PERISTALTIC PUMPING OF A FLUID OF VARIABLE VISCOSITY IN A NON-UNIFORM TUBE WITH PERMEABLE WALL

[1]K.VIJAYARAJ, [2]G. KRISHNAIAH, [3]M.M. RAVIKUMAR

[1]Professor, Department of Mech. Engg, Ranippettai Engg, College, Walaja.-632513, TAMILNADU, INDIA
[2]Professor, Department of Mech. Engg, S.V.U. College of Engg., S.V. University, Tirupati-517502, ANDHRA PRADESH, INDIA
[3] Head of the Department , Pallavan College of Engineering, Kanchipuram- 631502, TAMILNADU, INDIA
E mail: mmr.head @ gmail.com

ABSTRACT

Peristaltic pumping of a fluid of variable viscosity in a non-uniform tube / channel lined with porous material is investigated. The flow in the tube is governed by Navier- Stokes equation and the permeable boundary is described by Darcy law. It is observed that larger the permeability of the porous medium, greater the pressure rise against which the pump works, so the increase of permeability of the wall causes less frictional force and also observed that the frictional force shows opposite behaviour to that of pressure rise in peristalsis.

Keywords: *Peristaltic pump, Navier- Stokes equation, Darcy's Law*

1. INTRODUCTION

A peristaltic pump is a device for pumping fluids, generally from a region of lower to higher pressure, by means of a contraction wave traveling along a tube-like structure. This traveling-wave phenomenon is referred to as (peristaltic). This phenomenon is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. Peristalsis is the mechanism of fluid transport that occurs generally from a region of lower pressure to higher pressure when a progressive wave of area contraction or expansion travels along the flexible wall of the tube. This mechanism is applied not only by small blood vessels, ureter and stomach to pump various bio-fluids in a human body, but also by mechanical devices such as roller pumps and finger pumps. Peristaltic pumps are designed for various industries to transport corrosive fluids without contamination due to contact with the pumping machinery.

Most of the industrial fluids show variable viscosity behavior. Further the pump cannot always be designed as a uniform tube for serving the needs such as biomedical instruments. The pressure rise, the velocity field and stream functions are determined by the Navier–Stokes equations, named after Claude-Louis Navier and George Gabriel Stokes, describe the motion of fluid substances, that is substances which can flow. These equations arise from applying Newton's second law to fluid motion, together with the assumption that the fluid stress is the sum of a diffusing viscous term (proportional to the gradient of velocity), plus a pressure term and the results are discussed through graphs. Darcy's law states that the rate at which a fluid flows through a permeable substance per unit area is equal to the permeability, which is a property only of the substance through which the fluid is flowing, times the pressure drop per unit length of flow, divided by the viscosity of the fluid. In view of all these facts, an attempt is made to study the peristaltic pumping of a fluid with variable viscosity through a non-uniform tube lined with porous material.

Latham[6] made an experimental study on the mechanics of peristaltic transport. The results of experiments were found to be in good agreement with the theoretical studies of Shapiro[9]. Later Shapiro et al[8]. Jaffrin and Shapiro[5] Burns and Parkes[3], Yin[15], Barton and Raynor[1] Subba Reddy et al[11] and several others made fundamental contributions to peristaltic transport. All these works are based on the Newtonian behavior of the pumping fluid. Treating the fluid as non-Newtonian, good numbers of investigations are available. Some of them are Bohme and Fedrich [2], Shehawey and Mekheimer[7], Usha and Rama Chandra Rao[4], Vajravelu et al[12], Vajravelu et al[13], Vajravelu et al[14], Subba Reddy et al[11] and Sreenadh et al[10].

In the design of pumps, it is usual that the inner surface of the tube is not smooth. The roughness that arises due to corrugations plays an important role in pumping. Further in the human body, the biology-systems such as blood vessels contain tissue region, which surrounds the blood. A better understanding can be done in these situations by modeling the pump as a non-uniform tube with permeable wall.

In this paper peristaltic flow of a fluid of variable viscosity in a non-uniform tube/channel lined with porous material is investigated. The flow in the free flow of the tube is governed by Navier- Stokes equation. The flow in the permeable boundary is described by Darcy law. The velocity distribution, the pressure rise and frictional force are obtained and the results are discussed through graphs.

2. NOMENCLATURE

a : Half width of the channel or radius of the tube

a_0 : Half width of the channel or radius of the tube at inlet

a(z) : radius of the tube at any axial distance 'z' from the inlet 'a_0'.

B: Amplitude

C: Wave speed

D_a : Darcy number

E: Mechanical efficiency

ΔF: Dimensionless friction force

H: Wall co-ordinate of peristaltic wave

K: Constant = 3 a_0/L

L: Length of the channel or tube

P: Pressure rise

ΔP: Dimensionless pressure rise

Q: Instantaneous volume flow rate

$\overline{Q}$: Time mean volume rate of flow

t : Time

U_B : Slip velocity

U_{porous} : Velocity in the permeable boundary

V: Co-ordinate velocity

R, Θ, Z: Cylindrical polar co-ordinates in laboratory frame

r, θ,z : Cylindrical polar co-ordinates in wave frame

α: Slip parameter

δ: Viscosity parameter

ρ: density

μ_0: Viscosity at inlet

υ: Kinematic viscosity

μ: Dynamic viscosity

φ: amplitude ratio = b/ a_0

λ: Wave length

3. MATHEMATICAL FORMULATION AND SOLUTION

Consider the peristaltic transport of a fluid of variable viscosity in a non-uniform tube with permeable wall as shown in Figure (1).

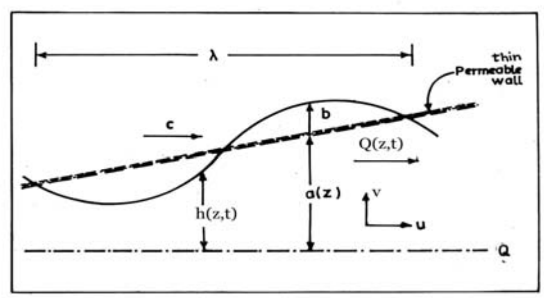

Fig. 1 Physical Model

The axisymmetric flow in the pump is governed by Navier-Stokes equation. The flow in the permeable wall is described by Darcy law. The cylindrical polar coordinate system (R,θ,Z) is used. The wall deformation due to the infinite train of peristaltic waves is represented by the geometry of the wall surface is defined as

$$R = H(z, t) = a(z) + b \sin \frac{2\pi}{\lambda}(z - ct)$$

(3.1)

$a(z) = a_0 + k(z)$

4. EQUATIONS OF MOTION

Under the assumptions that the tube length is an integral multiple of the wave length λ and the pressure difference across the ends of the tube is a constant, the flow is inherently unsteady in the laboratory frame (R,θ,Z) and becomes steady in the wave frame (r,θ,z) which is moving with the velocity c along the wave. The transformation between these two frames is given by

$r = R, \quad \theta = \theta, \quad z = Z - ct.$

(3.2)

Let us assume that

$$V_r = 0, \qquad V_\theta = 0, \qquad V_z = u(r, t)$$

(3.3)

we assume that the flow is inertia free and the wavelength is infinite.

The appropriate equation describing the flow in the wave frame under lubrication approach becomes =

$$\frac{\partial p^1}{\partial z^1} = \frac{\partial p^1}{\partial t^1} + \frac{\partial}{r^1 \partial r^1}\left(r^1 \mu_{(r)} \frac{\partial u^1}{\partial r^1}\right)$$

Using the non-dimensional quantities

$$r = \frac{r^1}{a_0}; \quad z = \frac{z^1}{\lambda}; \quad \bar{\mu} = \frac{\mu}{\mu_0}; \quad u = \frac{u^1}{c}; \quad p = \frac{p a_0^2}{\lambda c \mu_0}$$

(3.4)

p^1 (z, t) pressure $u^1 = u^1$ (z,r,t) axial velocity. The equations governing the motion in dimensionless form are

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\bar{\mu}\frac{\partial u}{\partial r}\right)$$

(3.5)

$$0 = \frac{\partial p}{\partial r}$$

(3.6)

The dimensionless boundary conditions are

$$\frac{\partial u}{\partial r} = \frac{\alpha}{\sqrt{Da}}\left(u_B - u_{porous}\right)$$

(3.7)

(Beauves and Joseph slip condition, 1967).

Where $u_{porous} = \dfrac{-Da}{\bar{\mu}_{(r)}}\dfrac{\partial P}{\partial z}$

(3.8)

(Darcy law)

Here u_{porous} is the velocity in the permeable boundary

5. SOLUTIONS

Solving the equations (3.5) & (3.6) subject to the conditions (3.7) and (3.8) we obtain the velocity as u(h) – u(r) = $\frac{1}{2}\ \frac{\partial p}{\partial z}\ \int_r^h \frac{r}{\overline{\mu}_{(r)}}\ dr$

(3.9)

u(r) = $-\frac{1}{2}\ \frac{\partial p}{\partial z}\ \int_r^h \frac{r}{\overline{\mu}_{(r)}}\ dr + u_B$

(3.10)

Where U_B is called slip velocity and has to be found using the condition (3.7)

Now we assume that viscosity decays exponentially with r

i.e. $\overline{\mu}_{(r)} = e^{-\delta r} \cong 1 - \delta r$ (for δ<<1)

(3.11)

The slip velocity u_B is obtained as

$$u_B = \frac{\sqrt{Da}}{2\alpha(1-\delta h)}\left[h - 2\alpha\sqrt{Da}\right]$$

(3.12)

The instantaneous flow rate Q(z, t) in the laboratory frame between the centre line and the wall is

Q (z, t) = $\int_0^h 2\pi r\, u dr$

(3.13)

From equation (3.9 and 3.13) we have

$$\frac{\partial p}{\partial z} = -\frac{2}{\pi}\left[\frac{Q(z,t) - \pi h^2 u_B}{I_1}\right]$$

(3.14)

where $\frac{\partial p}{\partial z}$ in pressure gradient and

$$I_1 = \int_0^h \frac{r^3}{\overline{\mu}_{(r)}}\ dr$$

6. THE PUMPING CHARACTERISTICS

Integrating the equation (3.14) w.r.t ‘z’ over one wavelength, we get the pressure rise (drop) over one cycle of the wave as

$$\Delta P_L(t) = \int_0^1 \left(\frac{dp}{dz}\right) dz$$

(3.15)

$$= -\frac{2}{\pi}\int_0^1 \frac{[\overline{Q} - \pi h^2 u_B]}{I_1}\ dz$$

(3.16)

7. THE FRICTIONAL FORCE

The dimensionless frictional force at the wall across one wave length in tube is given by

$$F_L(t) = \int_0^1 h^2\left(-\frac{dp}{dz}\right) dz \qquad (3.17)$$

$$= \frac{2}{\pi}\int_0^1 h^2 \frac{[\overline{Q} - \pi h^2 u_B]}{I_1}\ dz \qquad (3.18)$$

8. MECHANICAL EFFICIENCY OF PUMPING

$$E = \frac{\overline{Q}\Delta P}{\frac{1}{T}\int_0^T \int_0^\lambda P \frac{\partial h}{\partial t}\ dz\, dt} \qquad (3.19)$$

In equation 3.19 the numerator denotes the average rate of work done by the fluid over one wave length against the pressure rise, and the denominator denotes the average rate of work done by the wall over one wave length both being averaged over one period of the wave.

9. RESULTS AND DISCUSSIONS

From equation (3.16), we have calculated the pressure difference as a function of ‘t’ for different values of averaged mean flow rate ‘Q’ at an amplitude ratio ‘φ’ of 0.8 and is shown in fig. (2). It is observed that the peak occurs at‘t’ approximately 0.3 which corresponds to the instant when the maximum occlusion occurs at the entry to the channel. The magnitude of the peak pressure rise decreases with increasing flow rate.

From equation (3.16), we have calculated the pressure difference as a function of ‘t’ for different values of amplitude ratio ‘φ’at a given averaged mean flow rate ‘Q’ = 0.5 and is shown in fig. (3). It is observed that, the peak pressure rise increases with increasing amplitude ratio ‘φ’ and it occurs approximately at t = 0.3.

From equation (3.16), we have calculated the pressure difference as a function of ‘t’ for different values of Darcy number at an amplitude ratio ‘φ’of 0.7 and at a given averaged mean flow rate ‘Q’ = 0.5 and is shown in fig. (4). It is observed that, the peak pressure rise occurs at the same value of ‘t’ as in the earlier cases and it increases with increasing Darcy number ‘D_a’. This shows that the larger the permeability of the porous medium, the greater the pressure rise against which the pump works.

In fig. (5) The effect of variation of viscosity on the pressure rise is shown at an amplitude ratio ‘φ’of 0.5 and at a given averaged mean flow rate ‘Q’ = 0.5. It is observed that as the viscosity decreases the pressure rise increases.

We have calculated the frictional force from the equation (3.18) as a function of ‘t’ for different values of averaged mean flow rate ‘Q’ at an amplitude ratio ‘φ’ of 0.8 and is shown in fig. (6). It is observed that the frictional force exhibits opposite behavior to that of pressure rise, which is shown in fig. (2).

We have calculated the frictional force as a function of ‘t’ for different values of amplitude ratio ‘φ’as shown in fig. (7). It is observed that the friction force is negative and the trough in the frictional force increases with decreasing amplitude ratio.

From equation (3.18) we have calculated the frictional force as a function of ‘t’ for different Darcy number. It is observed that the trough in frictional force occurs approximately at ‘t’ = 0.3 and the magnitude of the trough frictional force decreases with increase in Darcy number. So the increase of permeability of the wall causes less frictional forces.

We have calculated the frictional force as a function of ‘t’ for different values of viscosity as shown in fig. (9). It is observed that the less the viscosity ratio smaller the frictional force in other words, decrease in viscosity makes the peristaltic pump to work under less frictional force.

From equation (3.16) we have calculated the pressure rise as a function of averaged mean flow rate 'Q' for different values of amplitude ratio 'φ' and is shown in fig. (10). It is observed that for a given ΔP, the flux increases with increasing amplitude ratio 'φ'. For free pumping the flux Q decreases with increasing amplitude ratio 'φ'.

The variation of pressure difference with time averaged flow rate Q, is calculated from equation (3.16) for different 'δ' and is depicted in fig. (11). It is observed that, the pressure rise decrease with increase in flow rate at a given 'δ'. For a given flow rate Q, the pressure rise increases with increasing 'δ' (i.e. decrease in viscosity).

From equation (3.18) we have calculated the frictional force as a function of time averaged flow rate Q for different values of amplitude ratio. It is observed that the frictional has opposite behavior to that of pressure rise (fig. 10). In order to study the effect of variation of viscosity on the variation of frictional force with flow rate Q, we have evaluated equation (3.18) numerically and is shown in figure (13). It is observed that the frictional force increases, with the volume flow rate Q, for a constant 'δ'. For a given frictional force, the flux increases with increasing 'δ' i.e. the decrease in viscosity.

It is clear that the frictional force shows opposite behavior to that of pressure rise in peristalsis.

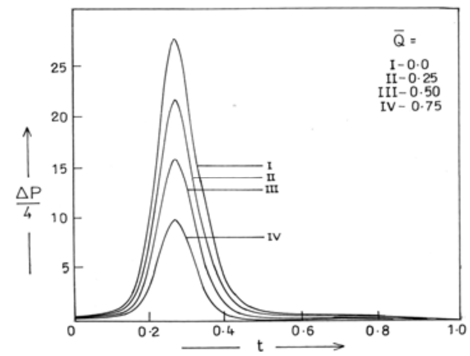

Fig. 2. The variation of Pressure rise with time't' for different flow rate $\overline{Q}$ with $\sqrt{D_a}$ = 0.1, α = 0.1, φ = 0.8, a = 0.012, δ = 0.1, λ = L = 20

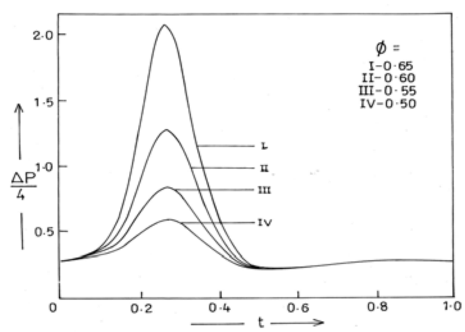

Fig. 3. The variation of Pressure rise with time't' for different Amplitude ratio φ, with $\sqrt{D_a}$ = 0.1, α = 0.1, $\overline{Q}$ = 0.5, a = 0.012, δ = 0.1, λ = L = 20

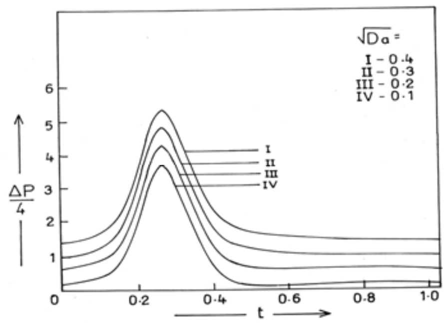

Fig. 4. The variation of Pressure rise with time'*t*' for different Darcy's number $\sqrt{D_a}$, with φ = 0.7, α = 0.1, $\overline{Q}$ = 0.5, a = 0.012, δ = 0.01, λ = L = 20

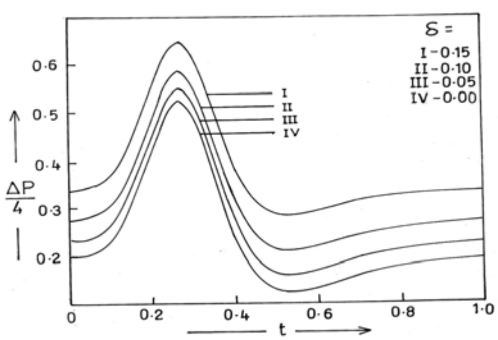

Fig. 5. The variation of Pressure rise with time '*t*' for different Viscosity co-efficient 'δ' with $\sqrt{D_a}$ = 0.1, φ = 0.5, α = 0.1, $\overline{Q}$ = 0.5, a = 0.012, λ = L = 20

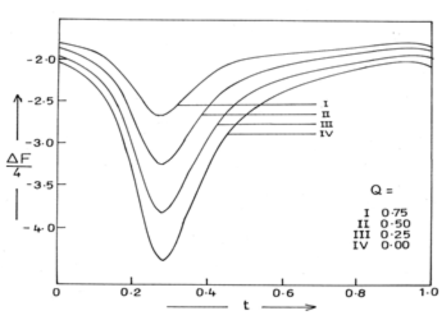

Fig. 6. The variation of Friction force with time '*t*' for different flow rate $\overline{Q}$ with $\sqrt{D_a}$ = 0.1, α = 0.1, φ = 0.8, a = 0.012, δ = 0.1, λ = L = 20

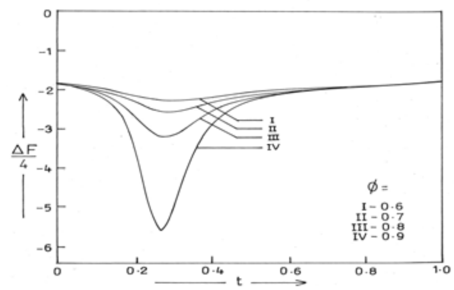

Fig. 7. The variation of Friction force with time'*t*' for different Amplitude ratioφ, with $\sqrt{D_a}$ = 0.1, α = 0.1, $\overline{Q}$ = 0.5, a = 0.012, δ = 0.1, λ = L = 20

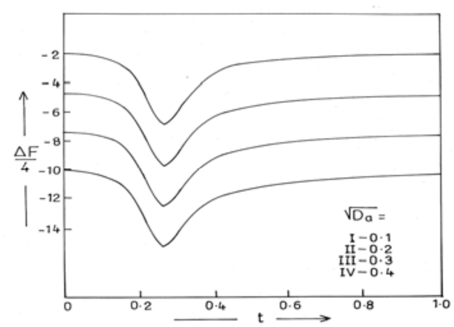

Fig. 8. The variation of Friction force with time'*t*' for different Darcy's number $\sqrt{D_a}$, with φ = 0.9, α = 0.1, $\overline{Q}$ = 0.25, a = 0.012, δ = 0.1, λ = L = 20

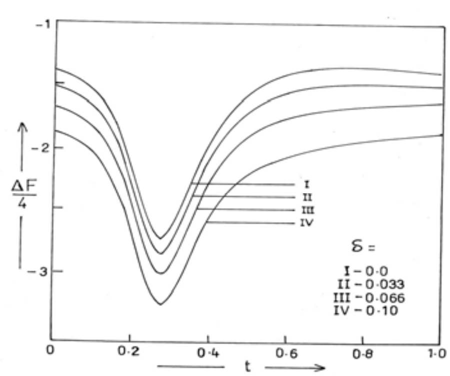

Fig. 9. The variation of Friction force with time't' for different Viscosity co-efficient 'δ' with $\sqrt{D_a}$ = 0.1, φ = 0.8, α = 0.1, $\overline{Q}$ = 0.25, a = 0.012, λ = L = 20

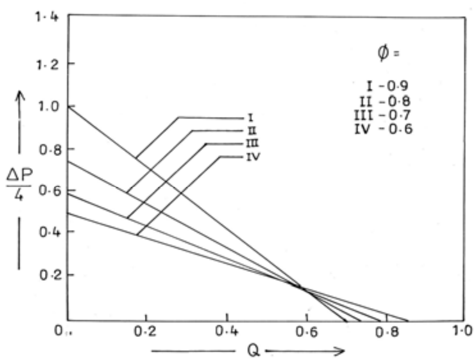

Fig. 10. The variation of Pressure rise with Averaged flow rate $\overline{Q}$ for different φ with $\sqrt{D_a}$ = 0.1, α = 0.1, a = 0.012, δ = 0.1, t = 0.5, λ = L = 20

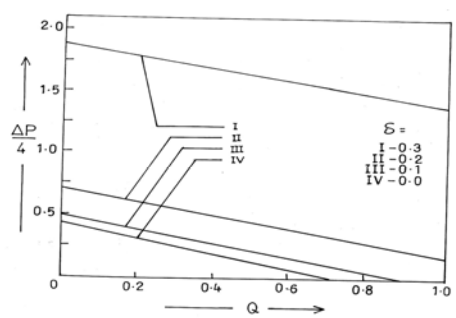

Fig. 11. The variation of Pressure rise with Averaged flow rate $\overline{Q}$ for different Viscosity co-efficient 'δ' with $\sqrt{D_a}$ = 0.1, α = 0.1, a = 0.012, φ = 0.6, t = 0.5, λ = L = 20

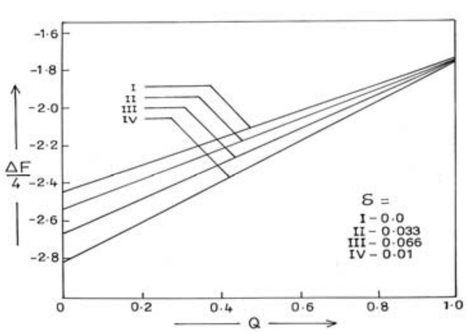

Fig. 12. The variation of Friction force with Averaged flow rate $\overline{Q}$ for different φ with $\sqrt{D_a}$ = 0.1, α = 0.1, a = 0.012, δ = 0.1, t = 0.5, λ = L = 20

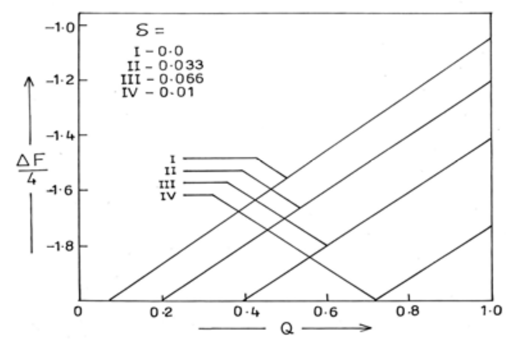

Fig. 13. The variation of Friction force with Averaged flow rate $\overline{Q}$ for different Viscosity co-efficient 'δ' with $\sqrt{D_a}$ = 0.1, α = 0.1, a = 0.012, φ = 0.8, t = 0.5, λ = L = 20

REFERENCES

1. BARTON. C. & RAYNOR. S, "Peristaltic flow in tubes", Bull. Math. Bio-physics. Vol. No 30, pg. no. 663-680, 1968.

2. BOHME. G. & FRIEDRICH. R., "Peristaltic flow of Visco elastic liquids", Journal of Fluid Mechanics", 1983, Vol No. 128, pg. no. 109-122.

3. BURNS. J. C. & PARKES. T, "Peristaltic motion:, Journal of Fluid Mechanics, 1967, Vol. no. 29, pg. no. 731-743.

4. HUGHES. W. F. & YOUNG. F. J,

"The Electromagnet dynamics of Fluids", John Willy and Sons, inc., New York. 1966, pg. no. 516-527.

5. JAFFRIN. M. Y. & SHAPIRO. A. H, "Peristaltic pumping", Ann. Rev. fluid Mech. Vol. no. 3, pg. no. 13-36.

6. LATHAM. T. W, " Fluid motions in a peristaltic pump" , M. S. Thesis, M.I.T., 1966

7. MEKHEIMER. K. S., EL SHEHAWEY. E. F & ELAW, A. M, "Peristaltic motion of a particle-fluid suspension in a planer channel", International. Journal of Theory Phys", 1998, Vol. No. 37, pg. no. 2895-2920.

8. SHAPIRO, A. H., JAFFRIN, M. Y. & WIENBERG, S. L, "Peristaltic pumping with long wavelengths at low Reynolds number", Journal of Fluid Mechanics, 1969, Vol. No.37, pg. no. 799-825.

9. SHAPIRO, A. H, "Pumping and retrograde diffusion in peristaltic waves", Procedings. Workshop in Ureteral Reflux in children, 1967, pg. no. 109-126.

10. SREENADH, S., NARAHARI, M. & RAMESH BABU, V, "Effect of yield stress on peristaltic pumping of non-Newtonian fluids in a channel", Proceedings. Int. Symp. On recent advances in Fluid Mech., Tata McGraw-Hill publishing company Ltd., New Delhi, 2005, pg. no. 234-247.

11. SUD, V. K., SEKHON, G. S. & MISHRA, R. K, "Pumping action on blood by a magnetic field", Bull. Math. Biol. 1977, Vol. No. 39, pg. no. 385-390.

12. VAJRAVELU, K., SREENADH, S. & RAMESH BABU, V, "Peristaltic transport of a Hershel-Bulkley fluid in an inclined tube", Int. J. Non linear Mech. 2005, Vol. No. 40, pg. no. 83-90.

13. VAJRAVELU, K., SREENADH, S. & RAMESH BABU, V, "Peristaltic transport of a Hershel-Bulkley fluid in a channel", Appl. Math. And Comput, 2005, Vol. No. 169, pg. no. 726-735.

14. VAJRAVELU, K., SREENADH, S. & RAMESH BABU, V, "Peristaltic transport of a Hershel-Bulkley fluid in contact with a Newtonian fluid", Quarterly J. of Appl. Math. 2006, Vol. no. 64, pg. no. 593-604.

15. YIN, C. C. & FUNG, Y. C., "Peristaltic waves in circular cylindrical tubes", Journal of Applied Mechanics", 1969, Vol .No. 36, pg. no. 579-587.