

RECONSTRUCTION OF THE HUMAN RETINAL BLOOD VESSELS BY FRACTAL INTERPOLATION

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ABSTRACT

This paper presents the fractal interpolation method of the human retina image. In the first part, we focus on the segmentation image with Skeletonization and identify different types of pixels. Secondly, using Douglas-Peucker algorithm to reduce the number of pixels in the image we try to keep a form close to the original. then, we used fractal interpolation (IFS) to decompress the encoded image. To evaluate the image quality of the methodology using the peak signal-to-noise (PSNR). The results obtained show that the method of Douglas-Peucker reduces the size of the image from 92 to 96 percent and the PSNR values of fractal interpolation are between 27 and 36.9 db. We conclude with fractal interpolation can have a better quality image.

Keywords: *Douglas-Peucker Algorithm, Fractal Interpolation, Retinal Blood Vessel Image.*

1. INTRODUCTION

With the progress of information, it requires information storage and faster communication links. Store images in a reduced memory leads to a direct reduction in storage costs and faster data transmissions. These facts support the efforts of private companies and universities on new image compression algorithms. Images are stored on computers as collections of bits (a bit is a binary unit of information that can answer "1" or "0") representing pixels or dots forming the pixels.

The current standard method most popular compression relies on eliminating high frequency components of the signal by storing only the low frequency components (Discrete Cosine Transform Algorithm). We find this method used in JPEG (still images) [1], MPEG (motion video), H.261 (video telephony on ISDN lines), and H.263 (video telephony over PSTN lines) compression algorithms [2].

Contrary for other conventional compression techniques, the fractal compression does not attempt conventionally compress byte composing the frame [3] [4]. The principle here is to replace the image with mathematical formulas [5] [6]. The fractal image compression has been proposed for the first time by Barnsley [3]. It shows that the fractal compression's policy that an image is a set of identical patterns in limited numbers [6] [7], to which we apply geometric transformations

(rotations, symmetries, enlargements, and reductions) [8] [9] [10]. This method, based on the theorem of collage, shows that it is possible to encode fractal images using transformations defining a few Contracting iterated function system (IFS) [11] [12] [13]. Barnsley proposed an algorithm to build, from a given image, a set of transformations the contracting representative [8] [9].

In the first part we looked skeletonization images of the retina, eventually, some behaviors of differentiation of the different classes of points: the end-points and the points of bifurcations (inner points, branches, cross over ...) [20] [21]. In the second part, are interested in the Douglas Peucker algorithm to determine the characteristic points in the human retina image used [22] [23] [24]. Then in another section we present the theory of affine transformation for compression and decompression fractal [14] [15] [16]. The proposed method to estimate the decoded image quality is explained in the next section [25]. Finally, the last section presents some results.

2. ALGORITHM

The different stages of compression and decompression fractal of the image used are shown in the following Figure 1 [13] [14] [15].

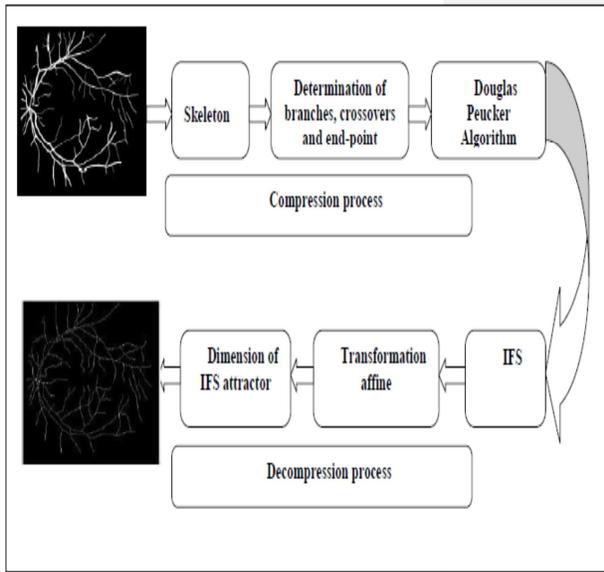


Figure 1: The Stages Of Fractal Compression And Decompression

3. IMAGE SEGMENTATION

2.1 Skeletonization of the human retina image

A real image is a very complex object to be manipulated without pretreatments. That is why it is often sought to simplify the information while retaining the most significant original image. A Skeletonization a binary image which corresponds to a kind of wire representation of the image elements, the aim is to represent a set with a minimum of information in a form that is both simple to extract and convenient to handle. It must account for geometrical properties and maintain relationships of connectivity: the same number of connected components, same number of holes each connected component which allows recovering the original shape [20] [21].

2.2 Detecting the endpoints and bifurcation point from the image

The first step is to remove all of the coordinates of all the pixels containing information in the image. Then, for each pixel containing information, a mask is applied to determine the coordinates of the eight neighboring pixels [20] [21].

- If only one neighbor pixel information so it is an end point (Figure 2.a).
- If two neighboring pixels of the information is such a developed interior (Figure 2.b).
- If three or more adjacent pixels with information so it is a point of intersections and branch (Figure 1.c).

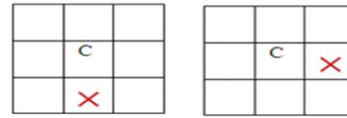


Figure 2.a: end point

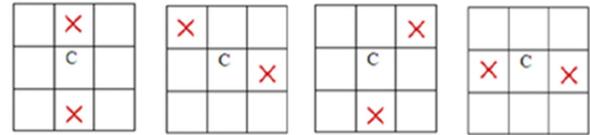


Figure 2.b: inner point

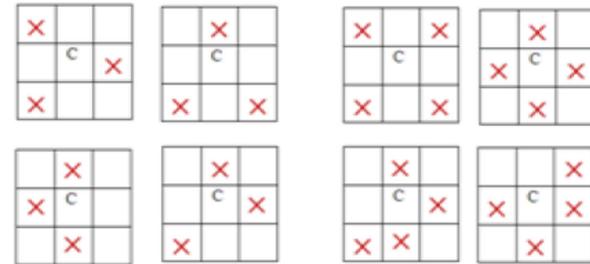


Figure 2.c: Crossovers And Branch Point

4. DOUGLAS-PEUCKER ALGORITHM

This is a simplification algorithm of line. It retains the critical points that describe the overall shape of a line and removes all other points. The algorithm begins by connecting the ends of a line with a trend. The distance of each vertex relative to the trend line is measured perpendicularly. Closest to the line vertices those tolerances are eliminated. The line is then divided by the far top of the trend line, which has the effect of creating two trends. The remaining vertices are measured against these curves and ongoing process until all vertices within the tolerance range are eliminated (Figure 3) [22] [23] [24].

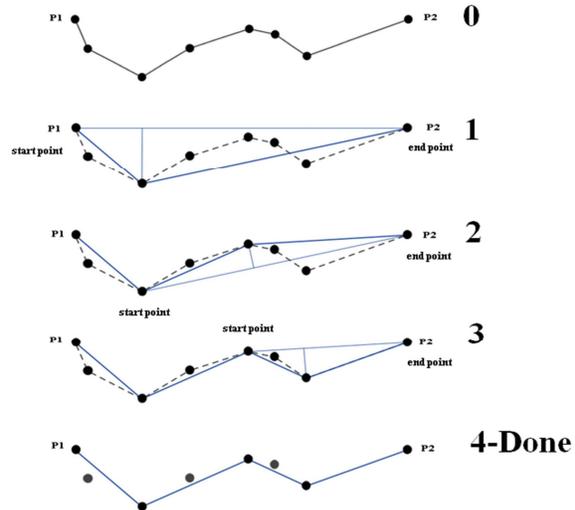


Figure 3: Douglas-Peucker simplification



5. THE THEORETICAL BASIS

In this section we explain the theoretical basis of our model, starting by recalling some definitions in the IFS theory [8] [9] [10].

5.1 Iterated Function System (IFS)

An IFS (iterated function system) which when applied to a geometric object provides a fractal. These functions are affine and contracting. The term contractor means that when we apply these functions to a figure, the points of it but do not stray approach; there is no difference, a figure contained in a finite space is obtained. In the cases of dimension 2, the resulting image fractal therefore occupies a finished surface, which is none other than the surface of the initial object to a contour of infinite length. In this study we used, as contracting functions, affine transformations [13] [14] [15].

5.2 Affine transformation

An affine transformation is defined by a translation vector T and a transformation matrix V to obtain the coordinates of the point P' image of a point P in this transformation, applying the formula Vector:

$$OP' = T.OP + V$$

If (x', y') and (x, y) represent the coordinates of P' and P, respectively, that can be translated by the following system of equations:

$$\begin{aligned} x' &= ax + by + e \\ y' &= cx + dy + f \end{aligned} \tag{1}$$

Where a, b, c and d are the coefficients of the matrix T and e and f are the components of the vector V [15] [16].

5.3 Iterated function system (IFS) and fractal interpolation

Let us represent the given set of data points as $\{(x_n, y_n) \in \mathbb{R}^2: n = 0, 1, \dots, N\}$. In general, the interpolation is applied to a subset of them, the interpolation points, represented as $\{(x_i, y_i) \in \mathbb{R}^2: i = 0, 1, \dots, N\}$. the interpolation points partition the set of data points into interpolation intervals and may be chosen equidistantly or not. The greater the number of interpolation points the better the fit of the data, but more interpolation points result in a smaller compression ratio since more information is required to describe the interpolation function. The fractal interpolation iterative function system (IFS) is of the form $\{\mathbb{R}^2; W_n, n = 1, 2, \dots, N\}$, where, W_n have the following affine transformation structure: Let y axis has no rotation term, i.e. parameter b = 0 in formula (1). For every $i = 1, 2, \dots, N$. Solving the above equations results in:

$$a_i = \frac{x_i - x_{i-1}}{x_N - x_1} \tag{2}$$

$$c_i = \frac{y_i - y_{i-1}}{x_N - x_1} - d_i * \frac{y_N - y_1}{x_N - x_1} \tag{3}$$

$$e_i = \frac{x_N x_{i-1} - x_1 x_i}{x_N - x_1} \tag{4}$$

$$c_i = \frac{x_N y_{i-1} - x_1 y_i}{x_N - x_1} - d_i * \frac{x_N y_1 - x_1 y_N}{x_N - x_1} \tag{5}$$

When $0 < d_i < 1$, IFS has a single attractor and this attractor must be the diagram of certain continuous functions, and crossing original data point. Let the starting point be $p_0 = p_i$ while [17] [18].

$p_i \in \{p_1, p_2, \dots, p_n\}$, the initial point can be selected at random, data series is $x_1 < x_2 < x_3 < \dots < x_n$, start iteration operation from the initial point, use the initial point and any one group of affine functions to the generate the first point, then use the first point and any one group of affine function to generate the second point, continue to generate iteration point till the data point generated conforms to user demands. $\{W_n (P'n) = (x'n, y'n) | n=1, 2, 3, \dots, n'\}$ is the fractal interpolation data point. Then N' is the total number of interpolation data points [15] [19].

5.4 Dimension of IFS attractor

Fractal dimension (denoted by DF) in IFS is associated with the perpendicular scaling factor d_i . It can be learned from the theorem proposed in [2] [3], that the total number of data points is n, $a_i (i = 2, 3, \dots, n)$ follows the above-mentioned affine transformation IFS, when $0 < d_i < 1$ and $\sum_{i=2}^N d_i > 1$, suppose interpolation data points are not collinear, then attractor fractal dimension is the only real solution of the following equation [15][16]:

$$\sum_{i=2}^N d_i a_i^{DF-1} \tag{6}$$

When x axis components of data points are equal-interval distributed, or $x_i - x_{i-1} = \text{constant}$, then $a_i = 1 / (N - 1)$. Let perpendicular scaling component $d_i = d$ be fixed. Then the equation above can be simplified as follows:

$$d = (N-1)^{DF-2} \tag{7}$$

6. PERFORMANCE EVALUATION

The PSNR computes the peak signal-to-noise ratio, in decibels, between two images. This ratio is used as a quality measurement between the original image and a compressed image.

The Mean Square Error (MSE) and the Peak Signal to Noise Ratio (PSNR) are the two error metrics used to compare image compression quality. The MSE represents the cumulative

squared error between the compressed and the original image, and the PSNR value represents a measure of the peak error [25].

For calculates the mean-squared error using the following equation:

$$MSE = \frac{1}{m*n} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \|f(i, j) - g(i, j)\|^2 \quad (8)$$

We confine our measurements to 1 bit per pixel binary images, so the peak signal-to-noise-ratio (PSNR) is computed as:

$$PSNR = 10 * \log_{10} \left(\frac{3*d^2}{MSE} \right) \quad (9)$$

f: represents the matrix data of our original image

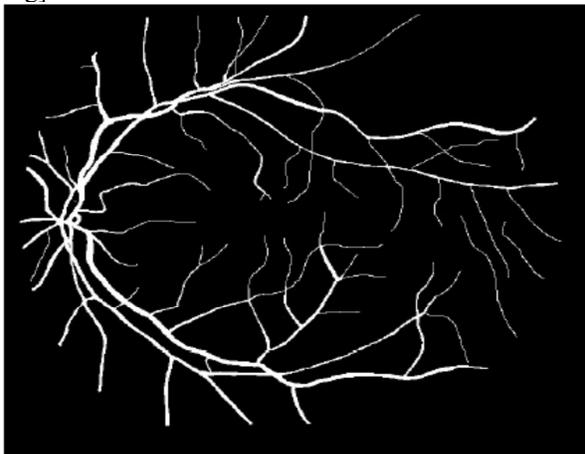
g: is the matrix of image data decompressed

m: represents the numbers of rows of pixels of the images and i represents the index of that row

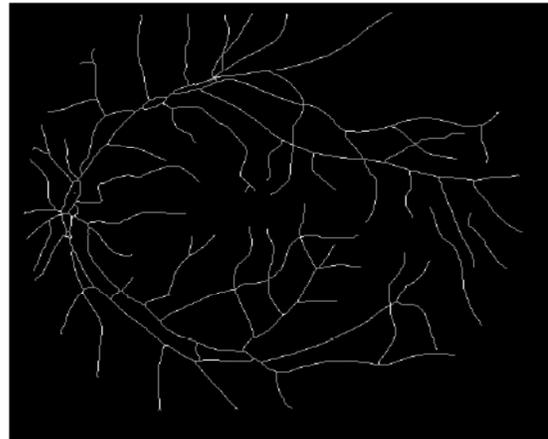
n: represents the number of columns of pixels of the image and j represents the index of that column.

7. RESULTS

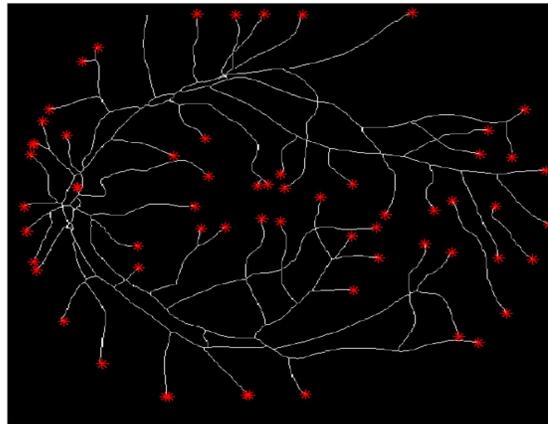
The skeleton reduces the number of image points worm a line while keeping its main aspect. It provides a simple and compact blood vessel of a shape that preserves many of the topological and size characteristics of the original shape. Thus, for instance, we can get a rough idea of the length of a shape by considering just the end points (and Figure 4.c) of the skeleton (Figure 4.b) and finding the maximally separated pair of end points on the skeleton. Similarly, we can distinguish many qualitatively different shapes from one another crossovers point (Figure 4.d). Figure 4.a shows one of the retinal images used from the STARE database[<http://www.parl.clemson.edu/stare/probing>].



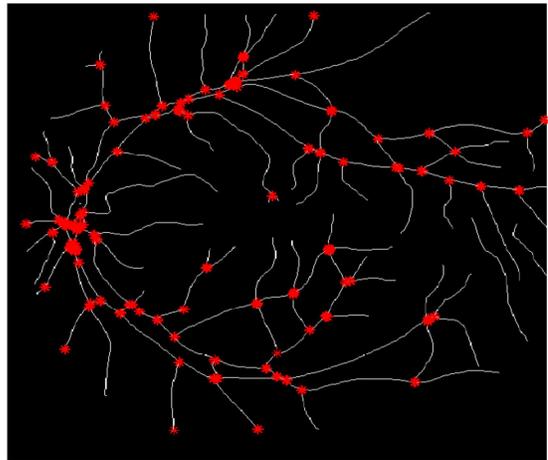
a



b



c



d

Figure 4.(A): Im0002.Jpg,(B): The Skeleton Of Im0002.Jpg,(C): Endpoint,(D): Crossovers Point

7.1 DP algorithm

The DP algorithm is used to simplify the 2D curve of the retina blood vessel, it has at the entrance of the much more detailed information (a set of the curve as shown in Figure .4b) and after

the execution of the algorithm DP the size of these information are reduced Figure 5.

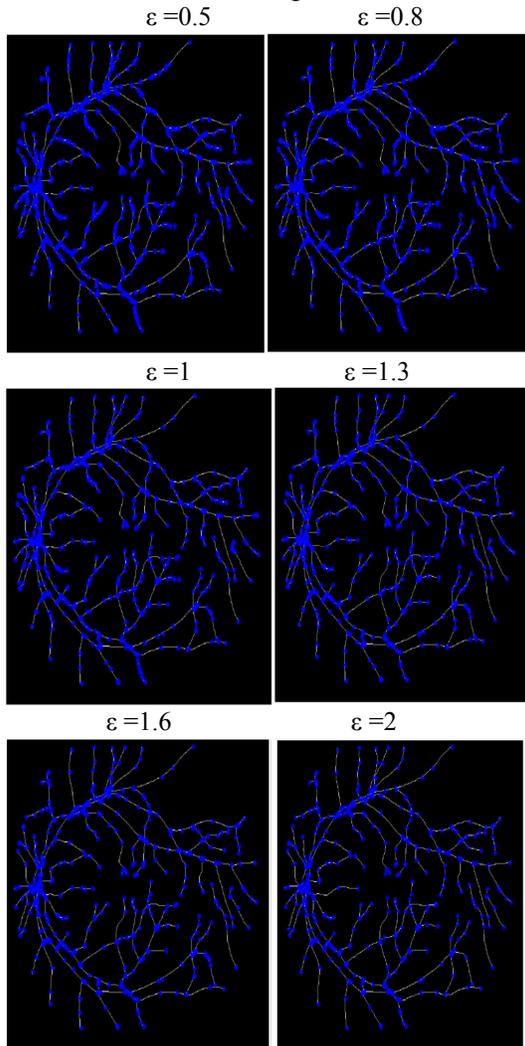


Figure 5: Characteristics Points With Douglas-Peucker Algorithm

The Data Compression Ratio (DCR) is defined as follows: $DCR = ((N-n)/N) * 100\%$.

Table 1: Number of characteristics pixels and DCR% FOR different ϵ

ϵ	2	1.6	1.3	1	0.8	0.5
n	237	263	292	340	401	545
n/N en %	3.11	3.45	3.8	4.4	5.2	7.1
Taux %	96.8	96.5	96.1	95.4	94.7	92.8

The Douglas-Peucker algorithm reduce the amount of data storage, it constrict the number of points of a given line while keeping its main geometrical and topological properties. We see in TABLE1 that the compression ratio is about 92%, and it can be improved up to 96.6%, so the Douglas Peucker algorithm has a compression ratio high. It is noted that the compression ratio and the number of feature points vary according to the value of ϵ . if ϵ is high the number of items found is small and vice versa.

7.2 Simulation of the algorithm

The following figure shows the simulation of the fractal interpolation to im0002.jpg image, we can say that are almost identical with the original image.

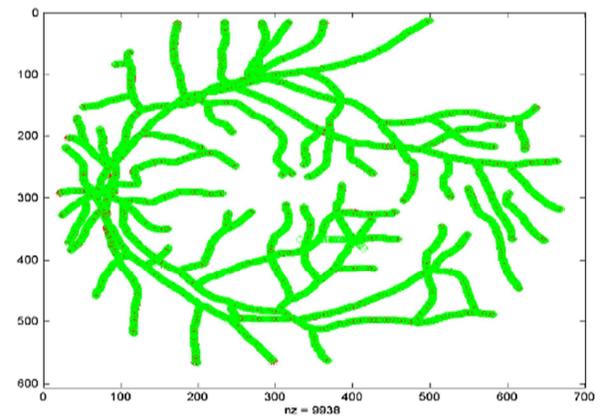


Figure 6: Fractal interpolation

7.3 Performance analysis

To assess the impact of fractal interpolation on image quality metrics used in this evaluation:

a. Calculates erroneous points

Table 2: Calculates erroneous points

fractal Dimension	1.541	1.645	1.747	1.857
Number of incorrect points	102	156	529	959
Number of the original points	7606	7606	7606	7606
Error (%)	1.341	2.051	6.955	12.608

b. The peak signal to noise ratio (PSNR)

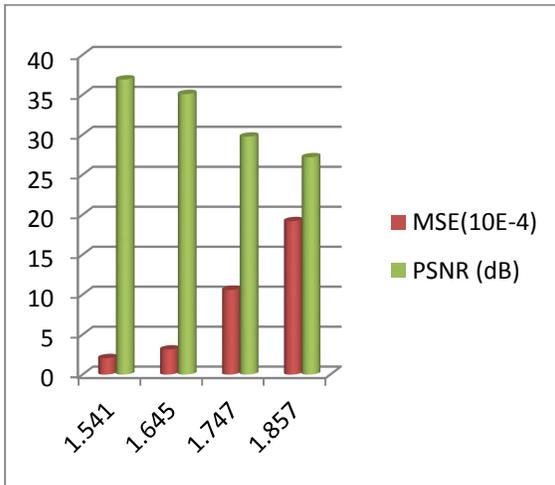


Figure 7. MSE And PSNR Values For Fractal Interpolation

Figure 7 gives the MSE values (8) and the values of PSNR (9) fractal interpolation for different values of the fractal dimension. We conclude that the values of PSNR are between 27 and 36.9 dB for the high value of the fractal dimension images, we have weak PSNR values (<30), therefore these images are of low quality. By against, for low values of their fractal dimension PSNR exceeds 30, as they are of acceptable quality images.

8. CONCLUSION

In this article, we studied the fractal interpolation of the image of the human retina. In the first part we interested in the image segmentation phase, we apply an image Skeletonization algorithm and determinate the different type of points (endpoint, bifurcation point, center line vessels). In the second part, using the Douglas-Peucker algorithm to determine the characteristic points from skeletons images, the algorithm advertised at a high compression rate can reach up to 96.6%. For the decompression phase us have used IFS fractal interpolation, the values of PSNR obtained between 27 and 36.9 dB in the high values of PSNR (PSNR > 30), the compressed images are acceptable quality images. To go further, we can go to the generation of blood vessels that are not included because of the limited resolution of the camera we use the L-system method

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