Journal of Theoretical and Applied Information Technology
30th September 2016. Vol.91. No.2

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

INVERSE OPTIMAL CONTROL OF SWITCHED DISCRETE NONLINEAR SYSTEMS USING CONTROL LYAPUNOV FUNCTION WITH ADJUSTED PARAMETER AND DISCRETE PARTICLE SWARM ALGORITHM

[1]MOUADH SAKLY, [2]ANIS SAKLY, [3]NESRINE MAJDOUB and [4]FAOUZI M'SAHLI

[1,2,3,4] Unit ESIER in the National School of Engineers of Monastir, University of Monastir, Tunisia.

[1]s.mouadh@gmail.com, [2]sakly_anis@yahoo.fr, [3]majnesrine@yahoo.fr, [4]faouzi.msahli@enim.rnu.tn

ABSTRACT

In this paper, the optimal control problem for switched discrete nonlinear systems, using Control Lyapunov Function with adjusted parameter, is studied to minimize the performance criterion. Two main stages are proposed, the first one is to find the optimal control of discrete systems using Control Lyapunov Function (CLF) and a speed gradient algorithm; whereas the second stage consists in applying the Particle swarm approach in order to come up with the optimal switching instants between the sub-systems.

Keywords: *Switched Discrete Nonlinear Systems, Particle Swarm Algorithm, Control Lyapunov Function (CLF), Speed Gradient Algorithm, Adjusted Parameter.*

1. INTRODUCTION

Switched systems constitute a particular class of complex systems that contain several subsystems and switching laws orchestrating the active subsystem at each time instant [1-3]. The control of such systems leads to a great energy consumption principally made at switching times. For this reason, we are interested in the optimal control problems of the switched systems which are considered as one of the most challenging classes.

Many researches treated the optimal control of linear switched systems in the continuous case [3-7] as well as in the discrete one [8-10]. Generally, the optimal command of such systems returns to solve the Algebraic Riccati Equation (ARE) either in the continuous time or in the discrete time (DARE). Compared to the linear switched systems, the optimal control of nonlinear switched systems is rarely studied. In fact, some researchers have dealt with the optimal control of nonlinear switched systems in the continuous case using the Hamiltonien-Jacobi-Bellman (HJB) to find the optimal input control and a metaheuristic algorithm, and to get the optimal switching instants [1,11]. The command of the discrete case is still difficult and limited. In fact, few researchers treated this case [12,13] using approximation, adaptive dynamic programming. Sakly et al., treated in [14,15] this issue using Control Lyaponuv function with a fixe parameter and a metaheurstic algorithm. In this paper we add the speed gradient algorithm to obtain an adjusted parameter which let us to overcome the problem of choosing the right fixe parameter wish lead to obtaining the needed CLF [14,15].

In fact, to avoid the resolution of discrete HJB equations whish seems inappropriate in the nonlinear discrete optimization, our technique uses the inverse optimal control via CLF in a quadratic form. In order to achieve stabilization, this method depends not only on the optimal input control of each sub-system, but also on an adjusted parameter by means of the speed-gradient algorithm. After that, the discrete particle swarm algorithm [16,17] is applied to find the global minimum of the switching instants sequence of the discrete main system.

This paper is organized as follows: firstly, we start by an introduction to the optimal control of switched systems. In the second section, we formulate the problematic and we present our method based on inverse optimal control using CLF with adjusted parameter as a solution for nonlinear discrete systems optimization. In the third part, we present a metaheuristic approach based on Discrete particle swarm optimization DPSO algorithm wish is used to find the optimal switching's instants between subsystems . Afterwards, numerical

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

example is given, and we finalize with a conclusion.

2. INVERSE DISCRETE TIME OPTIMAL CONTROL SYSTEMS VIA CLF USING SPEED GRADIENT ALGORITHM

2.1 Discrete Switched System

We consider the switched discrete system described by the following subsystems:

$$x(k+1) = f_i(x(k)) + g_i(x(k))u(k) \quad (1)$$

$$f_i : \mathrm{X} \to IR,\ i \in I = \{1,2,...,M\} \quad (2)$$

$$g_i : \mathrm{X} \to IR,\ i \in I = \{1,2,...,M\} \quad (3)$$

where $x(k)$ is the discrete state space vector, f_i and g_i are an indexed fields of vectors, and l is a set of finite discrete variables, which indicates that the system will be in M configurations.

We need to choose an input and a switching sequence to control a switched discrete system. Actually, a switching sequence in $n \in [N_0, N_f]$ regulates the sequence of active subsystems. It is defined as follows:

$$\sigma = ((n_0,i_0),(n_1,i_1),...,(n_k,i_k),...,(n_K,i_K)) \quad (4)$$

with $0 \le K \le \infty$, $N_0 \le n_1 \le ... \le n_k \le N_K$, and $i_k \in I$ for $k = 0,1,...,K$. It is worth-noting that (n_k, i_k) indicates that, at n_k, the system switches from subsystem i_{k-1} to sub-system i_k.

2.2 Inverse optimization with CLF

Many searcher tried to solve the discrete form of HJB equation [12, 18, 19,20], but this issue is not straightforward and still presents one of the main drawbacks of discrete-time optimal control for nonlinear systems [21, 22]. To overcome this problem, we propose to use inverse optimal control with CLF. In [14, 15] we introduced the uses of CLF with a fixe supposed parameter, in this paper we introduce the speed gradient algorithm wish generate an adjusted parameter leading to obtain the needed CLF.

Let us consider the affine discrete time nonlinear systems presented in [14,15].

$$x_{k+1} = f(x_k) + g(x_k)u_k, \text{with } x_0 = x(0) \quad (5)$$

where $x_k \in IR^n$ is the state of system at time $k \in \{0,1,2,...,\infty\}$, $u_k \in IR^m$ is the input, $f : IR^n \to IR^n$ and $g : IR^n \to IR^{nxm}$ are smooth mappings, $f(0)=0$ and $g(x_k) \ne 0$ for all $x_k \ne 0$.

For the presented system, it is desired to determinate a control law $u_k = \bar{u}(x_k)$ which minimizes the following cost functional:

$$V(x_k) = \sum_{n=k}^{\infty} \left(l(x_n) + u_n^T R u_n \right) \quad (6)$$

where $V : IR^n \to IR^+$ presents the performance measure, $l : IR^n \to IR^+$ denotes a positive semi-definite function weighting the performance of the state vector x_k, and $R : IR^n \to IR^{mxm}$ is a real symmetric and positive definite matrix weighting the control effort expenditure. The entries of R may be functions of the system state in order to vary the weighting on control effort according to the state value.

Consequently, equation (6) can be rewritten as follows:

$$V(x_k) = l(x_k) + u_k^T R u_k + V(x_{k+1}) \quad (7)$$

where

$$V(x_{k+1}) = \sum_{n=k+1}^{\infty} \left(l(x_n) + u_n^T R u_n \right) \quad (8)$$

with the boundary condition $V(0)$, $V(x_k)$ becomes a Lyapunov function.

From Bellman's optimality principle, we know that, for the infinite horizon optimization case, the value function $V(x_k)$ becomes time invariant and satisfies the discrete time Bellman equation solved backward in time.

$$V(x_k) = \min_{u_k} \left\{ l(x_k) + u_k^T R u_k + V(x_{k+1}) \right\} \quad (9)$$

In order to determine the conditions that the optimal control law must satisfy, we must define the discrete-time Hamiltonian $\hbar(x_k, u_k)$ as:

$$\hbar(x_k,u_k) = l(x_k) + u_k^T R u_k + V(x_{k+1}) - V(x_k) \quad (10)$$

which is used to obtain control law u_k by calculating $\min_{u_k} \hbar(x_k,u_k)$. The value of u_k, which leads to the desired minimum, is a feedback control law denoted as $u_k = \bar{u}(x_k)$. Then, $\min_{u_k} \hbar(x_k,u_k) = \hbar(x_k, \bar{u}(x_k))$.

This optimal control law must satisfy the necessary condition below:

$$\hbar(x_k, \bar{u}(x_k)) = 0 \quad (11)$$

where $\bar{u}(x_k)$ is obtained by calculating the gradient of the right-hand side with respect to u_k

$$0 = 2Ru_k + \frac{\partial V(x_{k+1})}{\partial u_k} \quad (12)$$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$0 = 2Ru_k + g^T(x_k)\frac{\partial V(x_{k+1})}{\partial x_{k+1}} \quad (13)$$

As a result, the optimal control law is presented in the equation below:

$$u_k^* = \bar{u}(x_k) = -\frac{1}{2}R^{-1}g^T(x_k)\frac{\partial V(x_{k+1})}{\partial x_{k+1}} \quad (14)$$

u_k^* is used to emphasize that u_k is optimal. Moreover, if $\hbar(x_k, u_k)$ is a quadratic form in u_k and $R > 0$ then $\frac{\partial^2 \hbar(x_k, u_k)}{\partial u_k^2} > 0$ holds as a sufficient condition such that optimal control law (14) minimizes $\hbar(x_k, u_k)$ and the performance index (6).

Substituting (14) into (9), we obtain:

$$V(x_k) = \left\{ \begin{array}{c} l(x_k) + \left(-\frac{1}{2}R^{-1}g^T(x_k)\frac{\partial V(x_{k+1})}{\partial x_{k+1}}\right)^T \\ \times R\left(-\frac{1}{2}R^{-1}g^T(x_k)\frac{\partial V(x_{k+1})}{\partial x_{k+1}}\right) + V(x_{k+1}) \end{array} \right\} \quad (15)$$

$$V(x_k) = \left\{ \begin{array}{c} l(x_k) + V(x_{k+1}) \\ +\frac{1}{4}\frac{\partial V^T(x_{k+1})}{\partial x_{k+1}} g(x_k)R^{-1}g^T(x_k)\frac{\partial V(x_{k+1})}{\partial x_{k+1}} \end{array} \right\} \quad (16)$$

which can be rewritten as follows:

$$\left\{ \begin{array}{c} l(x_k) + V(x_{k+1}) - V(x_k) \\ +\frac{1}{4}\frac{\partial V^T(x_{k+1})}{\partial x_{k+1}} g(x_k)R^{-1}g^T(x_k)\frac{\partial V(x_{k+1})}{\partial x_{k+1}} \end{array} \right\} = 0 \quad (17)$$

Equation (17) is known as the discrete time HJB equation. Solving this partial-differential equation for $V(x_k)$ is not straightforward. This is one of the main drawbacks of discrete-time optimal control for nonlinear systems . To overcome this problem, we propose using inverse optimal control. In fact, for this inverse approach, a stabilizing feedback control law was first developed. It was noticed that this control law optimizes a cost functional. Along the lines of the approach presented in [14, 15], the discrete time inverse optimal control law for the nonlinear discrete system can be defined as follows:

Definition 1:

The control law

$$u_k^* = -\frac{1}{2}R^{-1}g^T(x_k)\frac{\partial V(x_{k+1})}{\partial x_{k+1}} \quad (18)$$

is inverse optimal if :

(i) it achieves (global) exponential stability of the equilibrium point $x_k = 0$ for system (5)

(ii) it minimizes a cost functional defined as in (6), for which $l(x_k) = -\bar{V}$ with:

$$\bar{V} = V(x_{k+1}) - V(x_k) + u_k^{*T}Ru_k^* \le 0 \quad (19)$$

The inverse optimal control is based on knowing $V(x_k)$. Thus, we propose a CLF based on $V(x_k)$ such that (i) and (ii) can be guaranteed. That is to say, instead of solving (17) for $V(x_k)$, we propose a control Lyapunov as:

$$V(x_k) = \frac{1}{2}x_k^T P x_k, \text{ where } P = P^T > 0 \quad (20)$$

for control law (18), in order to ensure system stability (5), equilibrium point $x_k = 0$ which will be achieved by defining an appropriate matrix P. Moreover, it is obvious that control law (18) with (20), which is referred to as the inverse optimal control law, optimizes a cost functional of the form (6). Consequently, by considering $V(x_k)$ as in (20), control law (18) takes the following form:

$$u_k^* = -\frac{1}{2}R^{-1}g^T(x_k)(Px_{k+1}) \quad (21)$$

$$u_k^* = -\frac{1}{2}R^{-1}g^T(x_k)\left(Pf(x_k) + Pg(x_k)u_k^*\right) \quad (22)$$

Thus:

$$\left(I + \frac{1}{2}R^{-1}g^T(x_k)Pg(x_k)\right)u_k^* = -\frac{1}{2}R^{-1}g^T(x_k)Pf(x_k) \quad (23)$$

Multiplying (23) by R , we get

$$\left(R + \frac{1}{2}g^T(x_k)Pg(x_k)\right)u_k^* = -\frac{1}{2}g^T(x_k)Pf(x_k) \quad (24)$$

This leads to the following state feedback control law:

$$\alpha(x_k) = u_k^* = -\frac{1}{2}\left(R + P_2(x_k)\right)^{-1} P_1(x_k) \quad (25)$$

where $P_1(x_k) = g^T(x_k)Pf(x_k)$, and $P_2(x_k) = \frac{1}{2}g^T(x_k)Pg(x_k)$.

We note that $P_2(x_k)$ is a positive definite and symmetric matrix, which ensures the existence of the inverse matrix in (25). After proposing the CLF for solving the inverse optimal control, we present now the main contribution (Theorem 1):

Theorem 1: Considers the affine discrete-time nonlinear system (5). If we have a matrix $P = P^T > 0$, the following inequality holds:

$$V_f(x_k) - \frac{1}{4}P_1^T(x_k)\left(R + P_2(x_k)\right)^{-1} P_1(x_k) \le -\zeta_Q \|x_k\|^2 \quad (26)$$

where $V_f(x_k) = V(f(x_k)) - V(x_k)$, with $V(f(x_k)) = \frac{1}{2}f^T(x_k)Pf(x_k)$ and $\zeta_Q > 0$, where $P_1(x_k)$ and $P_2(x_k)$ are as defined in (25). Then, the equilibrium point $x_k = 0$ of system (5) is globally and exponentially stabilized by the control law

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

(25), with CLF (20). Moreover, with (20) as a CLF, this control law is inverse optimal, and it minimizes the cost functional given by:

$$V(x_k)=\sum_{k=0}^{\infty}\left(l(x_k)+u_k^T R u_k\right) \quad (27)$$

with

$$l(x_k)=-\bar{V}\Big|_{u_k^*=\alpha(x_k)} \quad (28)$$

and the optimal value function presented by $V(x_0)$. The proof of this theorem is presented in [20].

2.3 Speed-Gradient Algorithm for Inverse Optimal Control

To determine P, which guarantees the stability of the equilibrium point $x_k=0$ of system (5) with (25), we propose using the speed-gradient (SG) algorithm [20, 23] in order to ensure the following goal:

$$Q(x_{k+1})\le \Delta, \text{for } k\ge k^* \quad (29)$$

where Q is a positive definite goal function, Δ is a positive constant considered as a threshold, and k^* is the time step at which the goal is achieved.

Digressing from the SG application proposed in [23], in this paper, the control law is given by (25). Besides, in (29), Δ is a state dependent function $\Delta(x_k)$. At every time step, the control law (25) depends on the matrix P defined as $P=p_kP'$, where P' is a given constant matrix in which $P'=P'^T>0$, and p_k is a scalar parameter to be calculated by the SG algorithm. Then, (25) is transformed into:

$$u_k^*=-\frac{p_k}{2}\left(R+\frac{p_k}{2}g^T(x_k)P'g(x_k)\right)^{-1}\times g^T(x_k)P'f(x_k) \quad (30)$$

The SG algorithm is now reformulated for the inverse optimal control problem.

Definition 2:

Considering a time-varying parameter $p_k\in IR^+$, the positive definite function $Q: IR^n\times IR\to IR$ is given as

$$Q(x_k,p_k)=V_{sg}(x_{k+1}) \quad (31)$$

where $V_{sg}(x_{k+1})=\frac{1}{2}x_{k+1}^T P' x_{k+1}$, with $x_{k+1}=f(x_k)+g(x_k)u_k^*$ is named as the SG goal function for system (5) with control law (30). The SG goal function is defined as in (31) in such a way that the convexity property of $Q(x_k,p_k)$ for p_k is guaranteed. Thus, there exist an optimal value p^* of p_k and a positive constant ε^*, such that $Q(x_k,p^*)\le\varepsilon^*$ [23]. In Theorem 1 below, this SG goal function is used to construct a Lyapunov function for the closed-loop system.

Definition 3:

Considering a constant p^*, the SG control goal of system (5) with (30) is defined as finding p_k so that the SG goal function $Q(x_k,p_k)$, as given in (31), fulfills:

$$Q(x_k,p_k)\le\Delta(x_k) \text{ for } k\ge k^*, \quad (32)$$

where

$$\Delta(x_k)=V_{sg}(x_k)-\frac{1}{p_k}u_k^T R u_k \quad (33)$$

with $V_{sg}(x_k)=\frac{1}{2}x_k^T P' x_k$ and u_k are as defined in (30); k^* is the discrete time step at which the SG control goal is achieved.

Remark 1: Solution p_k must guarantee that $V_{sg}(x_k)>\frac{1}{p_k}u_k^T R u_k$ in order to obtain a positive definite function $\Delta(x_k)$.

Obviously, the SG algorithm is used to compute p_k in order to achieve the SG control goal defined above.

Proposition 1: We consider a discrete-time nonlinear system of the form (5) with (30) as input. Let Q be a SG goal function as defined in (31). Let $\bar{p}$ be a positive constant, and $\Delta(x_k)$ be a positive definite function with $\Delta(0)=0$. We assume that there exist a positive constant p^* and a sufficiently-small positive constant ε^* such that the following control goal is achievable [23].

$$Q(x_k,p^*)\le\varepsilon^*<<\Delta(x_k) \quad (34)$$

Then, for any initial condition $p_0>0$, there is a k^* such that the SG Control Goal (32) is attained by means of the following dynamic variation of parameter p_k:

$$p_{k+1}=p_k-\gamma_{d,k}\nabla_p Q(x_k,p_k) \quad (35)$$

with

$$\gamma_{d,k}=\gamma_c\delta_k\left|\nabla_p Q(x_k,p)\right|^{-2}, 0<\gamma_c\le 2\Delta(x_k) \quad (36)$$

and

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

$$\delta_k = \begin{cases} 1 & for\ Q(x_k, p) > \Delta(x_k) \\ 0 & \text{otherwise} \end{cases} \quad (37)$$

Finally, for $k \geq k^*$, p_k becomes a constant value denoted by $\bar{p}$ and the SG algorithm is terminated .

Proof: Along the lines of the journal presented in [23], the proof is based on the case for which $Q(x_k, p_k) > \Delta(x_k)$, and therefore $\delta_k = 1$. Let us consider the positive definite Lyapunov function $V_p(p_k) = |p_k - p^*|^2$. Then, the respective Lyapunov difference is given as follows:

$$\Delta V_p(p_k) = |p_{k+1} - p^*|^2 - |p_k - p^*|^2 \quad (38)$$

$$\Delta V_p(p_k) = (p_{k+1} - p_k)^T \left[(p_{k+1} - p_k) + 2(p_k - p^*)\right] \quad (39)$$

$$\Delta V_p(p_k) = \left\{ \begin{matrix} -\gamma_{d,k} \nabla_p Q(x_k, p) \\ \times\left[-\gamma_{d,k} \nabla_p Q(x_k, p_k) + 2(p_k - p^*)\right] \end{matrix} \right\} \quad (40)$$

Due to the SG goal function convexity (31) for p_k,

$$(p - p_k)^T \nabla_p Q(x_k, p_k) \leq \varepsilon^* - \Delta(x_k) < 0 \quad (41)$$

where $\nabla_p Q(x_k, p_k)$ denotes the gradient of $Q(x_k, p_k)$ with respect to p_k. Based on (41), (40) becomes

$$\Delta V_p(p_k) \leq -2\gamma_{d,k}\left(\Delta(x_k) - \varepsilon^*\right) + \gamma_{d,k}{}^2 \left|\nabla_p Q(x_k, p_k)\right|^2 \quad (42)$$

$$\Delta V_p(p_k) \leq \left\{ \begin{matrix} -2\gamma_c \delta_k \left(\Delta(x_k) - \varepsilon^*\right) \left|\nabla_p Q(x_k, p_k)\right|^{-2} \\ +\gamma_c^2 \delta_k^2 \left|\nabla_p Q(x_k, p_k)\right|^{-4} \left|\nabla_p Q(x_k, p_k)\right|^2 \end{matrix} \right\} \quad (43)$$

$$\Delta V_p(p_k) = -\frac{\gamma_c\left[2\Delta(x_k)\left(1 - \left(\varepsilon^* / \Delta(x_k)\right)\right) - \gamma_c\right]}{\left|\nabla_p Q(x_k, p_k)\right|^2} \quad (44)$$

From (34), $\left(1 - \left(\varepsilon^* / \Delta(x_k)\right)\right) \approx 1$ hence

$$\Delta V_p(p_k) = -\frac{\gamma_c\left[2\Delta(x_k) - \gamma_c\right]}{\left|\nabla_p Q(x_k, p_k)\right|^2} < 0 \quad (45)$$

Thus, the bounds of p_k is guaranteed if $0 < \gamma_c \leq 2\Delta(x_k)$. Finally, when $k \geq k^*$, $\delta_k = 0$, which means that the algorithm terminates; at this point $Q(x_k, p_k) \leq \Delta(x_k)$, then p_k becomes a constant value denoted by $p_k = \bar{p}$.

Since the parameter p_k is a scalar value, the gradient $\nabla_p Q(x_k, p)$ in (40) is reduced to be the partial derivative of $Q(x_k, p)$ with respect to p_k as $\frac{\partial}{\partial p_k} Q(x_k, p)$.

Remark 2: Parameter γc in (35) is selected such that solution p_k ensures the requirement $V_{sg}(x_k) > \frac{1}{p_k} u_k^T R(x_k) u_k$ in Remark 1. Then, we have a positive definite function $\Delta(x_k)$.

Remark 3: With $Q(x_k, p_k)$ as defined in (31), the dynamic variation of parameter p_k in (35) results in the following form:

$$p_{k+1} = p_k + 8\gamma_{d,k} \frac{f^T(x_k) P' g(x_k) R^2 g^T(x_k) f(x_k)}{\left(2R + p_k g^T(x_k) P' g(x_k)\right)^3} \quad (46)$$

which is positive for all time step k if $p_0 > 0$. Therefore, the positivity of p_k is ensured and requirement $p_k = p_k^T > 0$ is guaranteed. When SG Control goal (32) is achieved, then $p_k = \bar{p}$ for $k \geq k^*$. Thus, matrix p_k in (25) is considered constant and $P_k = P$ where P is computed as $P = \bar{p}P'$, with P' a design positive definite matrix. Under these constraints, we obtain:

$$\alpha(x_k) := u_k^* = -\frac{1}{2}\left(R(x_k) + \frac{1}{2} g^T(x_k) P g(x_k)\right)^{-1} g^T(x_k) P f(x_k) \quad (47)$$

The following diagram resumes the speed gradient algorithm (figure 1).

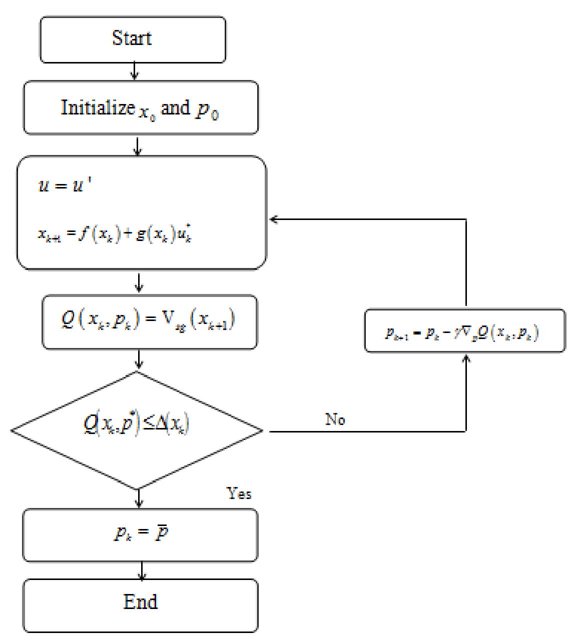

Figure1: Speed-Gradient Algorithm Flow Diagram

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

3. PARTICULE SWARM ALGORITHM FOR SWITCHED INSTANTS OPTIMIZATION

3.1 Standard PSO

Particle Swarm Optimization (PSO) is a modern optimization technique inspired by bird flocking and fish schooling originally designed and introduced by Kennedy and Eberhart [24] in 1995.

A PSO algorithm contains a swarm of particles in which each particle includes a potential solution. The particles fly through a multidimensional search space in which the position of each particle is adjusted according to its own experience and the experience of its neighbors. PSO system combines local search methods with global search methods attempting to balance exploration and exploitation [16]. It's applied to many research areas, such as clustering and classification, communication networks, and scheduling.

the particle in the swarm update it position according the following set of conditions:

$$V_i^{(k+1)}(j)=\begin{Bmatrix} W_i V_i^{(k)}(j)+c_1 r_1\left(pbest_i^k(j)-X_i^k(j)\right) \\ +c_2 r_2\left(pgbest_i^k(j)-X_i^k(j)\right)\end{Bmatrix} \quad (48)$$

$$X_i^{(k+1)}(j)=X_i^{(k)}(j)+V_i^{(k+1)}(j) \quad (49)$$

$$W_i^{(k)}(j)=W_{\max}-\left(\frac{W_{\max}-W_{\min}}{k_{\max}}\right).k \quad (50)$$

$X_i^k(j)$ present the candidate enclosed as the position of j^{th} element of i^{th} particle in k^{th} step of algorithm and $V_i^k(j)$ present the j^{th} element of the velocity vector of the i^{th} particle in the k^{th} step.

c_1 and c_2 are positive acceleration constants which control the influence of *pbest* and *pgbest* on the search process. Also r_1 and r_2 are random values in range [0, 1] sampled from a uniform distribution.

$pbest_i^k$ and $pgbest_i^k$ present respectively the best position reached by the i^{th} particle in the k^{th} step, and the best position between neighbors of each particle known as the global best.

$k_{\max}$ is the maximum number of iterations, W_i^k is the inertia weight making balance between exploration and exploitation .

3.2 Binary and Discrete PSO

Primarily PSO was successfully used to solve continuous problems whish positions are real valued, so standard PSO cannot be applied directly to binary/discrete problems.

Kennedy and Eberhart introduced the discrete binary version of PSO with a stochastic velocity model in 1997, which was the first PSO algorithm to be used in binary discrete space. In this method each particle is considered as a position in a *D*-dimensional space and each element of a particle position can take the binary value of 0 or 1 in which 1 means "included" and 0 means "not included" [16].

In the binary PSO the updated position equation takes the following form:

$$X_i^{(k+1)}(j)=\begin{cases}1\, if\ sig\left(V_i^{(k+1)}(j)\right)>r_{ij}\\ 0 \qquad otherwise\end{cases} \quad (52)$$

where $sig\left(V_i^{(k+1)}(j)\right)$ is a Sigmoid Function and r_{ij} is a random number in range [0, 1].

$$sig\left(V_i^{(k+1)}(j)\right)=\frac{1}{1+\exp\left(-V_i^{(k+1)}(j)\right)} \quad (53)$$

In our case, we need to ensure the connection between the "switching instants" and the "PSO particles". The decision variables, representing the discrete integer switching instant, are coded as the position vector of the particle and updated as follow:

$$X_i^{(k+1)}(j)=round\left(X_i^{(k)}(j)+V_i^{(k+1)}(j)\right) \quad (54)$$

We rounded each real value of position to the nearest integer value (discrete value), this is due to the fact that integer discrete solutions are needed for the optimization problem studied in this paper [17].

the associated code for the optimization of fitness function f is described below:

Create and Initialize a D-dimensional swarm Px particles
repeat
  for each particle $i=1,..,Px$ do
   if $f(X_i)>f(pbest_i)$ then $pbest_i=X_i$
   end
   if $f(pbest_i)>f(pgbest_i)$
then $pgbest_i=pbest_i$
   end
  end
  for each particle $i=1,..,Px$ do
  update the velocity vector using equation (48)
  update the position vector using equation (54)
  end
until the stopping criteria is true.

Stopping condition can either be the maximum number of iterations, or finding an acceptable solution, or having no improvement in a number of iterations.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

4. EXAMPLE

We consider the controlled nonlinear discrete switched system presented in [12]. It consists of

– subsystem 1:

$$x(k+1)=\begin{bmatrix}0.2\sin(x_1(k))x_2(k)\\0.5x_2(k)\end{bmatrix}+\begin{bmatrix}0\\0.5\end{bmatrix}u(k)$$

– subsystem 2:

$$x(k+1)=\begin{bmatrix}0.5x_1^3(k)x_2(k)\\0.3x_2(k)\end{bmatrix}+\begin{bmatrix}0\\0.5\end{bmatrix}u(k)$$

The discrete interval time is: $[N_0 \quad N_f]$ where $N_0=0$ and $N_f=20$. We want to find the optimal switching discrete instant N that minimizes the discrete criterion

$$V=\sum_{k=0}^{k=N_f}\left((x_1(k))^2+(x_2(k))^2+u^2(k)\right)$$

such that $R_1=R_2=1$.

and the initial point is $x_0=[-1.5 \quad 1.5]$

The program code used is formulated as below:

Step 1:

- Initialize $V=Q=0$
- Initialize P'=eye(2),
- N_1 = discrete instant to optimize with SFL algorithm
- $N_f=N_2=$ discrete limit of discrete space search
- Initialize $k=0$, x_0 and p_0

Step 2:
Repeat

$$u_k^*=-\frac{p_k}{2}\left(R(x_k)+\frac{p_k}{2}g_1^T(x_k)P'g_1(x_k)\right)^{-1}g_1^T(x_k)P'f_1(x_k)$$

$$x_{k+1}=f_1(x_k)+g_1(x_k)u_k^*$$

$$Q(x_k,p_k)=V_{sg}(x_{k+1})$$

$$p_{k+1}=p_k-\gamma\nabla_p Q(x_k,p_k)$$

until $Q(x_k,p^*)\le\Delta(x_k)$

Step 3:

$p_k=\bar{p}_1$

$P_1=\bar{p}_1\times P'$

Step 4:
Initialize counter $k=0$

Step 5:
Repeat

$$u_k^*=\left[-\frac{\bar{p}_1}{2}\left(R(x_k)+\frac{\bar{p}_1}{2}g_1^T(x_k)P'g_1(x_k)\right)^{-1}\times g_1^T(x_k)P'f_1(x_k)\right]$$

$$x_{k+1}=f_1(x_k)+g_1(x_k)u_k^*$$

until $k>N_1$

then , SOMME 1= $\sum_{k=0}^{N_1}(x_k^2+u_k^{*2})$

Step 6:
We obtain x_{N_1}

Then, we initialize, again, $k=0$, $x_0=x_{N_1}$ and p_0

Step 7:
Repeat

$$u_k^*=\left[-\frac{p_k}{2}\left(R(x_k)+\frac{p_k}{2}g_2^T(x_k)P'g_2(x_k)\right)^{-1}\times g_2^T(x_k)P'f_2(x_k)\right]$$

$$x_{k+1}=f_2(x_k)+g_2(x_k)u_k^*$$

$$Q(x_k,p_k)=V_{sg}(x_{k+1})$$

$$p_{k+1}=p_k-\gamma\nabla_p Q(x_k,p_k)$$

until $Q(x_k,p^*)\le\Delta(x_k)$

Step 8:

$p_k=\bar{p}_2$

$P_2=\bar{p}_2\times P'$

Step 9:
repeat

$$u_k^*=-\frac{\bar{p}_2}{2}\left(R(x_k)+\frac{\bar{p}_2}{2}g_2^T(x_k)P'g_2(x_k)\right)^{-1}g_2^T(x_k)P'f_2(x_k)$$

$$x_{k+1}=f_2(x_k)+g_2(x_k)u_k^*$$

until $k>N_2$

then, SOMME 2= $\sum_{k=N_1}^{N_2}(x_k^2+u_k^{*2})$

Step 10:
Function to optimize:

$$V=Q=\sum_{k=0}^{N_2}(x_k^2+u_k^{*2})=\text{SOMME 1+ SOMME 2.}$$

To find the optimum switching instants between the two subsystems, we use the DPSO algorithm with the following parameters:

- Number of populations = 30
- Acceleration coefficient relative to the best actual position of particle $C_1=1.5$

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

– Acceleration coefficient relative to the global best position of particle $C_2 = 1.5$
– Iterations' maximum number = 20
– $W_{max} = 0.9$ & $W_{min} = 0.4$

The programs have been developed under the MATLAB 7.1 on a PC whose features are:

– RAM : 3 GB
– Processor : Intel ® Core ® I5-5200U CPU 2.2 GHz
– Operating system : Windows 7.
– program execution time: 10.25 minutes.

Now, we begin by showing the results of our study. Firstly, we will present figure 3 & figure 4 depicting respectively the speed gradient evolution of subsystem 1 and subsystem 2. figure 5 demonstrates the discrete state x_1 evolution, while figure 6 presents the discrete state x_1 evolution. figure 7 clarifies the discrete control input evolution, while figure 8 presents the state evolution and figure 9 present the cost evolution.

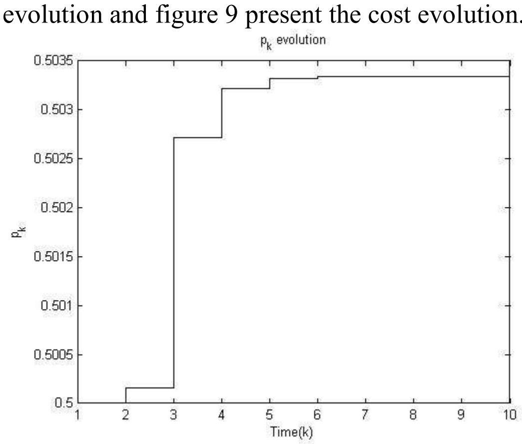

Figure 3: Speed gradient evolution for subsystem 1

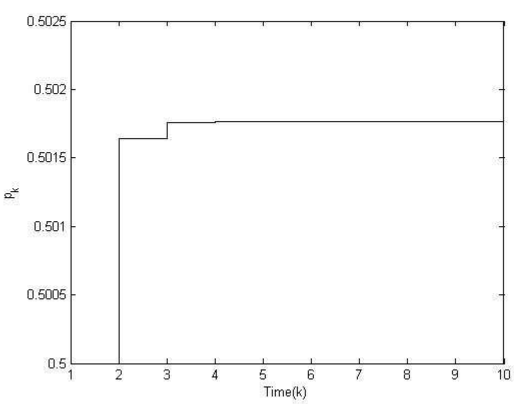

Figure 4: Speed gradient evolution for subsystem 2

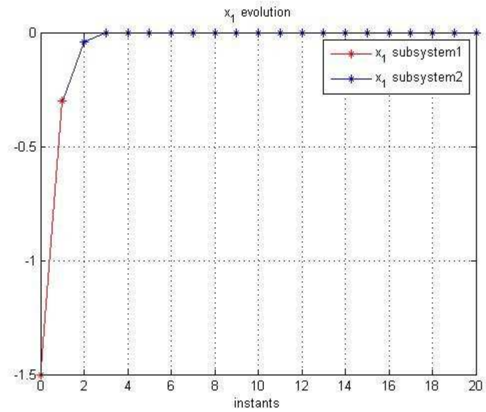

Figure 5: Discrete state x_1 evolution DPSO approach

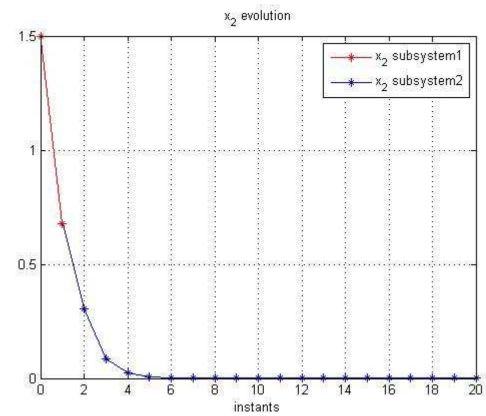

Figure 6: Discrete state x_2 evolution DPSO approach

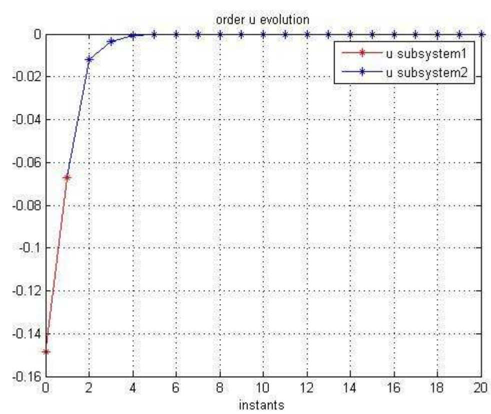

Figure 7: Control input u evolution using DPSO approach

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

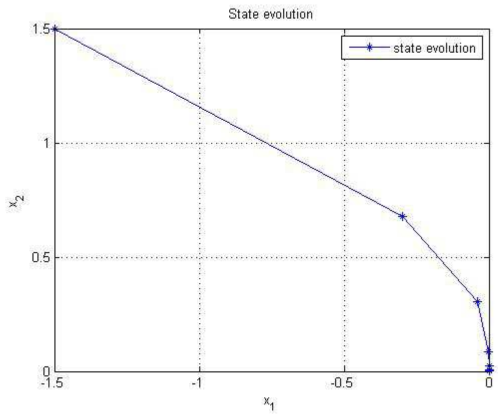

Figure 8: State evolution DPSO approach

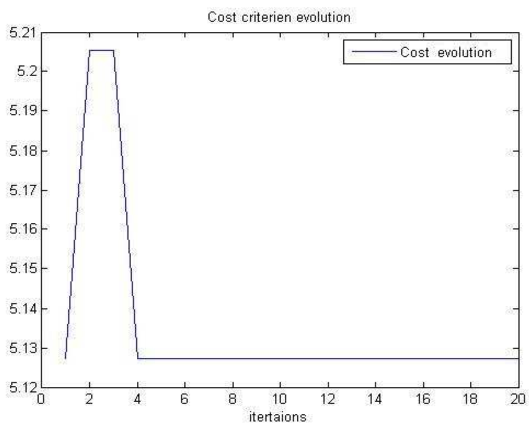

Figure 9: Cost criterien V evolution DPSO approach

After simulation, The numeric results give us 5.1229 as global minimum achieved in the first switching instant $N_1 = 1$ (The same result was obtained in [12] but with using two stages approximations). Here we introduced a direct CLF method wish ensure the stability of switched systems.

5. CONCLUSION

We tried, in this paper, to solve the problem of optimization of instants of nonlinear switching systems with a meta-heuristic approach. We used two stages. The first one, in which we utilized the CLF with an adjusted parameter to avoid the resolution of HJB equation, allows us to obtain the minimum control for the nonlinear system. For the second stage, we used the DPSO algorithm to find the minimum instant of switching system with optimization of the given cost criterion.

REFERENCES

[1] N. Majdoub, A. Sakly, M. Benrajeb, "ACO Based Optimization of Switching Instants for Nonlinear Switched Systems", *International Journal of Information Sciences and Computer Engineering*, Vol. 2, 2012, pp. 30-34.

[2] N. Majdoub, A. Sakly, M. Sakly, M. Benrjeb, "Ant Colony Based on Optimization Of Switching Instants for Autonomous Switched Systems", *International Review of Automatic Control*, Vol. 03, No. 01, 2010, p24.

[3] M. Sakly, A. Sakly, N. Majdoub, M. Benrjeb, "Optimal Switching Instants of Linear Switched Systems Based on Meta-Heuristic Approaches", *International Journal of Intelligent Control and Systems*, Vol. 16, No. 01, 2011, pp. 8-18.

[4] C. Aghayeva , M. Alpaslan, "Singular Optimal Control Problem of Stochastic Switching Systems", *Proceedings in the world congress on engineering and computer Science*, WCECS, San Francisco, USA Vol. 02, , 2014.

[5] R. Luus, Y.Q. Chen, "Optimal Switching Control via Direct Search Optimization", *Asian Journal of Control*, Vol. 06, No. 02, 2004, pp. 302-306.

[6] E. Mojica-nava, R. Meziat, N. Quijano, A. Gauthier, N. Rakoto-ravalontsalama, "Optimal Control of Switched Systems: A Polynomial Approach", *Proceedings in the 17th International Federation of Automatic Control Congress*, Seoul, Korea, July 6-11, 2008.

[7] X. Xu, P.J. Antsaklis, "Optimal Control of Switched Systems via Nonlinear Optimization Based on Direct Differentiations of Value Functions", *International Journal of control*, Vol. 75, 2002, pp. 1406-1426.

[8] M. Sakly, A.A. Kahloul, N. Majdoub, A. Sakly, F. M'sahli, "Optimization of Switching Instants For Optimal Control of Switched Discrete Systems Based on Particle Swarm Algorithm", *Proceeding of the 14th international conference on Sciences and Techniques of Automatic Control & Computer Engineering*, Sousse, Tunisia, 2013, pp.358-363.

ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195

[9] W. Zhang, J. Hu, "On Optimal Quadratic Regulation for Discrete-Time Switched Linear Systems", *Lecture Notes Computer Sciences*, Vol. 4981, pp.584-597.

[10] Q. Zhu, G. Xie, "Finite-Horizon Optimal Control of Discrete-Time Switched Linear Systems", *Mathematical Problems in Engineering* 2012, Article ID 483568.

[11] N. Majdoub, A. SAKLY, M. Benrjeb, "Hybrid Approach for Optimal Control Problem of Switched Systems", *Proceeding of Systems Man and Cybernetics (SMC), IEEE International Conference,* Istanbul, 2010, pp. 4161-4168.

[12] N. Cao, H. Zhang, Y. Luo, D. Feng, "Infinite Horizon Optimal Control of Affine Nonlinear Discrete Switched Systems using Two-Stage Approximate Dynamic Programming", *International Journal of System Science*, Vol. 43, No. 9, 2012, pp. 1673-1682.

[13] A. Heydari, S.N. Balakrishnan, "Optimal Switching and Control of Nonlinear Switching System Using Approximate Dynamic Programming", *Neural Networks and Learning Systems, IEEE Transaction on.*, Vol. 25, No. 06, 2014, pp. 1106-1117.

[14] M. Sakly, A. Sakly, F. M'sahli, "Inverse Optimal Control of Switched Discrete Nonlinear Systems Based on Control Lyapunov Function And Genetic Algorithm", *International Conference on Sciences and Techniques of Automatic Control & Computer Engineering*, Monastir, Tunisia, December 21-23, 2015, pp. 24-29.

[15] M. Sakly, A. Sakly, N. Majdoub, F. M'sahli, "Optimization of Switching Instants for Optimal Control of Switched Discrete Nonlinear Systems Based on Particle Swarm Algorithm", *International Conference on Sciences and Techniques of Automatic Control & Computer Engineering*, Monastir, Tunisia, December 21-23, 2014, pp. 105-111.

[16] H. Izakian, B.T. Ladani, A. Abraham, V. Snasel "A Discrete Particle Swarm Optimization Approach for Grid Job Scheduling", *International Journal of Innovative Computing, Information and Control*,Vol. 6, No. 9, 2010, pp. 1-09-0370.

[17] S. Boubaker, M. Djemai, N. Manamanni and F. M'sahli, "Active Modes and Switching identification for Linear Switched Systems Based on Discrete Particle Swarm Optimization", *Applied Soft Computing*, Vol. 14, Part C, January 2014, pp 482-488.

[18] Z. Chen, S. Jagannathan, "Neural Network Based Nearly Optimal Hamilton-Jacobi-Bellman Solution for Affine Nonlinear Discrete Time Systems", *Proceedings of the 44th IEEE Conference On Decision and Control & European Control Conference*, Seville, Spain, 2005, pp. 4123 - 4128.

[19] Z. Chen, S. Jagannathan, "Generalized Hamilton Jacobi Bellman Formulation Based on Neural Network Control of Affine Nonlinear Discrete Time Systems", *IEEE Transactions on Neural networks*, Vol. 19, No.1, 2008, pp. 90-106.

[20] E.N. Sanchez, F. Ornelas-tellez, "Discrete-Time Inverse Optimal Control for Nonlinear Systems", *CRC Press, Taylor& Francis*. 2013.

[21] A. Al-tamimi , F. Lewis, "Discrete Time Nonlinear HJB Solution Using Approximate Dynamic Programming: Convergence Proof", *IEEE Transaction on systems*, Vol. 38, No. 4 , 2008, pp. 943-949.

[22] T. Dierks, S. Jagannathan, "Optimal Tracking Control of Affine Nonlinear Discrete Time Systems with Unknown Internal Dynamics", *The 48th IEEE Conference On Decision and Control*, Shanghai, P.R China, 2009, pp. 6750 - 6755.

[23] F. Ornelas-tellez, E.N. Sanchez, A.G. Loukianov, E.M. Navarro-lopez, "Speed Gradient Inverse Optimal Control for Discrete Time Nonlinear Systems", *Proceeding of The 50th IEEE Conference On Decision and Control & European Control Conference*, Oriando, FL, USA, 2011, pp. 290-295.

[24] J. Kennedy and R.C. Eberhart, "Particle Swarm Optimization", *IEEE international conference on Neural Networks*, Perth, Australia, Vol.04, 1995, pp.1942-1948.