

# INTERACTIVE VISUALIZATION OF MULTI-DIMENSIONAL DATA ON HETEROGENEOUS INFORMATION OBJECTS IN THE DECISION SUPPORT SYSTEMS FOR SOLVING MULTICRITERIA CHOICE PROBLEM

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## ABSTRACT

In decision support systems, one of the tools for solving multicriteria choice problems is the analytic hierarchy process. This method uses a large amount of data. To improve the efficiency of decision-making, the method of interactive visualization of multidimensional data is proposed. The method makes it possible to change the hierarchical structure of objectives in the choice problem, set weights of objective functions at each of the hierarchy levels, and observe the quantitative dependencies of the objective function using graphs. The method is based on the combination and modification of charts: a bar graph, a Tree Map and a flat organizational chart. The modification involves adding interactive elements, making it possible to change the hierarchical structure, change the quantitative values of individual parameters and the weighting factors, select a range of values, as well as encoding quantitative values using the gradient transition of color to display the relationships between the values of the objective functions of different hierarchy levels. Continuous interactive actions of the user result in a corresponding change in graphically displayed items. It provides fast feedback between the user and the decision support system. The practical implementation of the method will provide ease of perception of choosing, setting rules for decision-making, ease of explaining the main reasons, influencing the choice of solutions, high-speed operation.

**Keywords:** *Decision Support System, Analytic Hierarchy Process, Choice Problem, Interactive Visualization, Treemap, Visualization Of Hierarchical Structure, Multi-Criteria Problem, Visualization Of Quantitative Values, Multidimensional Data.*

## 1. INTRODUCTION

### 1.1 The specific features of source data for a decision-making problem

Using decision support systems (DSS) [1], [2] or expert systems [3] makes it possible to significantly reduce the time of decision-making and improve the quality of decisions through the use of information, accumulated in the knowledge base. DSS are usually designed to aid in solving unformalized or poorly formalizable tasks in many application areas for different functional tasks (financial planning, identifying market trends, resource allocation, etc.). These systems include a set of software tools using modern methods of analysis, data processing [4], [5], decision-making [6] and information visualization [7], [8],[9] that make it easier for a user to understand the task in hand and carry out the necessary calculations. The most common decision-making problem is the choice problem, in which it is necessary to select one or more from among the admissible alternatives. For example, it may be the

problem of choice of preferred projects, in which it is expedient to invest, the problem of choice of the most creditworthy borrowers, the problem of choice of equipment from among the equipment available in the market, and many others. An instructive and easy-to-understand example of a choice problem, which can be used to illustrate the application of different methods of solving poorly formalizable tasks, is provided in [10]. This work illustrates the multi-criteria choice problem using the example of making a decision about buying a house. This is a hypothetical problem, in which it is necessary to choose a suitable house to purchase (the best one, from the perspective of a decision-maker). There are several sale offers, and it is necessary to choose one of them. Because of the uncertainty of the main goal (the notion of the best house), it is impossible to get a quantitative assessment of the degree of its achievement (using other terminology: the rating, the significance of the objective function). It results in the need for further refinement of the objective by resolving it into more simple components that

are easier to quantify. For this purpose, individual objectives or house properties are identified, which may affect the preferences in the process choosing a house, and characterize the parameters which are important for a person who is buying a house. These properties include: size, transportation, neighborhood, age, yard, etc. By means of such refinement of the objective it becomes possible to reduce the problem with an indefinite optimality principle to a multi-criteria problem, which corresponds to the idea of the Thomas Saaty's analytic hierarchy process [11]. The scheme of the problem, similar to one provided in [10], is shown in Figure 1.

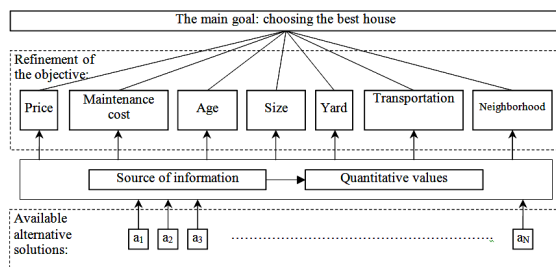


Figure 1. The Scheme Of The Task

If it is possible to assess the individual objectives (or properties) numerically, then the main goal can also be quantified, for example, as a linear convolution of quantitative values of specific properties with certain weighting factors. To calculate these weighting factors, the pairwise comparison method [6], [10] or more sophisticated methods outlined in [12] can be used. It was also suggested [10] apply the pairwise comparison method for the purpose of determination of quantitative values of specific properties. Such an approach doesn't involve any significant difficulties, if the number of alternatives is no more than a dozen. If there is a large number of alternatives, it becomes difficult to carry out a substantive analysis thereof, and if there is a large number of properties reflecting the main objective, it is also difficult to define weighting factors for calculating its quantitative value.

In general, for decision-making problems which are more comprehensive than the example under consideration, decision-making should be based on accurate and timely information on the quantitative values of the properties of alternatives. The need for obtaining information, which can be used as the source data for decision-making, suggests the need for the implementation of a subsystem of the DSS,

designed for the retrieval of data from heterogeneous sources. One of the ways of obtaining such information is the retrieval of the necessary data from news flows containing text messages, for example, reviews on alternatives, news, ads and other types of messages. The alternatives in a decision-making problem are formally presented as information objects. By an information object is meant the formalized description of some real object or phenomenon in the form of the structure of data describing the properties of the alternatives. The data on information objects are multidimensional, since they represent a comprehensive description of the multiple properties of the alternatives. To retrieve the data on informational objects, the methods of automatic processing of natural language can be applied [13]. The specifics of the data retrieved from news flows is that they can be: incomplete (an individual message does not contain all the data); not accurate (different authors can make different quantitative assessments regarding individual properties, and in some cases, these assessments may be not quantitative, but qualitative); multiple (several values can be obtained for each property of one and the same information object).

To improve the effectiveness of work with such data sets, it is necessary to use software solutions, which make it possible to carry out the analysis thereof within a reasonable period of time and solve the choice problem.

## 1.2 Interactive work of a user with a decision support system

By analogy with the scheme, provided in [14], interactive work of a user with a DSS is presented in the form of a scheme, shown in Figure 2.

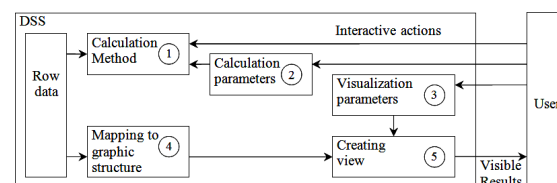


Figure 2. The scheme of the user-data interaction in a decision support system

A user through the implementation of interactive control actions selects: a calculation method (Unit 1), calculation parameters (Unit 2) and visualization parameters (Unit 3). A DSS, by using the data, stored in the knowledge base, carries out calculations (Unit 1), calculates a projection of the

obtained calculation results into a structure, containing the parameters of graphic display (Unit 4), and displays the obtained results (Unit 5), which are observed by the user. Thus, a feedback to DSS is implemented, which makes it possible for a user to find the correct solution. The effectiveness of work with a DSS depends on how quickly and accurately a user perceives the displayed results and sets the control commands, and also on the speed of a system's response to a user's actions. Therefore, in addition to the software implementation of the data processing methods, the improvement of interactive technologies of data management and data visualization [12], [15] is also of considerable practical importance for improving effectiveness of DSS. The basic ideas of interactive data visualization are outlined in [14], [15], [16], [18], [19], but the practical methods of interactive visualization in each subject area should take into account its specifics, therefore, they require a separate research and development of specific tools.

In the process of solving a choice problem, using the technologies of interactive visualization should ensure the ease of dealing with accumulated information, the ability to change the hierarchical structure of data and to calculate the rating (the objective function) for alternatives. In order to eliminate the need for engaging additional specialists, a user in the process of working with a DSS should have an opportunity to carry out the necessary calculations and the analysis of data on one's own.

### 1.3 Description of the task

The aim of this work is to propose a method of interactive visualization of multi-dimensional data on heterogeneous information objects, which facilitates the formation of hierarchical relationships between data and makes it possible to visualize and modify hierarchical dependencies of the rating of information objects, for using in decision support systems.

The content of the task is the following. To carry out interactive actions of data analysis, standard input devices should be used: a mouse with two buttons and a scroll wheel, and a keyboard. Visualization is performed on the screen of a computer monitor. A user deals with multidimensional data, represented by the values of a specified set of variables. For example, in the

problem of buying a house the variables are:  $p_1^1$  – price,  $p_2^1$  – maintenance cost,  $p_3^1$  – age,  $p_4^1$  –

size,  $p_5^1$  – yard,  $p_6^1$  – transportation,  $p_7^1$  – neighborhood. Each set of values of the variables is an information object, describing various properties of one of the alternatives. A user's task is to choose one of the alternatives on the basis of building a rule of calculating their rating (objective function) or identifying patterns in the form of the dependence of one of the parameters from others based on the statistical data.

To make a decision, a user formulates the goal of decision-making and presupposes that this goal can be formalized in the form of some numerical function, which is called the objective function. The value of this function determines the rating for each alternative, which is used to choose the best decision. The analytic hierarchy process is used for decision-making [11]. According to the rules of the analytic hierarchy process, a user carries out the decomposition of a goal, breaking it into simpler individual objectives, the achievement of which leads to the achievement of the main goal. At the same time, a user forms an objective function, which depends on several variables. These variables are, in turn, objective functions of more specific objectives. As a rule, the functional dependency is understandable, if there are from 3 to 5 variables, on which the objective function depends. The decomposition procedure ends at a stage where there is a possibility to obtain quantitative estimates of the values of the variables, corresponding to individual objectives. The result of the decomposition is a hierarchical objectives tree – a graph reflecting the dependence of the main goal from individual objectives. The analytic hierarchy process involves reasoning from the general to the specific. If there is already some data of information objects in a DSS, the objectives tree can be formed by aggregation of existing variables.

In the discussed example of the problem of buying a house, to get a clearer picture of the dependence of the main goal on individual objectives, it can be useful to group the properties into categories and present the dependence on three main factors, which in turn depend on more specific properties, as shown in Figure 3.

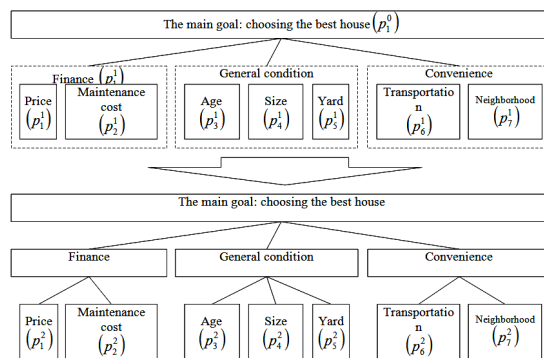


Figure 3. Illustration Of Forming A Hierarchical Objectives Tree

Informative examples of using the methods of solving this type of tasks are provided in [1], [6], [12]. The dependence of the values of an objective function on the specific properties of alternatives is built in the form of some function, as a rule, a multilevel linear convolution, the coefficients of which should be set by the user. When retrieving data from news flows, misrepresentation of information, obtaining inaccurate data or omissions of certain data are possible. As a result, with reference to the problem in hand, a lot of information on heterogeneous objects, representing the individual properties of selected alternatives, is accumulated.

A user can also try to group the individual objectives into other categories (for example, as in [6]: benefits, opportunities, costs, risks) and build an objectives tree, which is different from that shown in Figure 3.

The method of interactive visualization should provide an opportunity to explore a solution to the problem in different ways. Graphic display should make it possible to quickly see the entire map of dependencies and intuitively assess the degree of influence of variables of different hierarchical levels on the objective function. When developing the method of interactive visualization, it should be taken into account that a user is interested in issues related to consistent patterns in data: how much and what data is available for decision-making, what is the degree of its validity, what actions (processing) can be carried out in relation to existing data, how the different sides of the phenomenon are interrelated, etc.

## 2. THE METHOD OF THE INTERACTIVE VISUALIZATION OF DATA

### 2.1 General characteristics of the method

The variables, with which a user works, can be divided into groups according to different criteria. When forming a hierarchical structure, it is convenient to identify primary variables (or independent variables), the values of which already exist in the database or can be set by the user, and dependent (or aggregate) variables, the values of which are calculated as a functional dependency on other variables. Dependent variables are, in turn, objective functions of individual objectives. Independent variables correspond to the end vertices of an objectives tree.

The degree of the achievement of the main goal is represented in the form of a functional dependency of the objective function on the parameters. To enable the perception of the basic patterns, an objective function is built as dependent on no more than five variables, but this requirement is not a limitation of the method of visualization. The objective function of each knot of a hierarchical scheme depends linearly on variables.

Actions on data analysis and decision-making, carried out by the user, are the following:

- to choose variables that affect the objective function;

- to identify groups of interrelated parameters;

- to set a dependency between the groups of interrelated parameters and the structure of this dependency;

- to determine quantitative values of the obtained dependency between parameters;

- to assess the degree of dispersion of the values of the objective function, if there are random deviations in the source data.

In order to automate the above-listed steps, it is necessary to display the multidimensional data, arranged in a hierarchical structure, to carry out editing of the hierarchical structure, and to change the parameters of the functional dependency of the objective functions on variables. A good way of displaying the decision-making process using the analytic hierarchy process is proposed in [20]. In this work, for the purpose of the visualization and modification of the hierarchical data, the Tree Map chart is applied, in which quantitative data is presented in the form of partially filled rectangles that fill a rectangular area. This variation of the



Tree Map chart is useful for displaying and editing of quantitative data, but it is not suitable for interactive editing of the hierarchical structure of the objectives tree.

In view of the need for the active work with data in a DSS, the visualization method requires using the minimum amount of graphic elements, encoding the greatest possible number of multidimensional data. The visualization of data should be interactive, and should make it possible to modify the methods and parameters of data processing and observe the results without long waiting. To enable a user to build a hierarchy of an objectives tree on one's own, and, if necessary, quickly edit it, to set the parameters used for the calculation of the objective function, to determine the sensitivity of the values of the objective function in relation to the change of variables, it is advisable to use simultaneously the Tree Map and the Organization Chart, consistently displaying the same hierarchical data in different ways.

The display of multiple values is implemented in the form of a Row Graph. The resulting interactive chart is built through combination and modification of these three types of diagrams (Organization Chart, Row Graph, and Tree Map) with the addition of interactive elements, which make it possible to edit the presentation of data.

The proposed method of interactive visualization includes the following components: displaying a hierarchical structure based on the Organization Chart; displaying one-dimensional quantitative data based on the Row Graph; displaying multidimensional quantitative data based on the Tree Map; combination of the Tree Map and the Organization Chart.

## 2.2 Displaying the hierarchical structure of the variables.

The hierarchical structure of the variables is displayed in the form of an objectives tree, in which the hierarchy levels are clearly identified. The upper level has a zero number, and the numbers of all the lower levels are assigned in ascending order. Formally, the hierarchical structure is represented by a graph – a tree with one root vertex corresponding to an aggregate variable denoting the main objective function. The vertices of the graph are dependent (aggregated) and independent (primary) variables. Each vertex corresponds to the following main attributes: the number of the vertex; the number of the hierarchy level; the number of the vertex in the numbering of one hierarchy level; the number of the vertex of an upper hierarchy level;

the name of the individual objective, which is reflected by the variable. Each edge of the graph corresponds to the following attributes: the number of the vertex of an upper level; the number of the vertex of a lower level; level number; the value of the weighting factor.

At each of the hierarchy levels, aggregated variables denote an objective function for an individual objective and are presented in the form of a linear convolution of the lower-level variables:

$$p_n^{k-1} = \sum_{i=1}^{N^k} w_i^k \cdot p_i^k (n=1, \dots, N^{k-1}), \quad (1)$$

where the superscript denotes the level number, the subscript denotes the number of the parameter within one level;  $w_i^k$  denotes the weighting factors, satisfying the condition:

$$\sum_{i=1}^{N^k} w_i^k = 1, \quad (2)$$

where  $N^k$  is the number of variables at the hierarchy level with a number  $k$ ;  $p_i^k$  – variables belonging to the hierarchy level with a number  $k$ . The weighting factors are equal to zero for those variables of the lower level, on which the objective function does not depend.

The visualization of an objectives tree can be implemented in different ways. One of them is to present the hierarchy of objectives in the form of Ishikawa diagram [21], which is also called cause-and-effect diagram or fishbone diagram (Figure 4).

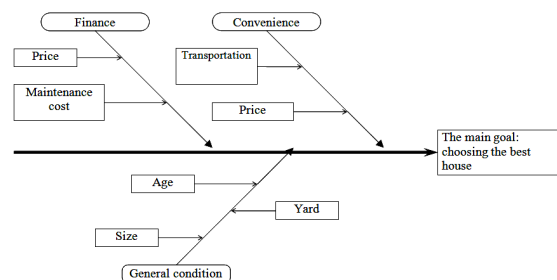


Figure 4. Ishikawa Diagram

This chart looks impressive and makes it possible to understand the dependency of the main goal on multiple secondary objectives. But the hierarchical levels of individual objectives in this diagram are located in different areas of the graphical representation, which makes it difficult to create

interactive elements and carry out interactive actions.

The other ways of displaying a hierarchical structure (Organization Chart, Tree Ring, Icicle Plot, and Tree Map) were studied in [22]. The results, obtained in this work, make it possible to conclude that the Organization Chart is more preferable for displaying a hierarchical structure with regard to several criteria, if there is no need for displaying quantitative values. If it is required to display quantitative values, for example, the disc space taken up for displaying the hierarchical file structure, Tree Map and Tree Ring are preferable. In [23] the advantages of the Tree Map for displaying the directory structure are demonstrated in comparison with the circular (Sunburst) chart. With regard to the criteria of clarity, simplicity, convenience of interactive action, the visualization of the objectives tree in the form of the Organization Chart is considered to be a preferred option.

The display of the objectives tree is build according to the following rules. The hierarchical levels are indicated by rectangles of equal width, spaced vertically. Inside each of such rectangles there are images in the form of smaller rectangles or circles, denoting the vertices of the objectives tree, belonging to one hierarchical level. The relationships between vertices of different levels are indicated by straight or broken lines. The original chart, showing only the main objective function and independent variables, consists of two levels (Figure 5). The inscriptions, indicating the names of individual objectives, are placed inside the respective rectangles. If the names of individual objectives are in natural language, the preferred form of graphic elements is a rectangle (Figure 5, a).

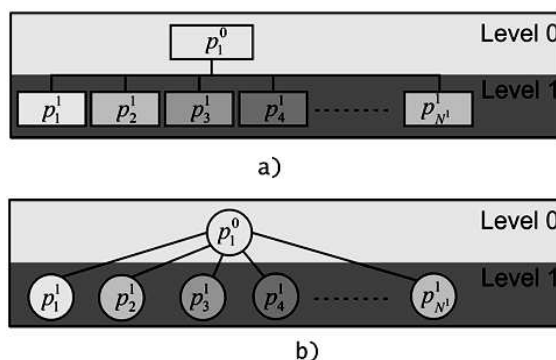


Figure 5. The Ways Of The Visualization Of The Objectives Tree On The Basis Of The Organization Chart

When displaying the relationships between variables of different levels, the encoding of weighting factors, with which the variables form a linear convolution, is possible through the thickness of lines. This method makes it possible to display the relevance of variables, but it is not convenient for the interactive modification of weighting factors.

Editing of this chart is implemented through interactive actions, which include: selecting, aggregating, refinement, changing a hierarchical level, adding a hierarchical level, adding a new variable, deleting a variable, setting a relationship between the variable of adjacent levels, zooming, changing the color coding of vertices.

Most interactive actions are carried out using common techniques, with the use of input devices, and include changing the parameters of the functional dependency (Figure 2, unit 2). Selection makes it possible to choose several vertices of a tree belonging to one level. Aggregation involves combining multiple vertices and forming a new aggregate variable (an individual objective function). Aggregation can be used, when the decision-making problem is solved according to the scheme “from the specific to the general” (when it is necessary to build an objective function on the basis of existing variables, the values of which are already known). The transformation of the objectives tree in the process of aggregation is shown in Figure 6.

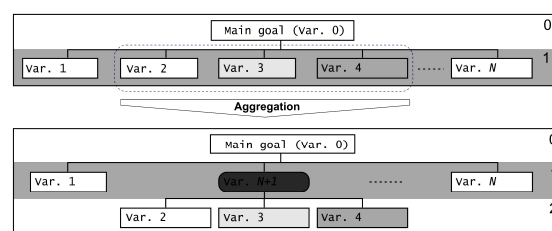


Figure 6. Interactive Aggregation Of The Variables Of The Objectives Tree

For the formal definition of this action, a new vertex is added to the objectives tree, which denotes an aggregate variable, belonging to the level, at which the aggregation of variables takes place. All vertices, corresponding to the grouped variables, are moved to a lower level. After this, the renumbering of variables takes place. Refinement is the operation inverse to aggregation, which involves deleting an aggregate variable and moving

all the variables, on which it depends, to an upper level.

Changing the hierarchical level is implemented through specifying a parameter, which should become subordinate to another aggregate parameter. An interactive action involves moving the graphical representation of the first parameter to the graphical representation of the second parameter (Figure 7).

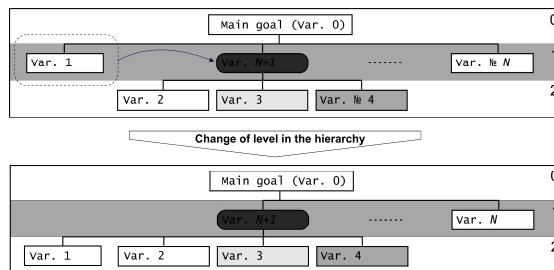


Figure 7. Changing The Hierarchical Level Of The Variable

Adding a hierarchical level is intended for the refinement of existing goals, if the decision-making problem is solved according to the scheme “from the general to the specific”. After adding a new hierarchical level, primary variables are added to this level, representing independent variables that may be obtained from the database, or representing more specific objectives. Deleting a variable is used in cases when it is necessary to modify the original hierarchical structure. In case of deleting an aggregate variable, all the variables, on which it depends, are also deleted. Setting a relationship between variables of adjacent levels makes it possible to set hierarchical dependencies for objective functions.

Scaling is changing the settings of display (Figure 2, unit 3), which involves changing the degree of refinement of the display of the objectives tree by identifying the number of displayed hierarchical levels (it is reasonable to display no more than 5 levels). Changing the color coding of vertices is used, when it is necessary to provide selection of individual vertices of the objectives tree, which is more convenient for the user.

The functional capabilities of the chart are not limited to the interactive building of the objectives tree. The aggregation of variables can be carried out by using statistical dimension-decreasing methods and identifying the main components [4], [5]. In this case, the hierarchical structure is built on the basis of the results of the appropriate calculations. Other data-processing algorithms (clustering,

correlation analysis, etc.) can also be used in the interactive chart for displaying and implementing of interactive actions.

This chart shows the structure of relationships between variables and makes it possible to build an objectives tree, but the quantitative values of variables and weighting factors are not shown clearly enough (they can be presented in the form of tooltips or inscriptions). The display of quantitative values of multidimensional data is based on the charts, similar to the Row Graph and the Tree Map.

### 2.3 The visualization of one-dimensional quantitative data

Quantitative values of individual variables are displayed on the basis of the RowGraph. Information is encoded by geometrical size, color, color transparency, the chart of the sample distribution function. To display one value of a variable, a chart in the form of a partially filled rectangle is used. Intuitively, such a chart represents the relative amount of fluid in a cylindrical vessel (a glass), which is not full to the brim; therefore, it is also called a Glassful.

First of all, quantitative values should be normalized by means of a monotone function that performs mapping of the range of values of the variable into an interval from 0 to 1. The worst (lowest) value corresponds to 0 in the relative scale, while the best (highest) value corresponds to 1. Normalization can be performed for both quantitative and qualitative data [24].

The rule of displaying the chart is the following. The elementary displayed item is a rectangle, divided into two parts (upper and lower) by a horizontal line. The height of the rectangle is taken to be equal to 1 according to the relative scale. The height of the lower part is equal to the displayed quantitative value in the relative scale and is filled with opaque color; the upper part is filled with the same color, but with transparency of about 90%, as shown in Figure 8.a.

If it is necessary to display several different values in one chart, the color transition is used with a change in the intensity of transparency from 0 to 90% (Figure 8). The horizontal line dividing the rectangle is equal to the average sample value. For a sample, a sample distribution function is built, and the transparency of the color is changed from the minimum value to the maximum. The minimum and maximum values are indicated by horizontal lines, and the sample distribution function is displayed as a chart in the same rectangle (Figure

8.c), which is turned through 90 degrees counter-clockwise. In some cases, it is important to differentiate the zero value of a parameter from the lack of data. The zero value corresponds to a filled rectangle with a transparency of 90%, and the missing value corresponds to a white rectangle.

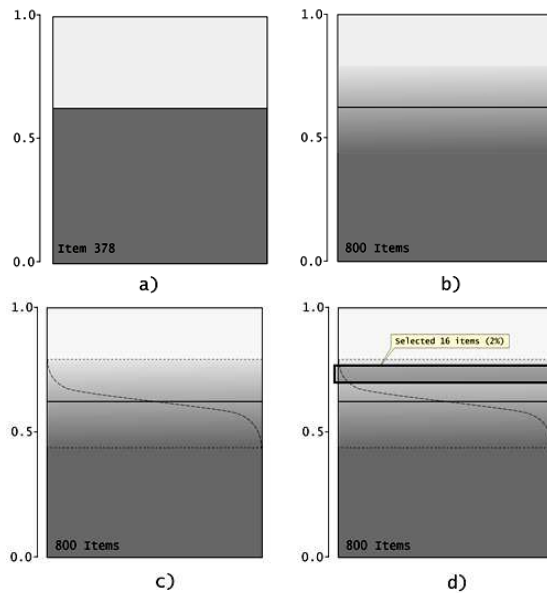


Figure 8. Displaying One-Dimensional Data On The Basis Of Row Graph

For this chart, the following interactive actions are used: changing the height of a filled rectangle, changing colors, changing the parameter spread, choosing displayed items, choosing a range of parameter values.

To choose a range of values of a variable, a more compact form is used, in comparison with the form used in the Attribute Explorer, proposed in [25], which is represented by a selection rectangle with interactively modifiable upper and lower boundaries (Figure 8.d). When the range of values is chosen, a tooltip is displayed, which contains the information on how many values correspond to the selected range (Figure 8.d).

This method of displaying quantitative values of one variable makes it possible to show multiple data on the values of this variable in one chart. Another interpretation of multiple values is applicable in case, if the user doesn't possess accurate information on the values of the variable. In this case, the set of possible values of a variable is regarded as a fuzzy set, and the graph of a

membership function is displayed instead of the graph of a sample distribution function.

## 2.4 The visualization of multidimensional quantitative data

There are many ways of displaying multidimensional hierarchical data. Most methods of visualization are based on a two-dimensional Tree Map, the analysis and a large number of examples of which are provided in [29], while the review and analysis of software, using three-dimensional variations of the Tree Map, are provided in [28]. But these methods don't provide adequate visual demonstration of the functional dependencies on the multidimensional data. The idea is based on the principles of building a Tree Map chart, as well as Parallel Coordinates and Heatmap charts, various ways of using which are shown in [26], [27], [30].

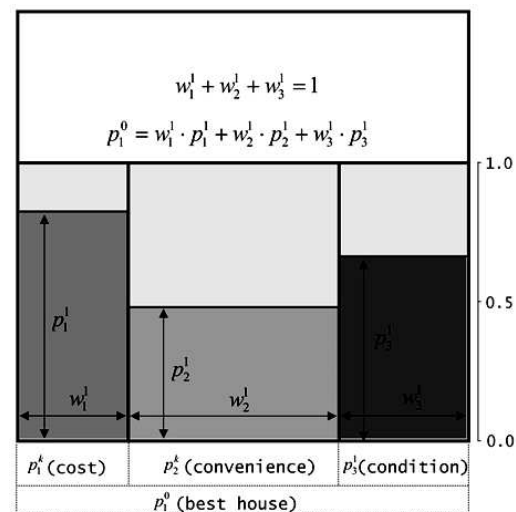
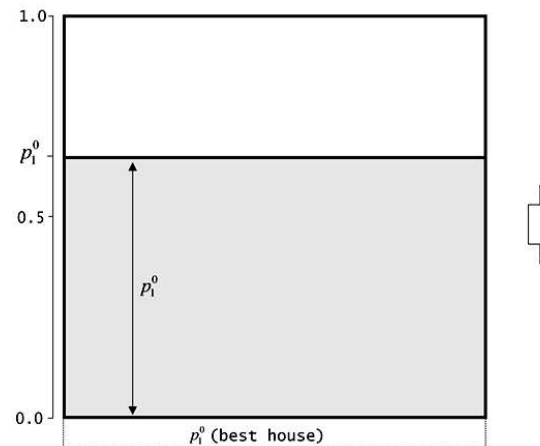




Figure 9. Displaying The Linear Dependence Of The Objective Function On Several Variables

For the purpose of the quantitative display of the structure of the dependency of the objective function on variables in the built hierarchical scheme, a recursive principle is applied. A number of glasses, standing in line, are placed in the filled part of the glass. Each of them displays the value of one of the variables, on which the objective function depends (Figure 9). In turn, these variables may also be individual objective functions and depend on the variables of another hierarchical level. Therefore, for the refinement of the dependency of their values on variables of the next hierarchical level by recursion the same method can be applied (on the left of Figure 10).

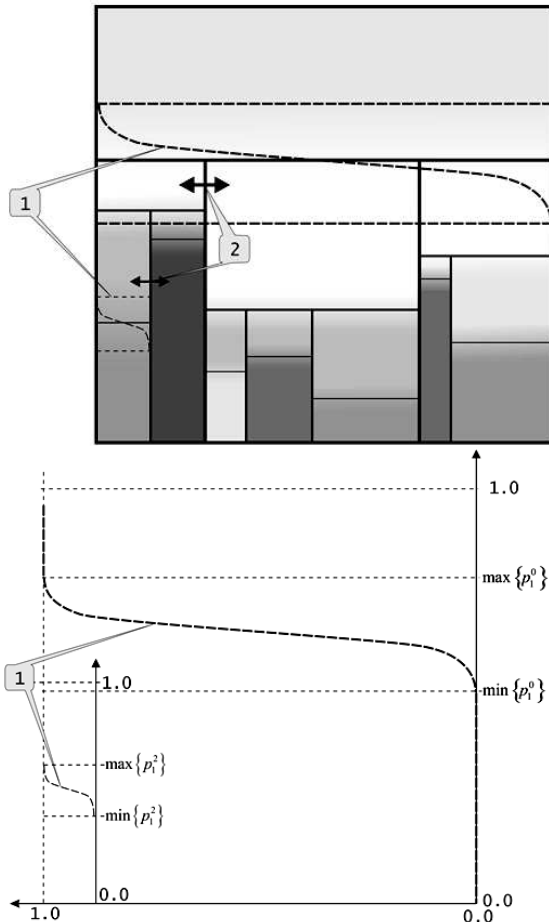


Figure 10. Displaying Multiple Data In The Form Of A Sample Distribution Function (1), And Interactive Changing Of Weighting Factors (2).

The rule of building the graphical representation is the following. Suppose that the value of an

individual objective function of an upper level

( $P_m^{k-1}$ ) is displayed in the form of a Glassful (on the left of Figure 9). The height of the filled part of

the rectangle is equal to the value  $P_m^{k-1}$ . If the

variable  $P_m^{k-1}$  is functionally dependent on several variables of a lower level, the lower part of the

chart is filled with the same height charts  $P_m^{k-1}$  that represent the values of those variables in a normalized scale from 0 to 1. In the process of building nested charts, the scale is changed in such

a way that the value of the objective function  $P_m^{k-1}$  is assumed to be equal to 1.0, as shown on the right of Figure 9. The horizontal size of the nested rectangles is selected in such a way as to maintain the proportion, set by weighting factors, with which the variables form a linear convolution. The visualization of the linear dependence of the objective function on the objective functions (variables) of lower levels is implemented by means of encoding the numerical values of parameters by the geometrical sizes of rectangles. The height of the column, indicating the value of the objective function, is equal to the sum of areas of rectangles, indicating the values of variables:

$$P_m^{k-1} = P_m^{k-1}(p_1^k, \dots, p_{N_k}^k) = \sum_{i=1}^{N_k} w_i^k \cdot p_i^k. \quad (3)$$

The inscriptions, indicating the names of variables, if necessary, may be placed in the lower part of the chart. In case of displaying multiple information objects or multiple values of variables, generalization is implemented through the gradient color transition, which is built on the basis of the sample distribution function. In contrast to the display of a single value, in this case a smooth color transition is used instead of a transparency effect. The color transition goes from the bottom upwards, from the color of a nested rectangle (the lower part) to the color of the outer rectangle (the upper part). This ensures the consistency of the display of the functional dependence of one parameter on several other parameters. When calculating the color transition, all the colors of nested rectangles converge in the upper part to the same color of the containing rectangle.

When working with this chart, the following basic interactive actions are carried out: changing the width of the rectangle, which corresponds to the changing of the weighting factors in the linear convolution of the parameters; changing the degree

of refinement of the display of quantitative values when it is necessary to see the bounds of the variable and the chart (Figure 10, (1)); scaling; changing the average value and threshold values of the level for primary variables, the values of which can be changed by the user; interactive graphic elements are vertical lines that separate single charts of Glassful type. Moving of the vertical lines is implemented using the computer mouse (Figure 10, (2)). The above-listed steps are accompanied by automatic recalculation of the values of the objective functions and redrawing the chart. This makes it possible to quickly set up a functional dependence and specify the most significant variables, the values of which constitute a substantial share in the value of the objective function. When displaying additional informative elements (the range of values, the sample distribution function), the display scale is scaled by the size of the rectangle, as shown on the left of Figure 10.

## 2.5 Combination of the objectives tree and the Tree Map

In the process of the active work on building the structure of the dependencies of the objective functions on variables and in the process of the analysis of the effectiveness of various goals, a user may need to carry out multiple modification of the objectives tree with the simultaneous viewing of quantitative values. To make it possible, the combination of the objectives tree and the Tree Map is applied. The charts are displayed in adjacent windows, as shown in Figure 11. The Glassful is shown on the left, the objectives tree – on the right. All the interactive changes, implemented in regard to the objectives tree, are also displayed in the Glassful. The color of the geometric figures, denoting the variables (the vertices of the objectives tree), makes it possible to compare the graphical representations of the variables from different windows.

Figure 11 shows the correlation of the hierarchy elements, displayed in these two types of charts. When the objectives tree is modified, it can result in the uncertainty in the choice of weighting factors for the display in the Tree Map. In this case, weighting factors should take the default values, or equal values, which can later be changed interactively by the user in the left diagram.

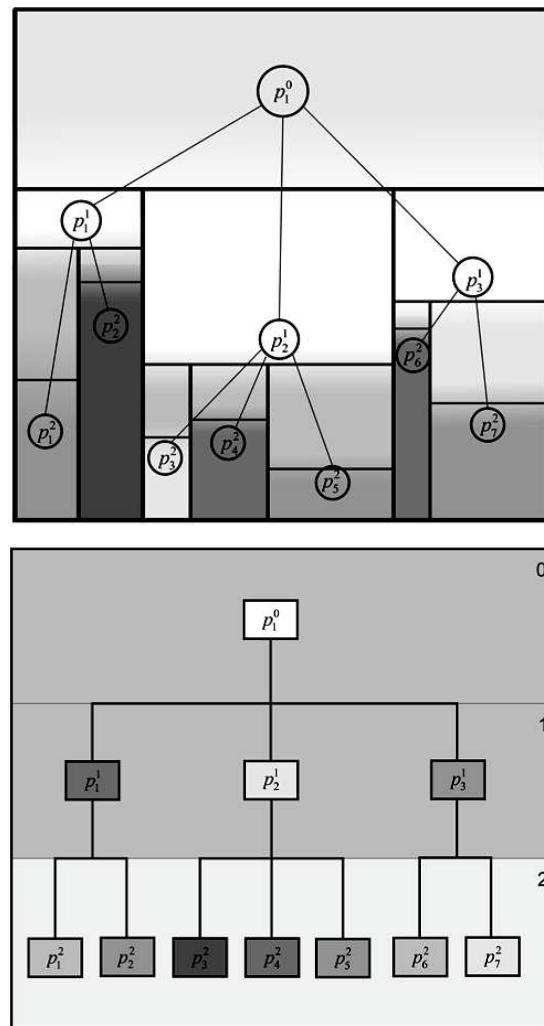


Figure 11. Joint Application Of The Tree Map And The Organization Chart

## 3. RESULTS

The result of this work is the software implementation of the described method. The software implementation provides for the connection of an interactive chart to the source of data on information objects, presented in the form of a table. The user has an opportunity to choose the name of the main goal and the attributes of information objects used as variables. The initial view of the chart is presented by two windows (Figure 11), reflecting a two-level hierarchical scheme. Further actions of a user include interactive work with this diagram and involve the formation of the structure of the objective function using the operations of grouping and editing the hierarchy, as well as the modification of the weighting factors. For comparative evaluation of the interactive chart and its counterparts the methods proposed in [14]

are applicable. The detailed evaluation methodology for other methods of visualization of a multi-criteria problem is provided in [30].

To assess the effectiveness of the method, the expert approach was applied, based on the comparison with the known methods of visualization of solving a multi-criteria decision-making problem, which were already assessed in [30]. It is also noted, that it is difficult to identify a universal method, as the quantitative results of the assessments are significantly affected by the characteristics of the user and the content of the multi-criteria problem in hand. The quality of the software implementation of the method also affects the choice of the preferred method. Due to the fact that an accurate quantitative assessment of the effectiveness of a method is a separate challenging problem [14], qualitative assessments were used, based on several most important criteria: visualization, functionality, accuracy, and completeness of the perception of the result of the calculation. The estimates were provided by test users in a qualitative form according to the principle of comparison (better or worse) with the explanation of the made estimates.

With regard to the criterion of the visualization of solving a multi-criteria problem and intuitive explanation of the reasons, significantly affecting the choice of the decision, the proposed method proved to be better than a table, a Heatmap or a Parallel coordinate plot. With regard to the criterion of functionality the method also proved to be better than other methods, as it provides an opportunity to modify weighting factors and the hierarchy diagram. With regard to the criterion of the accuracy and completeness of the perception of calculations the method is inferior to the visualization in the form of the Parallel coordinate plot due to the fact that it does not make it possible to identify all of the Pareto-optimal solutions. However, if there are a large number of variables, most solutions are Pareto optimal, so this disadvantage is not essential.

The obtained results show that the use of the proposed interactive chart will increase the effectiveness of working with large amounts of multidimensional data.

#### 4. DISCUSSION

The main objective of this work was to propose a method of interactive visualization of multidimensional data on heterogeneous information objects, which makes it easier for a

user to work with a decision support system. The content of the method is discussed for the case of a multi-criteria problem, which is solved using the analytic hierarchy process, but the scope of the method is not limited to this type of tasks. The method makes it possible to change the hierarchical structure of objectives in the choice problem, set weights of objective functions at each of the hierarchy levels, and observe the quantitative dependencies of the objective function using graphs. The method is based on the combination and modification of the charts, which are most commonly used for the purpose of visualization, and adding the minimum number of interactive elements, which contribute to the faster perception of information. Continuous interactive actions of a user result in a corresponding change in graphically displayed items, which ensures fast feedback between a user and a decision support system. The practical implementation of the method will provide ease of perception of choosing, setting rules for decision-making, ease of explaining the main reasons influencing the choice of solutions, high-speed operation.

The quantitative evaluation of the efficiency of the method of interactive visualization is hampered by the fact that its basic criteria [14] largely depend on the individual characteristics of a user, the degree of his/her preparedness and the content of the problem in hand. The software implementation of the proposed method provides a new tool for working with data in the process of decision-making, which expands a user's opportunities and thus contributes to the improvement of the efficiency of the DSS.

#### 5. CONCLUSIONS

The application of the proposed method of interactive visualization will provide to the user of a DSS a tool for on-line data processing, making it possible to solve tasks related to the revealing of consistent patterns in data and getting the data on the content of the available information on information objects. This makes it possible to improve the speed and quality of decision-making in the process working with a decision support system.

One of the directions of future work is related to overcoming a number of limitations of the proposed method, because it does not cover all the way of building the objective function. For example, the use of a linear objective function is not always feasible, in particular, when the criterion of the

minimum distance to the ideal point is used as a main criterion, or when the dependence of the objective function is built according to the neural network scheme. Therefore, the further research should involve extending the scope of application of the method for the cases, when different individual objective functions may depend on the same variables (in this case, the graph of the hierarchical structure of objectives will be more complicated than the objectives tree).

The other direction is improving graphical interactive elements, adding new functional capabilities to the interactive chart for the tasks of exploratory analysis of heterogeneous multidimensional data. One more direction of further research, leading to the improvement of the efficiency of decision support systems, involves providing a clearer relationship between the process of building an objectives tree and the process of obtaining information on the quantitative values of primary variables, for example, through the application of methods for extracting data from texts in natural language.

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