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CHARACTERIZATION AND DEVELOPMENT OF A NEW FAILURE MODEL

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ABSTRACT

In the present article a new failure model has been discussed with its necessary parameters and functions like mean, variance, median, moments generating function, characteristic function, reliability and hazard rate functions. A real life data from Davis (1952)[1] have been used to make numerical illustration of the proposed model. We have successfully established that our model fits well in a real life situation that endures the application of this model will definitely enlarge the family of failure / life testing models. The proposed model is a classic inverse J-shaped with constant failure rate. This model also satisfies the property of forgetfulness .Finally, the performance of the proposed model is illustrated with some graphs of different functions of this model such as pdf, cdf, reliability function and hazard rate function.

Keywords: *Failure Model, Reliability Function, Hazard Rate Function, Moments Generating Function, Characteristic Function*

1. INTRODUCTION

The parametric modeling is the important tool for any scientific analysis. Different probability distributions represent the different probabilistic populations. We need an appropriate probability distribution to analyze a specific probabilistic population.

There is a special class of distributions which are used in case of failure data analysis. Initially for failure data analysis the negative exponential distribution was the only distribution in use and a lot of early work is found through this model. Weibull(1951)[2]defined and established a new model, which is known after his name, the Weibull distribution. Thereafter a number of researchers introduced different models to contribute in the family of failure models.

Folks and Chhikara(1978)[3] have established the utilities of Inverted Gamma distribution in the field of life testing analysis. They discussed and established its different useful characteristics.

Now, as the area of applicability of life testing is enlarging day by day, the variety of situations and their related data are also growing up. So, it seems not logical that this variety of data should be dealt with only two or three types of the life testing models. It seems desirable to develop more and more realistic life testing models to deal with the variety of life testing data so that the proper inferences concerning with the reliability and other aspects can be drawn with more precision.

Some of the research scholars such as Mukheerji-Islam (1983)[4] and Siddiqui etal.[5,6,7,8] (1992[5,6], 1994[7], and 1995[8]) introduced a new concept of finite range models in life testing analysis. They also established the utility of their models in Inventory control and Survival analysis. Khan et al. (1989)[9] presented the discrete form of the well-known Weibull model.

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Cha and Mi(2007)[10]studied an stochastic failure model in a random situations. Mandel and Ritov(2010)[11] used an accelerated failure time model under biased sampling. Liu X (2012)[12] used gamma distribution as frailty model to study the accelerate life tests to study the failure modes.

It has been observed that in a number of situations the application of the existing failure models is used only after imposing a number of assumptions either on the parameters of the function or on the behavior of the test. So, there is a requirement of a relatively larger family of lifetime models, so that proper justification can be done with the data analysis.
In this paper, we have introduced a new model to be used as failure model. Different statistical constants have been obtained. We have proved numerically that our proposed model fits well for failure model.

2. MODEL DEVELOPMENT

The demanding situation is to develop a model considering the randomness of the situation. In deterministic modeling the randomness of the factors is not considered but it fixes some parameters affecting the failures. Though the phenomenon of failure is being studied either in any form.

In the probabilistic modeling approach, there exist three methodologies to study the same in the available literature so far. These three methodologies can be summarized as follows.

2.1 Through Existing Models:

In this approach the existing probability density functions in the available literature are studied and characterized to establish them as lifetime models. This approach was initiated by Weibull (1951)[2], when he established his own probability density function as lifetime model. Later on, in 1954, Epstein and Sobel[13]studied and characterized the well-known exponential probability density function as lifetime model. Also, Folks and Chhikara (1978)[3] worked on inverted gamma probability density function and justified its applicability in life testing theory.

2.2 Through Models Developed For Particular Situations:

The second approach is the development of probabilistic models for lifetime data based on one's requirement. In this approach a mathematical expression is being framed in such a manner that it satisfies the limits of the variable and parameters under study and also satisfies the requirements of being a probability density function. Using the same approach, Mukherjee-Islam (1983)[4] and Siddiqui et al. (1992[5,6], 1994[7], and 1995[8]) developed lifetime models to study a particular type of lifetime data.

2.3 Through Mathematical Derivation of Models:

This approach is based on the role of hazard rate function in life testing analysis and the fundamentals of the subject. This technique is not commonly known so far. An extensive discussion on the hazard rate function in Cox (1962)[14] straightens the idea of this approach. Krane(1963)[15] and Kodlin(1966)[16] considered polynomial of various degrees as hazard rate function and developed the lifetime models based on them.

The above technique is a direct implementation of the fact that if the hazard rate for any data is known in the form of a numerical equation, the lifetime model can uniquely be determined on the basis of given hazard rate. The procedure has been discussed in detail in the following section.

3. PROCEDURE

We consider an unknown differentiable function R(t), having the capacity to define the proportions of survivors out of the total population exposed to risk and we call this function as reliability function.

We check the function considered here as a reliability function; i.e. whether it satisfies the following conditions or not.

$R(0) = 1$; $R(\infty) = 0$;

and $R'(t) \le 0$ for all $t \ge 0$

$$.....(1.1)$$

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If it satisfies the above conditions then the corresponding probability density function can be obtained as;

$$f(t) = -d(R(t)) \qquad .....(1.2)$$

And the corresponding hazard rate function is;

$$h(t) = \frac{f(t)}{R(t)} \qquad .....(1.3)$$

From above equations we may have;

$$h(t) = \frac{-d(R(t))}{R(t)}$$
$$= -d(\ln(R(t)) \qquad .....(1.4)$$

In order to obtain R(t), equation (1.4) can be reformulated as follows

$$R(t) = e^{-\int_0^t h(u)du}$$
$$= e^{-h(t)} \qquad .....(1.5)$$

Putting value of R(t), from (1.5) to (1.2), finally the following result can be obtained;

$$f(t) = h(t)e^{-\int h(u)du} \qquad .....(1.6)$$

The equation (1.4) is exponentially distributed with mean unity (Krane,1963). Hence, the h(t) obtained in (1.5) should bind under the following conditions;

$$h(0) = 0, \qquad h(\infty) = \infty \qquad .....(1.7)$$

and $h^{'}(t) \geq 0$ for all $t \geq 0$

So, it can be stated in the light of above discussion and equation (1.6) and (1.7) that if the hazard rate $h(t)$ is known, the failure model or probability density function $f(t)$ can be defined uniquely.

Keeping the same concept of model development an attempt has been made in the following section. Constant hazard rate has been considered here. While working with the data of Davis (1952)[1], it seems that a modification can be made on the hazard rate function to increase the precision of the ultimate model. In this way, the accuracy of the decisions could be increased. For example in some situations, where the exponential model with constant hazard rate equals to the parameter say ‘θ‘ seems to be fitted well but it has been observed that if the hazard rate function is taken as $\ln\theta$, then the model developed for this hazard rate will be more appropriate to study the life time data.

4. THE PROPOSED INFINITE RANGE FAILURE MODEL

The proposed model has the following probability density function, pdf, graphic shape of the function has been shown in **graph (1.1)**

$$f(x) = \theta^{-x}\ln\theta\,; x, \theta \geq 0$$

Graph (1.1) is a mirror J – shaped curve and it is the most desirous property for a failure distribution. That confirms the idea behind the failure distribution that as the life is increasing the probability of survival is decreasing.

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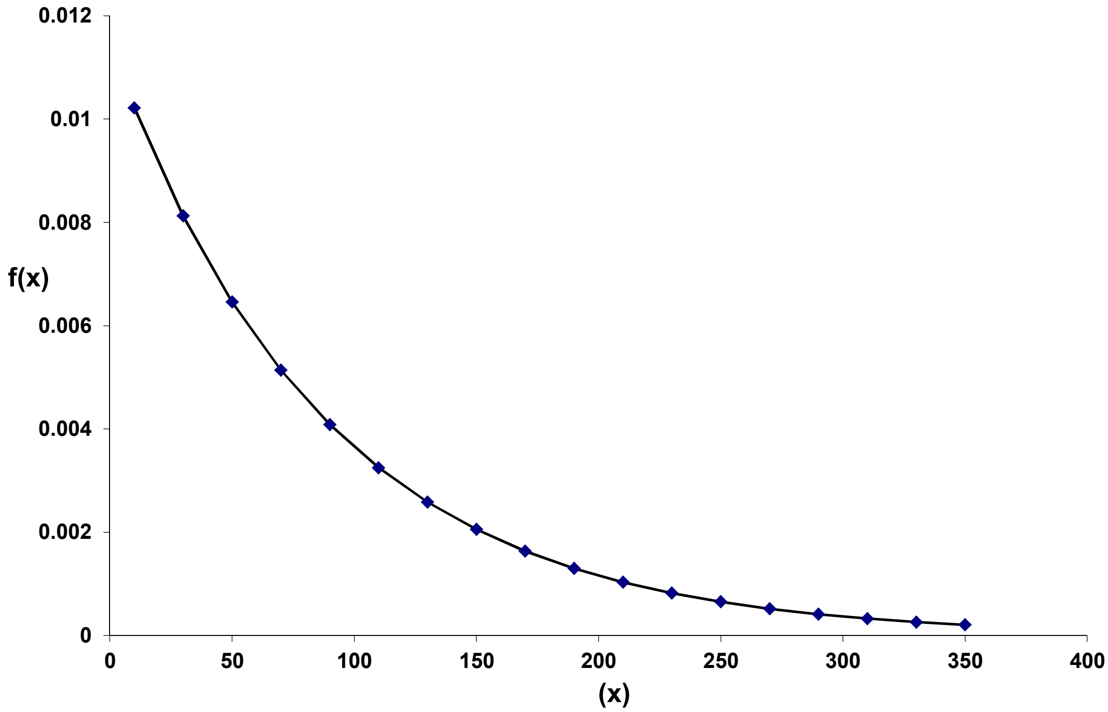
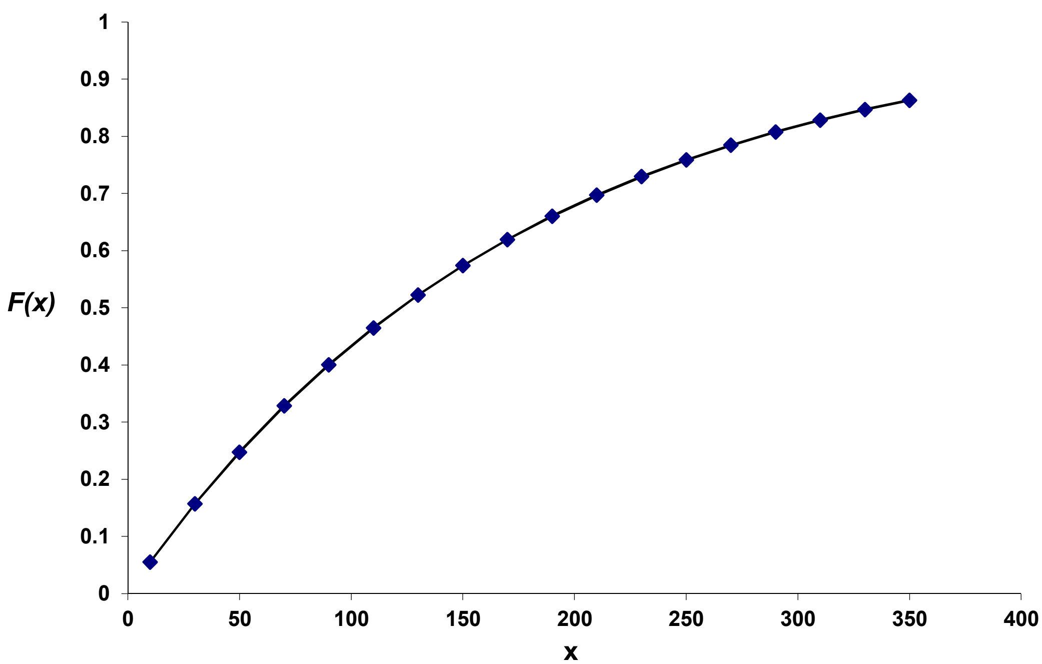


Graph 1.1. Probability Density Function

The cumulative distribution function, **graph(1.2),** is as follows;

$$F(x) = 1 - \theta^{-x}$$



Graph 1.2. Cumulative Density Function

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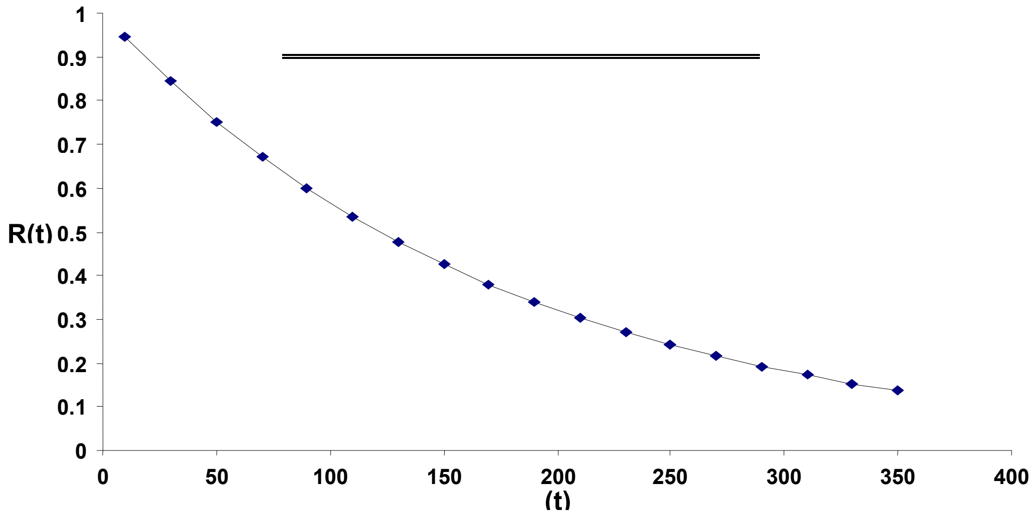
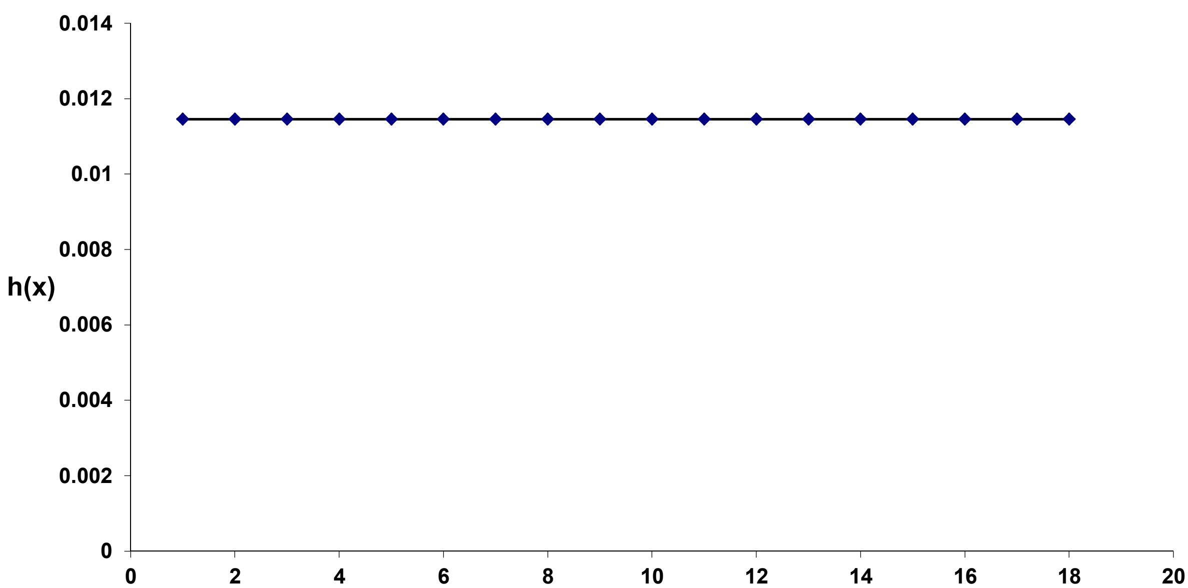
The reliability function **graph(1.3)** and hazard rate **graph (1.4)** functions are

$R(x) = \theta^{-x}$ and $h(x) = \ln\theta$



Graph (1.3) for Reliability Function



Graph 1.4.Hazard Rate Function

Graph (1.3) of the proposed distribution is clearly justifying the definition of reliability function. Graph (1.4) is a graph of a constant function.

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5. CHARACTERIZATION OF THE PROPOSED MODEL

Following are the main constants of the proposed model, which help in characterizing the model.

5.1Mean

$$E(X) = \int x f(x)dx$$

Which, comes as, $Mean = \dfrac{1}{\ln\theta}$

Similarly we can obtain

$$E(X^2) = \int x^2 f(x)dx$$

$$E(X^2) = \frac{2}{(\ln\theta)^2}$$

5.2Variance

Now the variance(X), can be obtained by using the expression

$$V(X) = E(X^2) - (E(X))^2$$

So, $V(X) = \dfrac{1}{(\ln\theta)^2}$

5.3Co-efficient Of Variation (CV)

$$CV = \frac{\text{Standard Deviation}}{Mean} = 1$$

The coefficient of variation is pure constant and independent of the parameterθ. Such an expression for CV is not seen in other lifetime models, and its utility is yet to be fully explored.

The proposed model has its mean and standard deviation equal.

5.4Median

$$\int_0^{Me} f(x)dx = \frac{1}{2}$$

This gives the value of median;

$$Median = \frac{\ln(\frac{1}{2})}{\ln\theta}$$

5.5 Moment Generating Function (m.g.f.)

The mgf about origin is defined as follows;

$$M_x(t) = \int e^{tx} f(x)dx$$

$$mgf = \ln\theta(\ln(\frac{\theta}{e^t}))^{-1}$$

5.6 Characteristic Function (c.f.)

The characteristic function of the proposed model is obtained as follows;

$$\phi_x(t) = E(e^{itx}) = \int e^{itx} f(x)dx$$

$$\phi_x(t) = \ln\theta[\ln(\frac{\theta}{e^{it}})]^{-1}$$

5.7Forgetfulness Property

The most desirable property among the lifetime models is the property of forgetfulness, especially when analysis of electronic items is to be done.

The proposed model follows forgetfulness property; the statement can be justified by the following proof.

$$P(Y \le x / X \ge a) = P(X \ge a)$$

Where, $Y = X - a$

Now

$$P(Y \le x \cap X \ge a) = P(X - a \le x \cap X \ge a)$$

$$= P(X \le x + a \cap X \ge a)$$

$$= P(a \le X \le x + a)$$

$$= \int_a^{x+a} f(x)dx$$

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$$= \theta^{-a}(1-\theta^{-x})$$

$$P(X \le x) = 1-\theta^{-x}$$

and

$$P(X \ge a) = \theta^{-a}$$

Now

$$P(Y \le x / X \ge a) = \frac{P(Y \le x \cap X \ge a)}{P(X \ge a)}$$

$$= 1-\theta^{-x}$$

$$= P(X \le x)$$

With this derivation, we can state that the proposed model possesses the forgetfulness property.

5.8 Maximum Likelihood Estimates (m.l.e) of Parameter

In order to obtain **mle** of the parameter θ, the likelihood function will be as

$$L(\theta) = \prod f(x_i, \theta) = (\log_e \theta)^n \theta^{-\sum x_i}$$

Taking log on both sides and differentiating partially with respect to θ and equating to zero; the **mle**of the parameter θ can be obtain as ;

$$\hat{\theta} = e^{(1/\bar{x})}$$

5.9Average Failure Rate

The average failure rate over the interval (0, t) can be obtained as

$$AFR(t) = \ln\theta$$

and the same over the interval (x_1, x_2) will be

$$AFR(x_1, x_2) = -\frac{\ln(\theta^{-x_1} - \theta^{-x_2})}{(x_2 - x_1)}$$

5.10 Series and Parallel Systems Lives

The two expressions for the distribution of $Y_1 = Min(x_1, x_2)$ and $Y_2 = Max(x_1, x_2)$ can be derived in order to represent the series and parallel system lives. It is assumed here that x_1 and x_2 are identically and independently distributed. Therefore,

$$f(y_1) = 2\theta^{-2x}\ln\theta$$

$$f(y_2) = 2\theta^{-x}\ln\theta(1-\theta^{-x})$$

5.11Model Acceleration

In a physically accelerated condition, the following expressions are required to be obtained.

Time to fail $t_u = AFxt_s$

Failure probability

$$F_u(t) = F_s\left(\frac{t}{AF}\right)$$

Density function

$$f_u(t) = \frac{1}{AF} F_s\left(\frac{t}{AF}\right)$$

Failure rate

$$h_u(t) = \frac{1}{AF} h_s\left(\frac{t}{AF}\right)$$

Where AF is the Acceleration Factor between two stresses i.e. usable (dented by suffix 'u') and stressed (denoted by 's').

6. NUMERICAL ILLUSTRATION

The data used to illustrate the applicability of the proposed model is given in the following table provided by Davis(1952).

Time Interval	Observed Frequency	Expected Frequency
0 - 50	82	89
50 - 100	75	67
100 -150	49	51
150 - 200	45	38
200 - 250	27	29
250 - 300	17	22
300 - 350	16	16
350 -400	13	12
400 - 500	11	16
500 - 600	10	9
600 & above	16	12
Total	**361**	**361**

For the given data the **mle** of the parameter θ is

$$\hat{\theta} = 1.0057$$

and the calculated value of chi-square is

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$\chi^2_{cal} = 7.0248$ with 9 degree of freedom.

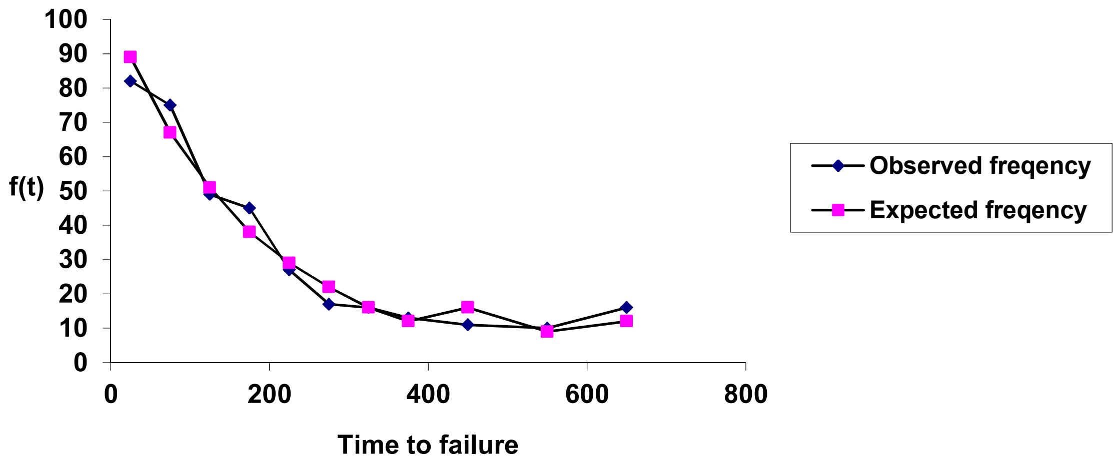
While;

$\chi^2_{tab} = 16.919$ at 5% level of significance

The fitting of the data to the proposed model is good.

This shows that the proposed model can be used as a failure model for these types of failure data.

Graph for the fitted data has been shown in Graph 1.5



Graph 1.5.Graph for Fitted Data

Graph(1.5) shows the closeness of real data and the fitted data which a good sign for the applicability of the proposed distribution as a failure distribution(model) to these types of data.

7. CONCLUSION

The proposed model is an effort towards increasing the family of life testing or failure models. The discussion so far given in the previous sections clearly gives an impression that the model has all desired characteristics of a good life-testing or failure model. The important features are;

- It satisfies the property of new better than used.
- Its co-efficient of variation is a pure constant.
- The model has the property of forgetfulness, which is the most desirous property among the failure models especially when we deal with life of electronic items.
- The graph of fitted data of proposed model clearly indicates that it is very close to the real data and hence it supports its practical importancein the field of reliability analysis.
- Mean and standard deviation of the proposed model are equal.

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