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# SIMULTANEOUS HYPOTHESIS TESTING OF SPLINE TRUNCATED MODEL IN NONLINEAR STRUCTURAL EQUATION MODELING (SEM)

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**ABSTRACT**

Model of spline truncated in structural equation modeling (SEM) is a nonlinear structural model of SEM that measures a nonlinear relationship between the latent variables. This report was developed a hypothesis testing for the parameter of spline truncated model in nonlinear SEM using likelihood ratio test (LRT). To test hypothesis $\mathrm{H}_0 : \mathbf{C}'\boldsymbol{\gamma} = \boldsymbol{\tau}$ versus $\mathrm{H}_0 : \mathbf{C}'\boldsymbol{\gamma} \neq \boldsymbol{\tau}$ in spline truncated model of nonlinear SEM for matrix **C**, vector parameter $\boldsymbol{\gamma}$ and a constant vector $\boldsymbol{\tau}$, is discovered that a statistical testing is in form of $\mathbf{W} = \dfrac{\mathbf{Q}_1 / d_1}{\mathbf{Q}_2 / d_2}$ which has a distribution $F(d_1, d_2)$, and rejects $\mathrm{H}_0$ in which $\mathbf{W}$ values are quite large.

**Keywords**: *Nonlinear SEM, Spline Truncated, Hypothesis Testing, Likelihood Ratio Test*

## 1. INTRODUCTION

Nonlinear SEM is developed from linear SEM studies, which have been previously reported by Joreskog [1], Bentler [2] and Bollen [3]. Several authors have worked about nonlinear SEM in a form of quadratic and interaction such as, Lee and Zhu [4], Lee and Song [5], Lee, Song and Lee [6], Lee, Song and Poon [7], Bollen and Current [8], Lee and Tang [9], Wall and Amemiya [10], Klein and Muthen [11], Mooijaart and Satorra [12]. The nonlinear SEM employing a model of polynomial was investigated by Wall and Amemiya [13], while, model of Griffiths-Miller between latent variable was introduced by Harring [14]. The Bayesian approach in SEM was investigated by Otok, Purnami and Andari [15]. Meanwhile, the current nonlinear structural model SEM using spline truncated was reported by Ruliana, Budiantara, Otok, and Wibowo [16]. Spline truncated model in nonlinear SEM contains knot, which is able to obtain information in the patterns of relationship between latent variable that keeps changing in particular interval. Some researchers who have especially investigated spline truncated and have published their works are Fernandes, Budiantara, Otok and Suhartono [17], Budiantara [18], Lestari, Budiantara, Sunaryo and Mashuri [19].

In the present study is conducted of hypothesis testing for spline truncated model in nonlinear SEM by applying the likelihood ratio test (LRT).

## 2. SPLINE TRUNCATED MODEL IN NONLINEAR STRUCTURAL MODEL SEM

The path diagram of spline truncated in nonlinear SEM is in Figure 1, The measurement model of exogenous latent variable and endogenous latent variable, also an estimation of factor score can be found in [16]. Mathematical formulation for structural model in Figure 1, can be written as

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$$\omega_{\eta i} = \sum_{r=0}^{m_1} \alpha_{1r}\omega_{\xi_1 i}^{r} + \sum_{s=1}^{K_1} \beta_{1s}(\omega_{\xi_1 i} - \kappa_s)_+^{m_1} + \sum_{t=0}^{m_2} \alpha_{2t}\omega_{\xi_2 i}^{t} + \sum_{u=1}^{K_2} \beta_{2u}(\omega_{\xi_2 i} - \kappa_u)_+^{m_2} + \zeta_i \quad (1)$$

In matrix form can be formulated as

$$\boldsymbol{\omega}_{\eta} = [\mathbf{T}(\boldsymbol{\omega}_{\xi 1}, \boldsymbol{\omega}_{\xi 2}, \boldsymbol{\kappa})]\boldsymbol{\gamma} + \boldsymbol{\zeta} \quad (2)$$

where $\mathbf{T}$ is the base of functionality that includes exogenous latent variables and knots, which follows previous study [16]. Because of the error $\boldsymbol{\zeta} \sim N(\mathbf{0}, \sigma^2\mathbf{I})$, then

$$\boldsymbol{\omega}_{\boldsymbol{\eta}} \sim N(\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\boldsymbol{\gamma}, \sigma^2\mathbf{I}) \quad (3)$$

also

$$\hat{\boldsymbol{\gamma}} \sim N(\boldsymbol{\gamma}, [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]^{-1}\sigma^2) \quad (4)$$

therefore

$$(\mathbf{C}'\hat{\boldsymbol{\gamma}}_{\Omega} - \boldsymbol{\tau}) \sim N(\mathbf{C}'\boldsymbol{\gamma}_{\Omega} - \boldsymbol{\tau}, \mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]\sigma^2) \quad (5)$$

to determine whether parameters

$$\boldsymbol{\gamma} = [\alpha_{10}, \alpha_{11}, \ldots, \alpha_{1m_1}, \beta_{11}, \ldots, \beta_{1K_1}, \alpha_{20}, \alpha_{21}, \ldots, \alpha_{2m_2}, \beta_{21}, \ldots, \beta_{2K_2}]'$$

influence model in Eq.(1), we will conduct hypothesis testing for $\mathrm{H}_0 : \mathbf{C}'\boldsymbol{\gamma} = \boldsymbol{\tau}$ versus $\mathrm{H}_1 : \mathbf{C}'\boldsymbol{\gamma} \neq \boldsymbol{\tau}$.

## 3. PARAMETER ESTIMATION UNDER SPACE $\Omega$ AND SPACE $\Psi$ OF SPLINE TRUNCATED IN NONLINEAR STRUCTURAL MODEL SEM

Parameter hypothetical test for model (1) as expressed below

$$\mathrm{H}_0 : \mathbf{C}'\boldsymbol{\gamma} = \boldsymbol{\tau} \text{ versus } \mathrm{H}_1 : \mathbf{C}'\boldsymbol{\gamma} \neq \boldsymbol{\tau} \quad (6)$$

where $\mathbf{C}'$ is matrix $s \times d_1$, $d_1 = m_1 + m_2 + k_1 + k_2 + 2$, $\boldsymbol{\gamma}$ is $d_1 \times 1$ vector parameter and $\boldsymbol{\tau}$ is vector $s \times 1$ that the elements are constant. Parameters set under population, or under the parameter space $\Omega$ is as follows:

$$\Omega = \{\boldsymbol{\gamma} = (\alpha_{10}, \alpha_{11}, \ldots, \alpha_{1m_1}, \beta_{11}, \ldots, \beta_{1K_1}, \alpha_{20}, \alpha_{21}, \ldots, \alpha_{2m_2}, \beta_{21}, \ldots, \beta_{2K_2}), \sigma^2_{\omega_\eta \Omega}\} \quad (7)$$

The set of parameters under $\mathrm{H}_0$ or parameter space $\Psi$ are formulated as follows:

$$\Psi = \{\boldsymbol{\gamma} = (\alpha_{10}, \alpha_{11}, \ldots, \alpha_{1m_1}, \beta_{11}, \ldots, \beta_{1K_1}, \alpha_{20}, \alpha_{21}, \ldots, \alpha_{2m_2}, \beta_{21}, \ldots, \beta_{2K_2}), \sigma^2_{\omega_\eta \Psi} \,|\, \mathbf{C}'\boldsymbol{\gamma} = \boldsymbol{\tau}\} \quad (8)$$

with

$$-\infty < \alpha_{1r} < \infty \; ; \; r = 0, 1, \ldots, m_1$$
$$-\infty < \alpha_{2t} < \infty \; ; \; t = 0, 1, \ldots, m_2$$
$$-\infty < \beta_{1s} < \infty \; ; \; s = 1, 2, \ldots, K_1$$
$$-\infty < \beta_{2u} < \infty \; ; \; u = 1, 2, \ldots, K_2$$

and

$$0 < \sigma^2_{\omega_\eta \Omega} < \infty \; ; \; 0 < \sigma^2_{\omega_\eta \Psi} < \infty$$

To obtain the parameter estimation of $\boldsymbol{\gamma}$ in the parameter space $\Omega$ and parameter space $\Psi$ for model (1), are available in the following lemma:

**Lemma 1**

If $\hat{\boldsymbol{\gamma}}_{\Omega}$ and $\hat{\boldsymbol{\gamma}}_{\Psi}$ are estimators of $\boldsymbol{\gamma}$ under space $\Omega$ and $\Psi$ for model (1), respectively, then:

$$\hat{\boldsymbol{\gamma}}_{\Psi} = \hat{\boldsymbol{\gamma}}_{\Omega} - [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C} \times [\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1} \times (\mathbf{C}'\hat{\boldsymbol{\gamma}}_{\Omega} - \boldsymbol{\tau})$$

**Proof**

To prove Lemma 1 as described above can be used Lagrange multipliers's method. The Lagrange function is assumed the following:

$$\mathrm{F}(\boldsymbol{\gamma}_{\Psi}, \boldsymbol{\theta}) = \mathrm{V}(\boldsymbol{\gamma}_{\Psi}) + 2\boldsymbol{\theta}'(\mathbf{C}'\boldsymbol{\gamma}_{\Psi} - \boldsymbol{\tau}) \quad (9)$$

where

$$\mathrm{V}(\boldsymbol{\gamma}_{\Psi}) = [(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\boldsymbol{\gamma}_{\Psi})' \times (\boldsymbol{\omega}_{\boldsymbol{\eta}} - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\boldsymbol{\gamma}_{\Psi}] \quad (10)$$

and $2\boldsymbol{\theta}'$ is Lagrange multipliers vector. Estimation of $\boldsymbol{\gamma}_{\Psi}$ is $\hat{\boldsymbol{\gamma}}_{\Psi}$ which minimizes an equation in form of:

$$\mathrm{V}(\hat{\boldsymbol{\gamma}}_{\Psi}) = [(\boldsymbol{\omega}_{\boldsymbol{\eta}} - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_{\Psi})' \times (\boldsymbol{\omega}_{\boldsymbol{\eta}} - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_{\Psi}] \quad (11)$$

with constrain of $\mathbf{C}'\hat{\boldsymbol{\gamma}}_{\Psi} = \boldsymbol{\tau}$. Estimator of $\hat{\boldsymbol{\gamma}}_{\Psi}$ is obtained by describing the Eq.(10) to be

$$\mathrm{V}(\boldsymbol{\gamma}_{\Psi}) = \boldsymbol{\omega}_{\boldsymbol{\eta}}'\boldsymbol{\omega}_{\boldsymbol{\eta}} - 2\boldsymbol{\gamma}_{\Psi}'\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\boldsymbol{\omega}_{\boldsymbol{\eta}} + \boldsymbol{\gamma}_{\Psi}'\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\boldsymbol{\gamma}_{\Psi} \quad (12)$$

By substituting Eq.(12) into Eq.(9), it generates an equation

$$\mathrm{F}(\boldsymbol{\gamma}_{\Psi}, \boldsymbol{\theta}) = \boldsymbol{\omega}_{\boldsymbol{\eta}}'\boldsymbol{\omega}_{\boldsymbol{\eta}} - 2\boldsymbol{\gamma}_{\Psi}'\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\boldsymbol{\omega}_{\boldsymbol{\eta}} + \boldsymbol{\gamma}_{\Psi}'\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\boldsymbol{\gamma}_{\Psi} + 2\boldsymbol{\theta}'(\mathbf{C}'\boldsymbol{\gamma}_{\Psi} - \boldsymbol{\tau}) \quad (13)$$

When the Eq.(13) is derived with respect to $\boldsymbol{\gamma}_{\Psi}$ and it results an equation:

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$$\frac{\partial F(\boldsymbol{\gamma}_\Psi,\boldsymbol{\theta})}{\partial \boldsymbol{\gamma}_\Psi} = -2\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\boldsymbol{\omega}_\eta + +2\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\boldsymbol{\gamma}_\Psi + 2\mathbf{C}\boldsymbol{\theta} \tag{14}$$

For the Eq.(14) is minimized, it is equal to zero and then by carrying out a simplification, it yields:

$$-2\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\boldsymbol{\omega}_\eta + 2\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2) \times \mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\hat{\boldsymbol{\gamma}}_\Psi + 2\mathbf{C}\boldsymbol{\theta} = \mathbf{0}$$

$$\hat{\boldsymbol{\gamma}}_\Psi = [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1} \times [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\boldsymbol{\omega}_\eta - \mathbf{C}\boldsymbol{\theta}]$$

$$= \hat{\boldsymbol{\gamma}}_\Omega - [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1}\mathbf{C}\boldsymbol{\theta} \tag{15}$$

And by deriving the Eq.(13) with respect to $\boldsymbol{\theta}$ and treat the result equals to zero; it yields a form

$$\frac{\partial F(\boldsymbol{\gamma}_\Psi,\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = 2\mathbf{C}'\boldsymbol{\gamma}_\Psi - 2\boldsymbol{\tau} \tag{16}$$

$$\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Psi = \boldsymbol{\tau} \tag{17}$$

By substituting Eq.(15) into Eq.(17), it generates an equation

$$\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega - [\mathbf{C}'\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{C}\boldsymbol{\theta}] = \boldsymbol{\tau}$$

Then by adding the both sides with vector $\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega$ generates a compiled equation

$$[\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1}\mathbf{C}\boldsymbol{\theta}] = (\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega - \boldsymbol{\tau})$$

$$\boldsymbol{\theta} = [\mathbf{C}'\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{C}]^{-1}(\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega - \boldsymbol{\tau}) \tag{18}$$

And by substituting the Eq. (18) into Eq. (15), is yielded:

$$\hat{\boldsymbol{\gamma}}_\Psi = \hat{\boldsymbol{\gamma}}_\Omega - [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1}\mathbf{C} \times [\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1} \times (\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega - \boldsymbol{\tau}) \tag{19}$$

Based on Eq.(19) as previously described, the Lemma 1 is proved. ■

## 4. STATISTICAL TEST OF SPLINE TRUNCATED IN NON LINEAR SEM

Statistical test for hypothesis testing parameter in the Eq.(6) by using likelihood ratio test is given into Theorem 1 the following :

**Theorem 1**

The spline truncated model in nonlinear SEM is given such as the Eq.(1) If the hypothesis in the Eq.(6) is tested, then the statistical test for this hypothesis is:

$$\mathbf{W} = \frac{\mathbf{Q}_1/d_1}{\mathbf{Q}_2/d_2} ,$$

where

$$d_1 = m_1 + m_2 + k_1 + k_2 + 2$$

$$d_2 = n - (m_1 + m_2 + k_1 + k_2 + 2)$$

$$\mathbf{Q}_1 = (\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega - \boldsymbol{\tau})'[\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1} \times (\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega - \boldsymbol{\tau})$$

$$\mathbf{Q}_2 = [\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_\Omega]'[\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_\Omega]$$

**Proof**

The Theorem 1 can be tested by employing a likelihood ratio test (LRT). Core method of LRT is purposed at searching for ratio results from the likelihood function which is the maximum value under parameter space of $\Psi$ against the likelihood function which is the maximum value under parameter space $\Omega$. By using parameter sets of space $\Omega$ and space $\Psi$ in the Eq.(7) and (8) respectively, can be written likelihood function for model (1) under $\Omega$ as follows:

$$L(\boldsymbol{\gamma}_\Omega, \sigma^2_{\omega_\eta\Omega}) = (2\pi\sigma^2_{\omega_\eta\Omega})^{-n/2} \exp\{-\frac{1}{2\sigma^2_{\omega_\eta\Omega}}[\boldsymbol{\omega}'_\eta\boldsymbol{\omega}_\eta + -2\boldsymbol{\gamma}'_\Omega\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\boldsymbol{\omega}_\eta + +\boldsymbol{\gamma}'_\Omega\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\boldsymbol{\gamma}_\Omega]\} \tag{20}$$

If the logarithm of the Eq.(20) is derived partially with respect to $\sigma^2_{\omega_\eta\Omega}$ and make the result equals to zero then it is obtained:

$$\hat{\sigma}^2_{\omega_\eta\Omega} = \frac{(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_\Omega)'(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_\Omega)}{n} \tag{21}$$

Authors, Ruliana et al (2015), showed that the estimator for $\boldsymbol{\gamma}$ under the space $\Omega$ was

$$\hat{\boldsymbol{\gamma}}_\Omega = [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})]^{-1} \times \mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\boldsymbol{\omega}_\eta \tag{22}$$

It is discovered that by using Eq.(22) and Eq.(21) the maximum value of likelihood function under parameter space $\Omega$ is yielded:

$$L(\hat{\Omega}) = L(\hat{\boldsymbol{\gamma}}_\Omega, \hat{\sigma}^2_{\omega_\eta\Omega}) = (2\pi\hat{\sigma}^2_{\omega_\eta\Omega})^{-n/2} e^{-n/2} \tag{23}$$

Subsequently, the likelihood function model (1) under the space $\Psi$ is:

$$L(\boldsymbol{\gamma}_\Psi, \sigma^2_{\omega_\eta\Psi}) = (2\pi\sigma^2_{\omega_\eta\Psi})^{-n/2} \exp\{-\frac{1}{2\sigma^2_{\omega_\eta\Psi}}(\boldsymbol{\omega}'_\eta\boldsymbol{\omega}_\eta + -2\boldsymbol{\gamma}'_\Psi\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\boldsymbol{\omega}_\eta + +\boldsymbol{\gamma}'_\Psi\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\boldsymbol{\gamma}_\Psi)\}$$

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(24)

By partially deriving the logarithm of Eq. (24), with respect to $\sigma^2_{\omega_\eta \Psi}$ and make the result equals to zero, it is resulted:

$$\hat{\sigma}^2_{\omega_\eta \Psi} = \frac{(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\gamma_\Psi)'(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\gamma_\Psi)}{n} \quad (25)$$

and based on Lemma 1, the mathematical form is obtained the following:

$$\hat{\gamma}_\Psi = \hat{\gamma}_\Omega - [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C} \times [\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1} \times (\mathbf{C}'\hat{\gamma}_\Omega - \boldsymbol{\tau})$$

by applying Eq.(19) and Eq.(25), maximum value of likelihood function under parameter space $\Psi$ is yielded

$$L(\hat{\Psi}) = L(\hat{\gamma}_\omega, \hat{\sigma}^2_{\omega_\eta \Psi}) = (2\pi\hat{\sigma}^2_{\omega_\eta \Psi})^{-n/2}\, \mathrm{e}^{-n/2} \quad (26)$$

Subsequently, by dividing of Eq.(26) toward Eq.(23) likelihood ratio is resulted as follows:

$$\mathrm{L}_{\text{ratio}} = \frac{L(\hat{\Psi})}{L(\hat{\Omega})} = \frac{(2\pi\sigma^2_{\omega_\eta \Psi})^{-n/2} e^{-n/2}}{(2\pi\sigma^2_{\omega_\eta \Omega})^{-n/2} e^{-n/2}} = \left(\frac{\mathbf{Q}_2}{\mathbf{A}}\right)^{\frac{n}{2}} \quad (27)$$

where

$$\mathbf{A} = (\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Psi)'(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Psi) \quad (28)$$

$$\mathbf{Q}_2 = (\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega)'(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega) \quad (29)$$

By elaborating Eq. (28) the form $\mathbf{A}$ can be written as follows:

$$\begin{aligned}\mathbf{A} &= (\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Psi)' \times (\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Psi) \\ &= [[\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega]'(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega] + \\ &\quad + [(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega]'\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})(\hat{\gamma}_\Omega - \hat{\gamma}_\Psi)] + \\ &\quad + (\hat{\gamma}_\Omega - \hat{\gamma}_\Psi)'\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega + \\ &\quad + (\hat{\gamma}_\Omega - \hat{\gamma}_\Psi)'\mathbf{T}'\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})(\hat{\gamma}_\Omega - \hat{\gamma}_\Psi)\end{aligned} \quad (30)$$

Because of

$$[(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega]'\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}) = \mathbf{0}$$

$$\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})[\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega] = \mathbf{0}$$

and

$$[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]^{-1}\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}) = \mathbf{I}$$

then Eq. (30) can be formulated to be

$$\mathbf{A} = (\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega)'(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega) + + (\hat{\gamma}_\Omega - \hat{\gamma}_\Psi)'\mathbf{T}'\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})(\hat{\gamma}_\Omega - \hat{\gamma}_\Psi) \quad (31)$$

Eq. (31) including previous Eq.(19) while the Eq.(19) can be decomposed into

$$\hat{\gamma}_\Omega - \hat{\gamma}_\Psi = [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C} \times [\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1} \times (\mathbf{C}'\hat{\gamma}_\Omega - \boldsymbol{\tau}) \quad (32)$$

So, by substituting the Eq.(32) into Eq.(31) expression of $\mathbf{A}$ becomes

$$\begin{aligned}\mathbf{A} &= (\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega)' \times (\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\gamma}_\Omega)(\mathbf{C}'\hat{\gamma}_\Omega - \boldsymbol{\tau})' \\ &\quad \times [\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1} \times (\mathbf{C}'\hat{\gamma}_\Omega - \boldsymbol{\tau}) \\ &= \mathbf{Q}_2 + \mathbf{Q}_1\end{aligned} \quad (33)$$

where

$$\mathbf{Q}_1 = (\mathbf{C}'\hat{\gamma}_\Omega - \boldsymbol{\tau})' \times [\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1} \times (\mathbf{C}'\hat{\gamma}_\Omega - \boldsymbol{\tau}) \quad (34)$$

Then by substituting the Eq.(33) into Eq.(27) and elaborating the likelihood ratio, the Eq.(27) can be written as follows:

$$\mathrm{L}_{\text{ratio}} = \left(1 + \frac{\mathbf{Q}_1}{\mathbf{Q}_2}\right)^{-n/2} \quad (35)$$

Based on the Eq.(35), and conducted a slightly elaboration , it is obtained that the statistical test for hypothesis $\mathrm{H}_0 : \mathbf{C}'\boldsymbol{\gamma} = \boldsymbol{\tau}$ versus $\mathrm{H}_0 : \mathbf{C}'\boldsymbol{\gamma} \neq \boldsymbol{\tau}$ from model of (1) is expressed in form of

$$\mathbf{W} = \frac{\mathbf{Q}_1 / d_1}{\mathbf{Q}_2 / d_2} \quad \blacksquare$$

## 5. DISTRIBUTION OF STATISTICAL TEST AND CRITICAL AREA OF HYPOTHESIS PARAMETER FOR SPLINE TRUNCATED MODEL IN NONLINEAR SEM

The distribution from statistical test resulted by Theorem 1 is derived and is written in Theorem 2 as follows:

**Theorem 2**

If $\mathbf{Q}_1$ and $\mathbf{Q}_2$, respectively, which are given in the Eq.(34) and Eq.(29), then

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$$\mathbf{W} = \frac{\mathbf{Q}_1 / d_1}{\mathbf{Q}_2 / d_2} \sim F(d_1, d_2)$$

**Proof**

To prove the Theorem 2, should be shows $\mathbf{Q}_1/\sigma^2 \sim \chi^2(d_1)$, $\mathbf{Q}_2/\sigma^2 \sim \chi^2(d_2)$, also $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are independent, with the description is stated below as follows:

From Eq.(34) with

$$(\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega - \boldsymbol{\tau}) \sim N(\mathbf{C}'\boldsymbol{\gamma}_\Omega - \boldsymbol{\tau}\ ,\ \mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2) \times \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]\sigma^2)$$

and $(\mathbf{AV})'(\mathbf{AV}) = (\mathbf{AV})$ is an idempotent matrix where

$$\mathbf{A} = \frac{[\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1}}{\sigma^2}$$

$$\mathbf{V} = \sigma^2[\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]$$

then

$$\frac{\mathbf{Q}_1}{\sigma^2} \sim \chi^2(\text{rank}(\mathbf{A}), \mathbf{D}) \qquad (36)$$

with

$$\mathbf{D} = (\mathbf{C}'\boldsymbol{\gamma}_\Omega - \boldsymbol{\tau})'[\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2) \times \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1} \frac{(\mathbf{C}'\boldsymbol{\gamma}_\Omega - \boldsymbol{\tau})}{2\sigma^2}$$

Since $\mathbf{A}$ is the full rank matrix with the order $d_1$ and it is under the hypothesis $\mathrm{H}_0$, also matrix $\mathbf{D} = \mathbf{0}$, then the form of Eq.(36) becomes

$$\frac{\mathbf{Q}_1}{\sigma^2} \sim \chi^2(d_1) \qquad (37)$$

Moreover, to show that $\mathbf{Q}_2/\sigma^2 \sim \chi^2(d_2)$ it was conducted a modification of Eq.(29) into the form of quadratic form with the following description:

$$\begin{aligned}\mathbf{Q}_2 &= [\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_\Omega]' \times [(\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_\Omega)] \\ &= \boldsymbol{\omega}'_\eta[\mathbf{J}\,\mathbf{J}]\boldsymbol{\omega}_\eta \end{aligned} \qquad (38)$$

where

$$\mathbf{J} = [\mathbf{I} - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]^{-1} \times \mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]$$

For $\mathbf{J}'\mathbf{J} = \mathbf{J}$ is symmetric and idempotent matrix, then the form of Eq.(38) can be rewritten as follows:

$$\mathbf{Q}_2 = \boldsymbol{\omega}'_\eta[\mathbf{I} - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]^{-1} \times \mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]\boldsymbol{\omega}_\eta \qquad (39)$$

with $\boldsymbol{\omega}_\eta \sim N(\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\boldsymbol{\gamma}, \sigma^2\mathbf{I})$

and $(\mathbf{A}_*\mathbf{V}_*)'(\mathbf{A}_*\mathbf{V}_*) = (\mathbf{A}_*\mathbf{V}_*)$ is idempotent matrix where

$$\mathbf{A}_* = \frac{[\mathbf{I} - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]^{-1}\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})}{\sigma^2}$$

$$\mathbf{V}_* = \sigma^2\mathbf{I}$$

Then it is obtained the following

$$\frac{\mathbf{Q}_2}{\sigma^2} \sim \chi^2(\text{rank}(\mathbf{A}_*), \mathbf{E}) \qquad (40)$$

with

$$\mathbf{E} = (\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\boldsymbol{\gamma})'[\mathbf{I} - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}) \times [\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]^{-1} \times \mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}) \frac{(\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\boldsymbol{\gamma})}{2\sigma^2}$$

since

$$\begin{aligned}\text{rank}(\mathbf{A}) &= \text{rank}(\mathbf{A}_*\mathbf{V}_*) \\ &= \text{tr}(\mathbf{AV}) \\ &= n - (m_1 + m_2 + k_1 + k_2 + 2)\end{aligned}$$

and by elaborating the matrix $\mathbf{E}$, it is yielded that $\mathbf{E} = \mathbf{0}$, then Eq.(40) becomes into

$$\frac{\mathbf{Q}_2}{\sigma^2} \sim \chi^2(d_2) \qquad (41)$$

Furthermore, to show whether $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are independent or not, it should be shown $\mathbf{GVF} = \mathbf{0}$, where $\mathbf{V} = \sigma^2\mathbf{I}$ is the variance of vector quadratic form $\mathbf{Q}_1$ and $\mathbf{Q}_2$; $\mathbf{G}$ and $\mathbf{F}$, respectively, are matrix quadratic form for $\mathbf{Q}_1$ and $\mathbf{Q}_2$. By modifying $\mathbf{Q}_1$ in the Eq.(34) and $\mathbf{Q}_2$ in the Eq.(29) into vector quadratic form as random variable distributed normally, $\mathbf{Q}_1$ and $\mathbf{Q}_2$ respectively are obtained in form of:

$$\begin{aligned}\mathbf{Q}_1 = &[\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{C}[\mathbf{C}'\mathbf{C}]^{-1}\boldsymbol{\tau}]' \\ &\times \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]^{-1}\mathbf{C} \\ &\times [\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}_1, \boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1} \\ &\times \mathbf{C}'[[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]^{-1}\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa}) \\ &\times [\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{C}[\mathbf{C}'\mathbf{C}]^{-1}\boldsymbol{\tau}]\end{aligned} \qquad (42)$$

and

$$\begin{aligned}\mathbf{Q}_2 = &[\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{C}[\mathbf{C}'\mathbf{C}]^{-1}\boldsymbol{\tau}]' \\ &\times [\mathbf{I} - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})]^{-1} \\ &\times \mathbf{T}'(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})][\boldsymbol{\omega}_\eta - \mathbf{T}(\boldsymbol{\omega}_{\xi_1}, \boldsymbol{\omega}_{\xi_2}, \boldsymbol{\kappa})\mathbf{C}[\mathbf{C}'\mathbf{C}]^{-1}\boldsymbol{\tau}]\end{aligned} \qquad (43)$$

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From the Eq.(42) and Eq.(43) are found that $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are quadratic form in vector $\mathbf{x}=[\boldsymbol{\omega}_{\boldsymbol{\eta}}-\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\mathbf{C}[\mathbf{C}'\mathbf{C}]^{-1}\boldsymbol{\tau}]$ distributed normally, where matrix quadratic form for $\mathbf{Q}_1$ is:

$$\mathbf{F}=\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})]^{-1}\mathbf{C}$$
$$\times[\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1}$$
$$\times\mathbf{C}'[[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})]^{-1}\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}) \quad (44)$$

and matrix quadratic form for $\mathbf{Q}_2$ is:

$$\mathbf{G}=[\mathbf{I}-\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})]^{-1}$$
$$\times\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}) \quad (45)$$

By using the results of the multiplication of the matrices in Eq.(45) and Eq.(44) also variance from $[\boldsymbol{\omega}_{\boldsymbol{\eta}}-\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\mathbf{C}[\mathbf{C}'\mathbf{C}]^{-1}\boldsymbol{\tau}]$, slightly Elaboration obtained:

$$\begin{aligned}\mathbf{GVF}&=\mathbf{G}(\sigma^2\mathbf{I})\mathbf{F}\\&=\sigma^2\mathbf{GF}\\&=\mathbf{0}\end{aligned} \quad (46)$$

It results that $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are independent. Based on the Eq.(37), Eq.(41) and Eq.(46) then the form of distribution from statistical test of spline truncated in nonlinear SEM in the equation of (1) is:

$$\mathbf{W}=\frac{\mathbf{Q}_1/d_1}{\mathbf{Q}_2/d_2}\sim F(d_1,d_2) \quad \blacksquare$$

The critical area of hypothesis of statistical test for spline truncated model in nonlinear SEM is described in Theorem 3 as follows:

**Theorem 3**

If given a hypothesis such as in Eq.(6), and statistical test as stated in Theorem 1 as well as the distribution of the statistical test, which is given by Theorem 2, then the critical areas of the test are obtained from completing the equation:

$$\mathrm{P}\left(\mathbf{W}>k^*\middle|\mathbf{C}\boldsymbol{\gamma}=\boldsymbol{\tau}\right)=\alpha$$

where $\alpha$ and $k^*$ are the significant level and a particular constant, respectively.

**Proof**

The critical area to test hypothesis $\mathrm{H}_0:\mathbf{C}'\boldsymbol{\gamma}=\boldsymbol{\tau}$ versus $\mathrm{H}_1:\mathbf{C}'\boldsymbol{\gamma}\neq\boldsymbol{\tau}$, is the sets of all points that satisfy the condition

$$\mathrm{L}_{\text{ratio}}=\frac{L(\hat{\Psi})}{L(\hat{\Omega})}<k \quad (47)$$

or

$$(1+\mathbf{Q})^{-\frac{n}{2}}<k \quad (48)$$

By decomposing the expression of inequality stated in Eq.(48) is obtained

$\mathbf{W}>k*$ (49)

where

$$k*=\frac{d_2}{d_1}\left(k^{-\frac{2}{n}}-1\right)$$

Then if given a significant level of $\alpha$ then $k^*$ is resulted from the equation of shown below: $\alpha=\mathrm{P}\left(\mathbf{W}>k^*\middle|\mathbf{C}\boldsymbol{\gamma}=\boldsymbol{\tau}\right)$, where $\mathbf{W}\sim F(d_1,d_2)$ $\blacksquare$

## 6. CONCLUSION

Hypothesis testing of parameter of spline truncated model in nonlinear SEM for $\mathrm{H}_0:\mathbf{C}'\boldsymbol{\gamma}=\boldsymbol{\tau}$ versus $\mathrm{H}_1:\mathbf{C}'\boldsymbol{\gamma}\neq\boldsymbol{\tau}$ using Likelihood Ratio Test (LRT), can be concluded as follows:

1. Estimation of parameter for spline truncated model in nonlinear SEM under parameter space $\Omega$ is:
$$\hat{\boldsymbol{\gamma}}_\Omega=[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})]^{-1}\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\boldsymbol{\omega}_{\boldsymbol{\eta}}$$
and under parameter space $\Psi$ (hypothesis $\mathrm{H}_0$) is:
$$\hat{\boldsymbol{\gamma}}_\Psi=\hat{\boldsymbol{\gamma}}_\Omega-[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1}$$
$$\times\mathbf{C}[\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1}$$
$$\times(\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega-\boldsymbol{\tau})$$

2. Statistical test for hypothesis test of parameter for spline truncated model in nonlinear SEM is:
$$\mathbf{W}=\frac{\mathbf{Q}_1/d_1}{\mathbf{Q}_2/d_2}$$
where
$$d_1=m_1+m_2+k_1+k_2+2$$
$$d_2=n-(m_1+m_2+k_1+k_2+2)$$
$$\mathbf{Q}_1=(\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega-\boldsymbol{\tau})'$$
$$\times[\mathbf{C}'[\mathbf{T}'(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa}_1,\boldsymbol{\kappa}_2)]^{-1}\mathbf{C}]^{-1}$$
$$\times(\mathbf{C}'\hat{\boldsymbol{\gamma}}_\Omega-\boldsymbol{\tau})$$
$$\mathbf{Q}_2=(\boldsymbol{\omega}_{\boldsymbol{\eta}}-\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_\Omega)'$$
$$\times(\boldsymbol{\omega}_{\boldsymbol{\eta}}-\mathbf{T}(\boldsymbol{\omega}_{\xi_1},\boldsymbol{\omega}_{\xi_2},\boldsymbol{\kappa})\hat{\boldsymbol{\gamma}}_\Omega)$$

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3. Distribution of statistical test for hypothesis testing of parameter for spline truncated model in nonlinear SEM is in form of:
   $\mathbf{W} \sim F(d_1, d_2)$
4. Critical area for hypothesis test of parameter for spline truncated model in nonlinear SEM is obtained from the solving of equation
   $$\mathrm{P}\left(\mathbf{W} > k^* \middle| \mathbf{C}\boldsymbol{\gamma} = \boldsymbol{\tau}\right) = \alpha, \text{where } \mathbf{W} \sim F(d_1, d_2)$$

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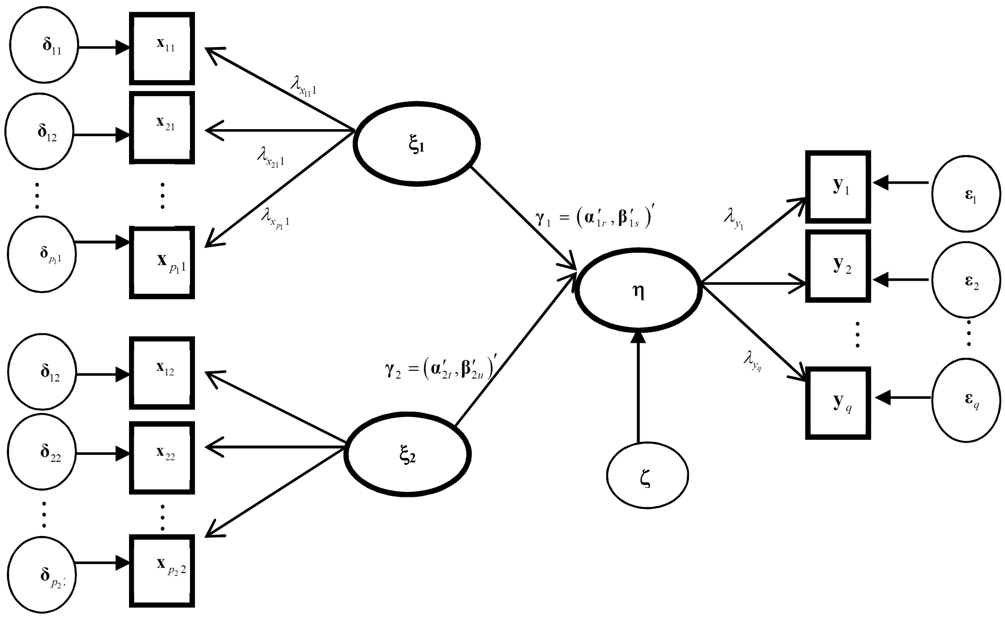


*Figure 1: Path Diagram of Spline Truncated Model in Nonlinear SEM for Two Exogenous Latent Variables and One Endogenous Latent Variable*