PERFORMANCE MEASURES OF A COMPUTER NETWORK SYSTEM

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ABSTRACT

This paper analyses the reliability of computer network system consists of a server and two subsystems connected in parallel. In this way whole system will down only when both the subsystems are down or server fails. By supplementary variable technique it is concluded that Steady state transition probability is around 99.55%, system will degrade slowly with time shown in fig-4. It is also shown that availability and reliability also decreases with time as shown in fig-5. This study also describes that how MTTF will effect with respect to failure rate as shown in fig-6.

1. INTRODUCTION

The IEEE defines reliability as "The ability of a system or competent to perform its required functions under stated conditions for a specified period of time". A large number of papers in the field of reliability theory have analyzed a two unit standby system with three modes-operative at full capacity (normal mode), operative at reduced efficiency (partial mode or degraded mode) and failed completely. Dhillon (1981), Gupta et al. (1982,1983) and Goel et al.(1983) have studied redundant system models under different set of operating units etc.

This paper analyses a computer network system having one server and two subsystems, namely Switch (I) and Switch (II). The system has three modes normal, degraded and failed. The system can fail in two cases:

(i) if server fails
(ii) both switches fail

All the failure rates are assumed to be exponential whereas the repair rates follow general distribution. In this paper, the authors employ the method of supplementary variables to obtain the probabilities of the system being in various states. Several reliability characteristics of interest of system designers as well as operation managers have been computed. Results obtained are verified as particular cases.

The authors, in the present paper investigate a system which consists of one server and two subsystems - SWITCH(I) and SWITCH (II) connected in parallel with the server having Ethernet standard. SWITCH(I) consists of n-units say 1,2,3…i…n in parallel and SWITCH(II) consists of m-units say 1,2,…j…m in parallel as shown in fig-1 below.

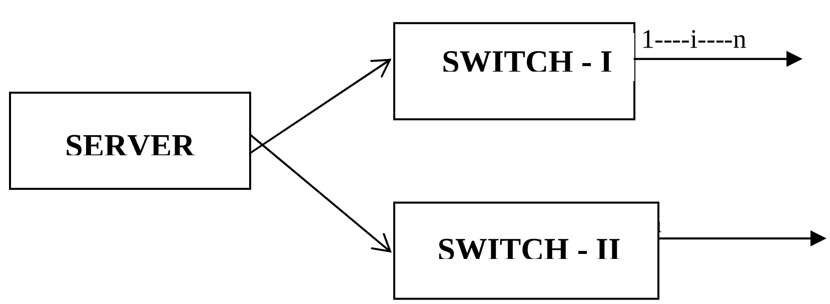

Fig- 1: System configuration

Although HUBs and switches both glue the PC,s in a network together, but network built with switches is faster than one built with hubs. When a hub receives a packet (chunk) of data (a frame in Ethernet lingo) at one of its ports from a PC on the network, it transmits (repeats) the packet to all of its ports and, thus, to all of the other PCs on the network. If two or more PCs on the network try to send packets at the same time a collision occurs. Then all of the PCs have to go through a routine to resolve the conflict. The process/routine is to resolve the conflict is Ethernet Carrier Sense Multiple Access with collision Detection **(CSMA/CD)** protocol.

Each Ethernet Adapter has both a receiver and a transmitter. A crossover cable is used for connecting two computers directly together in an Ethernet. A crossover cabled does not cause collision problem. It hardwires the Ethernet transmitter on one computer to the receiver on the other. Alternatively switches make multiple temporary crossover cable connections between pairs of computers.

An Ethernet switch automatically divides the network into multiple segments, acts as a high-speed, selective bridge between the segments, and supports simultaneous connections of multiple pairs of computers that don't compete with other pairs of computers for network bandwidth. It accomplishes this by maintaining a table of each destination address and its port. When the switch receives a packet, it reads the destination address from the header information in the packet, establishes a temporary connection between the source and destination ports, sends the packet on its way, and then terminates.

Fig-2 illustrates a LAN switch providing dedicated bandwidth to devices and illustrates the relationship of Layer 2 LAN switching to the OSI data link layer. Fig-3 represents the state-transition diagram of considered system.

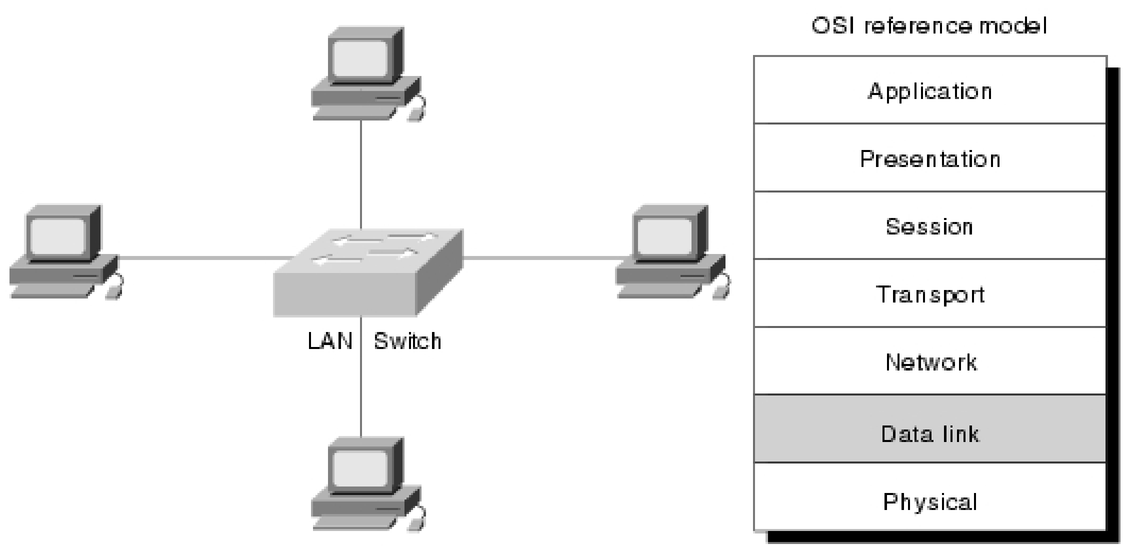

Fig- 2

COMPONENTS OF THE WHOLE SYSTEM

SWITCH A switch uses store-and-forward techniques to support multilayer switching. The switch must receive the entire frame before it performs any protocol-layer operations is a relatively simple multiport device rebroadcasts all data it receives on each port to all remaining ports. It operates at the physical layer of OSI network model.

SERVER A server collects data from the entire local networks and creates a substation database. Often a local human machine interface graphics package uses data from this database. Servers function at the application layer of the OSI model. If Ethernet servers are based on industrial personal computers, they have an MTBF of 14.3 years.

MEDIA In this system, Ethernet uses specialized copper unshielded twisted pair (UTP) cable connections.

NODES Each HUB consists of norm nodes connected in parallel. IEEE 802.3 supports the system. Ethernet having broad band signal, analog signal. Each station on an Ethernet network has its own Network Interface Card (NIC).

SIGNALING The base band systems use Manchester digital encoding. Manchester code is widely-used in Ethernet. It provides simple encoding with no long period without a level transition.

FRAME FORMAT IEEE 802.3 specifies one type of frame containing seven fields Preamble, SFD, DA, SA, length/ type of PDU, 802.2 frame and CRC.

Preamble	SFD	Destination Address	Source Address	Length PDU	DSAP	SSAP	Control	Information	CRC
7 bytes	1 byte	6 bytes	2 bytes		Data and Padding				4 bytes

The current system can fail completely only due to failure of server or both the switches. The failure rates are assumed to be exponential whereas repair rates follow general distribution. Employing the supplementary variables technique, we obtain the following performance measures.

(i) Steady-State Transition Probabilities.
(ii) Availability and Reliability.
(iii) MTTF.
(iv) Steady-State Availability of the System.

2. ASSUMPTIONS

1. Initially at $t = 0$, the system starts from state S_0.
2. The system has three states viz., good, degraded and failed.
3. The whole system works in degraded state on failure of any one switch.
4. The system fails only when both the switches fail and Server fails.
5. Common cause failure can occur only in state S_0.
6. Failures are statistically independent.
7. The system will be repaired only when it is failed completely.
8. All the repairs follow general distribution.
9. Repaired subsystem or unit works like new.

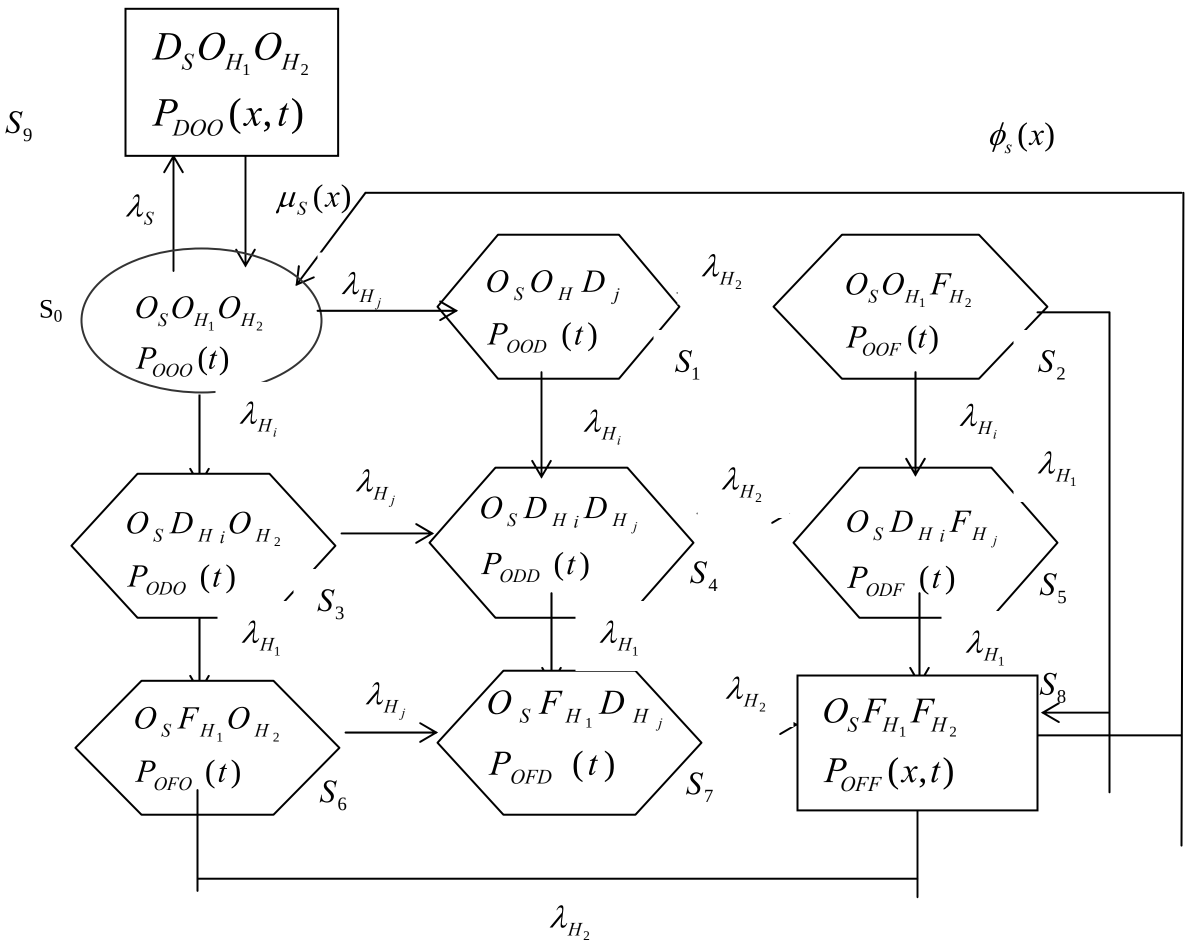

Fig-3: STATE-TRANSITION DIAGRAM

3. NOTATIONS

$D / D_x / D_t$	$\frac{d}{dt} / \frac{\partial}{\partial x} / \frac{\partial}{\partial t}$
n	Total number of units in SWITCH(I)
m	Total number of units in SWITCH(II)
$\lambda_{H_i} / \lambda_{H_j}$	Constant failure rate due to failure of $i^{\text{th}} / j^{\text{th}}$ unit of SWITCH(I)/ SWITCH (II)
$\lambda_{H_1} / \lambda_{H_2}$	Constant failure rate due to failure of SWITCH(I)/ SWITCH (II).
λ_S	Constant failure rate due to failure of server.
$\mu_s(x)\Delta / \phi_s(x)\Delta$	First order probabilities that the system repaired in the time interval $\{x, x+\Delta\}$.
$P_{OOO}(t)$	P_r {System is in state S_0 at time t}.
$P_{OOD}(t) / P_{OOF}(t) /$	P_r {System is in state S_k due to failure of j^{th} unit/ SWITCH(II)/ i^{th} unit}.

$P_{ODO}(t) / P_{OOD}(t) / P_{ODF}(t)$ — Where k =1,2,3,4,5.

$P_{OFO}(x,t)\Delta / P_{OFD}(x,t)\Delta / P_{OFF}(x,t)\Delta$ — P_r {System is in state S_k at time t due to failure of SWITCH(I) and elapsed repair time lies in the interval $(x, x+\Delta)$ where k = 6,7,8}.

$P_{DOO}(x,t)\Delta$ — P_r {System is in state S_9 at time t due to failure of server and elapsed repair time lies in the interval $(x, x+\Delta)$}.

$\overline{S}_i^K(s) = k(x)e^{-\int_0^x K(x)dx}$ where $k = \mu_s, \phi_s$

$\sum_i = \sum_{i=1}^{n}, \int = \int_0^\infty$ unless otherwise stated.

The possible transitions between states along with transition rates are shown in Fig.4.

4. FORMULATION OF MATHEMATICAL MODEL:

Probabilistic considerations and limiting procedure yield the following integral differential equations.

$$\left[D + \sum \lambda_{H_j} + \sum \lambda_{H_i} + \lambda_s\right] P_{OOO(t)} = \int_0^\infty P_{DOO}(x,t)\mu_s(x)dx + \int_0^\infty P_{OFF}(x,t)\phi_s(x)dx \qquad 4.1$$

$$\left[D + \lambda_{H_2} + \sum \lambda_{H_i}\right] P_{OOD}(t) = \sum \lambda_{H_j} P_{OOO}(t) \qquad 4.2$$

$$\left[D + \sum \lambda_{H_j} + \lambda_{H_1}\right] P_{OOF}(t) = \lambda_{H_2} P_{OOD}(t) \qquad 4.3$$

$$\left[D + \lambda_{H_1}\right] P_{ODF}(t) = \sum \lambda_{H_i} P_{OOF}(t) + \lambda_{H_2} P_{ODD}(t) \qquad 4.4$$

$$\left[D + \lambda_{H_1} + \lambda_{H_2}\right] P_{ODD}(t) = \sum \lambda_{H_J} P_{ODO}(t) + \sum \lambda_{H_i} P_{OOD}(t) \qquad 4.5$$

$$\left[D + \sum \lambda_{H_j} + \lambda_{H_2}\right] P_{ODO}(t) = \sum \lambda_{H_i} P_{OOO}(t) \qquad 4.6$$

$$\left[D + \sum \lambda_{H_j} + \lambda_{H_2}\right] P_{OFO}(t) = \lambda_{H_1} P_{ODO}(t) \qquad 4.7$$

$$\left[D + \lambda_{H_2}\right] P_{OFD}(t) = \lambda_{H_1} P_{ODD}(t) + \sum \lambda_{H_j} P_{OFO}(t) \qquad 4.8$$

$$\left[D_x + D + \phi_S(x)\right] P_{OFF}(x,t) = 0 \qquad 4.9$$

$$\left[D_x + D + \mu_s(x)\right] P_{DOO}(x,t) = 0 \qquad 4.10$$

BOUNDARY CONDITIONS

$$P_{OFF}(0,t) = \lambda_{H_1}\left[P_{OOF}(s) + P_{ODF}(s)\right] + \lambda_{H_2}\left[P_{OOF}(s) + P_{ODF}(s)\right] \qquad 4.11$$

$$P_{DOO}(0,t) = \lambda_s P_{OOO}(t) \qquad 4.12$$

INITIAL CONDITION

$$P_{OOO}(0) = 1 \qquad 4.13$$

5. SOLUTION OF THE MODEL

Taking Laplace transforms of equations (4.1) to (4.10) and solving one using boundary and initial conditions, we get

$$\overline{P}_{OOO}(s) = \frac{1}{D(s)} \quad 5.1$$

$$\overline{P}_{OOD}(s) = \frac{\sum \lambda_{H_j}}{(s + \lambda_{H_2} + \sum \lambda_{H_i})} \frac{1}{D(s)} \quad 5.2$$

$$P_{OOF}(s) = \frac{C(s)}{D(s)} \quad 5.3$$

$$P_{ODO}(s) = \frac{\sum \lambda_{H_i}}{(s + \lambda_{H_i} + \sum \lambda_{H_j})} \frac{1}{D(s)} \quad 5.4$$

$$P_{ODD}(s) = \frac{A(s)}{D(s)} \quad 5.5$$

$$P_{ODF}(s) = \frac{1}{(s + \lambda_{H_1})} \left[B(s) + \lambda_{H_2} A(s) \right] \frac{1}{D(s)} \quad 5.6$$

$$P_{OFO}(s) = \frac{\lambda_{H_1}}{(s + \sum \lambda_{H_j +} \lambda_{H_1})} \frac{\sum \lambda_{H_i}}{(s + \lambda_{H_1} + \sum \lambda_{H_j})} \frac{1}{D(s)} \quad 5.7$$

$$\overline{P}_{OFD}(s) = \frac{1}{s + \lambda_{H_2}} \left[\lambda_{H_1} A(s) + K(s) \right] \frac{1}{D(s)} \quad 5.8$$

$$\overline{P}_{OFF}(s) = \frac{1 - \overline{S}_{\varphi}(s)}{s} \left[\lambda_{H_1} \left(C(s) + \frac{1}{s + \lambda_{H_1}} [B(s) + \lambda_{H_2} A(s)] \right) \right.$$

$$\left. + \lambda_{H_2} \left(\frac{\lambda_{H_i} \sum \lambda_{H_i}}{(s + \sum \lambda_{H_j} + \lambda_{H_2})(s + \sum \lambda_{H_j} + \lambda_{H_1})} + \frac{1}{s + \lambda_{H_2}} \left[\lambda_{H_1} A(s) + K(s) \right] \right) \right] \frac{1}{D(s)} \quad 5.9$$

$$\overline{P}_{DOO}(x, s) = \frac{1 - \overline{S}_{\mu}(x)}{S} \lambda_s \frac{1}{D(s)} \quad 5.10$$

The Laplace transforms of the probabilities that at time t system is in up and down state are given by

$$\overline{P}_{up}(s) = \overline{P}_{OOO}(s) + \overline{P}_{OOD}(s) + \overline{P}_{OOF}(s) + \overline{P}_{ODO}(s) + \overline{P}_{ODD}(s) + \overline{P}_{ODF}(s) + \overline{P}_{OFD}(s) + \overline{P}_{OFO}(s)$$

$$= \left[\left[1 + \frac{\sum \lambda_{H_j}}{s + \lambda_{H_2} + \sum \lambda_{H_j}} + C(s) + \frac{\sum \lambda_{H_i}}{s + \lambda_{H_1} + \sum \lambda_{H_j}} + A(s) + \frac{1}{s + \lambda_{H_1}}\left[B(s) + \lambda_{H_2} A(s)\right] + \frac{1}{s + \lambda_{H_2}}\left[\lambda_{H_1} A(s) + K(s)\right] + \frac{\lambda_{H_i}}{\left(s + \sum \lambda_{H_j} + \lambda_{H_i}\right)\left(s + \sum \lambda_{H_j} + \lambda_{H_1}\right)} \sum \lambda_{H_j}\right]\right] \frac{1}{D(s)} \quad 5.11$$

$$\overline{P}_{down}(s) = \overline{P}_{OFF}(s) + \overline{P}_{DOO}(s)$$

$$= \left\{\left[\lambda_{H_1}\left(C(s) + \frac{1}{s + \lambda_{H_1}}\left[B(s) + \lambda_{H_2} A(s)\right]\right) + \lambda_{H_2}\left(\frac{\lambda_{H_1} \sum \lambda_{H_i}}{(s + \sum \lambda_{H_j} + \lambda_{H_2})(s + \sum \lambda_{H_j} + \lambda_{H_1}} + \frac{1}{s + \lambda_{H_2}}\left[\lambda_{H_1} A(s) + K(s)\right]\right)\right] \frac{1 - \overline{S}_{\phi}(x)}{s} + \frac{1 - \overline{S}_{\mu}(s)}{s} . \lambda_s \right\} \frac{1}{D(s)} \quad 5.12$$

It is worth noticing that

$$\overline{P}_{up}(s) + \overline{P}_{down}(s) = \frac{1}{s} \quad 5.13$$

Where

$$D(s) = s + \sum \lambda_{H_j} + \sum {}_{H_i} + \lambda_s - \lambda_s \overline{S}_{\mu_s}(s) - \left\{\lambda_{H_1}\left[C(s) + \frac{1}{s + \lambda_{H_1}}\left(B(s) + \lambda_{H_2} A(s)\right)\right] - \lambda_{H_2}\left[\frac{\lambda_{H_1} \sum \lambda_{H_i}}{\left(s + \sum \lambda_{H_j} + \lambda_{H_2}\right)\left(s + \sum \lambda_{H_j} + \lambda_{H_1}\right)} + \frac{1}{s + \lambda_{H_2}}\left(\lambda_{H_1} A(s) + K(s)\right)\right]\right\} \overline{S}_{\phi}(s) \quad 5.14$$

$$A(s) = \frac{1}{s + \lambda_{H_1} + \lambda_{H_2}}\left[\frac{\sum \lambda_{H_j} \sum \lambda_{H_i}}{\left(s + \lambda_{H_2} + \sum \lambda_{H_i}\right)} + \left[\frac{\sum \lambda_{H_j} \sum \lambda_{H_i}}{s + \lambda_{H_1} + \sum \lambda_{H_j}}\right]\right] \quad 5.15$$

$$B(s) = \frac{\sum \lambda_{H_j} \lambda_{H_2} \sum \lambda_{H_j}}{\left(s + \sum \lambda_{H_j} + \lambda_{H_1}\right)\left(s + \sum \lambda_{H_j} + \lambda_{H_2}\right)} \qquad 5.16$$

$$C(s) = \frac{\lambda_{H_2} \sum \lambda_{H_j}}{\left(s + \lambda_{H_i} + \sum \lambda_{H_1}\right)\left(s + \lambda_{H_i} + \sum \lambda_{H_2}\right)} \qquad 5.17$$

$$K(s) = \frac{\sum \lambda_{H_j} \lambda_{H_1} \sum \lambda_{H_i}}{\left(s + \sum \lambda_{H_j} + \lambda_{H_1}\right)\left(s + \sum \lambda_{H_j} + \lambda_{H_2}\right)} \qquad 5.18$$

6. ERGODIC BEHAVIOUR OF THE SYSTEM

Using Abel's Lemma in Laplace transform, viz.

$$\lim_{s \to 0} s\, \bar{f}(s) = \lim_{t \to \infty} f(t) = f, \text{ say}$$

Provided that the limit of the light hand side exists, the time independent probabilities are obtained as follows:

$$\bar{P}_{OOO} = \frac{1}{D'(0)} \qquad 6.1$$

$$\bar{P}_{OOD} = \frac{\lambda'_H}{\lambda_{H_2} + \lambda_H} \frac{1}{D(0)} \qquad 6.2$$

$$P_{OOF} = \frac{C(0)}{D'(0)} \qquad 6.3$$

$$P_{ODO} = \frac{\lambda'_H}{\lambda_{H_2} + \lambda_H} \frac{1}{D(0)} \qquad 6.4$$

$$P_{ODD} = \frac{A(0)}{D'(0)} \qquad 6.5$$

$$P_{ODF} = \frac{1}{\lambda_{H_1}} \left[B(0) + \lambda_{H_2} A(0)\right] \frac{1}{D'(0)} \qquad 6.6$$

$$P_{OFO} = \frac{\lambda_{H_1}}{\lambda'_H + \lambda_{H_2}} \frac{\lambda_H}{\lambda'_H + \lambda_{H_1}} \frac{1}{D'(0)} \qquad 6.7$$

$$\bar{P}_{OFD} = \frac{1}{\lambda_{H_2}} \left[\lambda_{H_1} A(0) + K(0)\right] \frac{1}{D'(0)} \qquad 6.8$$

$$\bar{P}_{OFF} = M^{\phi} \left[\left[\lambda_{H_1} \left[C(0) + \frac{1}{\lambda_{H_1}} (B(0) + \lambda_{H_2} A(0))\right]\right] + \lambda_{H_2} \left[\frac{\lambda_{H_1} . \lambda_H}{\left(\lambda'_H + \lambda_{H_2}\right)\left(\lambda'_H + \lambda_{H_1}\right)} + \frac{1}{\lambda_{H_2}} (\lambda_{H_1} A(0) + K(0))\right]\right] \frac{1}{D'(0)} \qquad 6.9$$

$$P_{DOO} = M^{\mu_s} \lambda_s \frac{1}{D'(0)} \qquad 6.10$$

$$P_{up} = \left[1 + \frac{\lambda'_H}{\lambda_{H_2+}\lambda_H} + C(0) + \frac{\lambda_H}{\lambda_{H_1+}\lambda'_H} + A(0) + \frac{1}{\lambda_{H_1}}\left[B(0) + \lambda_{H_2} A(0)\right]\right.$$

$$\left. + \frac{1}{\lambda_{H_2}}\left[\lambda_{H_1} + A(0) + K(0)\right] + \frac{\lambda_{H_1}}{(\lambda'_H + \lambda_H)}\frac{\lambda_H}{\left(\lambda_{H_j} + \lambda_{H_1}\right)}\right]\frac{1}{D'(0)} \qquad 6.11$$

$$P_{down} = \left[\left(\lambda_{H_1}\left[C(0) + \frac{1}{\lambda_{H_1}}\left[B(0) + \lambda_{H_2} A(0)\right]\right]\right)\right.$$

$$\left. + \lambda_{H_2}\left(\frac{\lambda_{H_1}.\lambda_H}{(\lambda'_H + \lambda_{H_2})(\lambda'_H + \lambda_{H1})} + \frac{1}{\lambda_{H_2}}\left[\lambda_{H_1} A(0) + K(0)\right]\right)\right] M^{\phi}\frac{1}{D'(0)} \qquad 6.12$$

$$-\lambda_s M^{\mu_s}\frac{1}{D'(0)}$$

Also $\quad P_{up} + P_{down} = 1$

where

$$D'(0) = \frac{d}{ds}D(s)\bigg|_{s=0}, \sum \lambda_{H_i} = \lambda_{H_1}, \sum \lambda_{H_j} = \lambda_H \text{ and}$$

$$M^K(K = \phi_s, \mu_s) = -\bar{S}_i'^K(0)$$

7. SOME SPECIAL CASES

7.1 When repair rates are constant i.e. repair times follow exponential distribution, setting $\bar{S}_{(s)}{}^{\theta} = \dfrac{\theta}{s+\theta}$ where $\theta = \mu_S, \phi_s$ in relation to 5.1 to 5.12, one may get

$$\bar{P}_{OOO}(s) = \frac{1}{E(s)} \qquad 7.1.1$$

$$\bar{P}_{ODO}(s) = \frac{\sum \lambda_{H_i}}{s + \lambda_{H_1} + \lambda_{H_j}}\frac{1}{E(s)} \qquad 7.1.2$$

$$\bar{P}_{OOD}(s) = \frac{\sum \lambda_{H_j}}{s + \lambda_{H_2} + \sum \lambda_{H_j}}\frac{1}{E(s)} \qquad 7.1.3$$

$$\bar{P}_{OOF}(s) = \frac{C(s)}{E(s)} \qquad 7.1.4$$

$$\bar{P}_{ODD}(s) = \frac{A(s)}{E(s)} \qquad 7.1.5$$

$$\overline{P}_{ODF}(s) = \frac{1}{s+\lambda_{H_1}}\left[B(s)+\lambda_{H_2}A(s)\right]\frac{1}{E(s)} \quad 7.1.6$$

$$\overline{P}_{OFO}(s) = \frac{\lambda_{H_1}}{(s+\sum\lambda_{H_j}+\lambda_{H_2})}\frac{\sum\lambda_{H_i}}{(s+\sum\lambda_{H_j}+\lambda_{H_1})}\frac{1}{E(s)} \quad 7.1.7$$

$$\overline{P}_{OFD}(s) = \frac{1}{s+\lambda_{H_2}}\left[\lambda_{H_1}A(s)+K(s)\right]\frac{1}{E(s)} \quad 7.1.8$$

$$\overline{P}_{OFF}(s) = \frac{(1-\overline{S}_{\varphi}(s))}{s}\left[\lambda_{H_1}\left(C(s)+\frac{1}{s+\lambda_{H_1}}[B(s)+\lambda_{H_2}A(s)]\right)\right.$$
$$\left.+\lambda_{H_2}\left(\frac{\lambda_{H_i}\sum\lambda_{H_i}}{(s+\sum\lambda_{H_j}+\lambda_{H_2})(s+\sum\lambda_{H_j}+\lambda_{H_1})}+\frac{1}{s+\lambda_{H_2}}\left[\lambda_{H_1}A(s)+K(s)\right]\right)\right]\frac{1}{E(s)} \quad 7.1.9$$

$$\overline{P}_{DOO}(s) = \frac{1-\overline{S}_{\mu_s}(x)}{s}\lambda_S\frac{1}{E(s)} \quad 7.1.10$$

$$\overline{P}_{up}(s) = \left[1+\frac{\sum\lambda_{H_j}}{s+\lambda_{H_2+}\sum\lambda_{H_i}}+C(s)+\frac{\sum\lambda_{H_i}}{s+\lambda_{H_1}+\sum\lambda_{H_j}}+A(s)\right.$$
$$+\frac{1}{s+\lambda_{H_1}}\left[B(s)+\lambda_{H_2}A(s)\right]+\frac{1}{s+\lambda_{H_2}}\left[\lambda_{H_i}A(s)+K(s)\right] \quad 7.1.11$$
$$\left.+\frac{\lambda_{H_i}\sum\lambda_{H_i}}{(s+\sum\lambda_{H_j}+\lambda_{H_2})(s+\sum\lambda_{H_j}+\lambda_{H_1})}\right]\frac{1}{E(s)}$$

where,

$$E(s) = s+\sum\lambda_{H_j}+\sum\lambda_{H_i}+\lambda_S-\lambda_S\frac{\mu_S}{s+\mu_S}-\left\{\lambda_{H_1}\left[C(s)+\frac{1}{s+\lambda_{H_1}}\right.\right.$$
$$\left.[B(s)+\lambda_{H_2}A(s)]\right]-\lambda_{H_2}\left[\frac{\lambda_{H_1}\sum\lambda_{H_i}}{(s+\sum\lambda_{H_j}+\lambda_{H_2})(s+\sum\lambda_{H_j}+\lambda_{H_1})}\right. \quad 7.1.12$$
$$\left.\left.+\frac{1}{s+\lambda_{H_2}}\left(\lambda_{H_1}A(s)+K(s)\right)\right]\right\}\frac{\phi_S}{s+\phi_S}$$

7.2 NON-REPAIRABLE SYSTEM

The Laplace transform of the reliability when all repair rates are zero of the system is given by:

$$\overline{R}(s) = \frac{A_1}{\sum\lambda_{H_j}+\sum\lambda_{H_i}+\lambda_s}+\frac{A_2}{\lambda_{H_2}+\sum\lambda_{H_i}}+\frac{A_3}{\lambda_{H_1}+\sum\lambda_{H_j}} \quad 7.2.1$$
$$+\frac{A_4}{\lambda_{H_1}+\lambda_{H_2}}+\frac{A_5}{\lambda_{H_1}}+\frac{A_6}{\lambda_{H_2}}$$

The reliability of the system is

$$R(t) = A_1 e^{-\left(\sum \lambda_{H_j} + \sum \lambda_{H_i} + \lambda_S\right)t} + A_2 e^{-\left(\lambda_{H_2} + \sum \lambda_{H_i}\right)t} + A_3 e^{-\left(\lambda_{H_1} + \sum_{H_j}\right)t} + A_4 e^{-\left(\lambda_{H_1} + \lambda_{H_2}\right)t} + A_5 e^{-\lambda_{H_1}.t} + A_6 e^{-\lambda_{H_2}.t} \quad 7.2.2$$

Mean time to system failure is given as under

$$MTTF = \int_0^\infty R(t)dt \quad 7.2.3$$

$$= \frac{A_1}{\sum \lambda_{H_j} + \sum \lambda_{H_i} + \lambda_S} + \frac{A_2}{\lambda_{H_2} + \sum \lambda_{H_j}} + \frac{A_3}{\lambda_{H_1} + \sum \lambda_{H_j}} + \frac{A_4}{\lambda_{H_1} + \lambda_{H_2}} + \frac{A_5}{\lambda_{H_1}} + \frac{A_6}{\lambda_{H_2}}$$

$$A_1 = \Bigg[\frac{\lambda_{H_2} - \lambda_S}{\lambda_{H_2} - \sum \lambda_{H_j} - \lambda_S} + \frac{\lambda_{H_j} - \lambda_{H_j}}{(\lambda_{H_2} - \sum \lambda_{H_j} - \lambda_S)(\lambda_{H_1} - \sum \lambda_{H_j} - \sum \lambda_{H_i} - \lambda_S)}$$

$$+ \frac{\sum \lambda_{H_i} \sum \lambda_{H_j}}{(\lambda_{H_1} - \sum \lambda_{H_i} - \lambda_S)(\lambda_{H_1} - \sum \lambda_{H_j} - \sum \lambda_{H_i} - \lambda_S)} \quad 7.2.4$$

$$+ \frac{\lambda_{H_2} \sum \lambda_{H_j}}{(\lambda_{H_2} - \sum \lambda_{H_i} - \lambda_S)(\lambda_{H_1} - \sum \lambda_{H_i} - \sum \lambda_{H_j} - \lambda_S)}$$

$$+ \frac{\lambda_{H_j}\left(\lambda_{H_2} - \sum \lambda_{H_i} - \lambda_S + \lambda_{H_1}\right)}{(\lambda_{H_1} - \sum \lambda_{H_j} - \lambda_S)(\lambda_{H_2} - \sum \lambda_{H_i} - \lambda_S)}$$

$$+ \frac{\lambda_{H_1} \sum \lambda_{H_j} \sum \lambda_{H_i}}{\left(\lambda_{H_2} - \sum \lambda_{H_j} - \sum \lambda_{H_I} - \lambda_S\right)\left(\lambda_{H_2} - \lambda_{H_1} - \sum \lambda_{H_j} - \sum \lambda_{H_i} - \lambda_S\right)}$$

$$+ \frac{\sum \lambda_{H_j} \lambda_{H_1} \sum \lambda_{H_i}}{\left(\lambda_{H_2} - \sum \lambda_{H_j} - \sum \lambda_{H_j} - \lambda_S\right)\left(\lambda_{H_1} - \sum \lambda_{H_i} - \lambda_S\right)\left(\lambda_{H_2} - \sum \lambda_{H_i} - \lambda_S\right)} \Bigg]$$

$$A_2 = \frac{\lambda_{H_1} \sum \lambda_{H_j}\left(\lambda_{H_2} - \sum \lambda_{H_j}\right)}{\left(\lambda_{H_2} - \sum \lambda_{H_j} - \lambda_S\right)\left(\lambda_{H_1} - \lambda_{H_2}\right)\left(\sum \lambda_{H_i} - \lambda_{H_1}\right)} +$$

$$+ \frac{\sum \lambda_{H_i} \sum \lambda_{H_j}\left(\lambda_{H_1} - \sum \lambda_{H_i}\right)}{\left(\sum \lambda_{H_j} - \lambda_S - \lambda_{H_2}\right)\left(\lambda_{H_1} - \sum \lambda_{H_j}\right)\left(\lambda_{H_1} - \lambda_{H_2} - \sum \lambda_{H_i}\right)}$$

$$+ \frac{\sum \lambda_{H_i} \sum \lambda_{H_j} \lambda_{H_2}}{\left(\sum \lambda_{H_j} + \lambda_S - \lambda_{H_2}\right)\left(\lambda_{H_1} - \lambda_{H_2} - \sum \lambda_{H_j}\right)\left(\lambda_{H_1} - \lambda_{H_2}\right)}$$

7.2.5

$$A_3 = \frac{\lambda_{H_1}\lambda_{H_2}\sum\lambda_{H_i}}{\left(\sum\lambda_{H_i} + \lambda_S - \lambda_{H_1}\right)\left(\lambda_{H_2} - \lambda_{H_1} - \sum\lambda_{H_j}\right)\left(\lambda_{H_2} - \sum\lambda_{H_j}\right)} \quad 7.2.6$$

$$A_4 = \frac{\sum\lambda_{H_j}\sum\lambda_{H_i}\left(\lambda_{H_2} + \lambda_{H_1} - \sum\lambda_{H_j} - \sum\lambda_{H_i}\right)}{\left(\sum\lambda_{H_j} + \sum\lambda_{H_i} + \lambda_S - \lambda_{H_1} - \lambda_{H_2}\right)\left(\lambda_{H_1} - \sum\lambda_{H_i}\right)\left(\lambda_{H_2} - \sum\lambda_{H_j}\right)} \quad 7.2.7$$

$$A_5 = \frac{\sum\lambda_{H_j}\lambda_{H_2}}{\left(\sum\lambda_{H_i} + \sum\lambda_{H_j} + \lambda_S - \lambda_{H_1}\right)\left(\sum\lambda_{H_i} + \sum\lambda_{H_2} - \lambda_{H_1}\right)} \quad 7.2.8$$

$$A_6 = \frac{\sum\lambda_{H_j}}{\left(\sum\lambda_{H_j} + \sum\lambda_{H_i} - \lambda_S - \lambda_{H_2}\right)} + \frac{\sum\lambda_{H_j}\sum\lambda_{H_i}}{\left(\sum\lambda_{H_j} + \sum\lambda_{H_i} + \lambda_S - \lambda_{H_2}\right)\left(\sum\lambda_{H_j} - \lambda_{H_2} + \lambda_{H_1}\right)} + \frac{\sum\lambda_{H_i}\lambda_{H_1}}{\left(\sum\lambda_{H_j} + \sum\lambda_{H_i} + \lambda_S - \lambda_{H_2}\right)\left(\sum\lambda_{H_j} - \lambda_{H_2} + \lambda_{H_1}\right)} \quad 7.2.9$$

8. NUMERICAL COMPUTATION

8.1 AVAILABILITY ANALYSIS Setting $\sum\lambda_{H_i}$ = 0.2, $\sum\lambda_{H_j}$ =0.1, $\lambda_S = .1$, $\mu_S = 1$, $\lambda_{H_1} = 0.15$, $\lambda_{H_2} = 0.25$, $\phi_S = 1$ in equation (5.11), one may get the inverse Laplace transform of $\overline{P}_{up}(s)$.

$$P_{up}(t) = .7236e^{-1.2190t} - 6.6652e^{-.7095t}.\cos(.3934t) + .0527e^{-.7095t}\frac{1}{.3934}.\sin(.3934t) - .0222e^{-.1809t}.\cos(.3748)t - 5.5203e^{-1809}.\frac{1}{.3748}.\sin(3748t) + 6.9659e^{-.1098t}.\cos(.01t) + 2.3474.e^{-.1098t}\frac{.1}{.01}\sin(.01t) - .0174e^{-.0307t} \quad 8.1.1$$

Putting t = 0,1,2,3--- in 8.1.1 one may get Fig-4

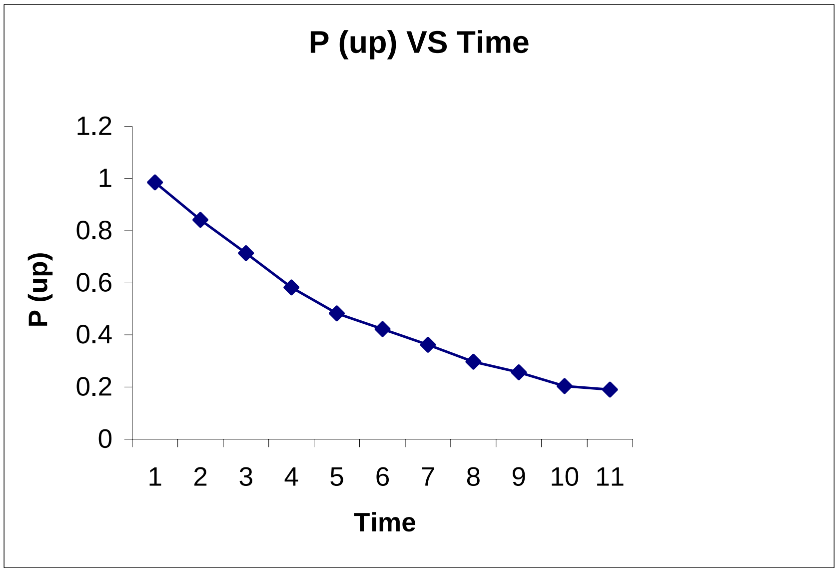

Fig-4

8.2 RELIABILITY ANALYSIS

For the same values setting as above except λ_{H_1} in equation 7.2.2

$$
\begin{aligned}
R(t) = & \left[3 + \frac{.9}{\lambda_{H_i} - .4} + \frac{0.02}{(\lambda_{H_1} - .3)(\lambda_{H_1} - .4)} - \frac{4(\lambda_{H_1} - .05)}{(\lambda_{H_1} - .3)} \right. \\
& \left. + \frac{2.6667.\lambda_{H_1}(\lambda_{H_1} - .25)}{(\lambda_{H_1} - .15)(\lambda_{H_1} - .3)} + \frac{2.6667\lambda_{H_1}}{\lambda_{H_1} - .3} \right] e^{-0.4t} \\
& + \left[\frac{.3\lambda_{H_1}}{(\lambda_{H_1} - .25)(.2 - \lambda_{H_1})} - \frac{.08(\lambda_{H_1} - .2)}{(\lambda_{H_1} - .1)(\lambda_{H_1} - .45)} \right. \\
& \left. - \frac{.01}{(\lambda_{H_1} - .45)(\lambda_{H_1} - .25)} \right] .e^{-.35t} + \left[\frac{.3333\lambda_{H_1}}{(.3 - \lambda_{H_1})(.15 - \lambda_{H_1})} \right] e^{-(\lambda_{H_1} - 0.1)t} \\
& + \left[\frac{.1333(\lambda_{H_1} - .05)}{(.15 - \lambda_{H_1})(\lambda_{H_1} - .2)} \right] .e^{(-\lambda_{H_1} + .25)t} + \left[\frac{0.45}{(.4 - \lambda_{H_1}).(.45 - \lambda_{H_1})} + \frac{.2}{.4 - \lambda_{H_1}} \right] e^{-\lambda_{H_1} t} \\
& + \left[.666 + \frac{.133}{\lambda_{H_1} - .15} + \frac{1.3\lambda_{H_1}}{\lambda_{H_1} - .15} \right] e^{-.25t}
\end{aligned}
\qquad 8.2.1
$$

For t = 0,1,2,3--- in 8.2.1 one can compute reliability for different value if λ_{H_1} which is shown in Fig-5.

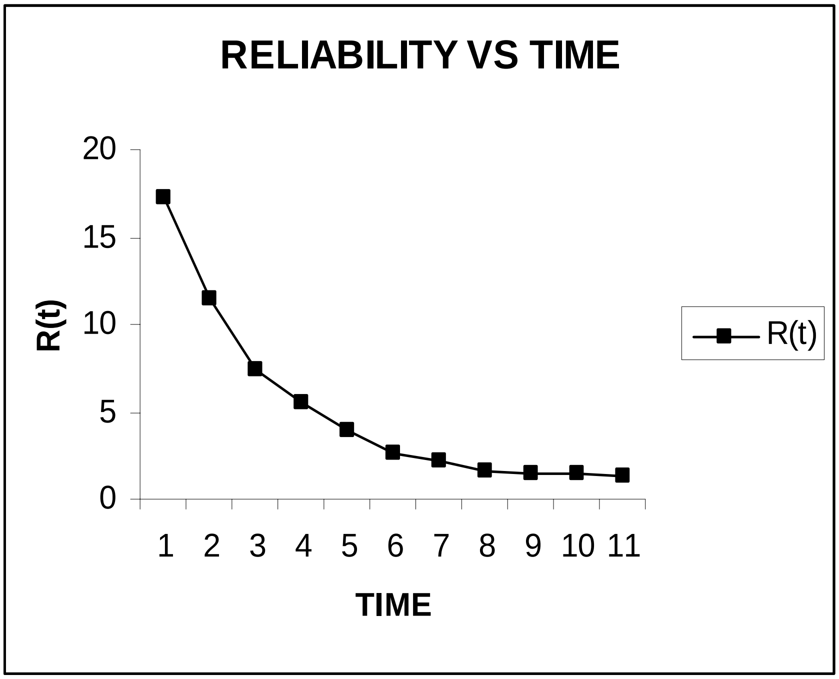

Fig-5

8.3 M.T.T.F. ANALYSIS

$$
\begin{aligned}
\text{MTTF} = {} & 2.5\left[3+\frac{.9}{\lambda_{H_1}-.4}+\frac{0.02}{(\lambda_{H_1}-.3)((\lambda_{H_1}-.4)}-\frac{4(\lambda_{H_1}-.05)}{(\lambda_{H_1}-.3)}\right. \\
& \left.+\frac{2.6667.\lambda_{H_1}(\lambda_{H_1}-.25)}{(\lambda_{H_1}-.15)(\lambda_{H_1}-.3)}+\frac{2.6667.\lambda_{H_1}}{\lambda_{H_1}-.3}\right]+2.222 \\
& \left[\frac{.3\lambda_{H_1}}{(\lambda_{H_1}-.25)(.2-\lambda_{H_1})}+\frac{.08(\lambda_{H_1}-.2)}{(\lambda_{H_1}-.1)(\lambda_{H_1}-.45)}\right. \\
& \left.-\frac{.01}{(\lambda_{H_1}-.45)(\lambda_{H_1}-.25)}\right]+\frac{1}{\lambda_{H_1}+0.1}\left[\frac{.3333\lambda_{H_1}}{(.3-\lambda_{H_1})(.15-\lambda_{H_1})}\right] \\
& +\frac{1}{\lambda_{H_1}+.25}\left[\frac{.1333(\lambda_{H_1}-.05)}{(.15-\lambda_{H_1})(\lambda_{H_1}-.2)}\right] \\
& +\frac{1}{\lambda_{H_1}}\left[\frac{.045}{(.4-\lambda_{H_1})(.45-\lambda_{H_1})}+\frac{.2}{.4-\lambda_{H_1}}\right] \\
& +4\left[.666+\frac{.133}{\lambda_{H_1}-.15}+\frac{1.3\lambda_{H_1}}{\lambda_{H_1}-.15}\right]
\end{aligned}
\tag{8.3.1}
$$

Fig-6 exhibits the variation of MTTF for different values of λ_{H_1}.

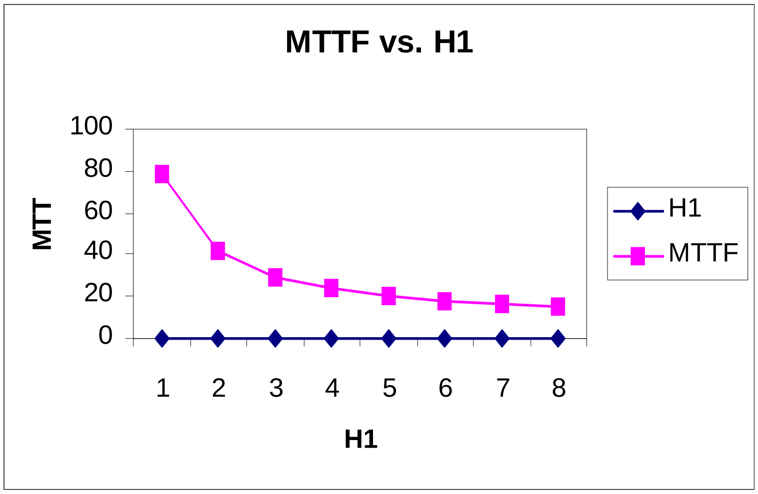

Fig-6

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