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VARIATIONAL ITERATION METHOD (VIM) FOR SOLVING PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS

(1,2)AMINA KASSIM HUSSAIN,(3)FADHEL SUBHI FADHEL,(1)ZAINOR RIDZUAN YAHYA, (1)NURSALASAWATI RUSLI

(1)Universiti Malaysia Perlis, Institute of Engineering Mathematics, 02600 Arau,Perlis, Malaysia

(2)University of Al-Mustansiriyah, Department of Material Engineering, 10052 Baghdad, Iraq

(3)Al-Nahrain University, College of Science, Department of Mathematics and Computer Applications,10070 Al-Jadriya, Baghdad, Iraq

E-mail:[1,2]amina1975_kas@yahoo.com,[3]dr_fadhel67@yahoo.com,[1]zainoryahya@unimap.edu.my, [1]nursalasawati@unimap.edu.my,

ABSTRACT

In this paper, two objectives will be achieved, the first one is to state and prove the existence of a unique solution of nonlinear partial integro-differential equations by using Banach fixed point theorem. The second objective is to apply He's variational iteration method for solving nonlinear partial integro-differential equations. This method is a very powerful method for solving a large amount of problems. It provides a sequence of iterated solutions which is converge to the exact solution of the problem. Also, in this work the derivation of the iteration formula using He's method have been presented and then prove the converge of the obtained sequence of iterated approximate solutions to the exact solution of the partial integro-differential equation. Finally, illustrative examples were presented to show the efficient of the new method and the proposed technique was programmed using Mathcade 15.0.

Keywords: *Variational Iteration Method, Nonlinear Partial integro-Differential Equation, Banach Fixed Point Theorem, Contraction Mapping Principle.*

1. INTRODUCTION

Mathematical modeling is the art of translating real life problems into tractable mathematical formulations, for example ordinary and partial differential equations, integral and integro-differential equations and others[1,2]. In recent years, there has been a growing interest in the integro-differential equations, in particular nonlinear partial integro-differential equations. Since there are many mathematical formulations of physical phenomena, such as nonlinear functional analysis and their applications in the theory of engineering, physics, mechanics, chemical kinetics, astronomy, economics, biology, potential theory and electro statistics contain partial integro-differential equations[3].

The problem of existence of a unique solution of a differential equation have been considered by many authors, such as, Momani [4], Momani and Hadid[5], Rabha and S. Momani[6], Hu et al. [7]; Shayma et al. [8];Karthikeyan and Trujillo [9]; ATari [10]. Partial integro-differential equations usually difficult to be solved analytically, therefore, numerical and approximate methods are required to solve such equations, and there are many such methods have been proposed previously, such as the method of successive approximations andHe's iteration method[11,12].

The variation iteration method (VIM) has established to be one of the useful techniques in solving many types of linear and nonlinear differential equations for finding both analytical and approximate solutions[13].This technique was developed by Chinese mathematician He. This method successfully applied to many situations, for example, He's proposed the VIM to solve Delay differential equations [14], linear and nonlinear differential equations[15],seepage flow equation with fractional derivatives in porous media [16], autonomous ordinary differential systems[17], following by Mommani and Abuasad used VIM to solve Helmholtz equation[18],Wazwaz used VIM for solving linear and nonlinear system of partial differential equations[19],Batiha et al. used VIM to

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ageneral Riccati equation [20],Hamida applied VIM to solve wave equations [21], Abbasbandy and Shivanian applied Variational Iteration Method for solving a system of nonlinear Volterra integro-Differential equations [22], Kurulay and Secer applied Variational iteration Method to solve nonlinear fractional order Integro-differential equations [23]. In this paper, our aim is to state and to prove the existence of a unique solution of partial integro-differential equation and then use the variation iteration method to solve such partial integro-differential equations, as well as, to prove the convergence of the iterated sequence of approximate solutions to the exact solution of the problem when it is assumed to be exist by the satisfaction of the conditions of the existence of a unique solution of such equations.

The form of the considered partial integro-differential equation is given by:

$$\frac{\partial u(x,t)}{\partial t}=g(x,t)+\int_a^x k(y,t,u(y,t))dy \quad , \qquad (1)$$

$x\in[a,b], t\in[0,T]$

By using the following initial condition:

$$u(x,0)=u_0(x) \qquad (2)$$

Where$k$ is the kernel function, g is given function and$u$ is the unknown real function to be evaluated.

2. BASIC CONCEPTS AND DEFINITIONS

In order to proceed, some fundamental concepts related to this work are given in this section.

Definitions 1,[24]:

Let $T:X\to X$ be a mapping on a normed space$(X,\|.\|)$.A point $x\in X$ such as $Tx=x$ is called a fixed point of T.

Definition 2,[24]:

A mapping T on a normed space $(X,\|.\|)$ is called contractive if there is a non-negative real number c, such that $0\le c<1$,and for each $x_1,x_2\in X$, implies that:

$$\|Tx_1-Tx_1\|\le c\|x_1-x_2\|$$

The next theorem is well known in analysis, which is of great importance for the existence of a unique solution of equations (1).

Theorem 1, (Banach Fixed Point Theorem),[25]

Let$(X,\|.\|)$ be a complete normed space and let $T : X\longrightarrow X$ be a contraction mapping, then T has exactly one fixed point.

Definitions 3,[26,27]:

Let $(X,\|.\|)$be a normed space, a function f$(x,t;y_1,y_2,\ldots,y_n)$ defined on the set:

$\Omega=\{(x,t;u_1,u_2,\ldots,u_m): a\le x, t\le b, -\infty<u_i<\infty$, for each $i=1,2,\ldots,m\}$

is said to satisfy Lipchitz condition on $\Omega$with respect to the variables $u_1,u_2,\ldots,u_m$ if a constant $L>0$ exists with the property that:

$$\|f(x,t;u_1,u_2,\ldots,u_m)-f(x,t;z_1,z_2,\ldots,z_m)\|\le L\sum_{i=1}^{m}\|y_i-z_i\|$$

for all $(x,t;u_1,u_2,\ldots,u_m)$ and $(x,t;z_1,z_2,\ldots,z_m)$ in Ω.

Remark 1:

The space $C_t^n([a,b]\times[0,\infty))$ will be considered in this work as the Banach space for all continuous real valued functions u defined on $[a,b]\times[0,T]$ with continuous n-th order partial derivative with respect to t.

3. THE EXISTENCE OF A UNIQUE SOLUTION FOR ONE-DIMENSIONAL PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS

In this section, the statement and the proof of the existence and uniqueness solution for equation (1) by using Banach fixed point theorem and contraction mapping principle.

Theorem 2:

Consider the partial integro-differential equation (1) with the initial condition equation (2) over the region:

$$Q=\{(x,t): a\le x\le b, 0\le t\le T\}$$

and suppose that k satisfies Lipschtiz condition with respect to u and costant M and

$MT(b-a)<1$, then equation(1) has a unique solution.

Proof:

By integrating both sides of equation (1)

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with respect to t, we get:

$$u(x,t)=u_0(x)+\int_0^t g(x,\xi)d\xi+\int_0^t\int_a^x k(y,\xi,u(y,\xi))dyd\xi \tag{3}$$

Since, it is known that the set of all continuous function defined on the region Q is a complete normed space with

$$\left\|u_1(x,t)-u_2(x,t)\right\|=\sup_{\substack{a\le x\le b\\0\le t\le T}}\left|u_1(x,t)-u_2(x,t)\right| \tag{4}$$

Rewrite equation (3) in operator forms as Nu=u

$$N.=u_0(\mathrm{x})+\int_0^t g(x,\xi)d\xi+\int_0^t\int_a^x k(y,\xi,.))dyd\xi \tag{5}$$

Next, to show that N is a contractive mapping and for this purpose, take $u_1,u_2\in C_t{}^n\left(([\mathrm{a},\mathrm{b}]\times[0,\infty)\right)$

$$\left\|Nu_1(x,t)-Nu_2(x,t)\right\|=\sup_{\substack{a\le x\le b\\0\le t\le T}}|u_0(x)+\int_0^t g(x,\xi)d\xi+\int_0^t\int_a^x k(y,\xi,u_1(y,\xi))\,dy\,d\xi-u_0(x)-\int_0^t g(x,\xi)d\xi+\int_0^t\int_a^x k(y,\xi,u_2(y,\xi))\,dy\,d\xi| \tag{6}$$

$$=\sup_{\substack{a\le x\le b\\0\le t\le T}}\left|\int_0^t\int_a^x k(y,\xi,u_1(y,\xi))dyd\xi-\int_0^t\int_a^x k(y,\xi,u_2(y,\xi))dyd\xi\right| \tag{6}$$

$$\le\sup_{\substack{a\le x\le b\\0\le t\le T}}\left|\int_0^t\int_a^x k(y,\xi,u_1(y,\xi))dyd\xi-\int_0^t\int_a^x k(y,\xi,u_2(y,\xi))\right|dyd\xi \tag{7}$$

$$\le\sup_{\substack{a\le x\le b\\0\le t\le T}}\int_0^t\int_a^x |M|\left|u_1(y,\xi)-u_2(y,\xi)\right|dyd\xi \tag{8}$$

$$\le M\sup_{\substack{a\le x\le b\\0\le t\le T}}|u1(x,y)-u2(x,y)|\int_0^t\int_a^x dyd\xi \tag{10}$$

$$=M\sup_{\substack{a\le x\le b\\0\le t\le T}}\left|u_1(x,t)-u_2(x,t)\right|t(x-a) \tag{9}$$

$$\le MT(b-a)\sup_{\substack{a\le x\le b\\0\le t\le T}}\left|u_1(x,t)-u_2(x,t)\right| \tag{10}$$

and since $MT(b-a)<1$, then N is a contraction mapping and therefore N has a unique fixed point, which means that equation (1) has a unique solution.

4. FORMULATION OF THE VARIATIONAL ITERATION METHOD FOR NONLINEAR PARTIAL INTEGRO-DIFFERENTIAL EQUATIONS

In this section before derivation the variation iteration formula for partial integro-differential equation will be made, the main aspects of the VIM will be given.

The Main Aspects of the VIM

As mentioned above, the VIM which was suggested by He in 1998 intensively studied by several scientists and engineers which is favorably applied to many kinds of linear and nonlinear problems. The method has been shown to solve a large class of linear and nonlinear problems effectively, easily and accurately. Generally, one or two iterations lead to high accurate solutions. This method which is a modification of the well-known general Lagrange multiplier method into an iteration method called correction functional. Generally speaking, the solution procedure of the VIM is very operative, straight forward and convenient [28].

To illustrate the basic idea of the VIM, consider the following general non-linear equation given in operator form:

$$\mathrm{L(u(x))+N(u(x))=g(x)},\ \mathrm{x}\in[\mathrm{a},\mathrm{b}] \tag{11}$$

where$L$ is a linear operator, N is a nonlinear operator and g is any given function which is called the non-homogeneous.

Now, rewrite equation (13) in as follows:

$$\mathrm{L(u(x))+N(u(x))-g(x)=0} \tag{12}$$

and let $\mathrm{u_n}$ be the $\mathrm{n^{th}}$ approximate solution of eq. (14), then it follows that:

$$\mathrm{L(u_n(x))+N(u_n(x))-g(x)\neq 0} \tag{13}$$

and therefore the correction functional for eq.(15), is given by:

$$\mathrm{u_{n+1}(x)=u_n(x)+\int_a^x\lambda(s)\{L(u_n(s)+N(\tilde{u}_n(s))-g(s)\}ds} \tag{14}$$

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$, n \geq 0$

Where λ is the general Lagrange multiplier, which can be identified optimally via the variational theory, and $\tilde{u}_n$ is considered as a restricted variation which means $\delta\tilde{u}_n = 0$, [1].

Generally speaking, it is obvious that the main steps of He's variational iteration method require the determination of the Lagrangian multiplier λ at first step that will be identified optimally.After determined the Lagrangian multiplier, the successive approximations u_{n+1}, $n \geq 0$ of the solution u will be readily obtained upon using any selective function u_0. Consequently, the solution u_n converge to the exact solution u

$$u(x) = \lim_{n\to\infty} u_n(x)$$

In the next theorem we will derive the general formula for solving eq.(1) using VIM which is based on the general form eq.(16) after evaluating the Lagrange multiplier related with the partial integro-differential equation (1).

Theorem 3:

Consider thenonlinear partial integro-differential equation (1) with initial equation (2). Then the related variational iteration formula is given by:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left[\frac{\partial u_n(x,\xi)}{\partial \xi} - g(x,\xi) - \int_a^x k(y,\xi,u(y,\xi))\,\mathrm{dy} \right] d\xi \tag{15}$$

For all n=0,1,…

Proof:

The correction functional (16) related to the partial integro-differential equation (1) is given by:

$$u_{n+1}(x,t) = u_n(x,t) + \int_0^t \lambda \left[\left(\frac{\partial u_n}{\partial \xi}(x,\xi) - g(x,\xi) - \int_a^x k(y,\xi,\tilde{u}(y,\xi)) \right) dy \right] d\xi \tag{16}$$

Where λ is the general Lagrange multiplier, which must be evaluated optimally, the subscript n denotes the nth approximation and $\tilde{u}_n(t)$ is considered as restricted variation.

Taking the first variation δ with respect to u_n to the both sides of equation (18) and setting $\delta u_n = 0$, yields to:

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \delta \int_0^t \lambda \left[\frac{\partial u_n}{\partial \xi}(x,\xi) - g(x,\xi) - \int_a^x k(y,\xi,\tilde{u}_n(y,\xi))\,dy \right] d\xi \tag{17}$$

where $\delta\tilde{u}_n = 0$ and consequently equation (19)will be reduced to

$$\delta_{n+1}(x,t) = \delta u_n(x,t) + \delta \int_0^t \lambda(\xi) \frac{\partial u_n}{\partial \xi}(x,\xi)\, d\xi \tag{18}$$

hence, upon using the method of integration by parts of equation (20) will give :

$$\delta u_{n+1}(x,t) = \delta u_n(x,t) + \lambda(\xi)\delta u_n(x,\xi)\big|_{\xi=t} - \int_0^t \delta u_n(x,\xi)\lambda'(\xi)\,d\xi \tag{19}$$

$$= \left(1+\lambda(\xi)\right)\delta u_n(x,\xi)\big|_{\xi=t} - \int_0^t \delta u_n(x,\xi)\lambda'(\xi)\,d\xi \tag{20}$$

as a result, the following necessary condition is obtained for an arbitrary δu_n :

$$\lambda'(\xi) = 0 \tag{21}$$

with initial condition:

$$1 + \lambda(\xi)\big|_{\xi=t} = 0 \tag{22}$$

solving the last ordinary differential equation will yields the general Lagrange multiplier to be defined as follows:

$$\lambda(\xi) = -1 \tag{23}$$

Hence, substituting $\lambda(\xi) = -1$ into the correction functional eq. (18) will results the following iteration formula:

$$u_{n+1}(x,t) = u_n(x,t) - \int_0^t \left[\frac{\partial u_n}{\partial \xi}(x,\xi) - g(x,\xi) - \int_a^x k(y,\xi,\tilde{u}_n(y,\xi))\,dy \right] d\xi$$

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5. ANALYSIS OF CONVERGENCE FOR NONLINEAR PARTIAL INTEGRO-DIFFERENTIAL EQUATION

In the next theorem, the convergence of the sequence of iterated approximate solution (17) of the partial integro-differential equation (1) to the exact solution will be proved.

Theorem (4)

Let $u, u_n \in C_t^n([a,b]\times[0,T])$ be the exact and approximate solutions of equation (1) and (17), respectively.

If $E_n(x,t)=u_n(x,t)-u(x,t)$ and the kernel k satisfies Lipschitz condition with constant M, then the sequence $\{u_n\}$ converge to the exact solution u.

Proof:

The approximate solution using the VIM is given by:

$$u_{n+1}(x,t)=u_n(x,t)-\int_0^t\left[\frac{\partial u_n}{\partial \xi}(x,\xi)-g(x,\xi)-\int_a^x k\left(y,\xi,\tilde{u}_n(y,\xi)\right)dy\right]d\xi \quad (17)$$

andsince u is the exact solution of equation (1), hence it satisfies the VIM formula, i.e.,

$$u(x,t)=u(x,t)-\int_0^t\left[\frac{\partial u}{\partial \xi}(x,\xi)-g(x,\xi)-\int_a^x k\left(y,\xi,u(y,\xi)\right)dy\right]d\xi \quad (24)$$

substracte u from u_{n+1} and recall that $E_n(x,t)=u_n(x,t)-u(x,t)$, implies to :

$$u_{n+1}(x,t)-u(x,t)=u_n(x,t)-u(x,t)-\int_0^t\left[\frac{\partial u_n(x,\xi)}{\partial \xi}-\frac{\partial u(x,\xi)}{\partial \xi}-g(x,\xi)+g(x,\xi)-\int_a^x k\left(y,\xi,u_n(y,\xi)\right)-k\left(y,\xi,u(y,\xi)\right)dy\right]d\xi \quad (27)$$

Hence

$$E_{n+1}(x,t)=E_n(x,t)-\int_0^t\left[\frac{\partial u}{\partial \xi}E_n(x,\xi)-\int_a^x k\left(y,\xi,u_n(y,\xi)\right)-k\left(y,\xi,u(y,\xi)\right)dy\right]d\xi \quad (25)$$

(26)

$$=E_n(x,t)-E_n(x,t)-E_n(x,0)+\int_0^t\int_a^x\left[k\left(y,\xi,u_n(y,\xi)\right)-k\left(y,\xi,u(y,\xi)\right)\right]dy\,d\xi$$

$$=\int_0^t\int_a^x\left[k\left(y,\xi,u_n(y,\xi)\right)-k\left(y,\xi,u(y,\xi)\right)\right]dy\,d\xi\ , \quad (27)$$

where $E_n(x,0)=0$

taking the norm to the both sides eq.(29), give

$$\|E_{n+1}(x,t)\|=\left\|\int_0^t\int_a^x\left[k\left(y,\xi,u_n(y,\xi)\right)-k\left(y,\xi,u(y,\xi)\right)\right]dy\,d\xi\right\| \quad (31)$$

$$\le\int_0^t\int_a^x\|k(y,\xi,u_n(y,\xi))-k(y,\xi,u(y,\xi))\|\,dy\,d\xi \quad (28)$$

$$\le M\int_0^t\int_a^x\|u_n(y,\xi)-u(y,\xi)\|dy\,d\xi \quad (29)$$

Hence

$$\|E_{n+1}(x,t)\|\le M\int_0^t\int_a^x\|E_n(y,\xi)\|dy\,d\xi\ , \text{ for all } n=0,1,2,\ldots \quad (30)$$

Now, if n=0, then :

$$\|E_1\|\le M\int_0^t\int_a^x\|E_0\|dy\,d\xi \quad (31)$$

$$=M\,\|E_0\|\int_0^t\int_a^x dy\,d\xi \quad (32)$$

$$=M\,\|E_0\|t(x-a) \quad (33)$$

While if n=1, then:

$$\|E_2\|\le M\int_0^t\int_a^x\|E_1\|dy\,d\xi \quad (34)$$

$$\le M\int_0^t\int_a^x M\,\|E_0\|\xi(y-a)dy\,d\xi \quad (35)$$

$$=M^2\|E_0\|t^2(x-a)^2 \quad (36)$$

also if n=2, then:

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$$\|E_3\| \le M \int_0^t \int_a^x \|E_2\| dy d\xi \qquad (37)$$

$$\le M \int_0^t \int_a^x M^2 \|E_0\| \xi^2 (y-a)^2 dy\, d\xi \qquad (38)$$

$$= \frac{L_3 \|E_0\|}{2^2} \frac{x^2 t^2}{3^2} \qquad (39)$$

$$= \frac{M^3 \|E_0\|}{2^2} \frac{t^3 (x-a)^3}{3^2} \qquad (40)$$

$$\vdots$$

$$\|E_n\| \le \frac{M^n \|E_0\|}{2^2 3^2 \ldots n^2} t^n (x-a)^n, a \le x \le b, 0 \le t \le T \qquad (41)$$

since $2^2 3^2 \ldots n^2 = (n!)^2$ and taking the supermom value of x and t over [a,b] and [0,T], respectively, to get

$$\|E_n\| \le \frac{M^n \|E_0\|}{(n!)^2} T^n (b-a)^n \qquad (42)$$

and it is clear that if $n \to \infty$ and M is not large in magnitude, then

$$\frac{M^n}{(n!)^2} \to 0$$

Hence $E_n \to 0$,i.e, $u_n \longrightarrow u$ as $n \longrightarrow \infty$ ■

6. NUMERICAL SIMULATION AND IILLUSTRATIVE EXAMPLES

In this section, two illustrative examples will be considered in order to examine the validity and illustrative the convergence of the variation iteration formula given by eq. (18). Two illustrative examples are considered, for linear and nonlinear partial integro-differential equations, in which the accuracy of the results aregiven by scheduling the absolute error between the exact solution (given here for comparison purpose) and each iterated solution.

Example (1):

Consider the following linear partial integro-differential equation

$$\frac{\partial u(x,t)}{\partial t} = -x^2 t + \int_0^x (yt+u) dy \text{ , where } (x,t) \in [0,1] \times [0,1]$$

with the initial condition

$$u(x,0) = u_0(x) = 1$$

The exact solution is given by

$$u(x,t) = 1 + xt$$

Starting with the initial approximation $u_0(x,t) = 1$, the first four approximate solutions using the VIM (18) are found to be:

$$u_1(x,t) \to 1 - \frac{tx(tx-4)}{4}$$

$$u_2(x,t) = 1 - \frac{tx(tx-4)}{4} - \frac{t^2 x^2 (tx-9)}{36}$$

$$u_3(x,t) \to 1 - \frac{t^3 x^3 (tx-16)}{576} - \frac{tx(tx-4)}{4} - \frac{t^2 x^2 (tx-9)}{36}$$

$$u_4(x,t) \to 1 - \frac{t^3 x^3 (tx-16)}{576} - \frac{t^4 x^4 (tx-25)}{14400} - \frac{tx(tx-4)}{4} - \frac{t^2 x^2 (tx-9)}{36}$$

Comparison between the exact and approximate solutions u_1, u_2, u_3 and u_4 using the absolute error are given in table (1).

TABLE 1: The Absolute Error Of Example (1)

(x,t)	A	B	C	D
(0,0)	0	0	0	0
(0.25,0.25)	9.766e − 4	6.782e − 6	2.649e − 8	6.623e − 11
(0.5,0.5)	0.016	4.34 e − 4	6.782e − 6	6.782e − 8
(0.75,0.75)	0.079	4.944e − 3	1.738e − 4	3.911e − 6
(1,1)	0.25	0.028	1.736 e − 3	6.944 e − 5

Where A: $|u(x,t) - u_1(x,t)|$

B: $|u(x,t) - u_2(x,t)|$

C: $|u(x,t) - u_3(x,t)|$

D: $|u(x,t) - u_4(x,t)|$

Example (2):

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(x,t)	A	B	C	D
(0,0)	0	0	0	0
(0.25,0.25)	4.883×10^{-5}	4.864×10^{-5}	4.864×10^{-5}	4.864×10^{-5}
(0.5,0.5)	1.563×10^{-3}	1.468×10^{-3}	1.468×10^{-3}	1.468×10^{-3}
(0.75,0.75)	0.012	8.287×10^{-3}	8.269×10^{-3}	8.269×10^{-3}
(1,1)	0.05	3.125×10^{-3}	2.154×10^{-3}	2.143×10^{-3}

Consider the following nonlinear partial integro-differential equation:

$$\frac{\partial u(x,t)}{\partial t} = 2tx^2 - \frac{t^4}{4} + \int_0^x yt^2 u(\mathrm{y},\mathrm{t})dy \ ,$$

Where $(x,t) \in [0,1]\times[0,1]$

with the initial condition:

$u(x,0) = u_0(x) = 1$

The exact solution is given by:

$u(x,t) = t^2 x^2$

Starting with the initial approximate solution $u_0(x,t) = 0$, the first four approximate solutions using the VIM eq. (18) are found to be:

$$u_1(x,t) \to -\frac{t^2(t^3-20x^2)}{20}$$

$$u_2(x,t) \to -\frac{t^2(t^3-20x^2)}{20} - \frac{t^5x^2(t^3-16x^2)}{320}$$

$$u_3(x,t) \to -\frac{t^2(t^3-20x^2)}{20} - \frac{t^5x^2(t^3-16x^2)}{320} - \frac{t^8x^4(3t^3-44x^2)}{42240}$$

$$u_4(x,t) \to -\frac{t^2(t^3-20x^2)}{20} - \frac{t^5x^2(t^3-16x^2)}{320} - \frac{t^{11}x^6(t^3-14x^2)}{1182720} - \frac{t^8x^4(3t^3-44x^2)}{42240}$$

Comparison between the exact and approximate solutions u_1, u_2, u_3 and u_4 using the absolute error are given in table (2).

TABLE 2: The absolute error of example (2)

Where A: $|u(x,t) - u_1(x,t)|$

B: $|u(x,t) - u_2(x,t)|$

C: $|u(x,t) - u_3(x,t)|$

D: $|u(x,t) - u_4(x,t)|$

7. CONCLUSIONS

The variation iteration method (VIM) has been shown to solve a large class of non-linear problems effectively,with the approximations which are convergent are rapidly to the exact solutions.In this work, the VIM has been successfully employed to obtain the approximate solution to analytical solution of linear and non-

linear partial integro-differential equations. For this purpose, we have shown that the VIM has rapid convergence by examples.

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