following way:

- N is the total number of system elements, it is a specified parameter, not changeable in the process of the epidemic, and does not have a probabilistic nature;
- $(1+n)$ is the average number of elements, which are in direct contact with each element; it is a probabilistic parameter in some extent that depends on the network topology;
- Q_i – evaluation of expectation of elements infecting at the i^{th} stage of the epidemic, according to the relevant probability distribution;
- P_i – evaluation of expectation of elements recovery at the i^{th} stage of the epidemic, according to the relevant probability distribution.

At the initial stage of the epidemic, the first element became infected, from which begins a viral epidemic in the system. Thus:

$$S[0] = 1, \quad (32)$$

$$Q_0 = 1, \quad (33)$$

$$I[0] = 1. \quad (34)$$

At the first stage, the number of susceptible and infected elements can be represented as:

$$S[1] = 1 + n, \quad (35)$$

$$I[1] = 1 + (Q_1 - P_1). \quad (36)$$

At the next stage, each of the $(Q_1 - P_1)$ infected elements interacts with n adjacent elements, so Q_2 become infected. Thus:

$$S[2] = 1 + n + (Q_1 - P_1)n, \quad (37)$$

$$I[2] = 1 + (Q_1 - P_1) + (Q_2 - P_2). \quad (38)$$

Similarly, for the k^{th} stage of the epidemic we get the following expressions:

$$S[k] = 1 + \sum_{i=0}^{k-1} n(Q_i - P_i), \quad (39)$$

$$I[k] = 1 + \sum_{i=1}^k (Q_i - P_i). \quad (40)$$

Let us modify the formulas (39) and (40) for different evaluations. For the average evaluations we get:

$$S[k] = 1 + \sum_{i=0}^{k-1} n(M_i(X) - A_i(X)), \quad (41)$$

$$I[k] = 1 + \sum_{i=1}^k (M_i(X) - A_i(X)), \quad (42)$$

where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic;

$A_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic.

For peak evaluation:

$$S[k] = 1 + \sum_{i=0}^{k-1} n(Mo_i(X) - Ao_i(X)), \quad (43)$$

$$I[k] = 1 + \sum_{i=1}^k (Mo_i(X) - Ao_i(X)), \quad (44)$$

where $Mo_i(X)$ is the mode of infected elements number at the i^{th} stage of the epidemic;

$Ao_i(X)$ is the mode of repaired elements number at the i^{th} stage of the epidemic.

In the case of interval estimation we have:

$$S[k] = 1 + \sum_{i=0}^{k-1} n \left((M_i(X) \mp Z_i(X)) - (A_j(X) \pm C_i(X)) \right), \quad (45)$$

$$I[k] = 1 + \sum_{i=1}^k \left((M_i(X) \mp Z_i(X)) - (A_j(X) \pm C_i(X)) \right), \quad (46)$$



where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic;
 $A_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic;
 $Z_i(X)$ is the SD of the finally infected elements number at the i^{th} stage of the epidemic;
 $C_i(X)$ is the SD of the restored infected elements number at the i^{th} stage of the epidemic.
 The number of infected and susceptible elements are in the intervals $(I_1[k], I_2[k])$ and $(S_1[k], S_2[k])$, respectively.

Risk analysis and evaluation of network epidemic resistance in terms of information epidemic dissemination by SIS model

Using the formulas (39) and (40), we obtain expressions for the ITN risk in case of viral epidemic by SIS model implementation:

$$\text{Risk}[k] = \frac{I[k]}{S[k]} = \frac{1 + \sum_{i=1}^k (Q_i - P_i)}{1 + \sum_{i=0}^{k-1} n(Q_i - P_i)} \quad (47)$$

Epidemic resistance of the system $L[k]$ can be evaluated as the ratio of the expected number of uninfected elements to the total number of elements involved in the process. As a result, we have:

$$L[k] = \frac{S[k] - I[k]}{S[k]} = 1 - \text{Risk}(k) = 1 - \frac{1 + \sum_{i=1}^k (Q_i - P_i)}{1 + \sum_{i=0}^{k-1} n(Q_i - P_i)} \quad (48)$$

Let us consider the formulas (47) and (48) for different evaluations. For average evaluation, the formulas of risk and epidemic resistance of the system are as follows:

$$\text{Risk}[k] = \frac{I[k]}{S[k]} = \frac{1 + \sum_{i=1}^k (M_i(X) - A_i(X))}{1 + \sum_{i=0}^{k-1} n(M_i(X) - A_i(X))} \quad (49)$$

$$L[k] = \frac{S[k] - I[k]}{S[k]} = 1 - \text{Risk}(k) = 1 - \frac{1 + \sum_{i=1}^k (M_i(X) - A_i(X))}{1 + \sum_{i=0}^{k-1} n(M_i(X) - A_i(X))} \quad (50)$$

where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic;
 $A_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic.
 In the case of peak evaluation, risk and epidemic resistance of the system can be estimated by the following formulas:

$$\text{Risk}[k] = \frac{I[k]}{S[k]} = \frac{1 + \sum_{i=1}^k (M_{o_i}(X) - A_{o_i}(X))}{1 + \sum_{i=0}^{k-1} n(M_{o_i}(X) - A_{o_i}(X))} \quad (51)$$

$$L[k] = \frac{S[k] - I[k]}{S[k]} = 1 - \text{Risk}(k) = 1 - \frac{1 + \sum_{i=1}^k (M_{o_i}(X) - A_{o_i}(X))}{1 + \sum_{i=0}^{k-1} n(M_{o_i}(X) - A_{o_i}(X))} \quad (52)$$

where $M_{o_i}(X)$ is the mode of infected elements number at the i^{th} stage of the epidemic;
 $A_{o_i}(X)$ is the mode of repaired elements number at the i^{th} stage of the epidemic.
 Similarly, for interval evaluation, we obtain the following formulas:

$$\text{Risk}[k] = \frac{I[k]}{S[k]} = \frac{1 + \sum_{i=1}^k ((M_i(X) \mp Z_i(X)) - (A_j(X) \pm C_i(X)))}{1 + \sum_{i=0}^{k-1} n((M_i(X) \mp Z_i(X)) - (A_j(X) \pm C_i(X)))} \quad (53)$$

$$L[k] = \frac{S[k] - I[k]}{S[k]} = 1 - \text{Risk}[k] = 1 - \frac{1 + \sum_{i=1}^k ((M_i(X) \mp Z_i(X)) - (A_j(X) \pm C_i(X)))}{1 + \sum_{i=0}^{k-1} n((M_i(X) \mp Z_i(X)) - (A_j(X) \pm C_i(X)))} \quad (54)$$

where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic,
 $A_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic;
 $Z_i(X)$ is the SD of finally infected elements number at the i^{th} stage of the epidemic;
 $C_i(X)$ is the SD of restored infected elements number at the i^{th} stage of the epidemic.
 The expressions (49)-(54) are appropriate to use for numerical evaluation of risk and epidemic resistance of the ITN.

5. EVALUATION OF RISK OF SPREAD PROCESS AND RISK ASSESSMENTS OF INFORMATION INFECTION FOR SEIS MODEL

Method of discrete SEIS model constructing

Let us consider the previous scenario of ITN attack by network virus considering the work of antiviral tools. Let us add additional ITN elements state and describe the resulting model [6].

The adding of an additional state allows reducing the modeling errors, thereby to obtain the results that are more close to real.

Let us add the state of the elements of this model, in which they are infected, but don't spread the viruses.

Note: the described case is typical for network viruses, the purpose of which is to obtain control over the system.

In accordance with the SEIS model, the system elements can be a part of one of the following subsets:

1. Susceptible (S) – elements that are susceptible to viral infection. As soon as they become



- infected, they move into the category of latent elements;
 - 2. Latent (E) – elements that are infected, but don't spread the virus yet. After the end of incubation period, they move into the category of infected elements;
 - 3. Infected (I) – elements that can spread malware information to susceptible objects.
- In this context, let us introduce the following indications:
- N is the total number of elements in the network structure;
 - (1+n) is the average number of elements interacting with each element of the network structure;
 - Q_i – evaluation of expectation of latent infection of the elements at the i^{th} stage of the epidemic, according to the relevant probability distribution;
 - P_i – evaluation of elements finally infecting expectation at the i^{th} stage of the epidemic, according to the relevant probability distribution;
 - W_i – evaluation of elements recovery expectation at the i^{th} stage of the epidemic, according to the relevant probability distribution.

At the initial stage of the epidemic, the first element became infected, from which begins a viral epidemic in the system. Thus:

$$S[0] = 1, \tag{55}$$

$$Q_0 = 1, \tag{56}$$

$$E[0] = 1. \tag{57}$$

$$P_0 = 1, \tag{58}$$

$$I[0] = 1. \tag{59}$$

At the first stage, the number of susceptible and infected elements can be represented as the following expressions:

$$S[1] = 1 + n, \tag{60}$$

$$E[1] = 1 + (Q_1 - P_1), \tag{61}$$

$$I[1] = 1 + (P_1 - W_1). \tag{62}$$

At the second stage, each of $(P_1 - W_1)$ infected elements interacts with n adjacent elements, so Q_2 become latent, P_2 become completely infected, and W_2 become restored. So we have:

$$S[2] = 1 + n + (P_1 - W_1)n, \tag{63}$$

$$E[2] = 1 + (Q_1 - P_1 - W_1) + (Q_2 - P_2 - W_2), \tag{64}$$

$$I[2] = 1 + (P_1 - W_1) + (P_2 - W_2). \tag{65}$$

Next, for the k^{th} stage we have the following expressions:

$$S[k] = 1 + \sum_{i=0}^{k-1} n(P_i - W_i), \tag{66}$$

$$E[k] = 1 + \sum_{i=1}^k (Q_i - P_i - W_i), \tag{67}$$

$$I[k] = 1 + \sum_{i=1}^k (P_i - W_i). \tag{68}$$

Let us consider the formulas (66), (67) and (68) for the various types of risk assessments. So for the average evaluation we have:

$$S[k] = 1 + \sum_{i=0}^{k-1} n(E_i(X) - A_i(X)), \tag{69}$$

$$E[k] = 1 + \sum_{i=1}^k (M_i(X) - E_i(X) - A_i(X)), \tag{70}$$

$$I[k] = 1 + \sum_{i=1}^k (E_i(X) - A_i(X)), \tag{71}$$

where $M_i(X)$ is the expected number of latent elements at the i^{th} stage of the epidemic;

$E_i(X)$ is the expected number of finally infected elements at the i^{th} stage of the epidemic;

$A_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic.

In turn, for peak evaluation we have:

$$S[k] = 1 + \sum_{i=0}^{k-1} n(Eo_i(X) - Ao_i(X)), \tag{72}$$

$$E[k] = 1 + \sum_{i=1}^k (Mo_i(X) - Eo_i(X) - Ao_i(X)), \tag{73}$$

$$I[k] = 1 + \sum_{i=1}^k (Eo_i(X) - Ao_i(X)), \tag{74}$$

where $Mo_i(X)$ is the mode of latent elements number at the i^{th} stage of the epidemic;

$Eo_i(X)$ is the mode of finally infected elements number at the i^{th} stage of the epidemic;

$Ao_i(X)$ is the mode of repaired elements number at the i^{th} stage of the epidemic.

The following expressions were obtained for interval evaluation:

$$S[k] = 1 + \sum_{i=0}^{k-1} n((E_i(X) \mp Z_i(X)) - (A_i(X) \pm C_i(X))), \tag{75}$$

$$E[k] = 1 + \sum_{i=1}^k ((M_i(X) \mp D_i(X)) - (E_i(X) \pm Z_i(X)) - (A_i(X) \pm C_i(X))), \tag{76}$$

$$I[k] = 1 + \sum_{i=1}^k ((E_i(X) \mp Z_i(X)) - (A_i(X) \pm C_i(X))), \tag{77}$$

where $M_i(X)$ is the expected number of latent elements at the i^{th} stage of the epidemic;

$D_i(X)$ is the SD for the number of latent elements at the i^{th} stage of the epidemic;



$E_i(X)$ is the expected number of finally infected elements at the i^{th} stage of the epidemic;
 $Z_i(X)$ is the SD for the number of finally infected elements at the i^{th} stage of the epidemic;
 $A_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic;
 $C_i(X)$ is the SD for the number of restored infected elements at the i^{th} stage of the epidemic.

Risk analysis and evaluation of network epidemic resistance in terms of information epidemic dissemination by the SEIS model

Using the formulas (66) and (68), we obtain expressions for the risk of ITN in case of viral epidemic implementation by the SEIS model:

$$\text{Risk}[k] = \frac{I[k]}{S[k]} = \frac{1 + \sum_{i=1}^k (P_i - W_i)}{1 + \sum_{i=0}^{k-1} n(P_i - W_i)} \tag{78}$$

So the epidemic resistance of the system can be evaluated as follows:

$$L[k] = \frac{S[k] - I[k]}{S[k]} = 1 - \frac{1 + \sum_{i=1}^k (P_i - W_i)}{1 + \sum_{i=0}^{k-1} n(P_i - W_i)} \tag{79}$$

The formulas for the average evaluation (78) and (79) take the following form:

$$\text{Risk}[k] = \frac{1 + \sum_{i=1}^k (E_i(X) - A_i(X))}{1 + \sum_{i=0}^{k-1} n(E_i(X) - A_i(X))} \tag{80}$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k (E_i(X) - A_i(X))}{1 + \sum_{i=0}^{k-1} n(E_i(X) - A_i(X))} \tag{81}$$

For peak evaluation we have:

$$\text{Risk}[k] = \frac{1 + \sum_{i=1}^k (E_{o_i}(X) - A_{o_i}(X))}{1 + \sum_{i=0}^{k-1} n(E_{o_i}(X) - A_{o_i}(X))} \tag{82}$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k (E_{o_i}(X) - A_{o_i}(X))}{1 + \sum_{i=0}^{k-1} n(E_{o_i}(X) - A_{o_i}(X))} \tag{83}$$

Interval evaluation gives the following formulas:

$$\text{Risk}[k] = \frac{1 + \sum_{i=1}^k ((E_i(X) \mp Z_i(X)) - (A_i(X) \pm C_i(X)))}{1 + \sum_{i=0}^{k-1} n((E_i(X) \mp Z_i(X)) - (A_i(X) \pm C_i(X)))} \tag{84}$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k ((E_i(X) \mp Z_i(X)) - (A_i(X) \pm C_i(X)))}{1 + \sum_{i=0}^{k-1} n((E_i(X) \mp Z_i(X)) - (A_i(X) \pm C_i(X)))} \tag{85}$$

The expressions (80)-(85) are appropriate to use for numerical evaluation of risk and epidemic resistance of ITN.

6. RISK ANALYSIS OF THE PROCESS OF INFORMATION INFECTION DISSEMINATION BY SIR MODEL

Method of discrete SIR model construction

Let us consider the scenario of a malware *simple virus* attack on the computer systems with working in the "on-the-fly" mode (resident) *antivirus*.

The principle of *simple virus* action: the virus records its own code to any place of the infected file (usually to the end) and transfers control to malware code; the result of this is infecting other files [7].

Thus, the infected file becomes a source of the epidemic.

Simple viruses are detected by their code, which they record into the infected file. Antivirus programs intercept all file operations (such as read, copy, run) and check the files that perform actions (life.sys driver of file antivirus). Detection of an infected file that was received via email or from the Internet occurs at the moment of saving the file to disk (in the case when only file antivirus is working and the mail and web antiviruses are disabled). It is impossible to run the file from the attachment or web page without saving to disk [8].

If during the file antivirus scan the file found to be infected, the operation is blocked and the file is treated or removed.

The delete operation applies to malware programs, the aim of which is the violation of the integrity – modification or destruction. If the goal is privacy (copying, disclosure) or availability (blocking) violation, the files can be cured.

Treatment of infected elements implies removing the malware parts of the code and transferring control to the initial block which is not removed by a simple virus but just moved to another location. Let us note that the file may become unusable after treatment. This occurs due to misidentification or restoring of the management part of the code. This case we will consider later because it does not correspond to this model.

Applicable to a simple virus, antiviral programs can apply immunization function. This function can be performed in two variants: immunization that reports about infection and immunization that blocks infection by any type of virus. The first is usually recorded to the end of the file (like file virus) and check the file to the presence of any changes each time when file runs.

The second type of immunization protects system from attack of some special type of virus. Files on the disk become modified in such a way that the virus takes them as the already infected. A program that simulates a copy of the virus is recorded into the computer memory for protection against the memory-resident virus. When run, the virus stumbles upon it and thinks that the system is already infected.



Thus, SIR model is applicable for this scenario, according to which the ITN elements can be a part of one of the following subsets:

1. Susceptible (*S*) – elements that are susceptible to virus infection, they are able to become infected. As soon as they become infected, they move to the category of the infected elements. $S[i]$ is the number of susceptible elements at the i^{th} stage of the epidemic;

2. Infected (*I*) – elements that can spread malware information to susceptible objects. $I[i]$ is the number of infected elements at the i^{th} stage of system infection;

3. Recovered (*R*) – elements that are completely free from malware information. $R[i]$ is the number of repaired elements at the i^{th} stage of the epidemic.

Let us consider a distributed computer system in which viral epidemic is evolving by the SIR model with the following parameters:

N is the total number of system elements, it is a specified parameter, not changeable in the process of the epidemic, and does not have a probabilistic nature;

$(1+n)$ is the average number of elements that are in direct contact with each element;

Q_i – evaluation of elements infecting expectation at the i^{th} stage of the epidemic, according to the relevant probability distribution of infection;

P_i – evaluation of elements recovery expectation at the i^{th} stage of the epidemic, according to the relevant probability distribution.

At the initial stage of the epidemic, the first element became infected, from which begins a viral epidemic in the system. Here we have:

$$S[0] = 1, \tag{86}$$

$$Q_0 = 1, \tag{87}$$

$$I[0] = 1, \tag{88}$$

$$R[0] = 0. \tag{89}$$

At the first stage, the number of susceptible and infected elements can be represented as:

$$S[1] = 1 + n - P_1, \tag{90}$$

$$I[1] = 1 + (Q_1 - P_1), \tag{91}$$

$$R[1] = P_1. \tag{92}$$

At the second stage, each of the $(Q_1 - P_1)$ infected elements interacts with n adjacent elements, so Q_2 become infected, and P_2 become restored. Thus:

$$S[2] = 1 + n - P_1 + (Q_1 - P_1)n - P_2, \tag{93}$$

$$I[2] = 1 + (Q_1 - P_1) + (Q_2 - P_2), \tag{94}$$

$$R[2] = P_1 + P_2. \tag{95}$$

At the third stage, each of the $(Q_2 - P_2)$ infected elements interacts with n adjacent elements.

Thus:

$$S[3] = 1 + n - P_1 + (Q_1 - P_1)n - P_2 + (Q_2 - P_2)n - P_3, \tag{96}$$

$$I[3] = 1 + (Q_1 - P_1) + (Q_2 - P_2) + (Q_3 - P_3), \tag{97}$$

$$R[3] = P_1 + P_2 + P_3. \tag{98}$$

Similarly, for the k^{th} stage we get the following expressions:

$$S[k] = 1 + \sum_{i=1}^k (n(Q_{i-1} - P_{i-1}) - P_i), \tag{99}$$

$$I[k] = 1 + \sum_{i=1}^k (Q_i - P_i), \tag{100}$$

$$R[k] = \sum_{i=0}^k P_i. \tag{101}$$

Let us consider the formulas (99), (100) and (101) for different types of risk assessments. For the average evaluation we get:

$$S[k] = 1 + \sum_{i=1}^k (n(M_{i-1}(X) - E_{i-1}(X)) - E_i(X)), \tag{102}$$

$$I[k] = 1 + \sum_{i=1}^k (M_i(X) - E_i(X)), \tag{103}$$

$$R[k] = \sum_{i=0}^k E_i(X), \tag{104}$$

where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic;

$E_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic.

For peak evaluation we have:

$$S[k] = 1 + \sum_{i=1}^k (n(Mo_{i-1}(X) - Eo_{i-1}(X)) - Eo_i(X)), \tag{105}$$

$$I[k] = 1 + \sum_{i=1}^k (Mo_i(X) - Eo_i(X)), \tag{106}$$

$$R[k] = \sum_{i=0}^k Eo_i(X), \tag{107}$$

where $Mo_i(X)$ is the mode of infected elements number at the i^{th} stage of the epidemic;

$Eo_i(X)$ is the mode of repaired elements number at the i^{th} stage of the epidemic.

For interval evaluation we obtain the following expressions:

$$S[k] = 1 + \sum_{i=1}^k [n((M_{i-1}(X) \mp D_{i-1}(X)) - (E_{i-1}(X) \pm Z_{i-1}(X))) - (E_i(X) \pm Z_i(X))], \tag{108}$$



$$S_1[k] = 1 + \sum_{i=1}^k [n((M_{i-1}(X) - D_{i-1}(X)) - (E_{i-1}(X) + Z_{i-1}(X))) - (E_i(X) + Z_i(X))], \quad (109)$$

$$S_2[k] = 1 + \sum_{i=1}^k [n((M_{i-1}(X) + D_{i-1}(X)) - (E_{i-1}(X) - Z_{i-1}(X))) - (E_i(X) - Z_i(X))], \quad (110)$$

$$I[k] = 1 + \sum_{i=1}^k ((M_i(X) \mp D_i(X)) - (E_i(X) \pm Z_i(X))), \quad (111)$$

$$I_1[k] = 1 + \sum_{i=1}^k ((M_i(X) - D_i(X)) - (E_i(X) + Z_i(X))), \quad (112)$$

$$I_2[k] = 1 + \sum_{i=1}^k ((M_i(X) + D_i(X)) - (E_i(X) - Z_i(X))), \quad (113)$$

$$R[k] = \sum_{i=0}^k (E_i(X) \pm Z_i(X)), \quad (114)$$

$$R_1[k] = \sum_{i=0}^k (E_i(X) + Z_i(X)), \quad (115)$$

$$R_2[k] = \sum_{i=0}^k (E_i(X) - Z_i(X)), \quad (116)$$

Where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic;

$D_i(X)$ is the SD of infected elements number at the i^{th} stage of the epidemic;

$E_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic;

$Z_i(X)$ is the SD of repaired elements number at the i^{th} stage of the epidemic.

While the number of infected, susceptible and recovered elements are in the intervals $(I_1[k], I_2[k]), (S_1[k], S_2[k])$ and $(R_1[k], R_2[k]),$ respectively.

Risk analysis and assessment of network epidemic resistance in terms of information infection dissemination by the SIR model

Using the formulas (99) and (100), let us evaluate the ITN risk in case of virus epidemic implementing by the SIR model:

$$Risk[k] = \frac{I[k]}{S[k]} = \frac{1 + \sum_{i=1}^k (Q_i - P_i)}{1 + \sum_{i=1}^k (n(Q_{i-1} - P_{i-1}) - P_i)}. \quad (117)$$

Epidemic resistance of the system can be evaluated as the ratio of the expected number of uninfected nodes to the total number of involved elements, thus:

$$L[k] = \frac{S[k] - I[k]}{S[k]} = 1 - Risk[k] = 1 - \frac{1 + \sum_{i=1}^k (Q_i - P_i)}{1 + \sum_{i=1}^k (n(Q_{i-1} - P_{i-1}) - P_i)}. \quad (118)$$

Let us consider the formulas (117) and (118) for average, peak and interval risk assessment. Formulas for the average evaluation of risk and epidemic resistance take the following form:

$$Risk[k] = \frac{1 + \sum_{i=1}^k (M_i(X) - E_i(X))}{1 + \sum_{i=1}^k (n(M_{i-1}(X) - E_{i-1}(X)) - E_i(X))}, \quad (119)$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k (M_i(X) - E_i(X))}{1 + \sum_{i=1}^k (n(M_{i-1}(X) - E_{i-1}(X)) - E_i(X))}, \quad (120)$$

where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic;

$E_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic.

For peak evaluation, risk and epidemic resistance can be determined by the following formulas:

$$Risk[k] = \frac{1 + \sum_{i=1}^k (Mo_i(X) - Eo_i(X))}{1 + \sum_{i=1}^k (n(Mo_{i-1}(X) - Eo_{i-1}(X)) - Eo_i(X))}, \quad (121)$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k (Mo_i(X) - Eo_i(X))}{1 + \sum_{i=1}^k (n(Mo_{i-1}(X) - Eo_{i-1}(X)) - Eo_i(X))}, \quad (122)$$

where $Mo_i(X)$ is the mode of the number of infected elements at the i^{th} stage of the epidemic;

$Eo_i(X)$ is the mode of the number of repaired elements at the i^{th} stage of the epidemic.

In the case of interval evaluation of risk and epidemic resistance, the functions take the following form:

$$Risk[k] = \frac{I[k]}{S[k]} = \frac{1 + \sum_{i=1}^k ((M_i(X) \mp D_i(X)) - (E_i(X) \pm Z_i(X)))}{1 + \sum_{i=1}^k [n((M_{i-1}(X) \pm D_{i-1}(X)) - (E_{i-1}(X) \mp Z_{i-1}(X))) - (E_i(X) \mp Z_i(X))]}, \quad (123)$$

$$Risk_1[k] = \frac{1 + \sum_{i=1}^k ((M_i(X) - D_i(X)) - (E_i(X) + Z_i(X)))}{1 + \sum_{i=1}^k [n((M_{i-1}(X) + D_{i-1}(X)) - (E_{i-1}(X) - Z_{i-1}(X))) - (E_i(X) - Z_i(X))]}, \quad (124)$$

$$Risk_2[k] = \frac{1 + \sum_{i=1}^k ((M_i(X) + D_i(X)) - (E_i(X) - Z_i(X)))}{1 + \sum_{i=1}^k [n((M_{i-1}(X) - D_{i-1}(X)) - (E_{i-1}(X) + Z_{i-1}(X))) - (E_i(X) + Z_i(X))]}, \quad (125)$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k ((M_i(X) \pm D_i(X)) - (E_i(X) \mp Z_i(X)))}{\sum_{i=1}^k [n((M_{i-1}(X) \mp D_{i-1}(X)) - (E_{i-1}(X) \pm Z_{i-1}(X))) - (E_i(X) \pm Z_i(X))]}, \quad (126)$$

$$L_1[k] = 1 - \frac{1 + \sum_{i=1}^k ((M_i(X) + D_i(X)) - (E_i(X) - Z_i(X)))}{1 + \sum_{i=1}^k [n((M_{i-1}(X) - D_{i-1}(X)) - (E_{i-1}(X) + Z_{i-1}(X))) - (E_i(X) + Z_i(X))]}, \quad (127)$$

$$L_2[k] = 1 - \frac{1 + \sum_{i=1}^k ((M_i(X) - D_i(X)) - (E_i(X) + Z_i(X)))}{1 + \sum_{i=1}^k [n((M_{i-1}(X) + D_{i-1}(X)) - (E_{i-1}(X) - Z_{i-1}(X))) - (E_i(X) - Z_i(X))]}, \quad (128)$$

Where $M_i(X)$ is the expected number of infected elements at the i^{th} stage of the epidemic;

$D_i(X)$ is the SD of infected elements number at the i^{th} stage of the epidemic;
 $E_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic;
 $Z_i(X)$ is the SD of repaired elements number at the i^{th} stage of the epidemic.
 In such assessments, the overall system risk and epidemic resistance of the system are evaluated in the intervals $(Risk_1[k], Risk_2[k])$ and $(L_1[k], L_2[k])$, respectively.

7. RISK ASSESSMENT OF THE PROCESS OF INFECTION DISSEMINATION BY THE SEIR MODEL

Method of discrete SEIR model constructing

Let us consider the previous scenario of computer systems malware attack with a *simple virus* with working in the "on-the-fly" mode (resident) *antivirus*. Let us introduce an additional possible state of the ITN elements and describe the resulting model.

For this model let us introduce the state of the elements in which they are infected, but don't spread the virus yet (latent state) [24].

This approach is used when the virus creators give up the speed, paying attention to the disguise. This virus is undetectable for some period, allows its creators to gain control over millions of machines – they can make compelling distributed DOS attacks, destroying ordinary servers and system nodes important for the functioning of the entire network [25].

SEIR model is applicable to this scenario, according to which the elements of ITN can be a part of one of the following subsets:

1. Susceptible (S) – elements that are susceptible to virus infection. As soon as they become infected, they move into the category of latent elements;
2. Latent (E) – elements that are infected, but don't spread the virus yet. After the end of incubation period, they move into the category of infected elements;
3. Infected (I) – elements that can spread malware information to susceptible objects.
4. Recovered (R) – elements that are completely free from any malware information and are immune to subsequent infection that affected them before.

Let us express the parameters as follows:
 N is the total number of system elements, it is specified parameter, not changeable in the

- process of the epidemic and does not have a probabilistic nature;
 $(1+n)$ – the average number of elements that are in direct contact with each element;
 Q_i – evaluation of expectation of latent elements infecting the i^{th} stage of the epidemic, according to the relevant probability distribution;
 P_i – evaluation of expectation of final elements infecting the i^{th} stage of the epidemic, according to the relevant probability distribution;
 W_i – evaluation of expectation of elements recovery at the i^{th} stage of the epidemic, according to the relevant probability distribution.

At the initial stage of the epidemic the first element became infected, from which begins a viral epidemic in the system, thus:

$$S[0] = 1, \tag{129}$$

$$Q_0 = 1, \tag{130}$$

$$E[0] = 1. \tag{131}$$

$$P_0 = 1, \tag{132}$$

$$I[0] = 1, \tag{133}$$

$$W_1 = 0. \tag{134}$$

At the first stage, the number of susceptible and infected elements can be represented as:

$$S[1] = 1 + n - W_1, \tag{135}$$

$$E[1] = 1 + (Q_1 - P_1), \tag{136}$$

$$I[1] = 1 + (P_1 - W_1), \tag{137}$$

$$R[1] = W_1. \tag{138}$$

At the second stage, each of $(P_1 - W_1)$ infected elements interacts with n adjacent elements, so Q_2 become latent, P_2 become completely infected, and W_2 become restored. Thus:

$$S[2] = 1 + n - W_1 + (P_1 - W_1)n - W_2, \tag{139}$$

$$E[2] = 1 + (Q_1 - P_1 - W_1) + (Q_2 - P_2 - W_2), \tag{140}$$

$$I[2] = 1 + (P_1 - W_1) + (P_2 - W_2), \tag{141}$$

$$R[2] = W_1 + W_2. \tag{142}$$

Normally, for the k^{th} stage we get the following expressions:



$$S[k] = 1 + \sum_{i=1}^k (n(P_{i-1} - W_{i-1}) - W_i), \quad (143)$$

$$E[k] = 1 + \sum_{i=1}^k (Q_i - P_i - W_i), \quad (144)$$

$$I[k] = 1 + \sum_{i=1}^k (P_i - W_i), \quad (145)$$

$$R[k] = \sum_{i=1}^k W_i. \quad (146)$$

Let us consider the formulas (143), (144), (145) and (146) for various types of assessments. For the average evaluation we get:

$$S[k] = 1 + \sum_{i=1}^k (n(E_{i-1}(X) - A_{i-1}(X)) - A_i(X)), \quad (147)$$

$$E[k] = 1 + \sum_{i=1}^k (M_i(X) - E_i(X) - A_i(X)), \quad (148)$$

$$I[k] = 1 + \sum_{i=1}^k (E_i(X) - A_i(X)), \quad (149)$$

$$R[k] = \sum_{i=1}^k A_i(X). \quad (150)$$

where $M_i(X)$ is the expected number of latent elements at the i^{th} stage of the epidemic;

$E_i(X)$ is the expected number of finally infected elements at the i^{th} stage of the epidemic;

$A_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic.

For peak evaluation we obtain:

$$S[k] = 1 + \sum_{i=1}^k (n(Eo_{i-1}(X) - Ao_{i-1}(X)) - Ao_i(X)), \quad (151)$$

$$E[k] = 1 + \sum_{i=1}^k (Mo_i(X) - Eo_i(X) - Ao_i(X)), \quad (152)$$

$$I[k] = 1 + \sum_{i=1}^k (Eo_i(X) - Ao_i(X)), \quad (153)$$

$$R[k] = \sum_{i=1}^k Ao_i(X), \quad (154)$$

where $Mo_i(X)$ is the mode of the number of latent elements at the i^{th} stage of the epidemic;

$Eo_i(X)$ is the mode of the number of finally infected elements at the i^{th} stage of the epidemic;

$A_i(X)$ is the mode of the number of repaired elements at the i^{th} stage of the epidemic.

For interval evaluation we have the following formulas:

$$S[k] = 1 + \sum_{i=1}^k \left[\frac{n((E_{i-1}(X) \mp Z_{i-1}(X)) - (A_{i-1}(X) \pm C_{i-1}(X))) - (A_i(X) \pm C_i(X))}{1} \right] \quad (155)$$

$$E[k] = 1 + \sum_{i=1}^k ((M_i(X) \mp D_i(X)) - (E_i(X) \pm Z_i(X)) - (A_i(X) \pm C_i(X))), \quad (156)$$

$$I[k] = 1 + \sum_{i=1}^k ((E_i(X) \mp Z_i(X)) - (A_i(X) \pm C_i(X))), \quad (157)$$

$$R[k] = \sum_{i=1}^k (A_i(X) \pm C_i(X)). \quad (158)$$

where $M_i(X)$ is the expected number of latent elements at the i^{th} stage of the epidemic;

$D_i(X)$ is the SD of latent elements number at the i^{th} stage of the epidemic;

$E_i(X)$ is the expected number of finally infected elements at the i^{th} stage of the epidemic;

$Z_i(X)$ is the SD of finally infected elements number at the i^{th} stage of the epidemic;

$A_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic;

$C_i(X)$ is the SD of restored infected elements number at the i^{th} stage of the epidemic.

Risk analysis and evaluation of network epidemic resistance in terms of information epidemic dissemination by the SEIR model

Using the formulas (143) and (143), we obtain the expression of the ITN risk in case of implementing viral epidemic by the SEIR model:

$$\text{Risk}[k] = \frac{I[k]}{S[k]} = \frac{1 + \sum_{i=1}^k (P_i - W_i)}{1 + \sum_{i=1}^k (n(P_{i-1} - W_{i-1}) - W_i)}, \quad (159)$$

Epidemic resistance of the system can be evaluated as follows:

$$L[k] = \frac{S[k] - I[k]}{S[k]} = 1 - \frac{1 + \sum_{i=1}^k (P_i - W_i)}{1 + \sum_{i=1}^k (n(P_{i-1} - W_{i-1}) - W_i)}. \quad (160)$$

Let us consider the formulas (159) and (160) for various types of assessments. Formulas for the average evaluation of risk and epidemic resistance of the system are as follows:

$$\text{Risk}[k] = \frac{1 + \sum_{i=1}^k (E_i(X) - A_i(X))}{1 + \sum_{i=1}^k (n(E_{i-1}(X) - A_{i-1}(X)) - A_i(X))}, \quad (161)$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k (E_i(X) - A_i(X))}{1 + \sum_{i=1}^k (n(E_{i-1}(X) - A_{i-1}(X)) - A_i(X))}. \quad (162)$$

For peak evaluation we obtain:

$$\text{Risk}[k] = \frac{1 + \sum_{i=1}^k (Eo_i(X) - Ao_i(X))}{1 + \sum_{i=1}^k (n(Eo_{i-1}(X) - Ao_{i-1}(X)) - Ao_i(X))}, \quad (163)$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k (Eo_i(X) - Ao_i(X))}{1 + \sum_{i=1}^k (n(Eo_{i-1}(X) - Ao_{i-1}(X)) - Ao_i(X))}. \quad (164)$$

For interval evaluation, we obtain the following formulas:

$$\text{Risk}[k] = \frac{1 + \sum_{i=1}^k ((E_i(X) \pm Z_i(X)) - (A_i(X) \mp C_i(X)))}{1 + \sum_{i=1}^k [n((E_{i-1}(X) \mp Z_{i-1}(X)) - (A_{i-1}(X) \pm C_{i-1}(X))) - (A_i(X) \pm C_i(X))]}, \quad (165)$$

$$L[k] = 1 -$$



$$\frac{1 + \sum_{j=1}^k ((E_i(X) \pm Z_i(X)) - (A_i(X) \mp C_i(X)))}{1 + \sum_{j=1}^k [n((E_{i-1}(X) \mp Z_{i-1}(X)) - (A_{i-1}(X) \pm C_{i-1}(X))) - (A_i(X) \pm C_i(X))]} \quad (166)$$

where $M_i(X)$ is the expected number of latent elements at the i^{th} stage of the epidemic;
 $D_i(X)$ is the SD of latent elements number at the i^{th} stage of the epidemic;
 $E_i(X)$ is the expected number of finally infected elements at the i^{th} stage of the epidemic;
 $Z_i(X)$ is the SD of finally infected elements number at the i^{th} stage of the epidemic;
 $A_i(X)$ is the expected number of repaired elements at the i^{th} stage of the epidemic;
 $C_i(X)$ is the SD of restored infected elements number at the i^{th} stage of the epidemic.

8. ASSESSMENT OF THE RISKS OF INFECTION DISSEMINATION PROCESS FOR THE MSEIR MODEL

Method of discrete MSEIR model constructing

Let us consider the previous scenario of a malware computer systems attack *simple virus* with working in the "on-the-fly" mode (resident) *antivirus*. Let us introduce an additional possible state of ITN elements and describe the resulting model.

Let us consider the process of MSEIR information epidemic implementation. Let us proceed from the fact that ITN is closed, i.e. there is no immigration or emigration processes. However, given the time frame of the information epidemic, the failure of system components is also not taken into account. To describe the models of viral epidemics implementation, let us consider the approach according to which the spread of the virus in distributed computer system is estimated using the mathematical expectation $M(X)$ (average evaluation), mode $Mo(X)$ (peak evaluation) or the mathematical expectation $M(X)$ with the variance $D(X)$ (range evaluation) of the number of infected elements X of the system depending on the analysis needs.

Let us consider a model of the development of the viral epidemic, in which spread begins with the single element infection. In this case, let us assume the worst variant when only uninfected and unrestored elements will be exposed to the impact at the each stage of the process.

In accordance with the MSEIR model, the elements of the system can be a part of one of the following:

1. Protected (M) – elements that are resistant to viral infection.

2. Susceptible elements (S) – elements that are susceptible to viral infection. As soon as they become infected, they move into the category of latent elements.

3. Latent (E) – elements that have been infected, but not spread software threats yet. At a time when they can infect other objects, they move to the category of infected elements.

4. Infected elements (I) – elements that can spread malware information to susceptible objects. The time that they spend in the infected state is an infectious period, after which they move to the category of recovered elements.

5. Recovered elements (R) – elements that are completely free from malware information.

Let us introduce parameters of the epidemic in the following way:

N is the total number of system elements, it is a specified parameter, not changeable in the process of the epidemic, and does not have a probabilistic nature;

n is the average number of elements that are in direct contact with each element.

T_i – evaluation of expectation for the number of elements that lost immunity at the i^{th} stage of the epidemic, according to the relevant probability distribution.

Q_i – evaluation of expectation of latent elements infecting at the i^{th} stage of the epidemic, according to the relevant probability distribution.

P_i – evaluation of expectation of final elements infecting the i^{th} stage of the epidemic, according to the relevant probability distribution.

W_i – evaluation of expectation of elements recovery at the i^{th} stage of the epidemic, according to the relevant probability distribution.

At the initial stage of the epidemic the first element became infected, from which begins a viral epidemic in the system, thus:

$$T_0 = 1, \tag{167}$$

$$S[0] = 1, \tag{168}$$

$$Q_0 = 1, \tag{169}$$

$$E[0] = 1. \tag{170}$$

$$P_0 = 1, \tag{171}$$

$$I[0] = 1. \tag{172}$$

At the first stage, the number of susceptible and infected elements can be represented as:

$$S[1] = 1 + T_1, \tag{173}$$

$$E[1] = 1 + (Q_1 - P_1), \tag{174}$$

$$I[1] = 1 + (P_1 - W_1). \tag{175}$$

$$R[1] = W_1 \tag{176}$$

At the second stage, each of $(P_1 - W_1)$ infected elements interacts with n adjacent elements, so T_2 elements lose their immunity, Q_2 become latent, P_2 become completely infected, and W_2 become restored. Thus:

$$S[2] = 1 + T_1 + (Q_1 - P_1)T_2, \tag{177}$$

$$E[2] = 1 + (Q_1 - P_1) + (P_1 - W_1)(Q_2 - P_2), \tag{178}$$

$$I[2] = 1 + (P_1 - W_1) + (P_1 - W_1)(P_2 - W_2). \tag{179}$$

$$R[2] = W_1 + W_2(P_1 - W_1) \tag{180}$$

Normally, for the k^{th} stage we get the following expressions:

$$S[k] = 1 + \sum_{i=1}^k T_i \prod_{j=0}^{i-1} (P_j - W_j), \tag{181}$$

$$E[k] = 1 + \sum_{i=1}^k (Q_i - P_i) \prod_{j=0}^{i-1} (P_j - W_j), \tag{182}$$

$$I[k] = 1 + \sum_{i=1}^k \prod_{j=0}^i (P_j - W_j), \tag{183}$$

$$R[k] = \sum_{i=1}^k W_i \prod_{j=0}^{i-1} (P_j - W_j). \tag{184}$$

Let us consider the formulas (181), (182), (183) and (184) for different types of assessments. For the average evaluation we get:

$$S[k] = 1 + \sum_{i=1}^k B_i(X) \prod_{j=0}^{i-1} (E_j(X) - A_j(X)), \tag{185}$$

$$E[k] = 1 + \sum_{i=1}^k (M_i(X) - E_i(X)) \prod_{j=0}^{i-1} (E_j(X) - A_j(X)), \tag{186}$$

$$I[k] = 1 + \sum_{i=1}^k \prod_{j=0}^i (E_j(X) - A_j(X)), \tag{187}$$

$$R[k] = \sum_{i=1}^k A_i(X) \prod_{j=0}^{i-1} (E_j(X) - A_j(X)). \tag{188}$$

where $M_j(X)$ is the expected number of latent elements at the j^{th} stage of the epidemic, $E_j(X)$ is the expected number of finally infected elements at the j^{th} stage of the epidemic, $A_j(X)$ is the expected number of repaired elements at the j^{th} stage of the epidemic, $B_i(X)$ is the expected number of elements that have lost their immunity at the j^{th} stage of the epidemic.

For peak evaluation we obtain:

$$S[k] = 1 + \sum_{i=1}^k B_{o_i}(X) \prod_{j=0}^{i-1} (E_{o_j}(X) - A_{o_j}(X)), \tag{189}$$

$$E[k] = 1 + \sum_{i=1}^k (M_{o_i}(X) - E_{o_i}(X)) \prod_{j=0}^{i-1} (E_{o_j}(X) - A_{o_j}(X)), \tag{190}$$

$$I[k] = 1 + \sum_{i=1}^k \prod_{j=0}^i (E_{o_j}(X) - A_{o_j}(X)), \tag{191}$$

$$R[k] = \sum_{i=1}^k A_{o_i}(X) \prod_{j=0}^{i-1} (E_{o_j}(X) - A_{o_j}(X)). \tag{192}$$

where $M_{o_j}(X)$ is the mode of the number of latent elements at the j^{th} stage of the epidemic, $E_{o_j}(X)$ is the mode of the number of finally infected elements at the j^{th} stage of the epidemic, $A_j(X)$ is the mode of the number of repaired elements at the j^{th} stage of the epidemic.

For interval evaluation, we obtain the following formulas:

$$S[k] = 1 + \sum_{i=1}^k (B_i(X) \pm V_i(X)) \cdot \prod_{j=0}^{i-1} ((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)})) \tag{193}$$

$$E[k] = 1 + \sum_{i=1}^k ((M_i(X) \pm \sqrt{D_i(X)}) - (E_i(X) \pm \sqrt{Z_i(X)})) \cdot \prod_{j=0}^{i-1} ((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)})), \tag{194}$$

$$I[k] = 1 + \sum_{i=1}^k \prod_{j=0}^i ((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)})), \tag{195}$$

$$R[k] = \sum_{i=1}^k A_i(X) \pm \sqrt{C_i(X)} \prod_{j=0}^{i-1} ((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)})) \tag{196}$$

where $M_j(X)$ is the expected number of latent elements at the j^{th} stage of the epidemic, $D_j(X)$ is the dispersion of the latent elements number at the j^{th} stage of the epidemic, $E_j(X)$ is the expected number of finally infected elements at the j^{th} stage of the epidemic, $Z_j(X)$ is the dispersion of the finally infected elements number at the j^{th} stage of the epidemic, $A_j(X)$ is the expected number of repaired elements at the j^{th} stage of the epidemic, $C_j(X)$ is the dispersion of the recovered infected elements number on the j^{th} stage of the epidemic, $B_i(X)$ is the expected number of elements that have lost the immunity at the i^{th} stage of the epidemic, $V_i(X)$ is the dispersion of the number of elements that have lost the immunity at the i^{th} stage of the epidemic.



Risk analysis and evaluation of network epidemic resistance in terms of information epidemic dissemination by the MSEIR model

Using the formulas (181) and (183) we obtain the formula of the ITN risk in case of viral epidemic implementation by the MSEIR model:

$$\text{Risk}[k] = \frac{I[k]}{S[k]} = \frac{1 + \sum_{i=1}^k \prod_{j=0}^i (P_j - W_j)}{1 + \sum_{i=1}^k T_i \prod_{j=0}^{i-1} (P_j - W_j)}, \tag{197}$$

Epidemic resistance of the system can be assessed as follows:

$$L[k] = \frac{S[k] - I[k]}{S[k]} = 1 - \frac{1 + \sum_{i=1}^k \prod_{j=0}^i (P_j - W_j)}{1 + \sum_{i=1}^k T_i \prod_{j=0}^{i-1} (P_j - W_j)}. \tag{198}$$

Let us consider the formulas (197) and (198) for various estimates. Formulas for average evaluation of risk and epidemic resistance of the system are as follows:

$$\text{Risk}[k] = \frac{1 + \sum_{i=1}^k \prod_{j=0}^i (E_j(X) - A_j(X))}{1 + \sum_{i=1}^k B_i(X) \prod_{j=0}^{i-1} (E_j(X) - A_j(X))}, \tag{199}$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k \prod_{j=0}^i (E_j(X) - A_j(X))}{1 + \sum_{i=1}^k B_i(X) \prod_{j=0}^{i-1} (E_j(X) - A_j(X))}. \tag{200}$$

For peak evaluation we obtain:

$$\text{Risk}[k] = \frac{1 + \sum_{i=1}^k \prod_{j=0}^i (E_{0j}(X) - A_{0j}(X))}{1 + \sum_{i=1}^k B_{0i}(X) \prod_{j=0}^{i-1} (E_{0j}(X) - A_{0j}(X))}, \tag{201}$$

$$L[k] = 1 - \frac{1 + \sum_{i=1}^k \prod_{j=0}^i (E_{0j}(X) - A_{0j}(X))}{1 + \sum_{i=1}^k B_{0i}(X) \prod_{j=0}^{i-1} (E_{0j}(X) - A_{0j}(X))}. \tag{202}$$

For interval evaluation, we obtain the following formulas:

$$\text{Risk}[k] = \frac{1 + \sum_{i=1}^k \prod_{j=0}^i ((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)}))}{1 + \sum_{i=1}^k (B_i(X) \pm V_i(X)) \prod_{j=0}^{i-1} ((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)})}), \tag{203}$$

$$L[k] = \frac{1 - \frac{1 + \sum_{i=1}^k \prod_{j=0}^i ((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)}))}{1 + \sum_{i=1}^k (B_i(X) \pm V_i(X)) \prod_{j=0}^{i-1} ((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)})})}{1 + \sum_{i=1}^k (B_i(X) \pm V_i(X)) \prod_{j=0}^{i-1} ((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)})})}. \tag{204}$$

where $M_j(X)$ is the expected number of latent elements at the j^{th} stage of the epidemic, $D_j(X)$ is the dispersion of the number of latent elements at the j^{th} stage of the epidemic, $E_j(X)$ is the expected number of finally infected elements at the j^{th} stage of the epidemic, $Z_j(X)$ is the dispersion of the number of finally

infected elements at the j^{th} stage of the epidemic, $A_j(X)$ is the expected number of repaired elements at the j^{th} stage of the epidemic, $C_j(X)$ is the dispersion of the number of recovered infected elements at the j^{th} stage of the epidemic, $B_i(X)$ is the expected number of elements that have lost the immunity at the i^{th} stage of the epidemic, $V_i(X)$ is the dispersion of the number of elements that have lost the immunity at the i^{th} stage of the epidemic.

9. ASSESSMENT OF THE RISK OF INFECTION DISSEMINATION PROCESS BY THE SIRM MODEL

Method of discrete SIRM model constructing

To describe the model of information epidemic implementation by the SIRM model, let us consider the ITN, in which the epidemic spread of malware information and system elements can be a part of one of the following sets:

1. Susceptible(S) – elements that are susceptible to receiving malware information, they are able to be infected. As soon as they become infected, they move to the category of the infected elements.
2. Infected(I) – elements that can spread malware information to susceptible objects. The time they spend in the infected state is an infectious period, after which they move to the category of restored elements.
3. Unworkable(R) – elements that after being infected with malware information become completely out of order and cannot bring benefit to the system.
4. Immunized (M) – elements that after being infected with malware information become completely restored and modified so that are not susceptible to this epidemic. Such elements cannot be infected again and cannot move to other states.

Let us consider the ITN, in which information epidemic is developing by SIRM model with the following parameters:

- N is the total number of system elements, it is a specified parameter, not changeable in the process of the epidemic, and does not have a probabilistic nature;
- n is the average number of elements that are in direct contact with each element, it is a probabilistic parameter in some extent; it depends on the network topology, which is beyond the consideration scope of this work.



Q_i – evaluation of elements infecting expectation at the i^{th} stage of the epidemic, according to the relevant probability distribution.

P_i – evaluation of elements failure expectation at the i^{th} stage of the epidemic, according to the relevant probability distribution.

W_i – evaluation of elements immunization expectation at the i^{th} stage of the epidemic, according to the relevant probability distribution.

At the initial stage of the epidemic, the first element became infected, from which begins a viral epidemic in the system, thus:

$$S[0] = 1, \tag{205}$$

$$Q_0 = 1, \tag{206}$$

$$I[0] = 1. \tag{207}$$

At the first stage, the number of susceptible and infected elements can be represented as:

$$S[1] = 1 + n, \tag{208}$$

$$I[1] = 1 + (Q_1 - P_1), \tag{209}$$

$$R[1] = (P_1 - W_1). \tag{210}$$

$$M[1] = W_1. \tag{211}$$

At the second stage, each of the $(Q_1 - P_1)$ infected elements interacts with n adjacent elements, so Q_2 become infected, P_2 become failed, while W_2 become immunized. Thus:

$$S[2] = 1 + n + (Q_1 - P_1)n, \tag{212}$$

$$I[2] = 1 + (Q_1 - P_1) + (Q_1 - P_1)(Q_2 - P_2), \tag{213}$$

$$R[2] = (P_1 - W_1) + (P_2 - W_2)(Q_1 - P_1), \tag{214}$$

$$M[2] = W_1 + W_2(Q_1 - P_1). \tag{215}$$

When continuing thinking, for the k^{th} stage we get the following expressions:

$$S[k] = 1 + n \sum_{i=1}^k \prod_{j=0}^{i-1} (Q_j - P_j), \tag{216}$$

$$I[k] = \sum_{i=0}^k \prod_{j=0}^i (Q_j - P_j), \tag{217}$$

$$R[k] = \sum_{i=1}^k (P_i - W_i) \prod_{j=0}^{i-1} (Q_j - P_j). \tag{218}$$

$$M[k] = \sum_{i=1}^k W_i \prod_{j=0}^{i-1} (Q_j - P_j). \tag{219}$$

Let us consider the formulas (216), (217), (218) and (219) for different types of assessments. For the average evaluation we get:

$$S[k] = 1 + n \sum_{i=1}^k \prod_{j=0}^{i-1} (M_j(X) - E_j(X)), \tag{220}$$

$$I[k] = \sum_{i=0}^k \prod_{j=0}^i (M_j(X) - E_j(X)), \tag{221}$$

$$R[k] = \sum_{i=1}^k (E_i(X) - A_j(X)) \prod_{j=0}^{i-1} (M_j(X) - E_j(X)) \tag{222}$$

$$M[k] = \sum_{i=1}^k A_j(X) \prod_{j=0}^{i-1} (M_j(X) - E_j(X)) \tag{223}$$

where $M_j(X)$ is the expected number of infected elements at the j^{th} stage of the epidemic, $E_j(X)$ is the expected number of failed elements at the j^{th} stage of the epidemic, $A_j(X)$ is the expected number of immunized elements at the j^{th} stage of the epidemic.

For peak evaluation we obtain:

$$S[k] = 1 + n \sum_{i=1}^k \prod_{j=0}^{i-1} (M_{0j}(X) - E_{0j}(X)), \tag{224}$$

$$I[k] = \sum_{i=0}^k \prod_{j=0}^i (M_{0j}(X) - E_{0j}(X)), \tag{225}$$

$$R[k] = \sum_{i=1}^k (E_{0j}(X) - A_{0j}(X)) \prod_{j=0}^{i-1} (M_{0j}(X) - E_{0j}(X)) \tag{226}$$

$$M[k] = \sum_{i=1}^k A_{0j}(X) \prod_{j=0}^{i-1} (M_{0j}(X) - E_{0j}(X)) \tag{227}$$

where $M_{0j}(X)$ is the mode of infected elements number at the j^{th} stage of the epidemic, $E_{0j}(X)$ is the mode of failed elements number at the j^{th} stage of the epidemic, $A_{0j}(X)$ is the mode of immunized elements number at the j^{th} stage of the epidemic.

For interval evaluation we obtain the following formulas:

$$S[k] = 1 + n \sum_{i=1}^k \prod_{j=0}^{i-1} ((M_j(X) \pm \sqrt{D_j(X)}) - (E_j(X) \pm \sqrt{Z_j(X)})), \tag{228}$$

$$S_1[k] = 1 + n \sum_{i=1}^k \prod_{j=0}^{i-1} ((M_j(X) - \sqrt{D_j(X)}) - (E_j(X) + \sqrt{Z_j(X)})), \tag{229}$$

$$S_2[k] = 1 + n \sum_{i=1}^k \prod_{j=0}^{i-1} ((M_j(X) + \sqrt{D_j(X)}) - (E_j(X) - \sqrt{Z_j(X)})), \tag{230}$$

$$I[k] = \sum_{i=0}^k \prod_{j=0}^i ((M_j(X) \pm \sqrt{D_j(X)}) - (E_j(X) \pm \sqrt{Z_j(X)})), \tag{230}$$

$$I_1[k] = \sum_{i=0}^k \prod_{j=0}^i ((M_j(X) - \sqrt{D_j(X)}) - (E_j(X) + \sqrt{Z_j(X)})), \tag{231}$$

$$I_2[k] = \sum_{i=0}^k \prod_{j=0}^i \left((M_j(X) + \sqrt{D_j(X)}) - (E_j(X) - \sqrt{Z_j(X)}) \right), \quad (232)$$

$$R[k] = \sum_{i=1}^k \left((E_j(X) \pm \sqrt{Z_j(X)}) - (A_j(X) \pm \sqrt{C_j(X)}) \right) \cdot \prod_{j=0}^{i-1} \left((M_j(X) \pm \sqrt{D_j(X)}) - (E_j(X) \pm \sqrt{Z_j(X)}) \right), \quad (233)$$

$$R_1[k] = \sum_{i=1}^k \left((E_j(X) - \sqrt{Z_j(X)}) - (A_j(X) + \sqrt{C_j(X)}) \right) \cdot \prod_{j=0}^{i-1} \left((M_j(X) + \sqrt{D_j(X)}) - (E_j(X) - \sqrt{Z_j(X)}) \right), \quad (234)$$

$$R_2[k] = \sum_{i=1}^k \left((E_j(X) + \sqrt{Z_j(X)}) - (A_j(X) - \sqrt{C_j(X)}) \right) \cdot \prod_{j=0}^{i-1} \left((M_j(X) - \sqrt{D_j(X)}) - (E_j(X) + \sqrt{Z_j(X)}) \right), \quad (235)$$

$$M[k] = \sum_{i=1}^k (A_j(X) \pm \sqrt{C_j(X)}) \cdot \prod_{j=0}^{i-1} \left((M_j(X) \pm \sqrt{D_j(X)}) - (E_j(X) \pm \sqrt{Z_j(X)}) \right), \quad (236)$$

$$M_1[k] = \sum_{i=1}^k (A_j(X) + \sqrt{C_j(X)}) \cdot \prod_{j=0}^{i-1} \left((M_j(X) - \sqrt{D_j(X)}) - (E_j(X) + \sqrt{Z_j(X)}) \right), \quad (237)$$

$$M_2[k] = \sum_{i=1}^k (A_j(X) - \sqrt{C_j(X)}) \cdot \prod_{j=0}^{i-1} \left((M_j(X) + \sqrt{D_j(X)}) - (E_j(X) - \sqrt{Z_j(X)}) \right), \quad (238)$$

where $M_j(X)$ is the expected number of infected elements at the j^{th} stage of the epidemic, $D_j(X)$ is the dispersion of the number of infected elements at the j^{th} stage of the epidemic, $E_j(X)$ is the expected number of failed elements at the j^{th} stage of the epidemic, $Z_j(X)$ is the dispersion of the number of failed elements at the j^{th} stage of the epidemic, $A_j(X)$ is the expected number of immunized elements at the j^{th} stage of the epidemic, $C_j(X)$ is the dispersion of the number of immunized elements at the j^{th} stage of the epidemic. In

this type of evaluations, the number of infected, susceptible, recovered and immunized elements are in the intervals $(I_1[k], I_2[k])$, $(S_1[k], S_2[k])$, $(R_1[k], R_2[k])$ and $(M_1[k], M_2[k])$, respectively.

Risk analysis and assessment of network epidemic resistance in terms of information epidemic dissemination by the SIRM model

Using the formulas (216), (217) and (218) we obtain the formula of ITN risk in case of viral epidemic implementation by the SIRM model:

$$\text{Risk}[k] = \frac{I[k] + bR[k]}{S[k]} = \frac{\sum_{i=0}^k \prod_{j=0}^i (Q_j - P_j) + b \sum_{i=1}^k (P_i - W_i) \prod_{j=0}^{i-1} (Q_j - P_j)}{1 + n \sum_{i=1}^k \prod_{j=0}^{i-1} (Q_j - P_j)}, \quad (239)$$

where b is the correction factor, showing the additional value of damages in case of failure of the elements, as the failed elements cause extra damage to the system.

Epidemic resistance of the system can be assessed as follows:

$$L[k] = \frac{S[k] - I[k] - bR[k]}{S[k]} = 1 - \frac{\sum_{i=0}^k \prod_{j=0}^i (Q_j - P_j) + b \sum_{i=1}^k (P_i - W_i) \prod_{j=0}^{i-1} (Q_j - P_j)}{1 + n \sum_{i=1}^k \prod_{j=0}^{i-1} (Q_j - P_j)}. \quad (240)$$

10. CONCLUSION AND DISCUSSION OF THE RESULTS

The proposed above epidemic models represent a real interest for the development of tactics and strategies of network warfare in the conditions of virus attacks. After analyzing homogenous part of the network, based on the developed model, we can estimate the number of failed $|X_R|$ network elements; then, knowing the degree of vertices k , we can find many others parameters:

- many of the lost arcs (links) of the network, as $|X_R| = k|X_R|$;
- starting a set of arcs of the network $|A| = \frac{k}{2}|X|$, where $|X|$ is a starting set of vertices of the network;
- starting potential of the network $P_{ot}[Net] = |X||A|$.

So the potential of the network at the considered s stage of the epidemic will be

$$P_{ot}[s] = X[s]A[s] = \{|X| - |X_R|\} \times \{|A| - k|X_R|\}.$$

In view of the above virus analysis, the network epidemic resistance at the s stage is equal to



$$L_3[s] = \frac{|X_R|}{|X|},$$

where the relative change of the potential will be

$$\frac{\Delta P_{ot}[s]}{P_{ot}} = 2L_3^2 - 3L_3 + 1.$$

The dependence of the network potential from the s step number is even more nonlinear, which opens up new prospects for epidemic process control by the criterion of potential dynamics of the attacked network.

These data can be very useful in the context of network warfare implementation, strategical and tactical solutions development during the implementation of the virus attack on the network.

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