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A SCALED WEIGHTED VARIANCE *S* CONTROL CHART FOR SKEWED POPULATIONS

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ABSTRACT

This paper proposes a new *S* control chart for monitoring process dispersion of skewed populations. This control chart, called Scaled Weighted Variance *S* control chart (SWV-*S*) hereafter, this new SWV-*S* control chart is an improvement of the Weighted Variance *S* control chart (WV-*S*) proposed by Khoo et al. [11]. The proposed control chart reduces to the Shewhart *S* control chart when the underlying distribution is symmetric. The proposed SWV-*S* control chart compared with the Shewhart *S* and WV-*S* control charts. An illustrative example is given to show how the proposed SWV-*S* control chart is constructed and works. Simulations study show that the proposed SWV-*S* control chart has the lower false alarm rates than the Shewhart *S* control chart and WV-*S* control chart when the underlying distributions are Weibull, lognormal and gamma. In terms of the probability of detection rates, the proposed SWV-*S* control chart is closer to *S* control chart with the exact method than those of the Shewhart *S* and WV-*S* control charts.

Keywords: *Control Chart, Weighted Variance, Scaled Weighted Variance, False Alarm Rates, Skewed Populations*

1. INTRODUCTION

In many situations, the normality assumption is usually violated. For example, the distributions of measurements from chemical and semiconductor processes are often skewed.

The control charts for variables data such as the $\bar{X}$, EWMA, CUSUM, *S* and *R* control charts all depend on the assumption that the distribution of a quality characteristic is normal or approximately normal. When the underlying distribution is non-normal, three approaches are presently employed to deal with this problem. The first approach is to increase the sample size until the sample mean is approximately normally distributed. The second approach is to transform the original data so that the transformed data have an approximate normal distribution. The third approach is to use heuristic methods to design control charts. This paper considers the use of scaled weighted variance (SWV) to compute the limits of *S*-chart. This method is found to perform well when the distribution is skewed. Unlike the Shewhart *S*- chart, the proposed SWV-*S* chart provide asymmetric limits in accordance with the direction and degree of skewness by using different variances in computing the upper and lower limits. Thus, the SWV-S chart has lower false alarm rates than the WV-S and Shewhart S- charts when the underlying distributions are skewed. Our objective in this research are: 1.To improve the performance of the Shewhart *S* and WV-*S* charts when the distributions are skewed.2.To overcome the problem of high false alarm rates faced by the Shewhart *S* chart when the underlying process has a skewed distribution. 3. To increase the probability of detection as well as reducing false alarm rate faced by Shewhart S and WV-S charts.

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Sections 2 review Shewhart S chart and heuristic methods. An example is provided to illustrate the construction of the proposed SWV-S control chart in section 3. In section 4, some discussions are given about the performance of the proposed SWV-S. In section 5, a performance comparison of the Shewhart S, WV-S and SWV-S control charts in terms of false alarm rate and probability of out-of-control detection will be conducted, when underling distributions are Weibull, Lognormal and gamma. Finally, conclusions are drawn in section 6.

2. AN OVERVIEW OF THE SHEWHAR *S* CHART AND HEURISTIC METHODS

The idea of using control charts to monitor process data was developed by Walter A. Shewhart of the Bell Telephone Laboratories in 1924 (Montgomery, 2009). The Shewhart control chart is based on the assumption that the distribution of the quality characteristic is normal or approximately normal.

2.1 Shewhart *S* Control Chart

The Shewhart control chart consists of three lines, the upper control limit, UCL, the center line, CL, and the lower control limit, LCL. These UCL and LCL are chosen so that the state of a process can be determined.

Assume that a process follows a normal distribution with in-control mean and standard deviation where both are known. The control limits of the Shewhart S control charts are [13]:

$$\text{UCL} = \mu_S + 3\sigma_S \tag{1}$$

$$\text{CL} = \mu_S \tag{2}$$

and

$$\text{LCL} = \mu_S - 3\sigma_S \,. \tag{3}$$

where μ_S and σ_s are the mean and standard deviation of S, respectively. If the process parameters are unknown, the limits of the Shewhart S are:

$$\text{UCL}_{SH-S} = \bar{S}\left[1+\frac{3\sqrt{1-\left(C_4'\right)^2}}{C_4'}\right] = B_4\bar{S} \tag{4}$$

and

$$\text{LCL}_{SH-S} = \bar{S}\left[1-\frac{3\sqrt{1-\left(C_4'\right)^2}}{C_4'}\right]^+ = B_3\bar{S} \tag{5}$$

Here, $C_4' = \dfrac{E\left(S\right)}{\sigma_X}$ is a constant computed using normal distribution, and $\bar{S} = \dfrac{\sum_{i=1}^{r} S_i}{r}$ is the average of the sample standard deviations estimated from r preliminary subgroups.

2.2 Methodologies of Heuristic Control Charts for Skewed Populations

The control charts such as the $\bar{X}$ and R charts based on the weighted variance (WV) method proposed by Bai and Choi [2], $\bar{X}$ control chart using scaled weighted variance (SWV- $\bar{X}$) chart proposed by Castagliola [3], the $\bar{X}$, EWMA and CUSUM charts based on the weighted standard deviation (WSD) method suggested by Chang and Bai [5], the $\bar{X}$ and R charts based on the skewness correction (SC) method presented by Chan and Cui [4], $\bar{X}_S$ and S control charts based on the weighted variance method proposed by Khoo et al. [11], a multivariate synthetic control chart for monitoring the process mean vector of skewed populations using weighted standard deviations suggested by Khoo et al.[9], a multivariate EWMA control chart using weighted variance method by Atta et al. [1], and comparing the median run length (MRL) performances of the Max-EWMA and Max-DEWMA control charts for skewed distributions by Teh et al. [12]. Other works that deal with univariate control charts for skewed distributions include that of Wu [15], Nichols and Padgett [13], Tsai [15], Dou and Sa

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[8], Chen [6], and Yourstone and Zimmer [17]. In this article, the S control chart is developed by using the scaled weighted variance (SWV) method suggested by Castagliola [3]. The proposed SWV-S control chart is an extension of the S control chart proposed by Khoo et al. [11]. The proposed control chart provides asymmetric limits in accordance with the direction and degree of skewness by using different variances in computing the upper and lower limits.

2.2.1 The weighted variance (WV) method

According to Choobineh and Ballard [7], the basic idea of the weighted variance (WV) method is that a skewed distribution can be split into two segments at its mean, where each segment is used to create a new symmetric distribution. The two new distributions created from the original skewed distribution have the same mean but different standard deviations.

The WV method uses the two created symmetric distributions to set up the limits of the WV control chart. Specifically, one of the two new distributions is used to compute the upper control limit, while the other is used to compute the lower control limit of the WV control chart. Since the WV method uses a multiple of the standard deviation to establish the control limits, it requires determination of the standard deviations of the two new symmetrical distributions. Choobineh and Ballard [7] developed a method to approximate the variance of the two distributions. This method of approximating the variance is summarized as follows:

Let $\phi(\cdot)$ and $\Phi(\cdot)$ denote the standard normal, $N(0,1)$, pdf and cdf, respectively. Let $f(x)$ in Figure 1 be the probability density function (pdf) of quality characteristic X, from a skewed distribution; μ_X and σ_X be the mean and standard deviation of X respectively, and $P_X = P(X \le \mu_X)$. The weighted variance method was initially proposed by Choobineh and Ballard [7]. This method is based on the idea that the probability density function $f(x)$ can be split into two new symmetrical functions, $f_L(x)$ and $f_U(x)$ having the same mean μ_X but different variances, σ_L^2 for $f_L(x)$ and σ_U^2 for $f_U(x)$ (see Figure 1). $f_L(x)$ and $f_U(x)$ are replaced by two normal distributions $\phi(x,\mu_X,\sigma_L) = \phi\left[(x-\mu_X)\sigma_L^{-1}\right]/\sigma_L$ and $\phi(x,\mu_X,\sigma_U) = \phi\left[(x-\mu_X)\sigma_U^{-1}\right]/\sigma_U$, having the same mean μ_X and variances σ_L^2 and σ_U^2, respectively (see Figure 1). This differs from the standard S control chart in that the standard deviation is multiplied by two different factors. One factor is used for the upper control limit (UCL), while the other is used for the lower control limit (LCL). Assume that, $P_X = P(X \le \mu_X)$, is the probability that random variable X is less than or equal to its mean μ_X. Then the UCL factor is $\sqrt{2P_X}$ and the LCL factor is $\sqrt{2(1-P_X)}$ (for more details see Choobineh and Ballard [7]). See Figure 1 for an illustration of the weighted variance method.

The WV-S control chart suggested by Khoo et al. [11] is set up by plotting the sample standard deviations, S_i for i =1, 2, …, based on the following limits in Khoo et al. [11]:

$$\mathrm{UCL}_{\mathrm{WV}-S} = \mu_S + 3\sigma_S\sqrt{2P_X} \tag{6}$$

and

$$\mathrm{LCL}_{\mathrm{WV}-S} = \mu_S - 3\sigma_S\sqrt{2(1-P_X)}, \tag{7}$$

where μ_S and σ_s are the mean and standard deviation of S, respectively. Note that when $P_X = \frac{1}{2}$ the WV-S control chart reduces to the standard S control chart. If the process parameters are unknown, the limits of the WV-S are:

$$\mathrm{UCL}_{\mathrm{WV}-S} = \bar{S}\left[1+\frac{3\sqrt{1-(C_4')^2}}{C_4'}\sqrt{2\hat{P}_X}\right] = B_U\bar{S} \tag{8}$$

and

$$\mathrm{LCL}_{\mathrm{WV}-S} = \bar{S}\left[1-\frac{3\sqrt{1-(C_4')^2}}{C_4'}\sqrt{2(1-\hat{P}_X)}\right]^+ = B_L\bar{S} \tag{9}$$

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Here, $C_4' = \frac{E(S)}{\sigma_X}$ is a constant for a given skewed population and $\bar{S} = \frac{\sum_{i=1}^{r} S_i}{r}$ is the average of the sample standard deviations estimated from r preliminary subgroups, while values of B_U and B_L are computed via simulation using SAS 9.3 and

$$\hat{P}_X = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} I\left(\bar{\bar{X}} - X_{ij}\right)}{m \times n}, \quad (10)$$

where m and n are the number of samples in the preliminary data set and the sample size, respectively, and $I(x) = 1$ if $x \geq 0$ or $I(x) = 0$, otherwise.

2.2.2 A scaled weighted variance (SWV) method

Castagliola [3] suggested a new approach, called the scaled weighted variance method to improve the performance of the weighted variance method. The functions $f_L(x)$ and $f_U(x)$ are not simply replaced by two normal probability density distributions $\phi(x, \mu_X, \sigma_L)$ and $\phi(x, \mu_X, \sigma_U)$, but are replaced by two "bell-shaped" functions $\phi(x, \mu_X, \sigma_L, 2P_X)$ and $\phi(x, \mu_X, \sigma_U, 2(1-P_X))$ centered on μ_X, having σ_L^2 and σ_U^2 for second central moments and $2P_X$ and $2(1-P_X)$ for areas. Castagliola [3] defined the function $\phi(x, \mu_X, t, k)$ as

$$\phi(x, \mu_X, t, k) = \frac{k^{3/2}}{t}\varphi\left(\frac{(x-\mu_X)\sqrt{k}}{t}\right).$$

This function has the following required properties (see Castagliola [3]) for more details about the derivations:

$$\int_{-\infty}^{+\infty} \phi(x, \mu_X, t, k)\,dx = k \quad (11)$$

$$\int_{-\infty}^{+\infty} (x-\mu_X)^2\, \phi(x, \mu_X, t, k)\,dx = t^2. \quad (12)$$

Using $\phi(x, \mu_X, t, k)$ instead of the probability density function $\phi(x, \mu_X, t)$ gives new limits for the weighted variance S control chart proposed by Khoo et al. [11].

Proposed scaled weighted variance S control chart (SWV-S) limits:

$$\text{UCL}_{\text{SWV}-S} = \mu_S + \Phi^{-1}\left(1 - \frac{\alpha}{4(1-P_X)}\right)\sqrt{\frac{P_X}{(1-P_X)}}\ \sigma_S \quad (13)$$

and

$$\text{LCL}_{\text{SWV}-S} = \mu_S - \Phi^{-1}\left(1 - \frac{\alpha}{4P_X}\right)\sqrt{\frac{(1-P_X)}{P_X}}\ \sigma_S. \quad (9)$$

Here, μ_S and σ_S are the mean and standard deviation of the S respectively, and α is Type I error rate (False alarm). Note that, we called this control chart a Scaled Weighted Variance S control chart or SWV-S control chart in short, because the function $\phi(x, \mu_X, t, k)$ is scaled by a factor $\frac{k^{3/2}}{t}$ (see Castagliola [3] for more details).

Note also that when $P_X = \frac{1}{2}$, the SWV-S control chart reduces to the standard t S control chart. If the process parameters are unknown, the control limits of the proposed SWV-S control chart are computed as follows:

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$$\mathrm{UCL}_{\mathrm{SWV}-S} = \bar{S}\left[1+\Phi^{-1}\left(1-\frac{\alpha}{4\left(1-\hat{P}_X\right)}\right)\frac{\sqrt{1-(C_4')^2}}{C_4'}\sqrt{\frac{\hat{P}_X}{\left(1-\hat{P}_X\right)}}\right] \tag{10}$$

and

$$\mathrm{LCL}_{\mathrm{SWV}-S} = \bar{S}\left[1-\Phi^{-1}\left(1-\frac{\alpha}{4\hat{P}_X}\right)\frac{\sqrt{1-(C_4')^2}}{C_4'}\sqrt{\frac{\left(1-\hat{P}_X\right)}{\hat{P}_X}}\right] \tag{11}$$

Here, $C_4' = \frac{E(S)}{\sigma_X}$ is a constant for a given skewed population and $\bar{S} = \frac{\sum_{i=1}^{r} S_i}{r}$ is the average of the sample standard deviations estimated from r preliminary subgroups.

3. AN ILLUSTRATIVE EXAMPLE

The data in table 1 are generated from a Weibull distribution for the purpose of illustration, the data consist of 200 skewed observations grouped into 40 subgroups of size n = 5 each. These data are supposed to correspond to an in-control process. The shape parameter, β , is chosen to be 0.9987 so that the skewness, α_3 is 2, and scale parameter, λ is chosen to be 30.50. From these data, we obtained $\hat{\mu}$ = 31.17, $\hat{\sigma}$ = 32.43 and C_4' = 0.8688. We also observe from the data that 125 observations fall below $\hat{\mu}$. Thus, $\hat{P}_X$ = 0.625 from Equation (5). By assuming $\alpha = 0.0027$, the SWV-*S* chart's control limits computed using Equations (10) and (11) are equal to $UCL_{\mathrm{SWV}-S}$ = 88.527 and $LCL_{\mathrm{SWV}-S}$ =-9.978. These limits are compared with those obtained for the WV-*S* control chart, ($UCL_{\mathrm{WV}-S}$ = 82.035 and $LCL_{\mathrm{WV}-S}$ = -13.545), and the standard *S* control chart ($UCL_{\mathrm{SH}-S}$ =76.349 and $LCL_{\mathrm{SH}-S}$ = -19.999).From this example, we note that, the upper control limits obtained for the SWV-*S* control chart is further away from the center line than the upper control limits obtained with other methods, and the lower control limit is closer to the center line than the lower limits obtained with the other methods. From Figure 2, we observe that all points fall within control limits of the SWV-*S* chart, indicating that the process is in-control. On the other hand, two points are on the board of $UCL_{\mathrm{SH}-s}$ of the standard *S* control, and these two points are plotted close to the $UCL_{\mathrm{SWV}-S}$ of the WV-*S* chart. We note that, the SWV-*S* control chart performs better than the WV-*S* and standard *S* control charts.

4. DISCUSSION

The false alarm rates of the SWV–S chart for many degree of skewness are lower than that of the normal theory value, (see Table 2).Table 3 shows that the probability of detection rates of the SWV–S chart for skewed populations are reasonably close to that of the exact –S chart for skewed distribution .These results show the robustness of the SWV–S chart to violations of the normality assumption. These results, combined with the fact that the SWV–S chart outperforms of the Shewhart S and WV-S control charts for skewed populations in detecting small, moderate and large shifts in the process standard deviation (see Tables 3) make the SWV–S chart appealing to practitioners. The chart with the lowest false alarm and the highest probability of out-of-control detections for most level of skewness and sample size, n is assumed to be have a better performance. Hence, our proposed SWV-S chart have this properties. However, the practitioners have confidence to choose this chart as a good alternative to the Shewhart S and WV-S control charts for monitoring process dispersion when the distributions are skewed.

5. PERFORMANCE EVALUATION OF THE PROPOSED SWV-*S* CONTROL CHART

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The SWV−S control chart is compared with the WV-S control chart for skewed data proposed by Khoo et al. [11] and standard S control chart, in terms of the false alarm rate. In terms of the Probabilities of out-of-control detections, the proposed SWV-S control chart is compared with the exact method, WV-S and standard S control charts. A Monte Carlo simulation is conducted using SAS 9.3 to compute the false alarm rates and Probabilities of out-of-control detections. The false alarm rate of a control chart is defined as the proportion of subgroup points plotting beyond the limits of the chart, given that the process is actually in-control. On the contrary, the probability of out-of-control detection measures the ability of a chart in responding to a shift in the process and it represents the proportion of subgroup points plotting beyond the limits of the chart when the process has shifted. All the charts considered in this paper are designed based on an in-control ARL of 370. A shift in the process standard deviation is represented by $\sigma_1 = \delta\ \sigma_X$, where $\delta \in \{1.1, 1.3, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0\}$ is the magnitude of a shift, in process standard deviation. The skewed distributions considered here, are Weibull, lognormal and gamma because they represent a wide variety of shapes from symmetric to highly skewed. For the sake of comparison, the standard normal distribution is also considered. For convenience, a scale parameter of one is used for the Weibull and gamma distributions, while a location parameter of zero is selected for the lognormal distribution since the skewness does not depend on the parameters of these distributions. Note that P_X for the Weibull, lognormal and gamma distributions are :

$$P_X = 1 - \exp\left[-\left(\Gamma\left(1 + \tfrac{1}{\beta}\right)\right)^{\beta}\right] \quad (14)$$

$$P_X = \Phi\left(\tfrac{\omega}{2}\right) \quad (15)$$

and

$$P_X = F(\gamma) \quad (16)$$

respectively, where β, ω and γ are the shape parameters (see Khoo et al., [10] and Khoo et al., [11]):. Here, $\Gamma(\cdot)$ is the gamma function, while $\Phi(\cdot)$ and $F(\cdot)$ are the lognormal and gamma distribution functions, respectively. In the case of the false alarm rates, the skewness coefficients considered are $\alpha_3 \in \{0.5, 1.0, 1.5, 2.0, 2.5, 3.0\}$, while skewness coefficient, α_3=2 is considered in the case of the probability of out-of-control detection. The sample sizes, $n \in \{5, 7, 10\}$ are considered for the false alarm rate and the probability of out-of-control detection. The false alarm rate and probability of out-of-control detection are obtained based on 10000 simulation trials. The simulated results are tabulated in Table 2 and 3 for the false alarm rate and probability of out-of-control detection, respectively. Table 2 shows that the proposed SWV-S control chart has lower false alarm rate than the WV-S control chart for almost all levels of skewnesses and sample sizes, when the distributions are Weibull, lognormal and gamma. Table 3 shows that the probabilities of out-of-control detections of the proposed SWV-S charts are closer to those of the exact S chart than those of the WV-S and standard S control charts. Figures 3 to 5 presented the false alarm rate when sample sizes, n 5, 7 and 10 for the Weibull, lognormal and gamma distributions respectively, the figures show that the false alarm rate of the proposed SWV-S control chart is lower than all the charts considered in this paper for all levels of skewnesses and sample sizes. In general, the proposed SWV-S control chart provides good performances in term of false alarm rate and probability of out-of-control detection for all levels of skewnesses, sample sizes and magnitudes of shifts.

6. CONCLUSIONS

In this paper, we have proposed the SWV-S control chart for skewed populations. This proposed chart based on the scaled weighted variance method suggested by Castagliola [3]. The proposed SWV-S control chart reduces to the standard S control chart when the underlying population has a normal distribution. Our simulation study on the false alarm rate indicates that the SWV-S control chart provides lower false alarm rates than those of WV-S and standard control charts for all levels of skewnesses and sample sizes. The proposed

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SWV-*S* control chart offers considerable improvement over the WV-*S* and standard *S* control charts when it is desirable for the false alarm rate to be closed to the conventional 0.0027. In the case of the probability of out-of-control detections, the simulation results show that the said probabilities of the proposed SWV-*S* control chart are closer to the chart constructed by exact *S* chart than the WV-*S* and standard *S* control charts. The findings are based on the SWV-*S* method instead of relying on the WV-*S*. Hence, the SWV-*S* chart can act as a favorable substitute to the existing WV-*S* and standard *S* control charts in the evaluation of the speed of a chart to detect shifts in process dispersion, when the underlying distribution is skewed. In conclusion, this study would help practitioners in deciding which type of chart to be used in process of monitoring as part of quality control procedures. Another focus for future research can deal with the construction of scaled weighted variance (SWV) method with EWMA and CUSUM control charts for skewed populations. Incorporating the scaled weighted variance (SWV) method to construct synthetic *S* chart for skewed populations is another potential topic for further research.

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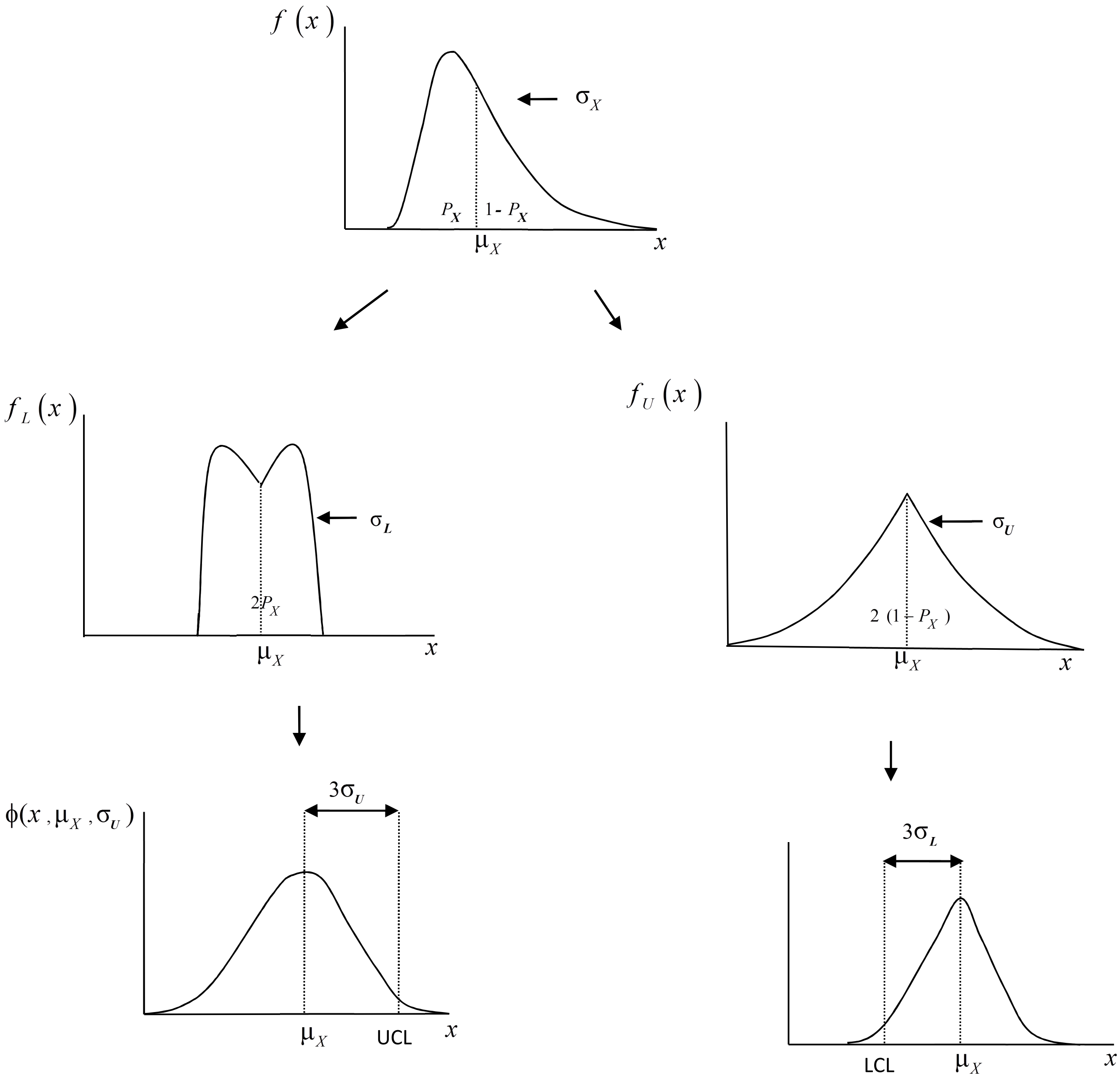

Figure. 1. An illustration of the weighted variance method

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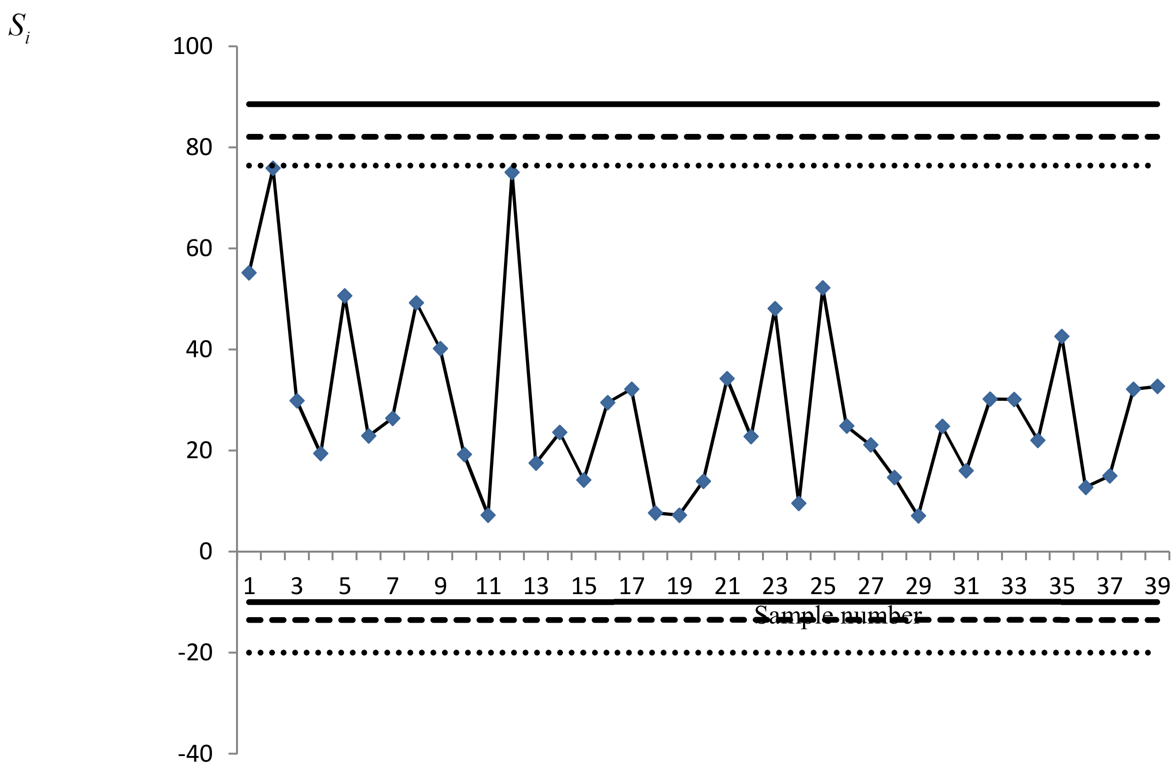

Figure2. S type control charts for the SWV, WV and standard S methods using simulated data from a skewed distribution

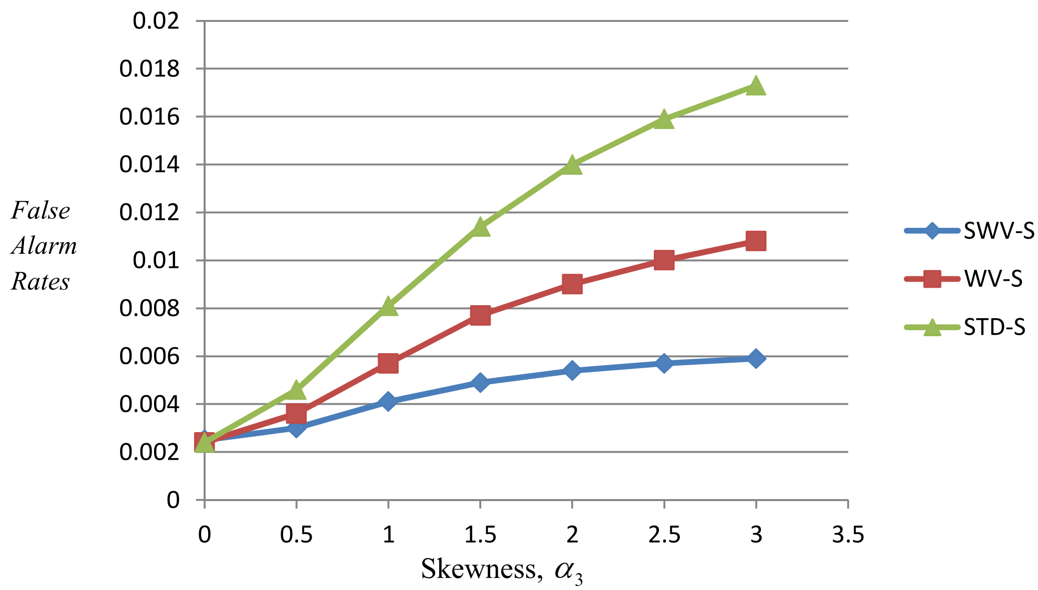

Figure 3. False alarm rates for SWV-S, WV-S and STD-S control charts for sample size, n=5 and various skewnesses, Weibull Distribution

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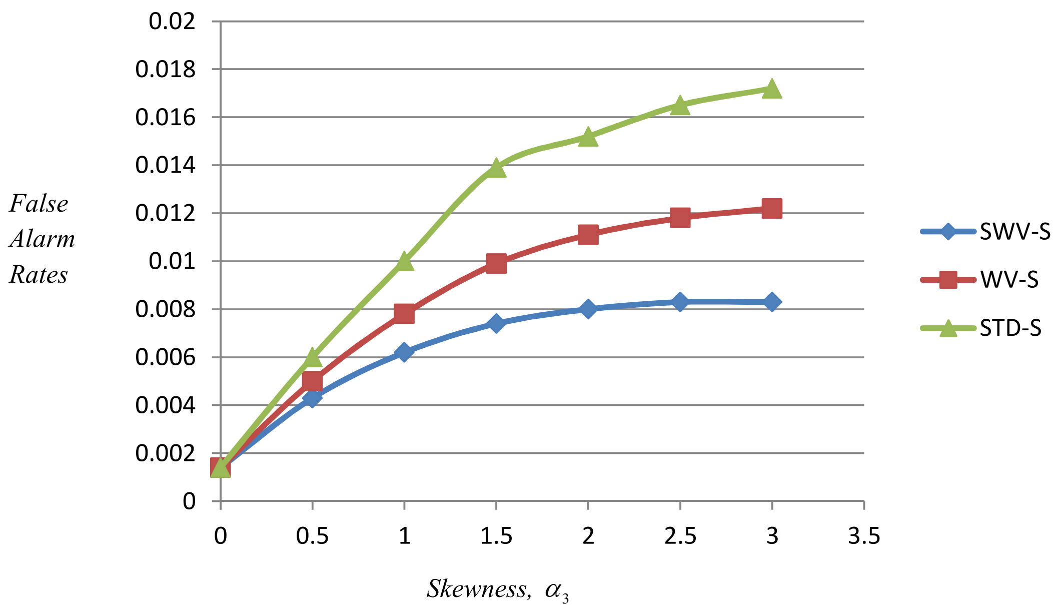

Figure 4. False alarm rates for SWV-S, WV-S and STD-S control charts for sample size, n=7 and various skewnesses, Lognormal Distribution

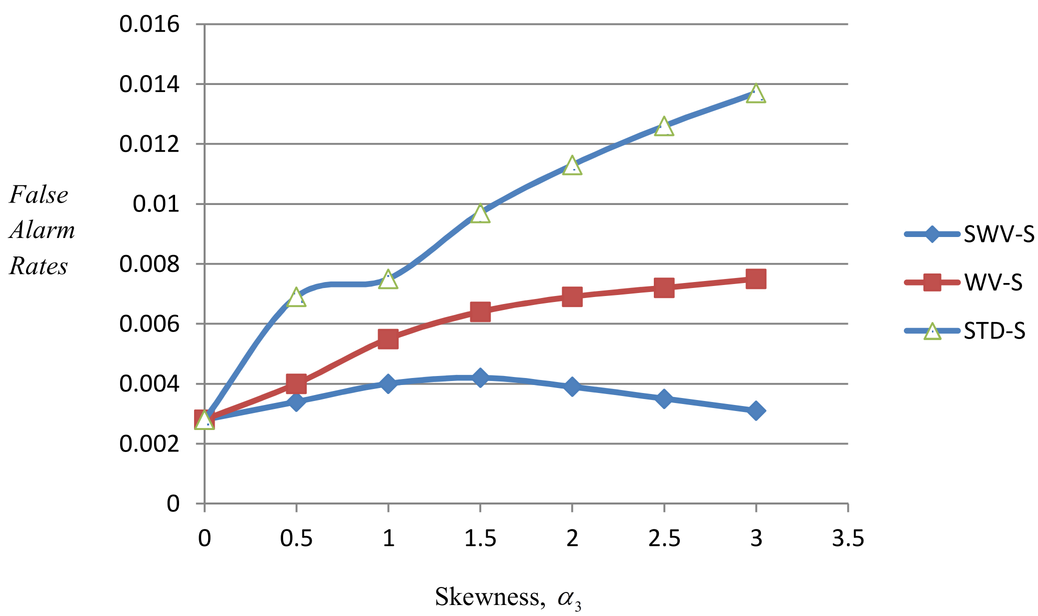

Figure 5. False alarm rates for SWV-S, WV-S and STD-S control charts for sample size, n=10 and various skewnesses, gamma distribution.

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Table 1: An example of illustration using simulated data from a skewed population (Weibull distribution)

	Observed values						
Sample number, i	X_1	X_2	X_3	X_4	X_5	$\bar{X}_i$	S_i
1	12.047	1.415	9.201	131.183	10.1457	32.79834	55.14764
2	16.301	186.55	7.385	37.432	14.3496	52.40352	75.82477
3	65.908	95.064	37.294	50.649	15.9247	52.96794	29.84449
4	8.423	49.401	8.283	31.655	4.6902	20.49044	19.39562
5	15.169	14.808	108.288	102.131	9.1358	49.90636	50.58816
6	9.705	36.436	65.255	20.001	12.6407	28.80754	22.86066
7	51	71.618	52.338	0.405	44.8413	44.04046	26.3747
8	114.697	90.067	2.609	19.298	23.2731	49.98882	49.22883
9	84.3	17.677	16.358	16.204	95.165	45.9408	40.16439
10	16.965	5.344	33.007	21.682	56.0763	26.61486	19.22718
11	26.815	14.227	13.95	14.051	6.9392	15.19644	7.193372
12	47.522	4.193	189.112	63.522	9.006	62.671	75.02858
13	26.05	14.524	12.992	40.213	53.9872	29.55324	17.48624
14	60.815	15.764	19.178	6.03	1.5032	20.65804	23.55637
15	40.344	13.188	13.478	10.292	35.442	22.5488	14.16895
16	75.525	31.102	6.69	1.482	36.1554	30.19088	29.4452
17	1.89	87.297	20.161	36.74	26.1829	34.45418	32.13535
18	14.883	22.683	16.254	14.316	1.6433	13.95586	7.649066
19	2.222	6.302	11.804	21.41	10.7152	10.49064	7.192407
20	41.74	23.349	29.183	4.229	31.9203	26.08426	13.90951
21	3.967	87.461	7.94	18.711	39.9235	31.6005	34.20247
22	2.647	7.643	30.265	3.945	55.3903	19.97806	22.74547
23	117.012	79.859	1.908	69.733	13.2122	56.34484	48.05324
24	24.569	18.992	13.665	22.738	0.8927	16.17134	9.504163
25	6.321	3.756	12.639	127.619	31.9564	36.45828	52.14306
26	25.511	39.771	67.108	87.18	39.7289	51.85978	24.83738
27	2.461	3.871	50.837	1.809	7.8113	13.35786	21.08061
28	7.101	38.319	37.136	10.246	26.3871	23.83782	14.645
29	18.008	26.145	10.619	11.85	8.9162	15.10764	7.059204
30	15.854	40.439	66.503	11.67	7.9733	28.48786	24.75521
31	40.679	6.863	21.177	7.269	36.9818	22.59396	15.95622
32	7.762	83.881	29.981	13.393	39.585	34.9204	30.18404
33	2.949	31.92	64.347	2.092	60.9962	32.46084	30.10192
34	1.243	22.756	5.563	13.903	56.5226	19.99752	22.01659
35	21.231	12.489	108.885	33.432	2.1206	35.63152	42.53482
36	4.63	32.31	4.456	23.315	7.2457	14.39134	12.70595
37	34.689	29.13	20.831	3.798	43.0136	26.29232	14.94974
38	33.858	0.364	55.423	72.747	1.6356	32.80552	32.1403
39	19.009	50.78	9.943	61.4	90.5286	46.33212	32.67596
40	32.125	2.549	42.875	2.303	12.1151	18.39342	18.28434

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Table. 2: False Alarm rates of the SWV-S, WV-S and standard –S control charts

				n						
		5			7			10		
Distribution	α_3	SWV-S	WV-S	SH-*S*	SWV-S	WV-S	SH-*S*	SWV-S	WV-S	SH-*S*
Normal	0.0	0.0039	0.0039	0.0039	0.0035	0.0035	0.0035	0.0031	0.0031	0.0031
Weibull										
3.6286	0.0	0.0025	0.0024	0.0024	0.0023	0.0023	0.0022	0.0023	0.0023	0.0022
2.2266	0.5	**0.0030**	0.0036	0.0046	**0.0027**	0.0032	0.0040	**0.0025**	0.0029	0.0035
1.5688	1.0	**0.0041**	0.0057	0.0081	**0.0034**	0.0049	0.0072	**0.0030**	0.0043	0.0063
β 1.2123	1.5	**0.0049**	0.0077	0.0114	**0.0041**	0.0067	0.0102	**0.0036**	0.0057	0.0090
0.9987	2.0	**0.0054**	0.0090	0.0140	**0.0046**	0.0080	0.0127	**0.0040**	0.0070	0.0114
0.8598	2.5	**0.0057**	0.0100	0.0159	**0.0049**	0.0091	0.0147	**0.0044**	0.0081	0.0134
0.7637	3.0	**0.0059**	0.0108	0.0173	**0.0052**	0.0099	0.0162	**0.0046**	0.0090	0.0151
Lognormal										
0.0010	0.0	0.0061	0.0061	0.0061	0.0014	0.0014	0.0014	0.0088	0.0088	0.0088
0.1656	0.5	**0.0046**	0.0054	0.0065	**0.0043**	0.0050	0.0060	**0.0038**	0.0045	0.0053
0.3170	1.0	**0.0066**	0.0084	0.0107	**0.0062**	0.0078	0.0100	**0.0056**	0.0072	0.0092
ω 0.4484	1.5	**0.0079**	0.0105	0.0138	**0.0074**	0.0099	0.0139	**0.0068**	0.0092	0.0124
0.5593	2.0	**0.0084**	0.0117	0.0158	**0.0080**	0.0111	0.0152	**0.0075**	0.0105	0.0144
0.6525	2.5	**0.0087**	0.0124	0.0169	**0.0083**	0.0118	0.0165	**0.0078**	0.0113	0.0157
0.7315	3.0	**0.0087**	0.0127	0.0176	**0.0083**	0.0122	0.0172	**0.0079**	0.0117	0.0166
Gamma										
38000	0.0	0.0039	0.0039	0.0040	0.0033	0.0033	0.0033	0.0028	0.0028	0.0028
15.4	0.5	**0.0044**	0.0051	0.0062	**0.0038**	0.0045	0.0054	**0.0034**	0.0040	0.0069
3.913	1.0	**0.0054**	0.0071	0.0095	**0.0047**	0.0063	0.0085	**0.0040**	0.0055	0.0075
γ 1.788	1.5	**0.0056**	0.0083	0.0119	**0.0049**	0.0074	0.0108	**0.0042**	0.0064	0.0097
0.983	2.0	**0.0053**	0.0089	0.0139	**0.0046**	0.0078	0.0125	**0.0039**	0.0069	0.0113
0.648	2.5	**0.0047**	0.0089	0.0148	**0.0041**	0.0080	0.0115	**0.0035**	0.0072	0.0126
0.442	3.0	**0.0044**	0.0094	0.0163	**0.0038**	0.0083	0.0123	**0.0031**	0.0075	0.0137

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Table 3: Probabilities of out-of-control detections for the various charts when the underlying distributions are Weibull, gamma and lognormal

			N											
			5				7				10			
			Exact	SWV-*S*	WV-*S*	SH-*S*	Exact	SWV-*S*	WV-*S*	SH-*S*	Exact	SWV-*S*	WV-*S*	SH-*S*
Shape	α_3	δ	Weibull											
$\beta = 0.9987$	2.0	1.1	0.9971	0.9903	0.9844	0.9769	0.9972	0.9910	0.9853	0.9800	0.9968	0.9917	0.9860	0.9782
		1.3	0.9914	0.9758	0.9644	0.9523	0.9906	0.9751	0.9625	0.9468	0.9887	0.9740	0.9598	0.9421
		1.5	0.9806	0.9533	0.9345	0.9127	0.9774	0.9482	0.9264	0.9010	0.9708	0.9410	0.9151	0.8849
		2.0	0.9293	0.8650	0.8274	0.7876	0.9091	0.8354	0.7898	0.7421	0.8724	0.7933	0.7368	0.6788
		2.5	0.8487	0.7502	0.6999	0.6501	0.7984	0.6878	0.6274	0.5687	0.7143	0.6000	0.5290	0.4639
		3.0	0.7526	0.6334	0.5774	0.5249	0.6705	0.5412	0.4777	0.4205	0.5428	0.4229	0.3558	0.2986
		3.5	0.6551	0.5275	0.4713	0.4199	0.5455	0.4165	0.3572	0.3054	0.3941	0.2865	0.2315	0.1871
		4.0	0.5637	0.4362	0.3821	0.3353	0.4368	0.3163	0.2643	0.2208	0.2782	0.1909	0.1490	0.1167
Shape	α_3	δ	Lognormal											
$\omega = 0.983$	2.0	1.1	0.9972	0.9904	0.9847	0.9771	0.9972	0.9913	0.9857	0.9780	0.9971	0.9919	0.9652	0.9786
		1.3	0.9917	0.9762	0.9647	0.9506	0.9910	0.9758	0.9634	0.9481	0.9894	0.9746	0.9158	0.9430
		1.5	0.9813	0.9539	0.9353	0.9135	0.9783	0.9499	0.9281	0.9029	0.9724	0.9423	0.8429	0.8867
		2.0	0.9314	0.8666	0.8291	0.7896	0.9122	0.8392	0.7937	0.7460	0.8777	0.7968	0.6097	0.6825
		2.5	0.8523	0.7538	0.7032	0.6539	0.8044	0.6937	0.6331	0.5748	0.7236	0.6061	0.3958	0.4698
		3.0	0.7584	0.6387	0.5822	0.5293	0.6783	0.5489	0.4851	0.4276	0.5549	0.4302	0.2437	0.3047
		3.5	0.6627	0.5329	0.4760	0.4248	0.5553	0.4249	0.3645	0.3126	0.4071	0.2939	0.1471	0.1923
		4.0	0.5722	0.4419	0.3878	0.3402	0.4473	0.3245	0.2718	0.2281	0.2906	0.1968	0.0886	0.1202
Shape	α_3	δ	Gamma											
$\gamma = 0.5593$	2.0	1.1	0.9958	0.9802	0.9741	0.9665	0.9955	0.9797	0.9733	0.9654	0.9949	0.9793	0.9725	0.9641
		1.3	0.9800	0.9373	0.9237	0.9083	0.9766	0.9284	0.9124	0.8946	0.9707	0.9176	0.8988	0.8777
		1.5	0.9478	0.8709	0.8500	0.8276	0.9325	0.8445	0.8196	0.7929	0.9114	0.8103	0.7804	0.7481
		2.0	0.7970	0.6681	0.6394	0.6105	0.7380	0.5902	0.5572	0.5246	0.6520	0.4931	0.4572	0.4221
		2.5	0.6302	0.4968	0.4703	0.4444	0.5315	0.3915	0.3640	0.3380	0.4056	0.2766	0.2507	0.2270
		3.0	0.4939	0.3774	0.3556	0.3345	0.3776	0.2668	0.2465	0.2274	0.2482	0.1607	0.1442	0.1296
		3.5	0.3939	0.2968	0.2790	0.2622	0.2747	0.1908	0.1759	0.1621	0.1576	0.0999	0.0897	0.0805
		4.0	0.3213	0.2412	0.2271	0.2138	0.2065	0.1430	0.1321	0.1219	0.1052	0.0666	0.0598	0.0536