ADAPTIVE NONLINEAR CONTROL OF DOUBLY-FED INDUCTION MACHINE IN WIND POWER GENERATION

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ABSTRACT

This paper presents an adaptive nonlinear controller for Doubly-Fed Induction Generator (DFIG) in wind power generation. The control objective is to extract the maximum power (Maximum Power Point Tracking) and have a specified reactive power at the generator terminal despite the variable wind speed and the unknown mechanical parameters and mechanical torque. To achieve such a decoupled control, the nonlinear Backstepping approach was combined with the field orientation principle. The induction machine is controlled in a synchronously reference frame oriented along the stator voltage vector position. The mathematical development of the nonlinear Backstepping controller design is examined in detail. The overall stability of the system is shown using Lyapunov approach. Application example on stabilizing a DFIG-based Wind Energy Conversion System (WECS) subject to large mechanical parameters uncertainties has been given to illustrate the design method and confirm the robustness and validity of the proposed control.

Keywords: Wind Energy Conversion System, Doubly-Fed Induction Generator, Unknown Mechanical Parameters and Torque, Adaptive Nonlinear control, Lyapunov Approach.

1. INTRODUCTION

Modern wind turbines generators can work at variable wind speed and produce fixed frequency electricity. These variable speed fixed frequency generators use DFIG with a partial converter interface or Permanent Magnet (PM) synchronous generator with a full-converter interface. For high-power wind generation, DFIG-based electric power conversion systems are currently dominating the market. A key DFIG benefit is that inverter rating is typically 25% of total system power which gives a substantial reduction in the power electronics cost as compared with direct-in-line synchronous generator systems. The power percentage fed through the rotor converter is proportional to the slip. At synchronous speed, the only power that flows through the converter is what is needed for ensuring the flow of DC excitation current through rotor windings. Additionally, DFIG systems offer the following advantages:

- The speed range is ±30% around the synchronous speed [1-2]. This allows to continuously operating the wind turbine at its optimum Tip Speed Ratio (TSR), which is specific to the aerodynamic design of a given turbine. This achieves maximum rotor efficiency and hence maximum power extraction.
- Reduced cost of the inverter filters and EMI filters, because filters are rated for 0.25 p.u. total system power.
- Power-factor control can be implemented at lower cost. The four-quadrant converter in the rotor circuit enables decoupled control of active and reactive power of the generator.

However, DFIG-based WECS control is a complex issue [3-8]. Indeed, this system is characterized by multiple variables and operates in environments with extreme variations in the operating conditions (wind speed variation, electrical energy consumption, etc.). On the other hand, DFIG-based drive system is a nonlinear system which is subject to parameter uncertainties and/or time-varying dynamics. The control challenge is to keep the generator at its maximum efficiency despite the wind speed variations and system’s parameters uncertainties. In this work, we propose an adaptive nonlinear Backstepping controller design to extract the maximum active power and have a specified reactive power at the generator terminal even in the presence of wind speed variations. The proposed controller also stabilizes the DFIG-based WECS...
even in the case of large changes in the damping coefficient and moment of inertia, and mechanical torque. In practice, the damping coefficient and moment of inertia are approximately known and the mechanical torque is a random process variable. To still ensure the stability condition despite parameter change, suitable update laws are derived.

A basic view of the DFIG-based WECS is illustrated in the figure 1. In this system, the power captured by the wind turbine is converted into electrical power by the generator and it is transmitted to the grid by the stator and the rotor windings. The rotor circuit is connected to the grid through a back-to-back converter. The capacitor connected on the DC side acts as the DC voltage source. The electrical grid frequency and voltage magnitude are assumed to be constant.

The control system generates the pitch angle command and the voltage command signals for converters. The grid-side converter (GSC) is controlled to have unity power factor and a constant voltage at the DC-link. The rotor-side converter (RSC) is controlled to have optimal power extraction from the wind and a specified reactive power at the generator terminal.

The paper is structured as follows. The second section presents a state-space modelling of the DFIG-based wind power conversion system. The induction machine is controlled in a rotating reference frame oriented along the stator voltage vector position. The third section describes the proposed nonlinear controller design and gives control and update laws for both RSC and GSC converters. An application example on stabilizing a DFIG-based wind power conversion system subject to large mechanical parameters variations will be given in the last section to show the merits of the proposed controller. Simulation results and analysis confirm the effectiveness of the proposed approach.

Figure 1: DFIG-based wind power conversion system

2. SYSTEM MODEL

2.1. Modelling of the Wind Turbine

The aerodynamic turbine power $P_t$ depends on the power coefficient $C_p$ as follows [9]:

$$ P_t = \frac{1}{2} \rho \pi R_t^2 C_p(\lambda, \beta) v^3 $$  \hspace{1cm} (1a)

where:

- $\rho$: Specific mass of the air (kg/m$^3$);
- $v$: Wind speed (m/s);
- $R_t$: Radius of turbine (m);
- $C_p$: Power coefficient;
- $\beta$: Blade pitch angle (deg);
- $\Omega$: Generator speed (rad/s);
- $\lambda$: Tip Speed Ratio (TSR) of the rotor blade tip speed to wind speed.

The TSR is given by:

$$ \lambda = \frac{R_t \Omega}{G v} $$  \hspace{1cm} (1b)

where $G$ is mechanical speed multiplier. A generic equation is used to model $c_p(\lambda, \beta)$, based on the modeling turbine characteristics of [9]:

$$ C_p(\lambda, \beta) = c_1 \left( \frac{c_2}{\lambda} - c_3 \beta - c_4 \right) \exp\left( \frac{c_5}{\lambda} + c_6 \lambda \right) $$  \hspace{1cm} (1c)

Where

$$ \frac{1}{\lambda_i} = \frac{1}{\lambda + 0.08 \beta} - \frac{0.035}{\beta^3 + 1} $$

and the coefficients $c_1$ to $c_6$ are: $c_1 = 0.5176$, $c_2 = 116$, $c_3 = 0.4$, $c_4 = 5$, $c_5 = 21$ and $c_6 = 0.0068$.

Note that the maximum value of $c_p (c_{p\text{max}} = 0.48)$ is achieved for $\beta = 0^\circ$ and $\lambda = \lambda_{\text{opt}} = 8.1$.

In this work, we assume that the wind turbine operates with $\beta = 0$. To capture the maximum power, a speed controller must control the mechanical speed $\Omega$ so as to track a speed reference $\Omega_c$ that keeps the system at $\lambda_{\text{opt}}$ (Maximum Power Point Tracking strategy). According to (1b), the optimal mechanical speed is:

$$ \Omega_c = \frac{G v}{R_t \lambda_{\text{opt}}} $$  \hspace{1cm} (1d)

2.2. Induction Generator Model

The induction machine is controlled in a synchronously rotating $dq$ axis frame, with the $d$-axis oriented along the stator-voltage vector position ($v_{sd}=V$, $v_{sq}=0$). In this reference frame, the electromechanical equations are [10]:

$$ [V] = [R] [I] + \frac{d[\phi]}{dt} + [\omega][\phi] $$  \hspace{1cm} (2a)

$$ [\phi] = [M] [I] $$  \hspace{1cm} (2b)
where:

\[
\begin{bmatrix}
R_s & 0 & 0 & 0 \\
0 & R_s & 0 & 0 \\
0 & 0 & R_r & 0 \\
0 & 0 & 0 & R_r
\end{bmatrix}
\begin{bmatrix}
\omega_s \\
\omega_s \\
0 \\
0
\end{bmatrix}
= \\
\begin{bmatrix}
L_s & 0 & L_m & 0 \\
0 & L_s & 0 & L_m \\
L_m & 0 & L_r & 0 \\
0 & L_m & 0 & L_r
\end{bmatrix}
\begin{bmatrix}
\omega_s \\
\omega_s \\
\omega_r \\
\omega_r
\end{bmatrix}
\]

\[\omega = \begin{bmatrix}
\omega_s \\
\omega_s \\
\omega_r \\
\omega_r
\end{bmatrix}
\]

\[\eta = \frac{1}{J}(-\delta \dot{T} + T_{em})
\]

\[= \frac{1}{J}(-\delta \dot{T} + h(x))
\]

Where

\[
x = (x_1, x_2, x_3, x_4)' = (i_{ad}, i_{aq}, i_{rd}, i_{rq})', \quad \eta = \Omega
\]

\[h(x) = T_{em} = a(x_2 x_3 - x_1 x_4)
\]

\[u_1 = v_{rd}, \quad u_2 = v_{rq}
\]

\[\alpha = \frac{L_m}{\sigma L_s L_r}, \quad \beta = \frac{L_m}{\sigma L_s L_r}
\]

By selecting currents and speed as the state variables, the system (2) can be put into the following state-space form:

\[\dot{x}_1 = g_1(x) - \beta u_1
\]

\[\dot{x}_2 = g_2(x) - \beta u_2
\]

\[\dot{x}_3 = g_3(x) + \alpha u_1
\]

\[\dot{x}_4 = g_4(x) + \alpha u_2
\]

\[\dot{\eta} = \frac{1}{J}(-\delta \dot{T} + T_{em})
\]

Equations (3a) to (3.d) can be put into the following compact form:

\[\dot{x} = g(x) + Au
\]

where

\[g(x) = (g_1(x), g_2(x), g_3(x), g_4(x))', \quad u = (u_1, u_2)', \quad A = \begin{bmatrix}
-\beta & 0 & \alpha & 0 \\
0 & -\beta & 0 & \alpha
\end{bmatrix}
\]

2.3. DC-link and Coupling Filter R-L Models

The GSC converter is connected to the grid through an R-L filter (fig 2) [11].

\[v_{ad} = R_i i_{ad} + L \frac{di_{ad}}{dt} - \delta \omega_i i_{dq} + v_{td} = V
\]

\[v_{aq} = R_i i_{aq} + L \frac{di_{aq}}{dt} + \omega_i i_{dq} + v_{td} = 0
\]
Neglecting losses, the DC-link dynamic equations are:

\[
\begin{align*}
2C \frac{dV_{dc}}{dt} &= i_g - i_r \\
v_{sd}i_{sd} + v_{sq}i_{sq} &= V_{dc}i_g \\
u_1x_3 + u_2x_4 &= V_{dc}i_r \\
\end{align*}
\]  \tag{4b}

Equations (4a) and (4b) can be put into the following state-space form:

\[
\begin{align*}
\dot{\zeta}_1 &= -b\zeta_1 + c\zeta_2 - \mu + \lambda v_1 = f_1(\zeta_1) + \lambda v_1 \\
\dot{\zeta}_2 &= -c\zeta_1 - b\zeta_2 + \lambda v_2 = f_2(\zeta_2) + \lambda v_2 \\
\dot{\zeta}_3 &= \alpha(x,u) + \gamma \zeta_1 \\
\end{align*}
\]  \tag{4c, 4d, 4e}

where

\[
\begin{align*}
\zeta_1 &= i_{od} ; \zeta_2 = i_{oq} ; \zeta_3 = V_{dc}^2 ; v_1 = v_{od} ; v_2 = v_{oq} ; \\
b &= \frac{R}{L} ; c = \frac{\omega_0}{L} , \mu = -\frac{V}{L} , \lambda = -\frac{1}{L} ; \\
\alpha(x,u) &= -\frac{1}{C}(u_1x_3 + u_2x_4) , \gamma = \frac{V}{C} \\
\end{align*}
\]

3. CONTROL DESIGN

3.1. RSC Control

3.1.1. Backstepping control strategy

The RSC converter enables decoupled control of stator reactive power and speed. Indeed, the following expression of the stator reactive power

\[ Q_{st} = \text{Im}(V_{st}^\top) = v_{oq}i_{od} - v_{od}i_{oq} = -Vx_2 \]  \tag{5a}

shows that the current variable \(x_2\) can be considered as a virtual control of \(Q_{st}\). According to equation (3b), it is clear that the control variable \(u_2\) can be designed so that the current variable \(x_2\) tracks its reference. On the other hand, equation (3e) shows that the electromagnetic torque \(T_{em}\) can be considered as a virtual control for the speed \(\eta\). The following dynamic equation

\[ \dot{T}_{em} = u_1 \\
= \psi . g + a (\alpha x_2 + \beta x_4) u_1 - a (\alpha x_1 + \beta x_3) u_2 \]  \tag{5b}

shows that the control variable \(u_1\) can be designed so that the electromagnetic torque \(T_{em}\) tracks its reference.

3.1.2. Control and update laws

In the case of uncertain model where system parameters are not known with enough accuracy, suitable choice of control variables and update laws must be done to still ensure the stability condition [11-14]. In this study, it is assumed that the mechanical parameters \(F, J\) and mechanical torque \(T_t\) are unknown. \(F\) and \(J\) are constant parameters whilst the torque \(T_t\) is assumed to vary slowly in time. Estimates of \(F, J\) and \(T_t\) are denoted \(\hat{F}, \hat{J}\) and \(\hat{T}_t\), respectively. \(\eta, T_{em}\) and \(x_{2c}\) are references of the variables \(\eta, T_{em}\) and \(x_2\), respectively.

We define the following error variables:

\[
\begin{align*}
\gamma_0 &= x_2 - x_{2c} \\
\gamma_1 &= \eta - \eta_c \\
\gamma_2 &= T_{em} - T_{emc} \\
\gamma_3 &= J_t - \hat{J}_t \\
\gamma &= 0 - \hat{\theta} \quad \text{with} \quad \theta = (F, J, T_t)^t , \hat{\theta} = (\hat{F}, \hat{J}, \hat{T}_t)^t , \\
\hat{\theta} &= (\hat{F}, \hat{J}, \hat{T}_t)^t \quad \text{with} \quad \theta = (F, J, T_t)^t \\
\end{align*}
\]

**Proposition 1:** The following control and update laws:

\[
\begin{align*}
u_1 &= \frac{\gamma - (k_2 + k_1)x_2 + au_2(\beta x_1 + \alpha x_3)}{a(\alpha x_2 + \beta x_4)} \\
u_2 &= \frac{k_3x_3 + g(x)}{\beta} \\
\end{align*}
\]

\[
\begin{align*}
\dot{\hat{F}} &= \gamma_1(z_2^2 - \eta z_1 + \eta z_2 \psi(\hat{\theta})) \\
\dot{\hat{J}} &= \gamma_2(z_1 - z_2 \psi(\hat{\theta})) \\
\end{align*}
\]

with

\[
\begin{align*}
\psi(\xi, z_1) &= T_{emc} \\
&= -J(\eta_c - k_1z_1) + \hat{T}_t - (z_1 + \eta_c)\hat{F} \\
\xi &= (\eta_c, \eta_c, \hat{J}, \hat{F}, \hat{T}_t) \\
\end{align*}
\]

\(k_0, k_1, k_2, \gamma_1, \gamma_2\) and \(\gamma_3\) are positive design constants

achieve speed and current tracking objectives and ensure asymptotic stability despite the changes in mechanical parameters.
Proof:

a- Control law $u_2$

Taking the derivative of $z_0$ and using (3b) gives:

$$
\dot{z}_0 = g_2(x) - \beta u_2 - \dot{x}_{2c}
$$

(6)

Let $V_0 = -\frac{1}{2} z_0^2$ be the Lyapunov candidate function; the choice $\dot{z}_0 = -k_0 z_0$, where $k_0$ is a positive design constant, makes negative the derivative $\dot{V}_0 = z_0 \dot{z}_0$ since $\dot{V}_0 = -k_0 z_0^2 \leq 0$. With this choice, the control law $u_2$ can be obtained from equation (6):

$$
u_2 = g_2(x) + k_0 z_0 - \dot{x}_{2c}
$$

(7)

b- Control $u_1$ and update laws

Step 1: Virtual control $T_{emc}$

Using equation (3e), the derivative of $z_1$ is written as:

$$
\dot{z}_1 = -F \eta + T_i - T_{emc} - \dot{\eta}_c
$$

(8)

Since $J$, $F$ and $T_i$ are unknown; it will be replaced with their estimates $\hat{F}$, $\hat{J}$ and $\hat{T}_i$, respectively. Let $V_1 = \frac{1}{2} z_1^2$ be the Lyapunov candidate function; the choice $\dot{z}_1 = -k_1 z_1$, where $k_1$ is a positive design constant, makes negative the derivative $\dot{V}_1 = z_1 \dot{z}_1$ since $\dot{V}_1 = -k_1 z_1^2 \leq 0$.

Thus, equation (8) gives:

$$
-k_1 z_1 = -\frac{\hat{F} \eta + \hat{T}_i - T_{emc}}{J} - \dot{\eta}_c
$$

(9)

which yields the following expression of the virtual control $T_{emc}$:

$$
T_{emc} = -\hat{J}(\dot{\eta}_c - k_1 z_1) + \hat{T}_i - \hat{F}(z_1 + \eta_c)
$$

(10)

where $\eta$ has been replaced by $z_1 + \eta_c$ and $\xi = (\eta_c, \dot{\eta}_c, \hat{J}, \hat{F}, \hat{T}_i)$.

Step 2: Control $u_1$ and update laws

Equation (8) can be written as:

$$
\dot{z}_1 = -\frac{\hat{F} \eta + \hat{T}_i - z_2}{J} - \frac{\hat{F} \eta + \hat{T}_i - T_{emc}}{J} - \dot{\eta}_c
$$

(11)

Subtracting (9) from (11) yields:

$$
\dot{z}_1 = -k_1 z_1 - \frac{z_2}{J} + \hat{A}
$$

(12)

where $\hat{A} = \frac{\hat{T}_i}{J} - \eta \frac{\hat{F}}{J} + (k_1 z_1 - \dot{\eta}_c) \frac{\hat{J}}{J}$.

On the other hand, the derivative of $z_2$ is:

$$
\dot{z}_2 = \nu - \dot{T}_{emc}
$$

Using (10) and (12):

$$
\dot{T}_{emc} = \nabla \psi \dot{\xi} + \frac{\partial \psi}{\partial z_1} z_1
$$

$$
= \nabla \psi \dot{\xi} - k_1 z_1 \psi(\hat{\theta}) - \frac{\phi(\hat{\theta})}{J} z_2 + \dot{\phi}(\hat{\theta}) \dot{\hat{A}}
$$

where $\phi(\hat{\theta}) = \frac{\partial \psi}{\partial z_1} = k_1 \dot{\hat{J}} - \hat{F}$.

Since:

$$
\frac{\phi(\hat{\theta})}{J} = -\frac{\phi(\hat{\theta})}{J} + \frac{\phi(\theta)}{J} = -\frac{\phi(\hat{\theta})}{J} + k_1 - \frac{F}{J}
$$

The derivative $\dot{z}_2$ becomes:

$$
\dot{z}_2 = \nu - \nabla \psi \dot{\xi} + k_1 z_1 \psi(\hat{\theta}) - \frac{\phi(\hat{\theta})}{J} z_2 - \frac{F}{J} z_2
$$

$$
+ k_1 z_1 - \phi(\hat{\theta}) \dot{\hat{A}}
$$

$$
= B - \frac{\nabla \psi \dot{\xi} + k_1 z_1 - \phi(\hat{\theta}) \dot{\hat{A}}}{J} + \dot{\hat{A}}
$$

(13)

with

$$
B = \nu - \nabla \psi \dot{\xi} + k_1 z_1 - \phi(\hat{\theta}) \dot{\hat{A}}
$$

$$
\dot{\hat{C}} = -\frac{\phi(\hat{\theta})}{J} z_2 - \phi(\hat{\theta}) \dot{\hat{A}}
$$

To design the control $u_1$ and update laws, we consider the following Lyapunov candidate function:

$$
V_1 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} \frac{\hat{F}^2}{J} + \frac{1}{2} \frac{\hat{T}_i^2}{J} + \frac{1}{2} \frac{\hat{J}^2}{J}
$$

Its derivative is given by:
Using (12) and (13), \( \hat{V}_1 \) can be written as follows:

\[
\hat{V}_1 = -k_1z_1^2 - \frac{Z_z z_2}{J} - \frac{F}{J} z_2^2 + z_2 B + \hat{D}
\]

with

\[
\hat{D} = z_2 \ddot{\overline{z}} + z_2 \frac{C}{J} - \frac{F}{J} \frac{\ddot{z}}{\gamma_1} - \frac{T_i}{\gamma_2} \frac{\ddot{z}}{J} + \frac{k}{\gamma_3} \frac{\ddot{z}}{J}
\]

\[
\hat{D} = \left[ z_2^2 - \eta(z_1 - z_2 \phi(\hat{\theta})) \right] \frac{\ddot{z}}{\gamma_1} + \left[ z_1 - z_2 \phi(\hat{\theta}) \right] \frac{\ddot{z}}{\gamma_2} + \left[ -k_1 z_1^2 + (z_1 - z_2 \phi(\hat{\theta}))(z_1 - z_2 \phi(\hat{\theta})) \right] \frac{\ddot{z}}{\gamma_3}
\]

The update laws are obtained by cancelling the terms with \( \frac{\ddot{z}}{J} \), \( \frac{F}{J} \), and \( \frac{T_i}{J} \):

\[
\dot{F} = \gamma_1(z_2^2 - \eta c_1 + \eta z_2 \phi(\hat{\theta}))
\]

\[
\dot{T}_i = \gamma_2(z_1 - z_2 \phi(\hat{\theta}))
\]

\[
\dot{J} = -\gamma_3(k_1 z_1^2 + (\eta_1 - k_1 z_1)(z_1 - z_2 \phi(\hat{\theta}))
\]

Subsequently, the derivative \( \dot{V}_1 \) reduces to:

\[
\dot{V}_1 = -k_1z_1^2 - \frac{Z_z z_2}{J} - \frac{F}{J} z_2^2 - k_2 z_2^2
\]

We choose \( B = -k_2 z_2 \), with \( k_2 \) is a positive design constant. Since \( -z_2 z_2 \leq \frac{z_1^2 + z_2^2}{2} \) for all \( z_1, z_2 \), we have:

\[
\dot{V}_1 = -k_1z_1^2 - \frac{Z_z z_2}{J} - \frac{F}{J} z_2^2 - k_2 z_2^2 \leq -(k_1 - \frac{1}{2k})z_1^2 - (k_2 + \frac{F}{J} - \frac{1}{2k})z_2^2
\]

By selecting \( k_1 \) and \( k_2 \) such that the inequalities \( k_1 > \frac{1}{2k} \) and \( k_2 > \frac{1}{2k} \) are verified, the derivative \( \dot{V}_1 \) becomes negative and so we ensure the stability condition.

The control variable \( \nu \) can then be obtained using equation (13) and the choice \( B = -k_2 z_2 \):

\[
\nu = \nabla \psi \hat{\xi} - k_1 z_1 \phi(\hat{\theta}) - (k_1 + k_2) z_2
\]

Finally, equation (5b) gives the control law \( u_1 \):

\[
u_1 = \frac{\nu - (k_1 + k_2) z_2 + a u_1 (\beta x_3 + \alpha x_1)}{a (\alpha x_2 + \beta x_4)}
\]

Where

\[
\nabla \psi \hat{\xi} = \frac{\partial \psi}{\partial \eta_{c_1}} \dot{\eta}_{c_1} + \frac{\partial \psi}{\partial \eta_{c_2}} \dot{\eta}_{c_2} + \frac{\partial \psi}{\partial \eta_{c_3}} \dot{\eta}_{c_3} + \frac{\partial \psi}{\partial \eta_{c_4}} \dot{\eta}_{c_4} + \frac{\partial \psi}{\partial \eta_{c_5}} \dot{\eta}_{c_5} + \frac{\partial \psi}{\partial \eta_{c_6}} \dot{\eta}_{c_6}
\]

and

\[
V h g = a(x_2 g_1(x) + x_3 g_2(x) - x_4 g_4(x) - x_5 g_5(x))
\]

**Remark:**

The term \( a(\alpha x_2 + \beta x_4) \) is proportional to q-axis flux \( \phi_{sq} \):

\[
a(\alpha x_2 + \beta x_4) = a \left( \frac{1}{\sigma L_r} I_{sq} + \frac{L_m}{\sigma L_r} I_{iq} \right)
\]

Since the effect of the stator resistance is negligible especially in high power, and the stator voltage vector is aligned along the d-axis of the reference frame, the amplitude \( \phi_{sq} \) is proportional to voltage \( V \). Hence, the term \( a(\alpha x_2 + \beta x_4) \) becomes different from zero as soon as the machine is connected to the grid.

### 3.2. GSC Control

#### 3.2.1. Backstepping control strategy

The GSC converter enables decoupled control of rotor reactive power and DC-link voltage. Indeed, the following expression of the rotor reactive power

\[
Q_{rg} = \text{Im}(V L^*) = v_{sq} L_{od} - v_{sd} L_{os} = -V _2 \zeta_2
\]

shows that the current variable \( \xi_2 \) can be considered as a virtual control of \( Q_{rg} \). According to equation (4d), it is clear that the control variable \( v_2 \)
can be designed so that the current variable $\zeta_2$ tracks its reference. On the other hand, equation (4e) shows that $\zeta_1$ can be considered as a virtual control for DC-link voltage. Equation (4c) shows that the control variable $v_1$ can be designed so that $\zeta_1$ tracks its reference.

### 3.2.2. Control laws

We define the following error variables:

\[ e_1 = \zeta_1 - \zeta_{1c} \]  
\[ e_2 = \zeta_2 - \zeta_{2c} \]  
\[ e_3 = \zeta_3 - \zeta_{3c} \]

where $\zeta_{1c}$, $\zeta_{2c}$, and $\zeta_{3c}$ are references of $\zeta_1$, $\zeta_2$, and $\zeta_3$, respectively.

**Proposition 2:** The following control laws:

\[
v_1 = \frac{\zeta_{1c} + (q_1^2 - 1)e_3 - (q_3 + q_4)e_1 - \alpha(x,u) - \gamma f_1(\zeta)}{\gamma k}
\]

\[
v_2 = -q_2e_2 - f_2(\zeta) + \dot{\zeta}_{2c}
\]

*With $q_1$, $q_2$ and $q_3$ are positive design constants, achieve the DC-link voltage and rotor reactive power control objective and ensure the asymptotic stability.*

**Proof:**

**a- Control law $v_2$**

According to (4d), the dynamic equation of the error $e_2$ is:

\[
\dot{e}_2 = f_2(\zeta) + \lambda v_2 - \dot{\zeta}_{2c}
\]

To reduce the tracking error, we use the following Lyapunov candidate function $V_2 = \frac{1}{2}e_2^2$. The choice $\dot{e}_2 = -q_2e_2$, where $q_2$ is a positive design constant, makes negative the derivative $\dot{V}_2 = -q_2e_2^2 \leq 0$. With this choice, the control law $v_2$ can be obtained from equation (17):

\[
v_2 = -q_2e_2 - f_2(\zeta) + \dot{\zeta}_{2c}
\]

**b- Control law $v_1$**

**Step 1:** Virtual control $\zeta_{1c}$

According to (4e), the dynamic equation of the error $e_3$ is:

\[
\dot{e}_3 = \alpha(x,u) + \gamma e_3 - \zeta_{1c}
\]

We consider the Lyapunov candidate function $V_3 = \frac{1}{2}e_3^2$. The choice $\dot{e}_3 = -q_3e_3$, where $q_3$ is a positive design constant, makes negative the derivative $\dot{V}_3 = e_3e_3$ and hence reduces the tracking error $e_3$. With this choice, the virtual control law $\zeta_{1c}$ can be obtained from equation (19):

\[
\dot{\zeta}_{1c} = -\alpha(x,u) - q_3e_3 + \dot{\zeta}_{3c}
\]

**Step 2:** control law $v_1$

According to (4c), the dynamic equation of the error $e_1$ is:

\[
\dot{e}_1 = f_1(\zeta) + \lambda v_1 - \dot{\zeta}_{1c}
\]

The derivative of $e_3$ can be re-written as:

\[
\dot{e}_3 = \alpha(x,u) + \gamma(\zeta_1 - \zeta_{1c}) + \gamma \zeta_{1c} - \zeta_{3c}
\]

From (20) and (22), we obtain the derivative $\dot{\zeta}_{1c}$:

\[
\dot{\zeta}_{1c} = -\alpha(x,u) - q_3e_3 + \dot{\zeta}_{3c}
\]

We now consider the Lyapunov candidate function:

\[
V_g = \frac{1}{2}e_1^2 + \frac{1}{2}e_3^2
\]

Its derivative is:

\[
\dot{V}_g = e_1\dot{e}_1 + e_3\dot{e}_3
\]

\[
= e_1\dot{e}_1 + e_3(-q_1e_1 + \gamma\zeta_1)
\]

\[
= -q_1e_1^2 + e_3(\dot{e}_1 + \gamma e_3)
\]

With the choice $\dot{e}_1 + \gamma e_3 = -q_1e_1$, we ensure the negativity of $\dot{V}_g$ since $\dot{V}_g = -q_1e_1^2 - q_3e_3^2 \leq 0$.

Also, equation (21) yields:

\[
f_1(\zeta) + \lambda v_1 - \dot{\zeta}_{1c} + \gamma e_3 = -q_1e_1
\]

From (23) and (24), we obtain the control law $v_1$:

\[
v_1 = \frac{\dot{\zeta}_{1c} + (q_1^2 - 1)e_3 - (q_3 + q_4)e_1 - \alpha(x,u) - \gamma f_1(\zeta)}{\gamma k}
\]
4. SIMULATION RESULTS

To demonstrate the effectiveness of the proposed controller, two SIMULINK models were constructed which correspond respectively to the conventional (without adaptation) and adaptive (unknown mechanical parameters) Backstepping controllers. The tracking capability was verified for the adaptive controller in the case of time-varying wind speed (Fig. 3). To show the robustness against mechanical parameters and mechanical torque change speed regulation performances were compared at constant wind speed (Fig. 4). Constant values for DC-link voltage, filter and stator quadratic currents references were considered. Design parameters used in the simulation are listed in Table 1.

<table>
<thead>
<tr>
<th>Table1: Design Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$k_0$</td>
</tr>
<tr>
<td>$k_1$</td>
</tr>
<tr>
<td>$k_2$</td>
</tr>
<tr>
<td>$\gamma_1, \gamma_2, \gamma_3$</td>
</tr>
<tr>
<td>$q_1, q_2, q_3$</td>
</tr>
</tbody>
</table>

Figures 3a and 3b confirm the effectiveness of the MPPT strategy since the rotor speed vary in accordance with wind speed so that the turbine operates at optimal TSR.

To have a unity power factor, the references of $q$-axis currents $i_{sq}$ and $i_{oq}$ have been set to zero. Figures 3c and 3d show the good tracking capability of the $q$-axis current controller. The same good performance is achieved in the case of DC-link regulation (Figure 3e) and torque controller (Figure 3f).

Figures 3g to 3j illustrate the two operating modes of the generator. At time $t=70s$, we can see in figure 3b that the induction machine operates at sub-synchronous mode ($N<1500$ rpm). Figure 3i shows that grid voltage and stator current are in phase, thus the active power is transmitted from the stator to grid. Figure (3h) shows that the grid voltage and filter current are in phase, thus the active power is transmitted from the grid to rotor. This result is compatible with the super-synchronous speed operation.

Figure 4 shows speed regulation performance for both conventional and adaptive controllers in the case of change in the coefficients $F$, $J$ and the mechanical torque $T_t$. The simulation assumes that changes occur first in the system model (at 100s, $\Delta F=200\%$, $\Delta J=-50\%$), and next in the mechanical torque (at 150s, $\Delta T_t=300\%$). Unlike the conventional backstepping controller (fig.4.b), the adaptive controller tracks the constant speed reference despite the changes (Fig. 4a). This confirms the robustness of the proposed controller.

5. CONCLUSION

A stable and robust Backstepping controller has been successfully applied to a DFIG-based WECS to extract the maximum power and have a unity power factor despite the wind speed variation and system parameter uncertainties. Simulation results showed the effectiveness of the proposed controller and its superiority over conventional controller. Good speed and current tracking performances were achieved even in the case of large change in the mechanical torque.

Appendix: Characteristics and Parameters

<table>
<thead>
<tr>
<th>Induction Generator</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated power</td>
<td>3MW</td>
</tr>
<tr>
<td>Rated stator voltage</td>
<td>690V</td>
</tr>
<tr>
<td>Nominal frequency</td>
<td>50Hz</td>
</tr>
<tr>
<td>Number of pole pairs</td>
<td>$p=2$</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>$R_r=2.97e-3\Omega$</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>$R_s=3.82e-3\Omega$</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>$L_s=0.0122H$</td>
</tr>
<tr>
<td>Rotor inductance</td>
<td>$L_r=0.0122H$</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>$L_m=12.12e-3H$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Wind Turbine</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Blade Radius</td>
<td>$R_f=45m$</td>
</tr>
<tr>
<td>Power coefficient</td>
<td>$C_{pmx}=0.48$</td>
</tr>
<tr>
<td>Optimal TSR</td>
<td>$\lambda_{opt}=8.14$</td>
</tr>
<tr>
<td>Mechanical speed multiplier</td>
<td>G=100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Generator and Turbine</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of inertia</td>
<td>J=254Kg .m$^2$</td>
</tr>
<tr>
<td>Damping coefficient</td>
<td>F=0.24</td>
</tr>
</tbody>
</table>
Bus DC

- C = 38 mF
- $v_{dc} = 1200$ V

Filter RL

- R = 0.075 Ω
- L = 0.75 mH

Electrical grid

- U = 690 V
- $f = 50$ Hz

REFERENCES:


Figure 3a: Time-varying wind speed

Figure 3b: Rotor speed and its reference

Figure 3c: Stator q-axis current and its reference

Figure 3d: Filter q-axis current and its reference

Figure 3e: DC-link voltage and its reference

Figure 3f: Electromagnetic torque and its reference
Figure 3: Tracking capability under time-varying wind speed

Figure 3g: Phase a grid voltage and filter current (sub-synchronous speed operation)

Figure 3h: Phase a grid voltage and filter current (super-synchronous speed operation)

Figure 3i: Phase a grid voltage and stator current (Sub-synchronous speed operation)

Figure 3j: Phase a grid voltage and stator current (Super-synchronous speed operation)

Figure 4: Speed Regulation Performance Under Unknown Mechanical Parameters

Figure 4a: Rotor speed with adaptation

Figure 4b: Rotor speed without adaptation